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Article

Cavity-Assisted Spin-Orbit Coupling of Ultracold atoms

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Abstract: We investigate dynamical and static properties of ultracold atoms confined in an optical cavity, where two photon Raman process induces effective coupling between atom's pseudo-spin and center-of-mass momentum. In the meantime, atomic dynamics exerts a back action to cavity photons. We adopt both mean field and master equation approach to tackle the problem and found surprising modifications to atomic dispersions and dynamical instabilities, arising from the intrinsic nonlinearity of the system. Correspondence between semi-classical and quantum limits is analyzed as well.

Keywords: cavity quantum electrodynamics; cold atoms; spin-orbit coupling

1. Introduction

Laser light is a versatile and standard experimental tool in the forefront field of modern quantum optics and ultracold atomic gases. Spatial coherence allows a laser beam to be focused to a tight spot and stay narrow over long distances. Extremely narrow energy spectrum, given by high temporal coherence, makes it ideal to create highly controllable and tunable optical potentials, when far red detuned from an atomic resonances. The pioneering experimental achievement of Bose-Einstein condensation (BEC) [1–3], and of Fermi degenerate dilute gases [4–6] has offered a plethora of opportunities to study quantum properties of light and ultracold matter. A major focus in recent years has been the exploration of quantum properties of many-body strongly correlated systems (for reviews, cf [7,8]) , by trapping coherent matter waves in optical potentials, e.g. harmonic trap, optical lattices, etc. A high level of microscopic understanding sheds light on some of long sought problems in condensed matter and high energy physics, e.g. Bose-Hubbard (BH) model [9,10], universal physics of