Spin-Orbit Coupled Quantum Gases: New Physics and Challenges

Hui Zhai

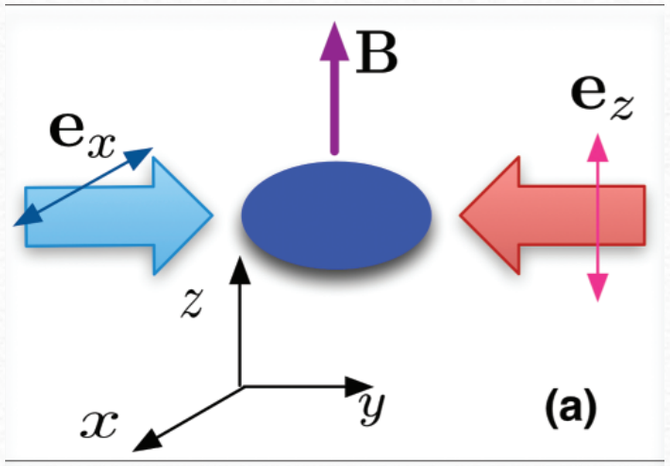
Institute for Advanced Study
Tsinghua University
Beijing, China

NSFC-ISF:

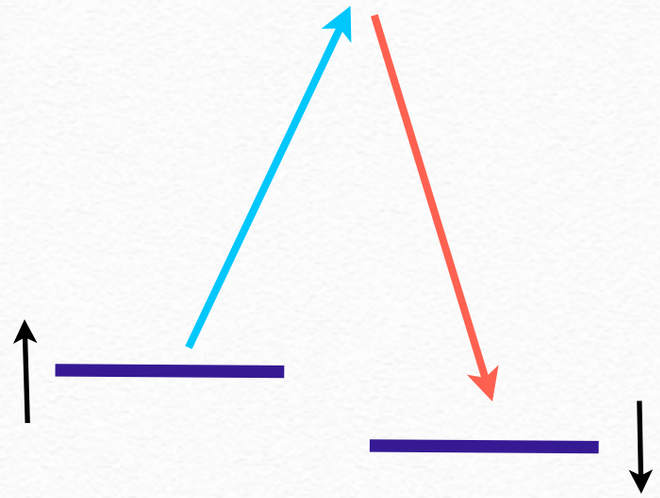
Joint Workshop on Bose Einstein Condensate and Ultracold Phenomena

2013.9

How to generate SO coupling for neutral atoms



Two-photo Raman process

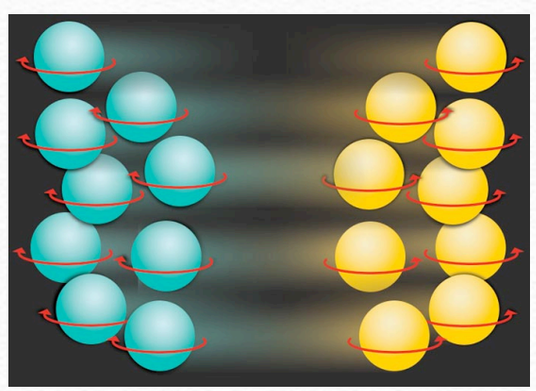


$$\hat{H}_0 = \frac{(k_x - k_{\rm r}\sigma_z)^2}{2m} + \frac{\delta}{2}\sigma_z + \frac{\Omega}{2}\sigma_x.$$

$$\hat{H}_0 = \frac{k_x^2}{2m} + \vec{h}_k \cdot \vec{\sigma} \qquad \vec{h}_k = \left(\frac{\Omega}{2}, 0, -\frac{k_x k_0}{m} + \frac{\delta}{2}\right)$$

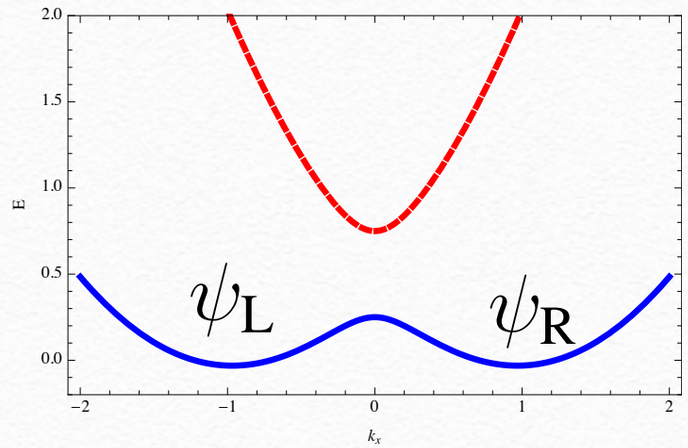
Spin-orbit coupling
= A momentum dependent Zeeman field
= Spin-momentum locking

Spielman, NIST, 2011, Nature



New Physics with Spin-Orbit Coupling

Single-Particle Physics



Single particle ground state with degeneracy

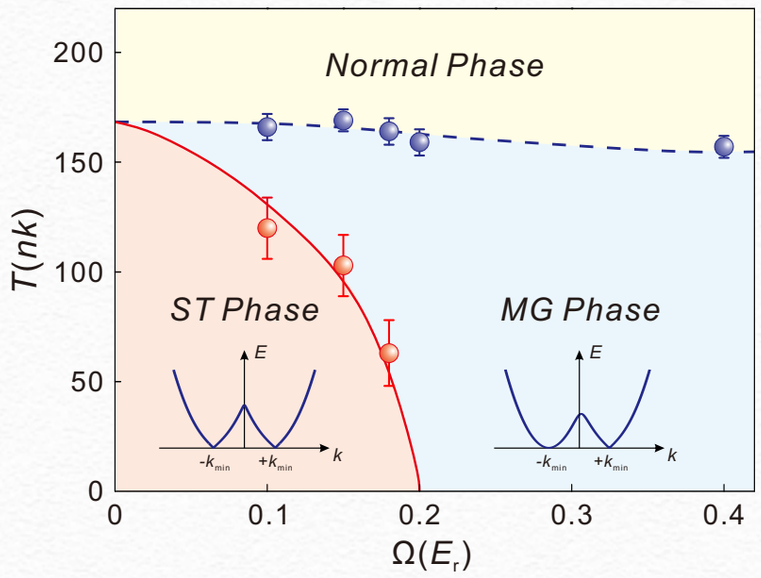
Questions: Bosons with degenerate ground state ?

New Physics with Spin-Orbit Coupling

Single-Particle Physics

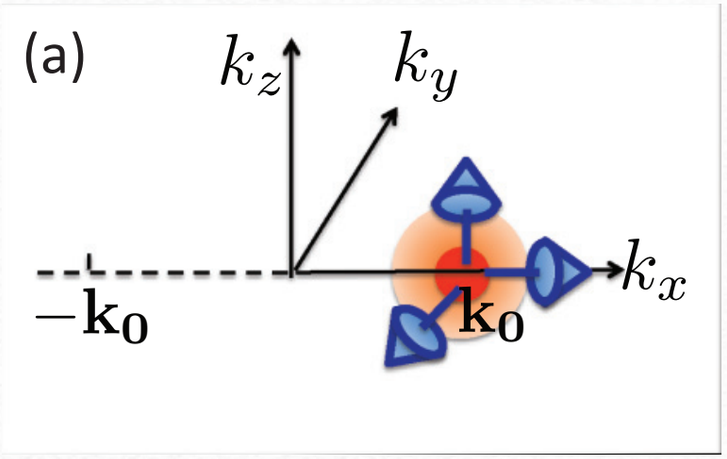
Superfluid with density stripe order

Wang, Gao, Jian and Zhai, 2010
Zhang and Ho, 2011
Spielman, 2012
Li, Pitaevskii, Stringari, 2012



Non-trivial phase diagram

Shuai Chen's talk



Enhancement of quantum fluctuation

Xiaoling Cui's talk

Jian and Zhai, 2011

New Physics with Spin-Orbit Coupling

Single-Particle Physics

Spin-momentum locking

Lack of Galilean invariance

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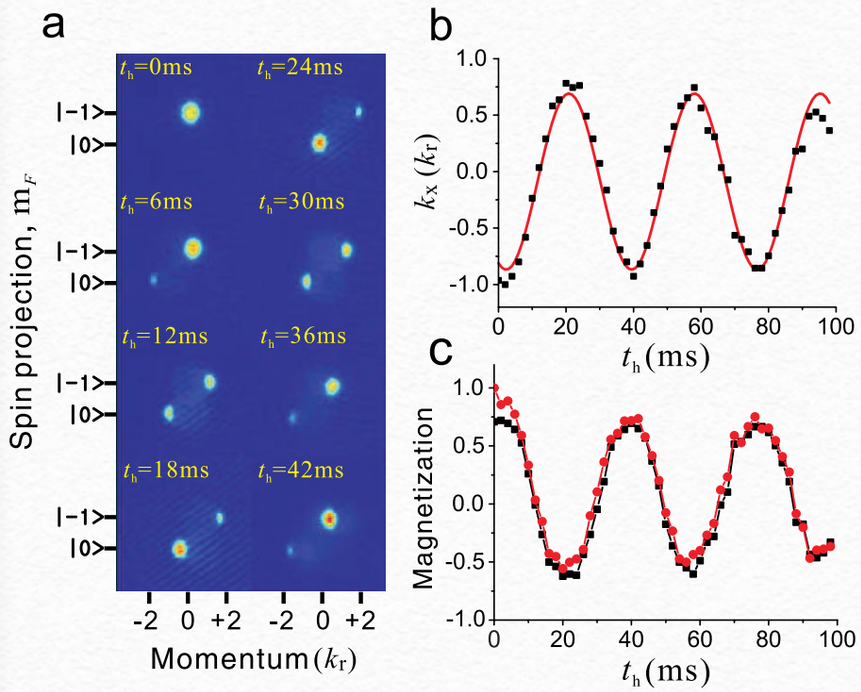

Non-trivial collective mode

Shuai Chen's talk
Spielman 2012

New Physics with Spin-Orbit Coupling

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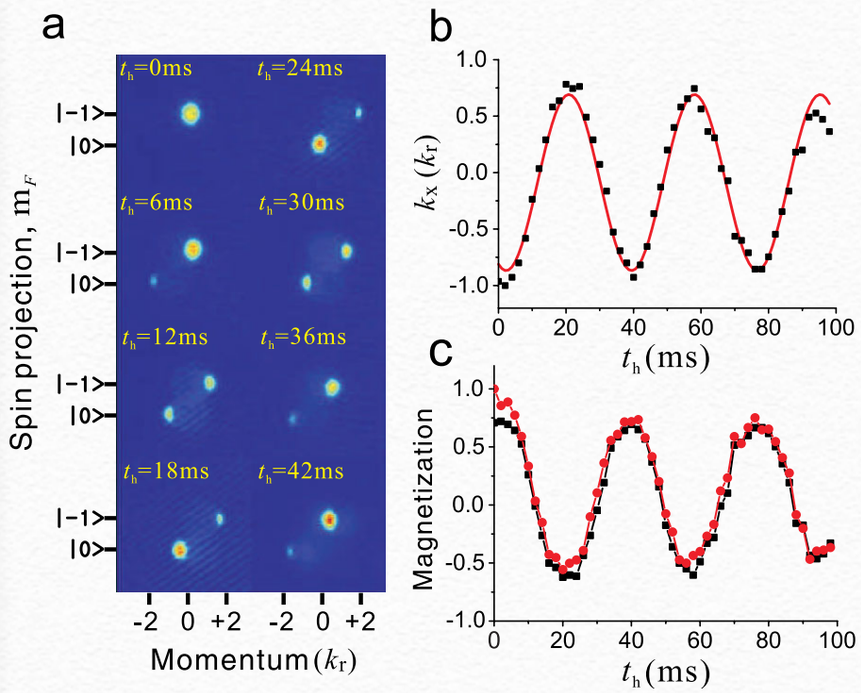
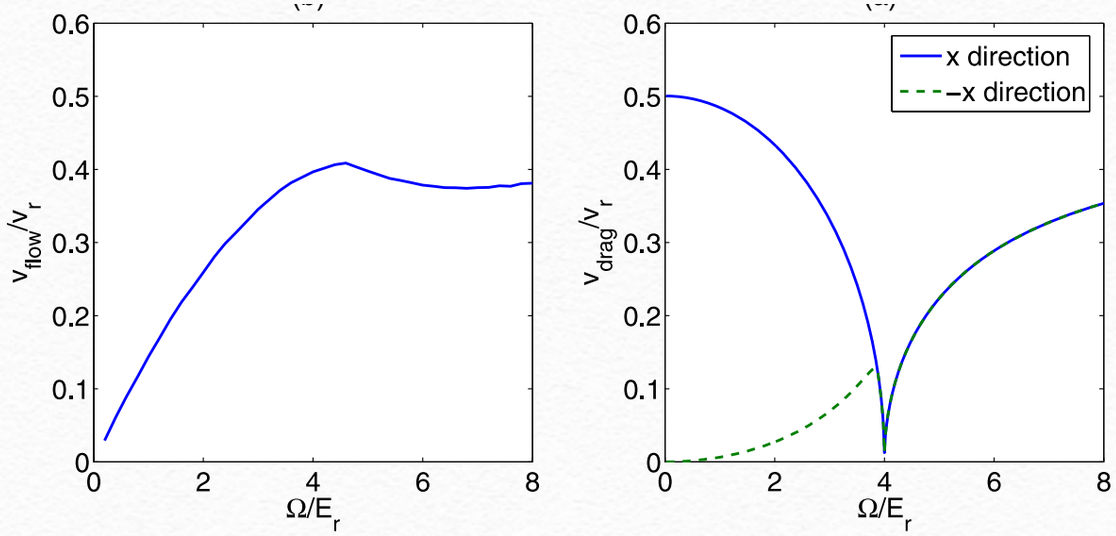


Lack of Galilean invariance



Non-trivial collective mode

Shuai Chen's talk
Spielman 2012

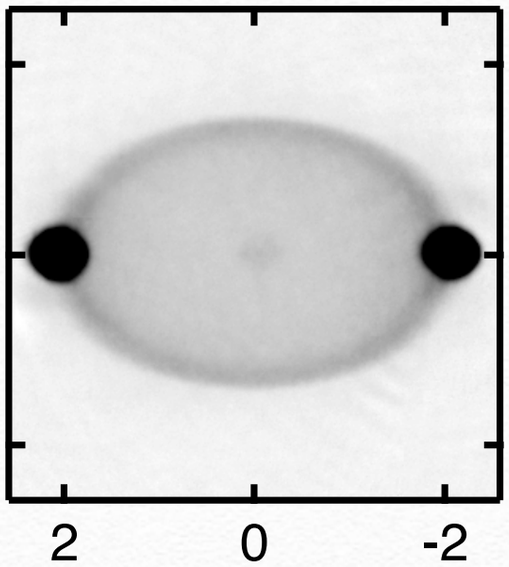
Critical velocity and superfluidity

Zhu, Zhang and Wu, 2012
Zheng, Yu, Cui and Zhai, 2013

New Physics with Spin-Orbit Coupling

Two-body Physics

Change of Collisional Properties
Mix different partial waves

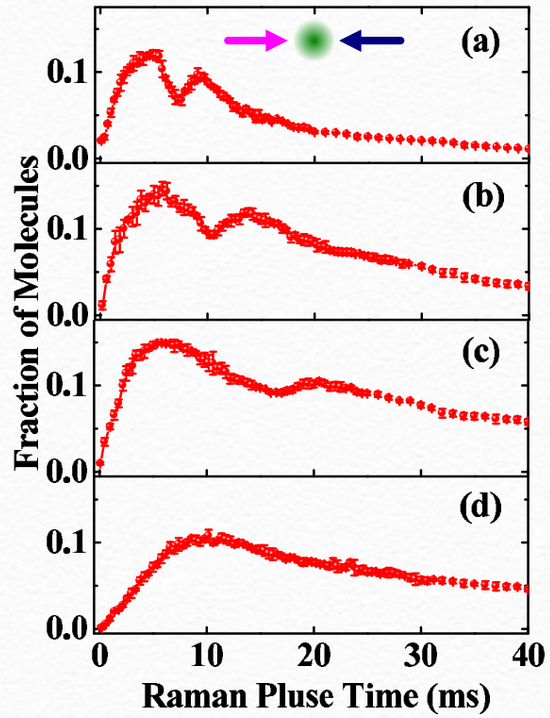


Spielman 2012

Li You's talk

Xiaoling Cui 2011
Peng Zhang 2012
Zhenhua Yu 2012

Molecule Formation:
Mix singlet with triplet



Jing Zhang's talk

Vijay Shenoy 2011
Hu and Pu 2011
Peng Zhang 2012

New Physics with Spin-Orbit Coupling

Impact of Two-body Physics on Fermi Gas

- **Enhance Superfluid transition temperature**

  Yu and Zhai, 2011; Hu and Pu, 2011

- **Lead to FFLO State**

  Wei Yi's talk, ...

- **Lead to Topological Superfluid**

  Chuan-wei Zhang, Hu and Pu, Wei Yi and Wei Zhang, ...

New Physics with Spin-Orbit Coupling

New Physics with Spin-Orbit Coupling

Three-body Physics

Importance of three-body physics

- **Classical Physics: for instance, Sun-Earth-Moon**

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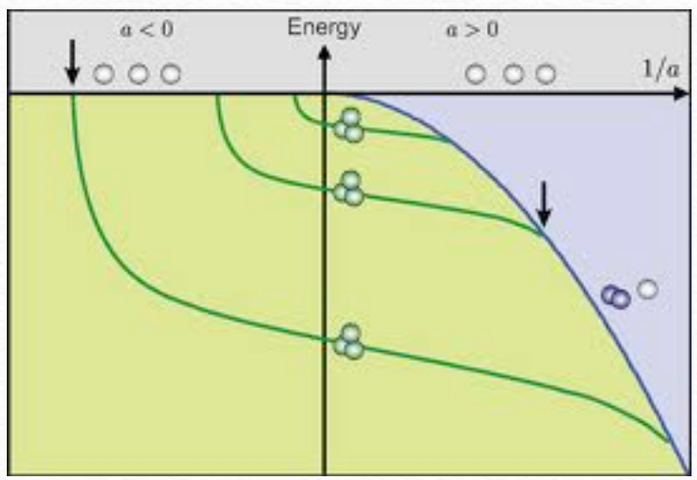
- **Quantum Mechanics: for instance, in nuclear physics**

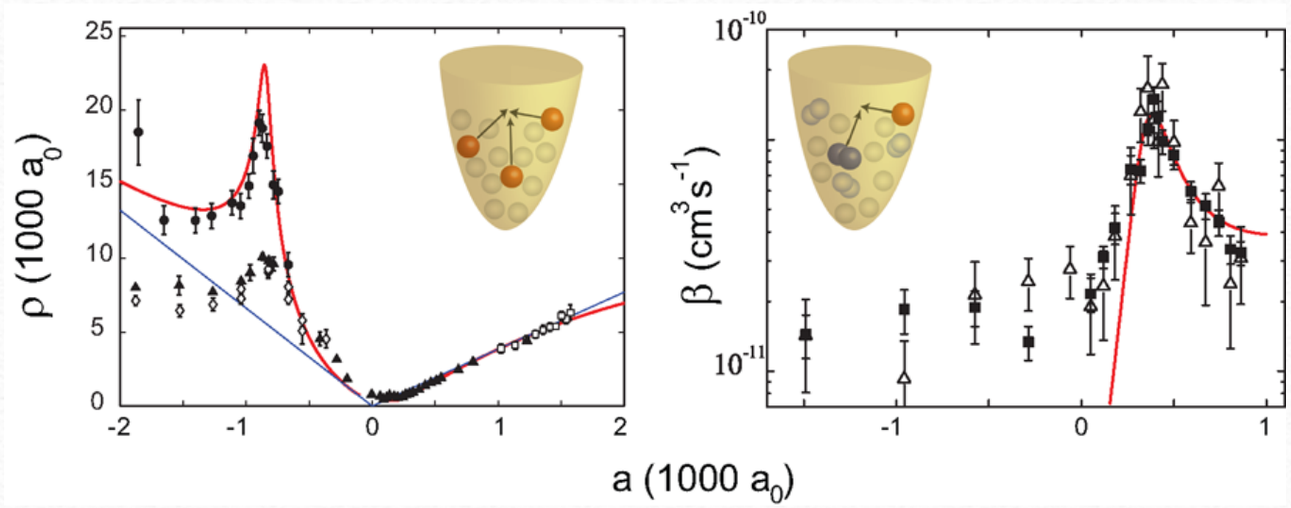


Efimov state predicated by Efimov in 1970 with
Universal Discrete Scaling Symmetry

Importance of three-body physics for cold atoms

Cold atom system is an idea system for studying few-body physics because of its diluteness !

- **Efimov physics has been observed and studied**



Innsbruck group 2004

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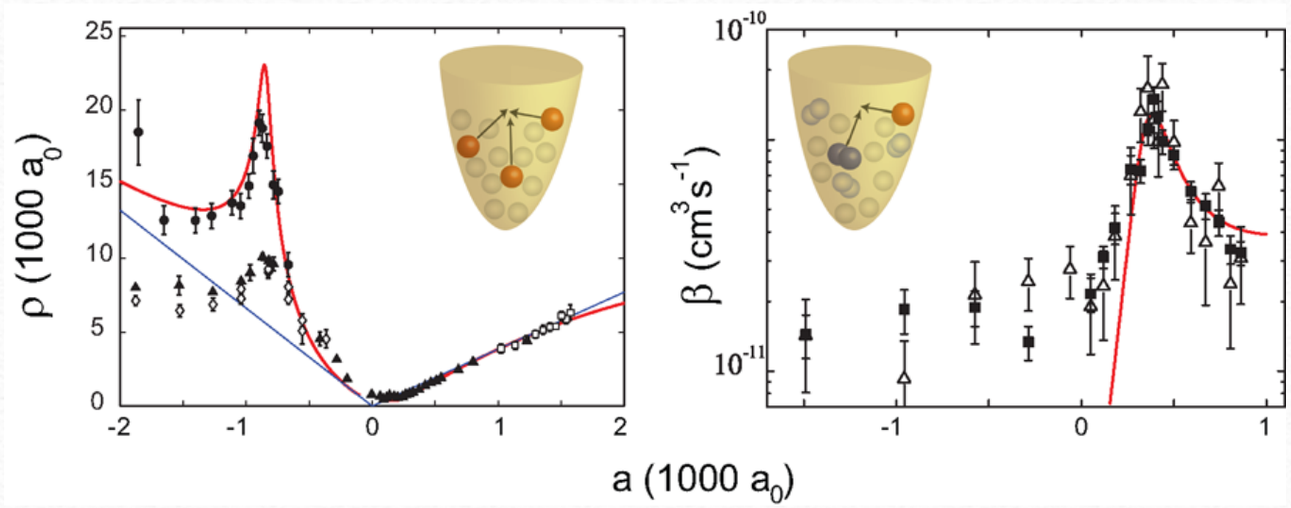

Innsbruck group 2004

Few-body physics plays a key role in determining basic properties of an atomic gas !

- **Determine atom-dimer scattering length**
- **Determine the loss rate and lifetime of atomic gas**

Importance of three-body physics for cold atoms with SO coupling

Practical:

- Determine atom-dimer scattering length
- Determine life time of Fermi gas at resonance regime

Theoretical:

- Whether SO coupling can give rise to new universal physics ?

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Complicated !!

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- ❒ **Determine life time of Fermi gas at resonance regime**

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Choose 3-dimensional symmetric form of SO coupling

$$(\lambda / m) p \cdot \sigma$$

Use the symmetry to simplify the calculation

Zheyu Shi, Xiaoling Cui and HZ, arXiv: 1309.1925

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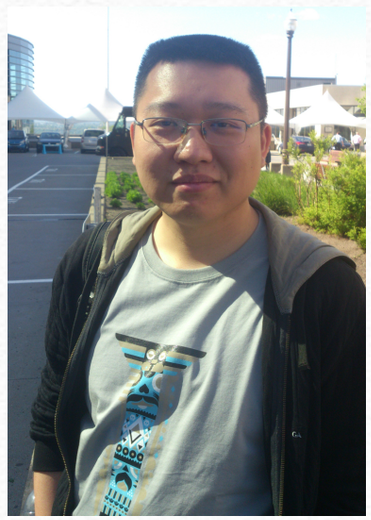

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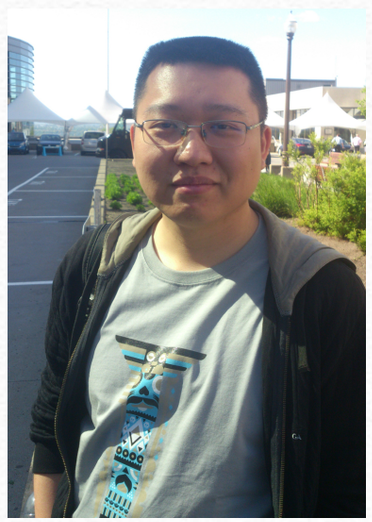
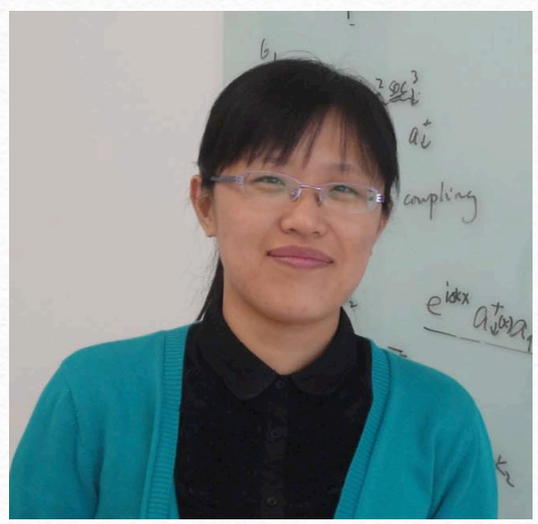
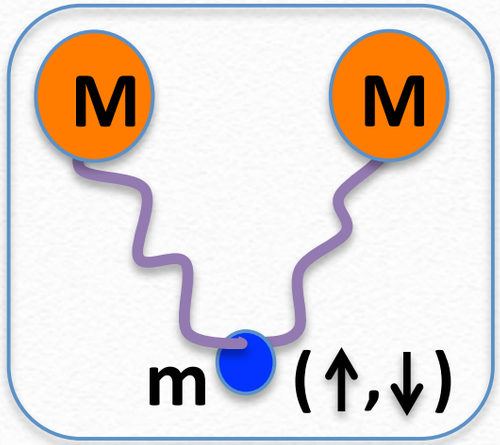




Zheyu Shi, Xiaoling Cui and HZ, arXiv: 1309.1925

The Three-Body System: Model



Two heavy fermions plus one light spin-1/2 fermion

$$\hat{H}_0 = \frac{\mathbf{p_1}^2}{2M} + \frac{\mathbf{p_2}^2}{2M} + \frac{(\mathbf{p_3} - \lambda\hat{\sigma})^2}{2m}$$

$$\hat{U} = [g\delta(\mathbf{r_1} - \mathbf{r_3}) + g\delta(\mathbf{r_2} - \mathbf{r_3})]\mathbf{I},$$

Isotropic SO coupling for light particle

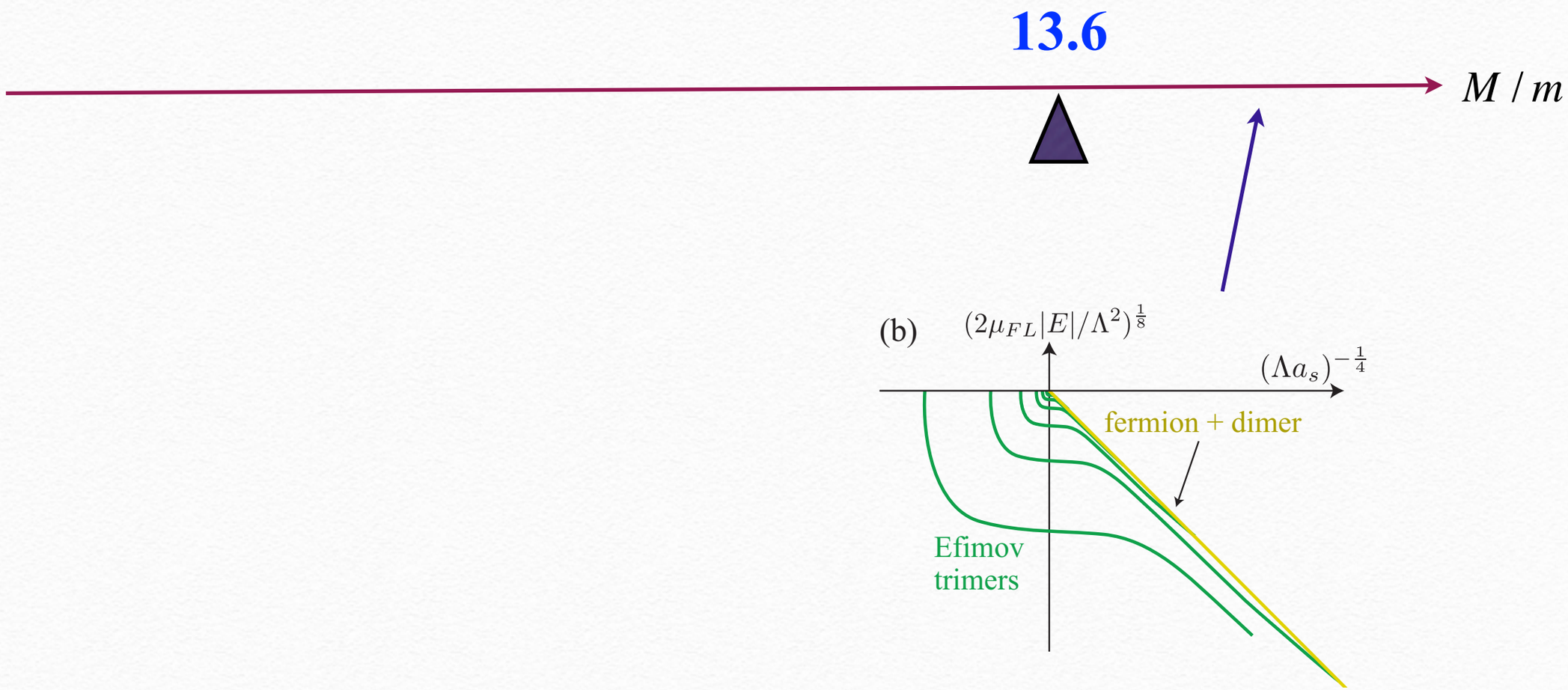
$$(\lambda / m) p \cdot \sigma$$

Resonant interaction between heavy and light fermions, independent of spin-degree of freedom

Three-body physics without SO coupling

Two-types of Universal Behaviors

Three-body physics without SO coupling

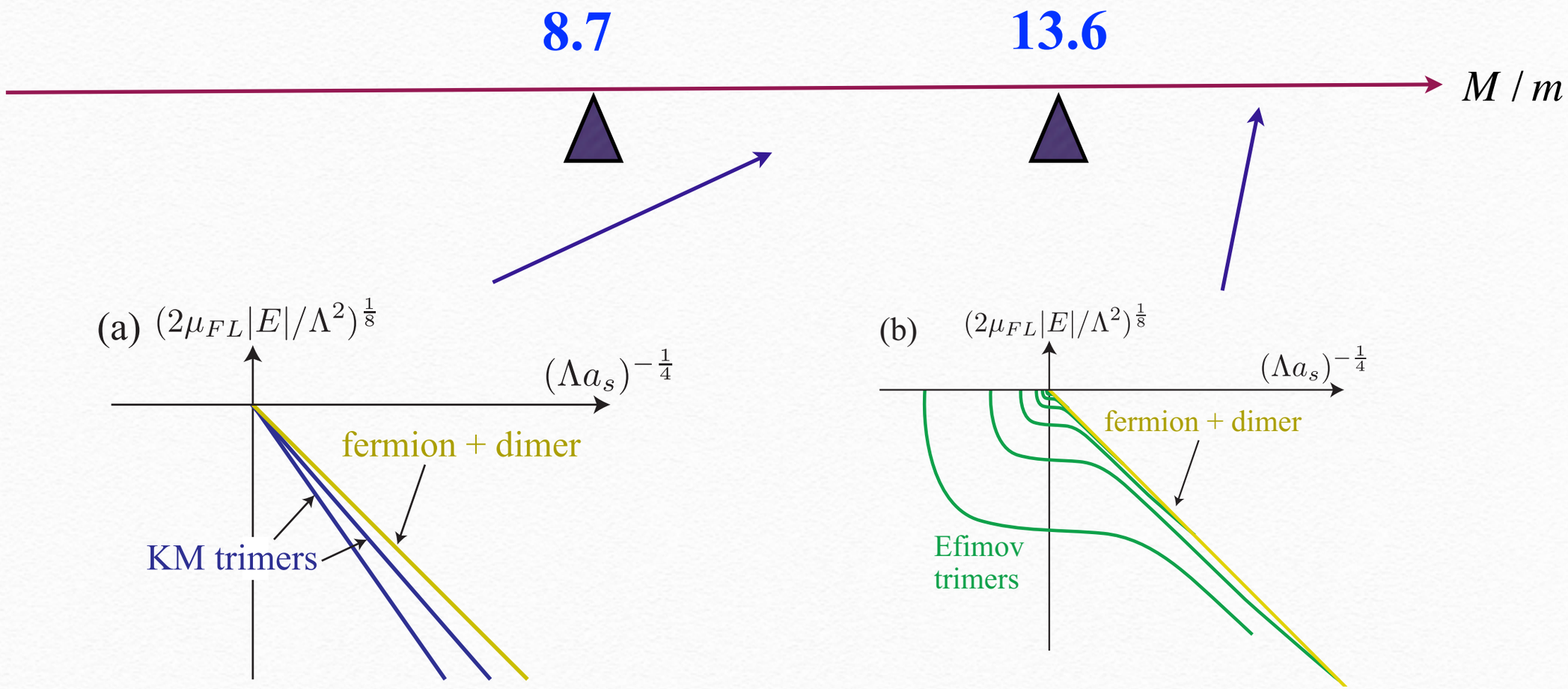


$M/m > 13.6$

Efimov trimer
Universal Scaling

Two-types of Universal Behaviors

Three-body physics without SO coupling



$8.17 < M/m < 13.6$

Kartavtsev-Malykh trimer
Universal energy
(independent of high-energy cutoff)

Kartavtsev and Malykh, 2007

$M/m > 13.6$

Efimov trimer
Universal Scaling

Two-types of Universal Behaviors

Outline of how to solve the problem

Wave-function assumption:

$$|\Psi\rangle = \sum_{\mathbf{p},\mathbf{q},\sigma} \Psi_\sigma(\mathbf{q}, \mathbf{K_0} - \mathbf{p}, \mathbf{p} - \mathbf{q}) \hat{\alpha}^\dagger_{\mathbf{q}} \hat{\alpha}^\dagger_{\mathbf{K_0}-\mathbf{p}} \hat{\beta}^\dagger_{\sigma,\mathbf{p}-\mathbf{q}} |0\rangle,$$

Auxiliary function:

$$f_\sigma(\mathbf{p}) = g \sum_{\mathbf{q}} \Psi_\sigma(\mathbf{q}, \mathbf{K_0} - \mathbf{p}, \mathbf{p} - \mathbf{q})$$

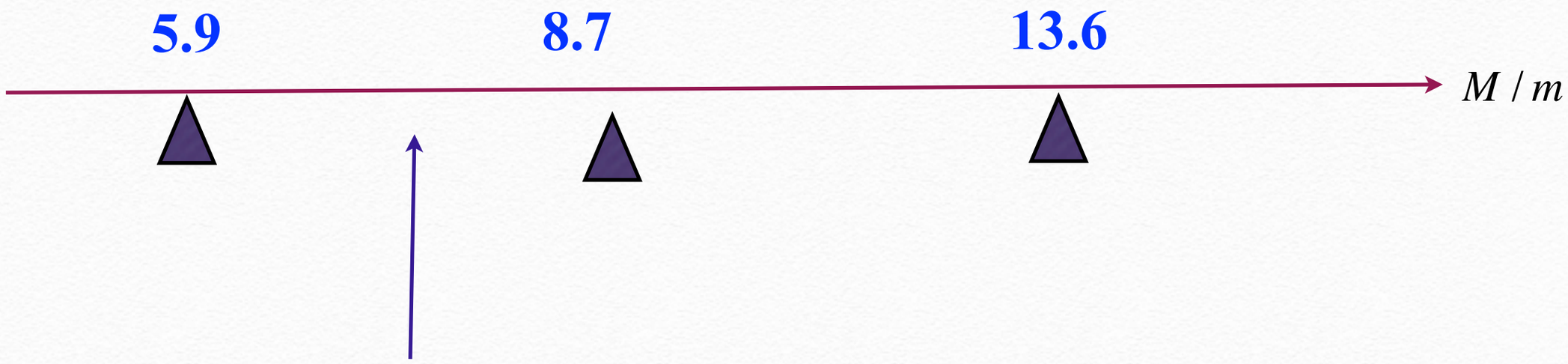
Central self-consistent equation:

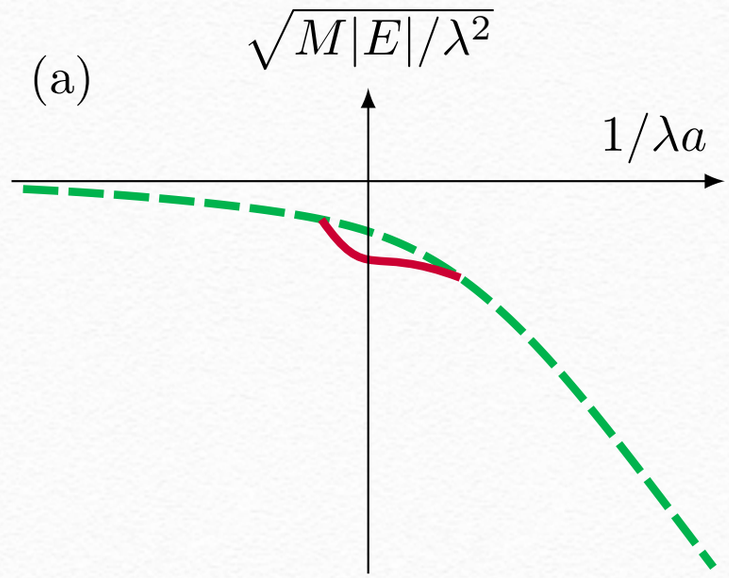
$$f_\sigma(\mathbf{k}) = g \sum_{\mathbf{p},\sigma'} G_{\sigma\sigma'}(\mathbf{p}, \mathbf{K_0} - \mathbf{k}, \mathbf{k} - \mathbf{p}) [f_{\sigma'}(\mathbf{k}) - f_{\sigma'}(\mathbf{K_0} - \mathbf{p})].$$

Key: use symmetry to simplify this equation

$$Z(k) \begin{pmatrix} f_0(k) \\ f_1(k) \end{pmatrix} = \int_0^\infty dp K_j(k,p) \begin{pmatrix} f_0(p) \\ f_1(p) \end{pmatrix}$$

Three-body physics with SO coupling

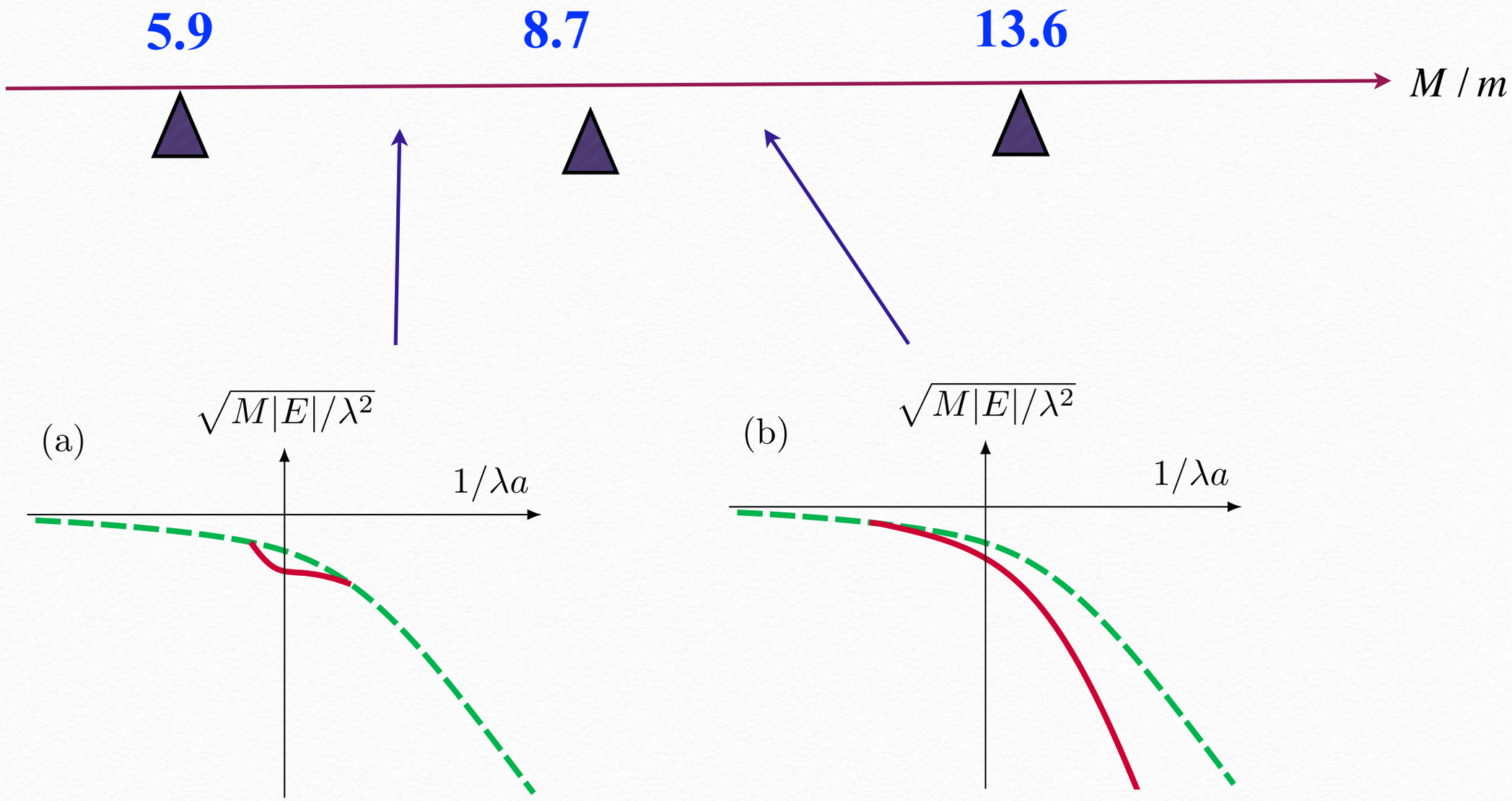




One Universal KM trimer around unitary regime

Lower bound 5.9 can be satisfied by Li-K mixture

Three-body physics with SO coupling

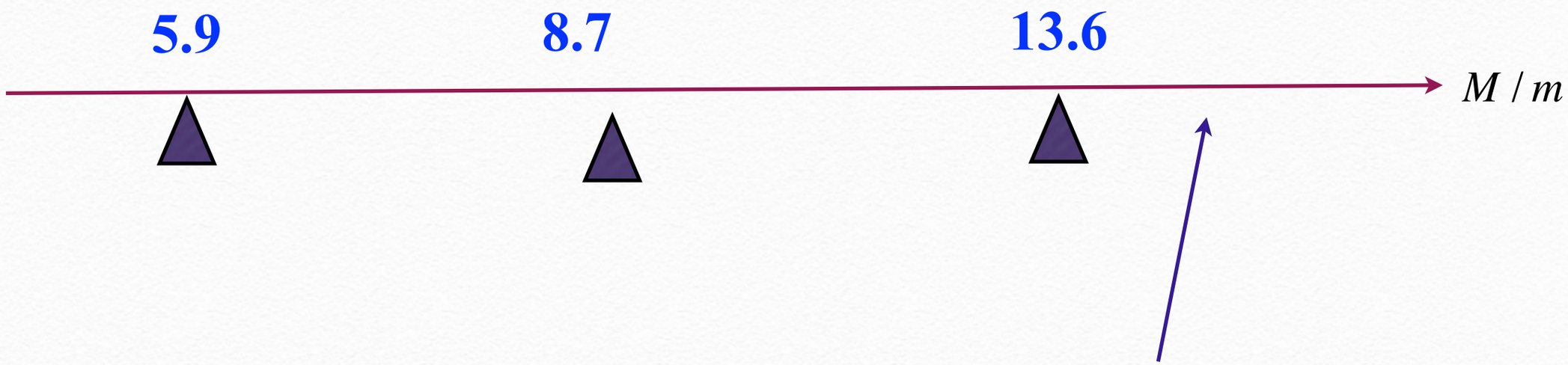


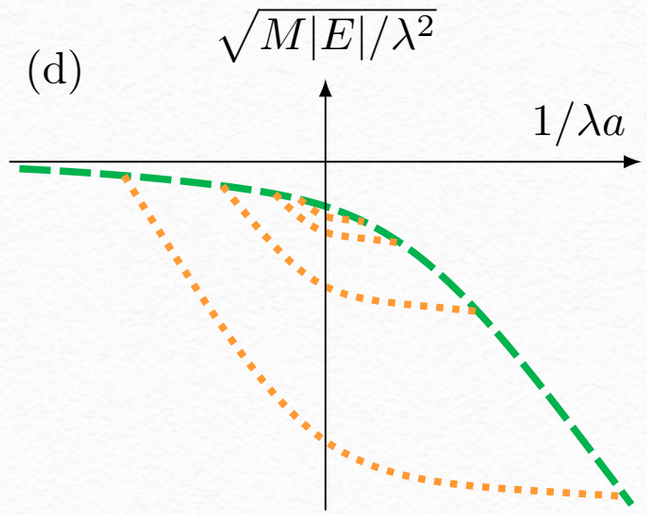
Universal KM trimer around unitary regime

Universal KM trimer extended to BEC side

New trimer merges into KM trimer

Three-body physics with SO coupling

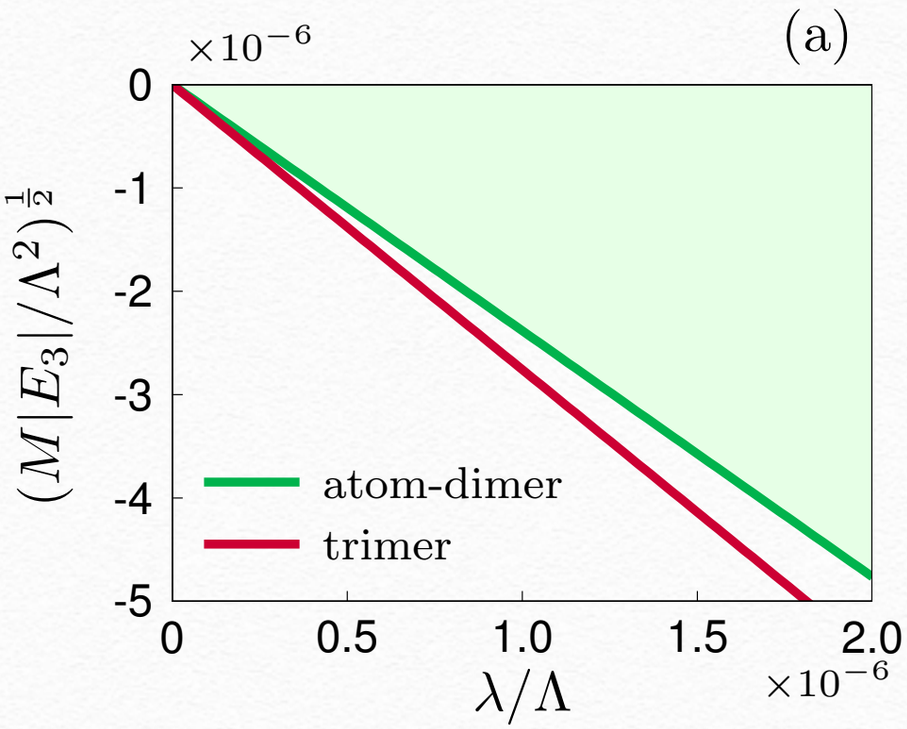


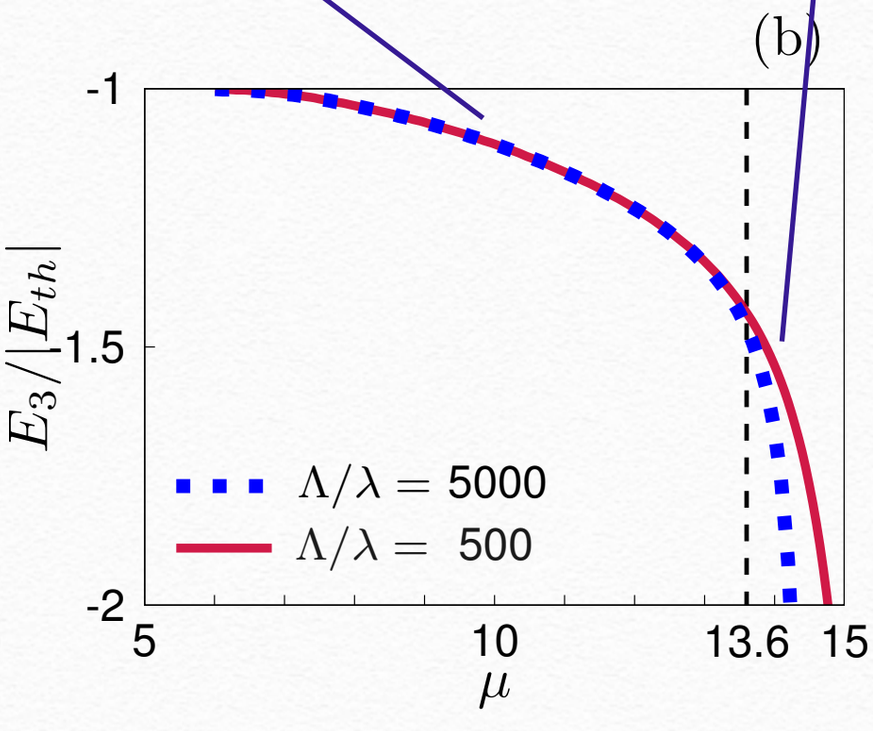


“ Efimov trimer ”
No Universal Scaling

SO coupling introduces an additional length scale that breaks Efimov universal scaling

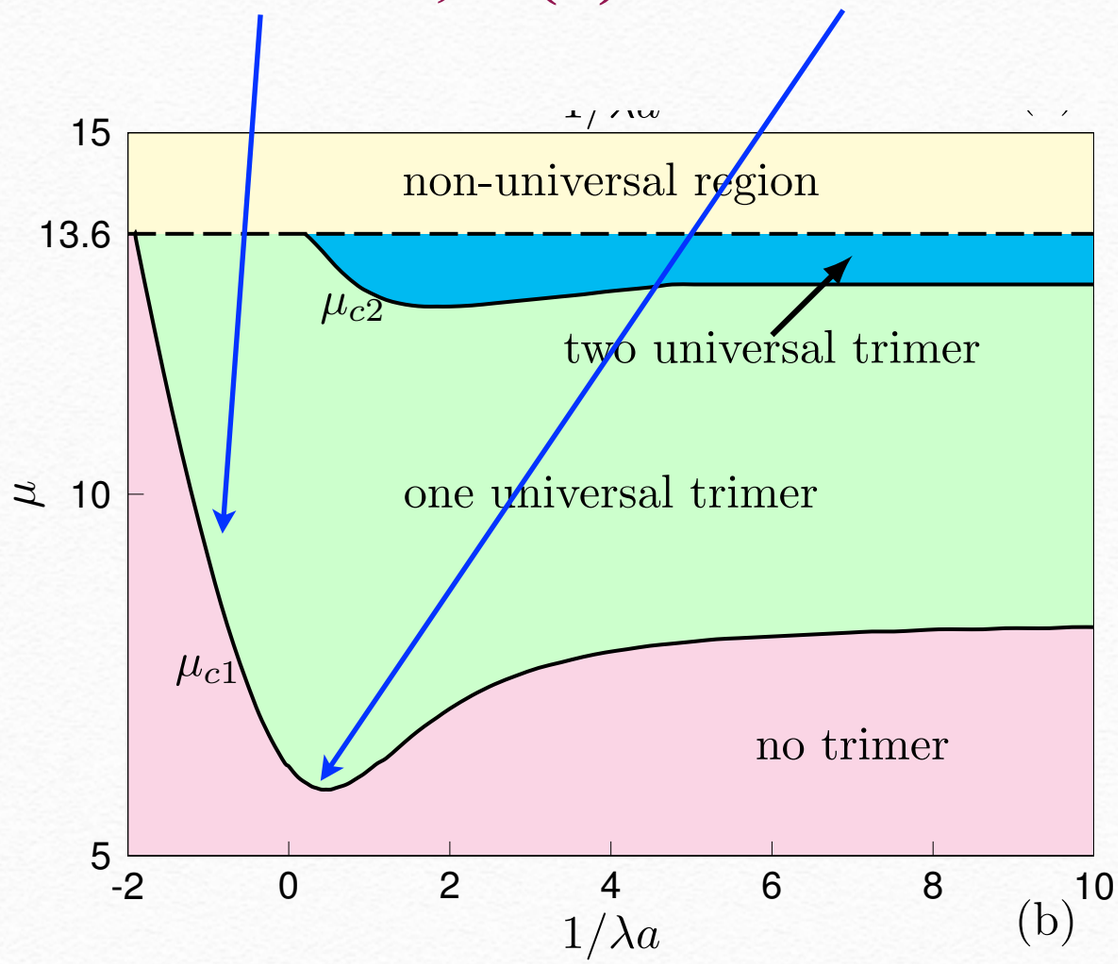
Three-body physics with SO coupling





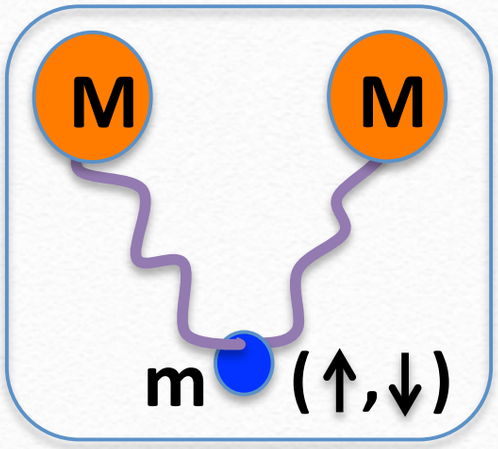
“ Phase diagram ” for three-body problem

Take-home message:
Universal trimer is induced by SO coupling
(i) at BCS side also; (ii) for smaller mass ratio



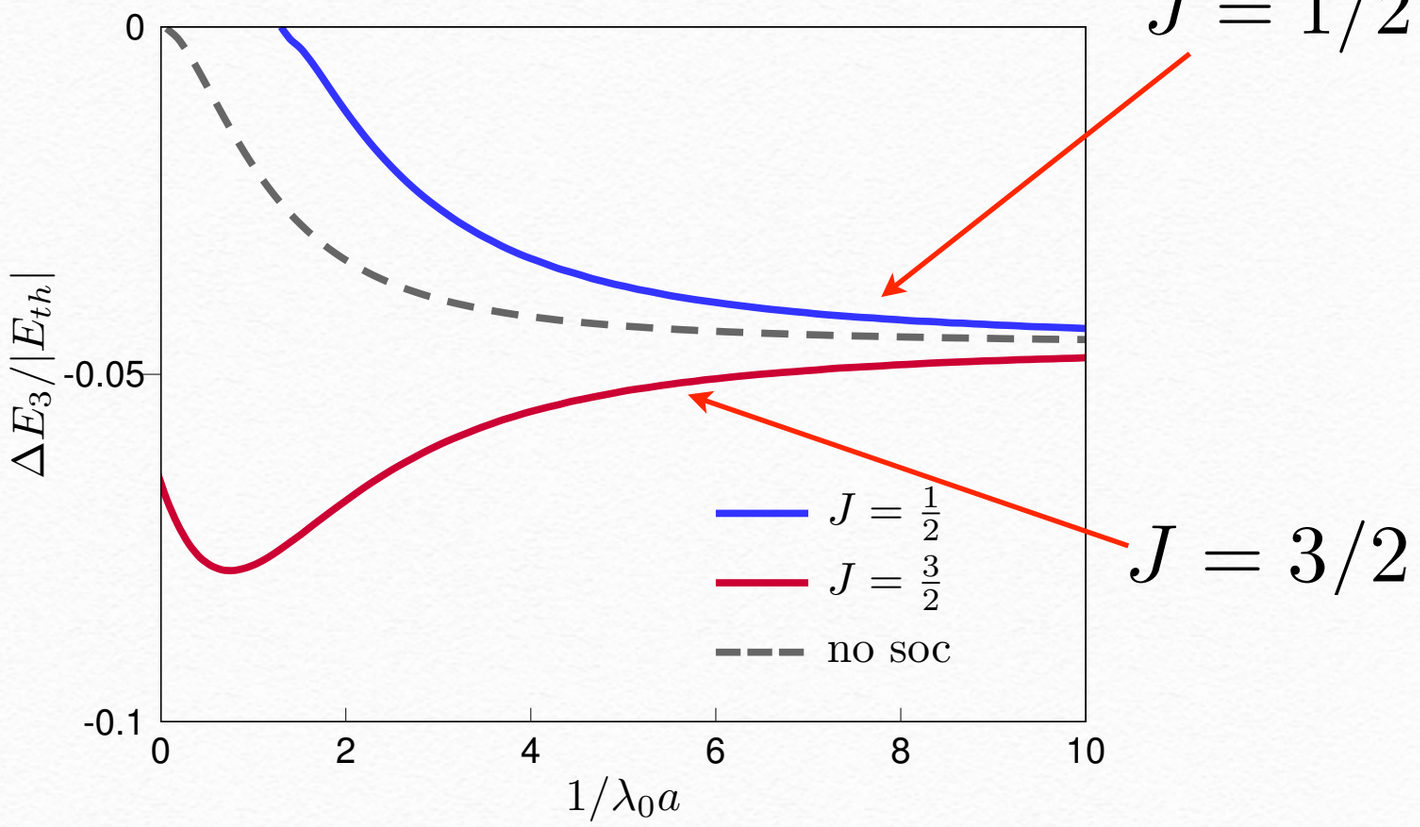
Physical Understanding

How general are our results ?



$L = 1$

$S = 1/2$



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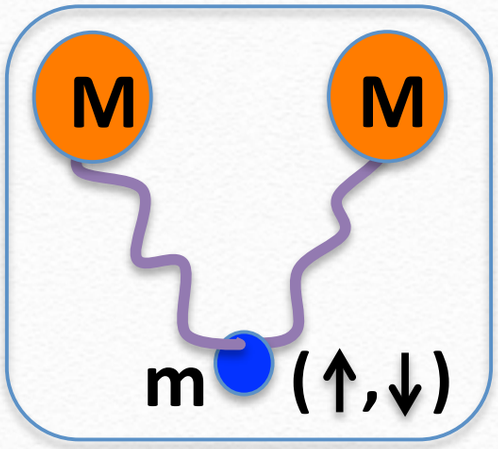

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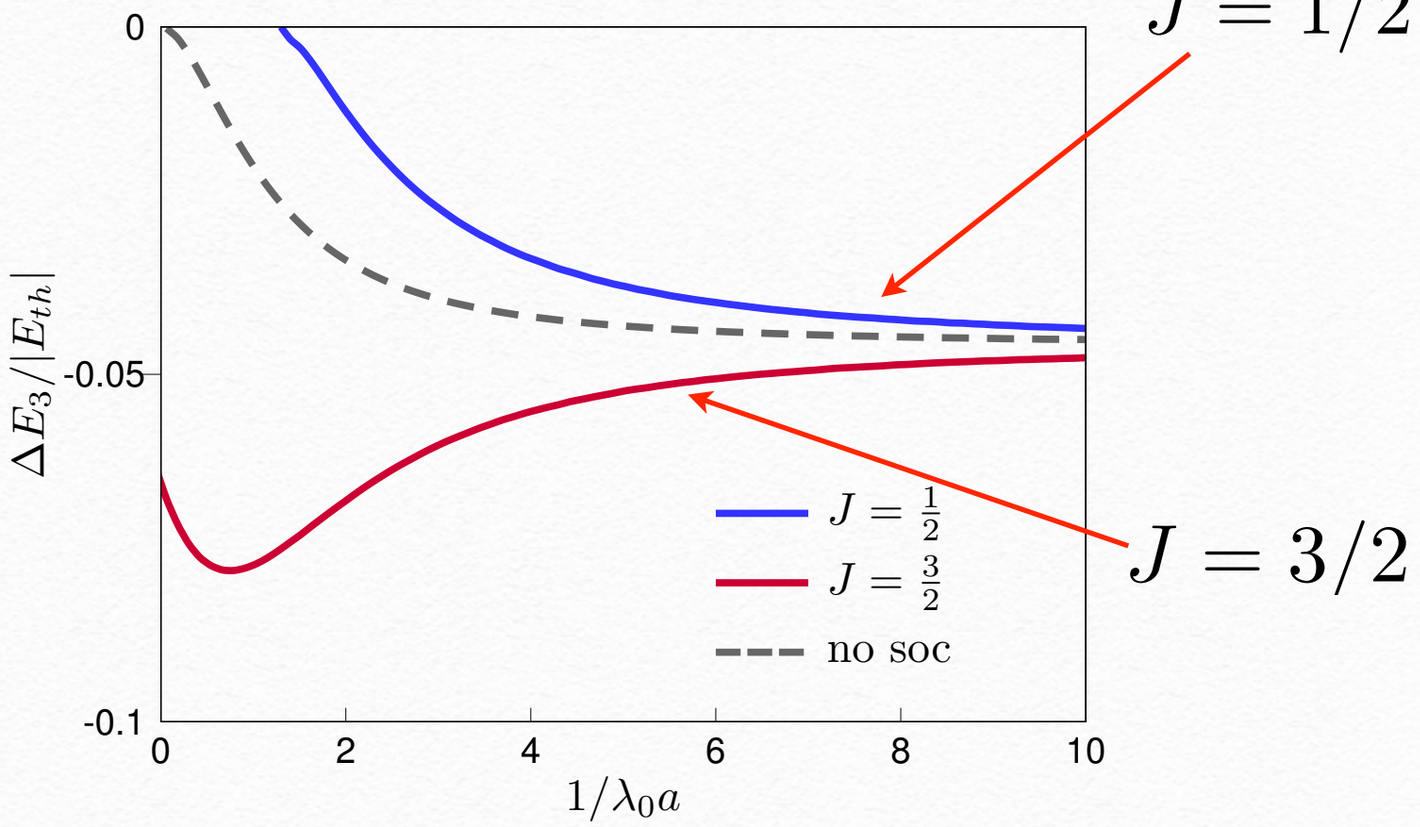


Spin-orbit coupling couples L and S,
lower energies of certain states

Expert: our results hold for general SO coupling

Three-body physics for cold atoms with SO coupling

Theoretical:

- **Whether SO coupling can give rise to new universal physics ?**

Efimov trimer with Universal Scaling → **with SO coupling** → **Broken**

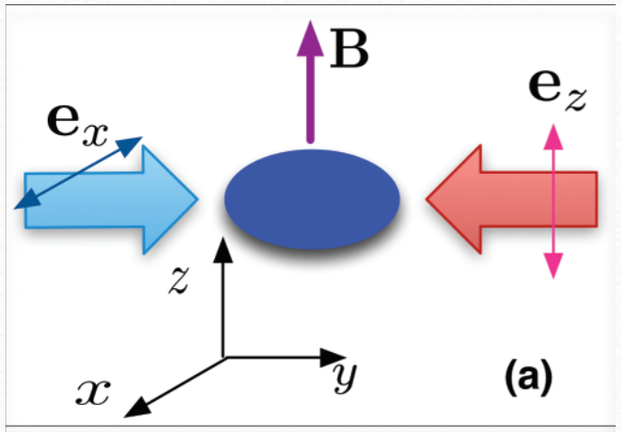
KM trimer with Universal Scaling → **with SO coupling** → **Enhanced**

Practical:

- **Atom-dimer scattering length can be controlled by SO coupling**
- **Life time of Fermi gas at resonance regime maybe shorter**

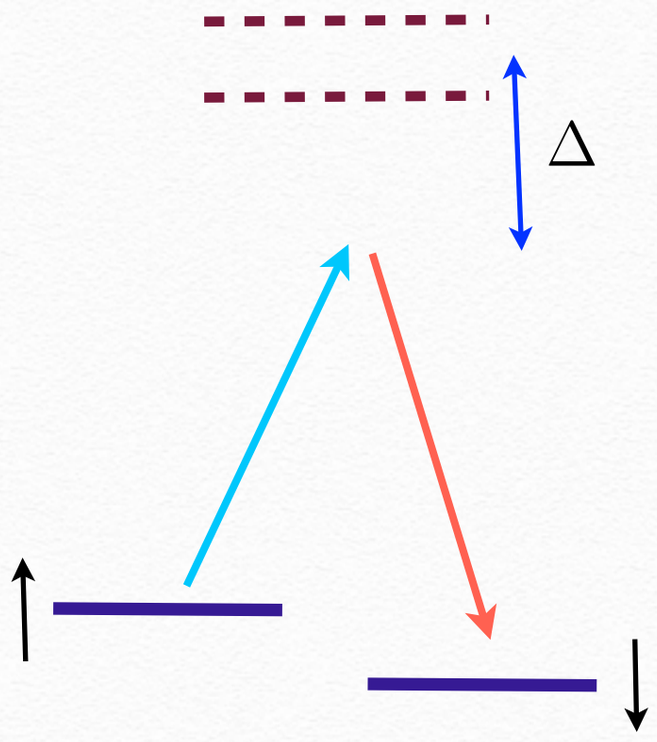
Zheyu Shi, Xiaoling Cui and HZ, arXiv: 1309.1925

Challenges in Spin-Orbit Coupling



Heating !

The source of heating

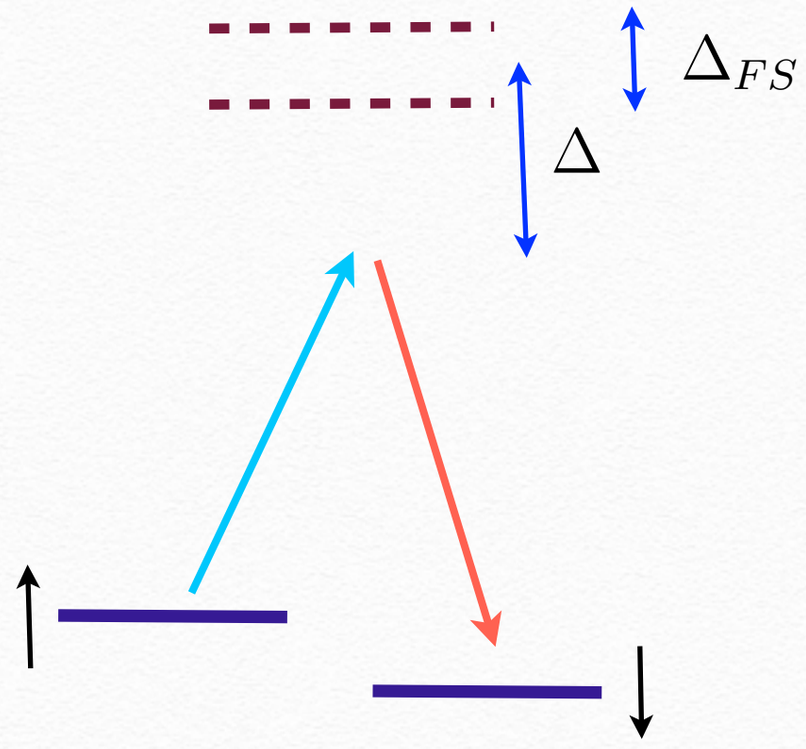


Raman coupling: $\frac{W}{\Delta}$

Heating rate: $\frac{W\Gamma}{\Delta^2}$

Heating can always be suppressed by increasing detunning !

The source of heating



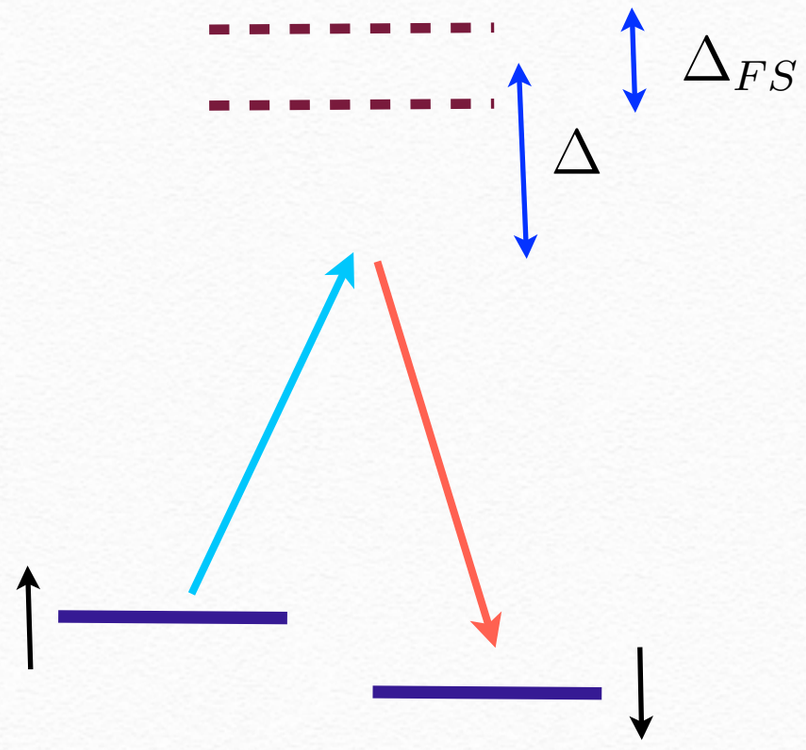
Alkali atoms

$$L = 0$$

Raman coupling: $\dfrac{W}{\Delta}$

Heating rate: $\dfrac{W\Gamma}{\Delta^2}$

The source of heating



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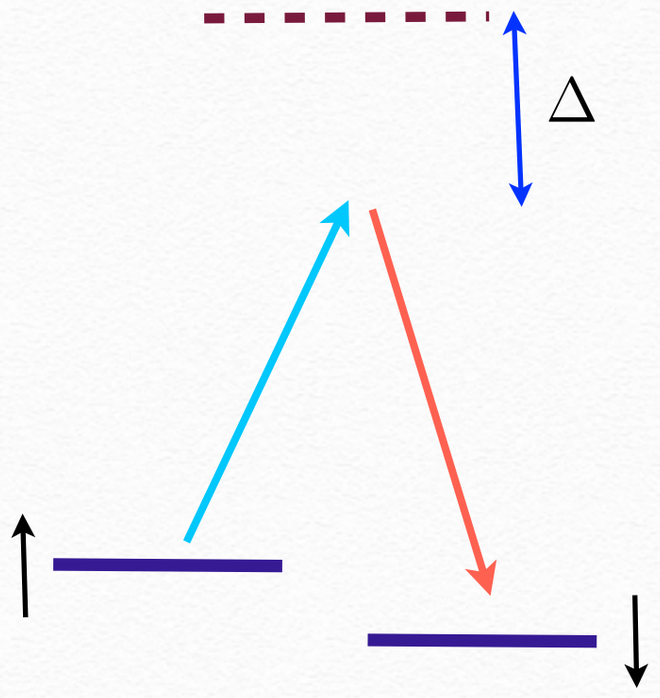


Open shell lanthanide atom

Dy (Stanford 2011)

$^5I_8 \qquad L=6 \quad S=2$

Er (Innsbruck 2012)

$^3H_6 \qquad L=5 \quad S=1$

Raman coupling: $\dfrac{W}{\Delta}$ $\qquad \dfrac{W}{\Delta}\dfrac{\Delta_{FS}}{\Delta}$

Heating rate: $\dfrac{W\Gamma}{\Delta^2}$

Heating can always be suppressed by increasing detunning !

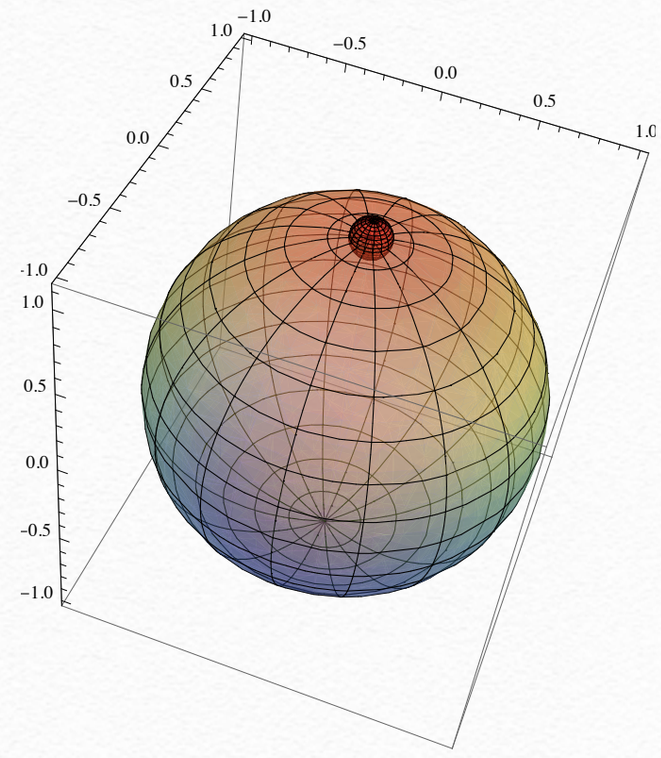
Xiaoling Cui, Biao Lian, Tin-Lun Ho, Benjamin L Lev and HZ, PRA, 88, 011601(R) (2013)

Difference between spin-1/2 and higher spins

Spin-1/2

$$\begin{pmatrix} \alpha_1 + i\beta_1 \\ \alpha_2 + i\beta_2 \end{pmatrix}$$

4 real number - 1 total density
-1 total phase = 2 real number



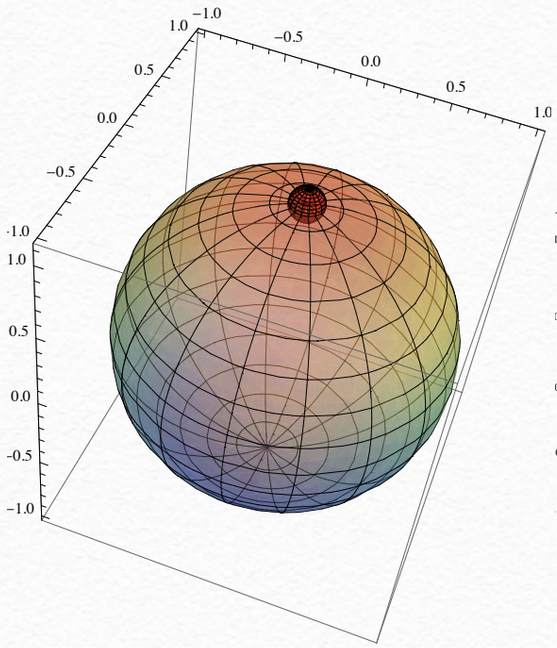
Higher spin - n/2

$$\begin{pmatrix} \alpha_1 + i\beta_1 \\ \alpha_2 + i\beta_2 \\ \cdots \\ \alpha_n + i\beta_n \end{pmatrix}$$

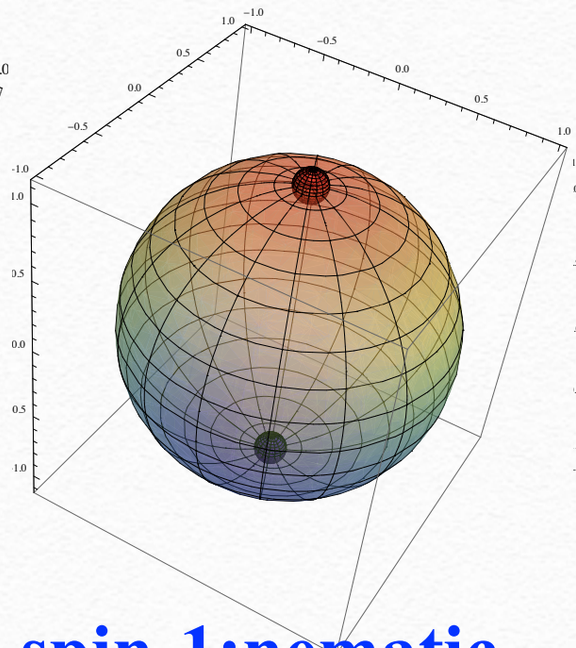
2(n+1) real number - 1 total density -1 total phase = 2n real number

A single Bloch spin is insufficient, is there an alternative geometric representation

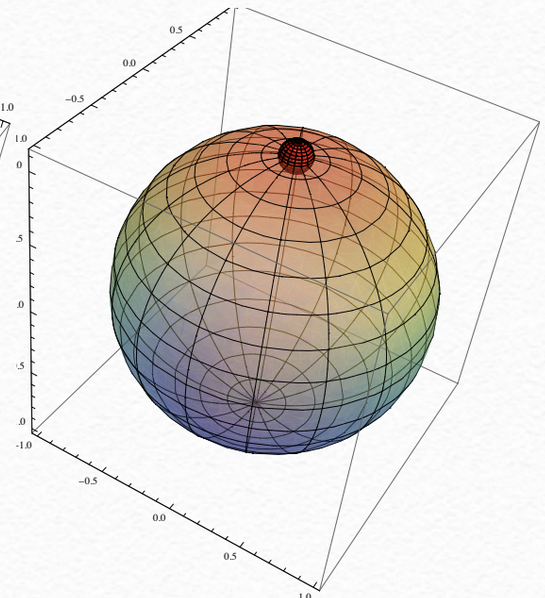
Majorana representation: 2S points on the sphere



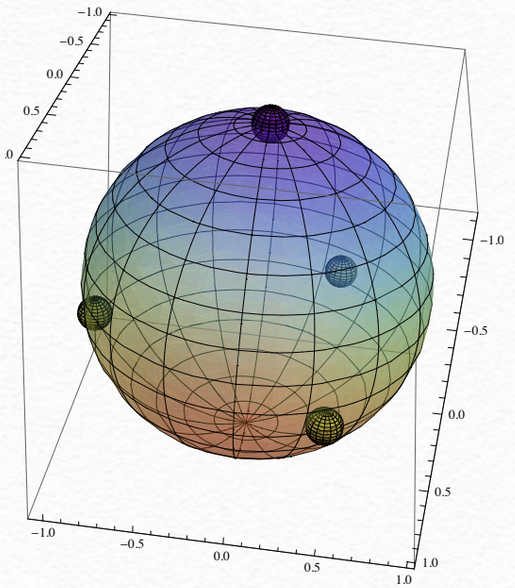
spin-1/2



spin-1:nematic



spin-1:FM

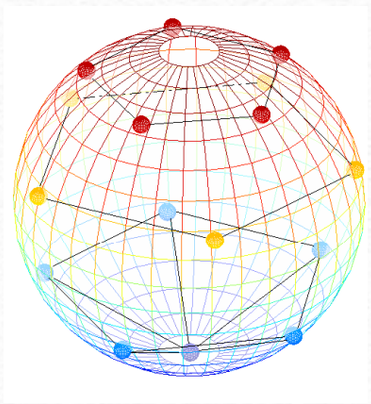


spin-2: tetrahedron

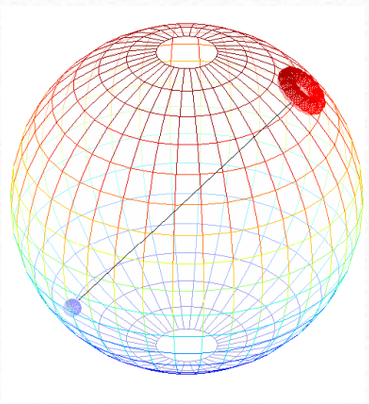
(i) under spin rotation, n-vector rotates as Cartestian vectors;
(ii) under time reversal, n => -n

Barnett, Turner and Demler, PRL, 97, 180412 (2006)
Barnett, Podolsky and Refael, PRB, 80, 024420 (2009)
Lamacraft, PRB, 81, 184526 (2010)
Kawaguchi and Ueda, PRA, 84, 053616 (2011)

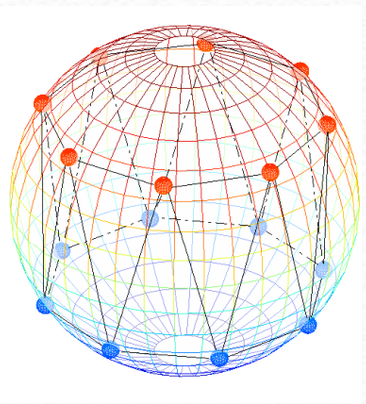
Majorana representation for Dy (J=8) Condensate



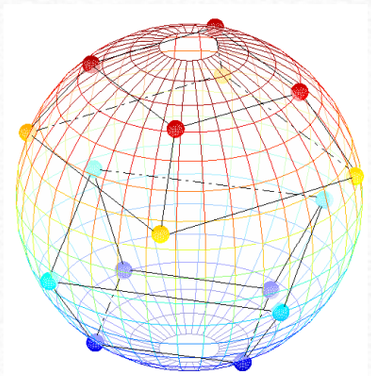
(a) C_{5v}



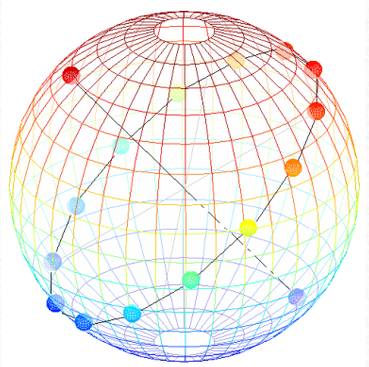
(b) C_{∞}



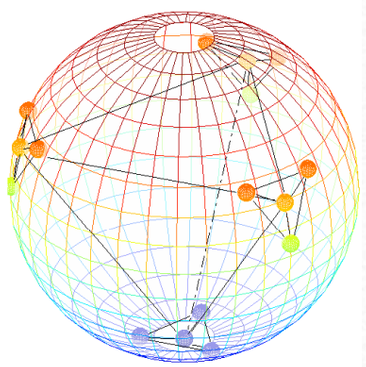
(c) D_{8d}



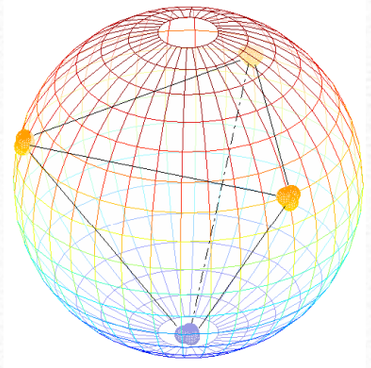
(d) D_{4d}



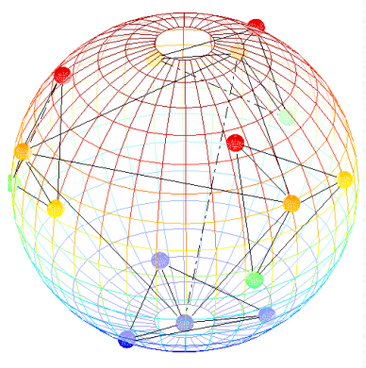
(e) D_{14h}



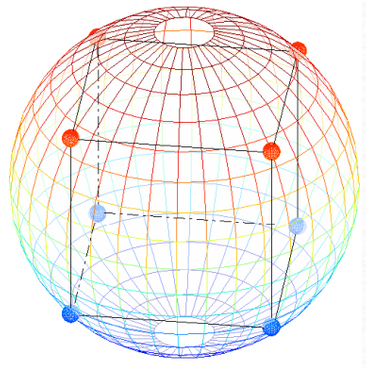
(f) T_d



(g) T_d



(h) T



(i) O_h

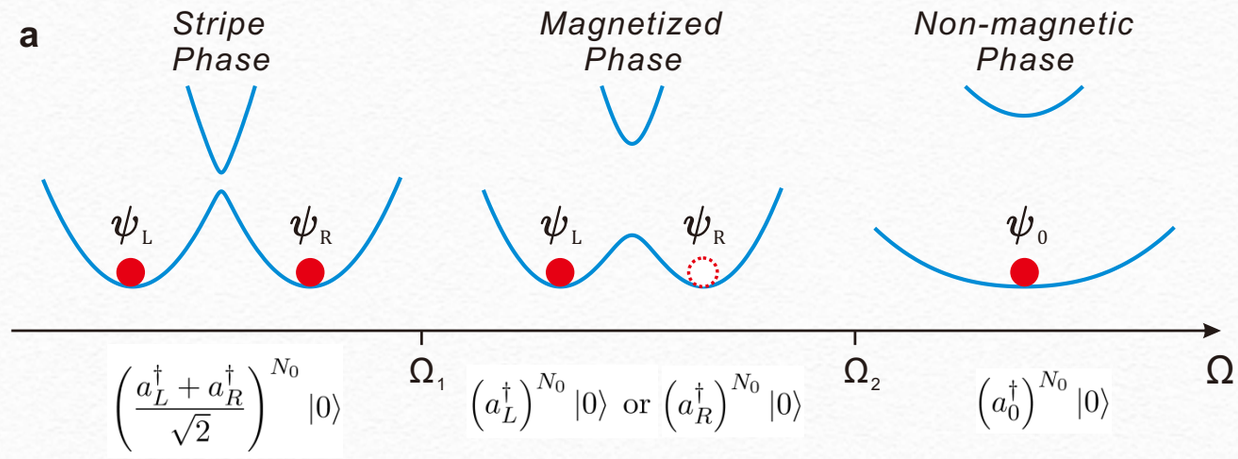
SU(2) symmetry is broken to point group:
Most of them are non-abelian !

Unique for high spin systems !

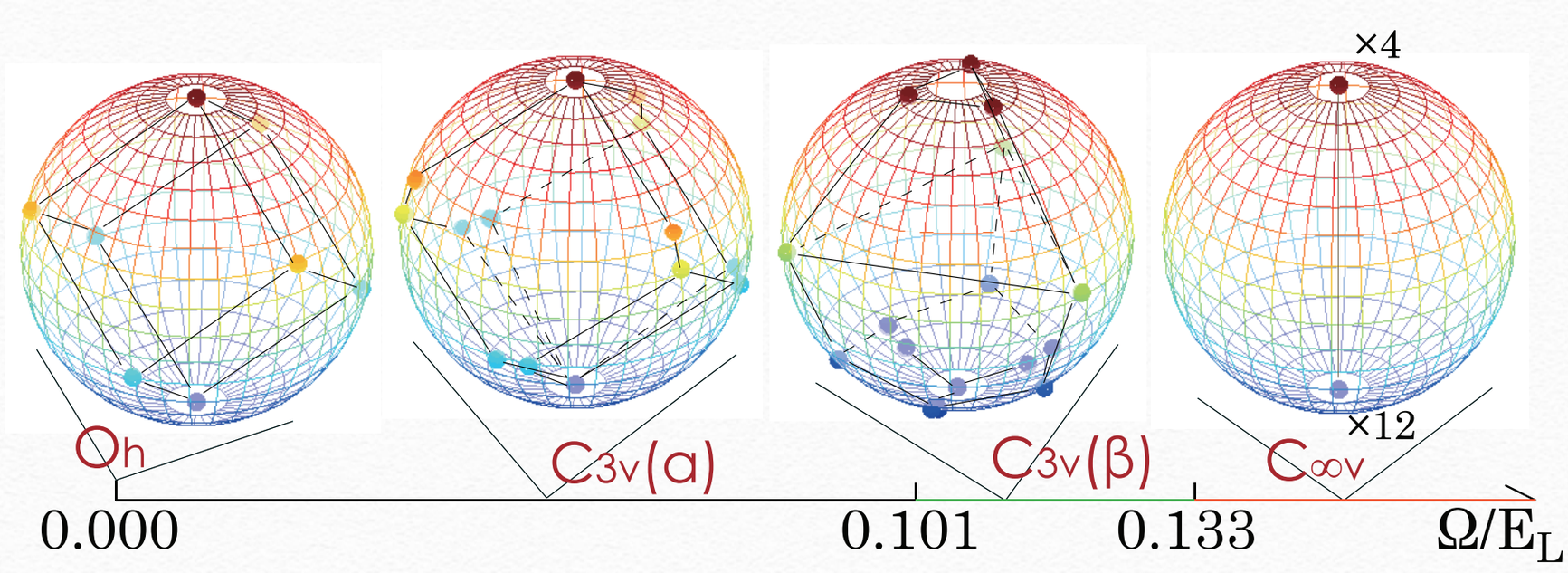
Bian Lian, Tin-Lun Ho and HZ, Phys. Rev. A 85, 051606(Rapid Communication) (2012)

Richer Physics of High Spin Bosons with Spin-Orbit Coupling

Spin-1/2



Spin-8



Xiaoling Cui, Biao Lian, Tin-Lun Ho, Benjamin L Lev and HZ, PRA, 88, 011601(R) (2013)

Spin-Orbit Coupling: New Physics and Challenges

Spin-orbit coupling in cold atom system brings about new physics

- New phase/phenomena of superfluids
- Universal behavior in few-body physics
- Spin-orbit coupled higher spin system

Great Challenges:

Heating problem ! But we have various ways to solve it.

Open shell lanthanide atom
Pure magnetic method, such as in atomic chip (Spielman, You...)

Thank you for your attention !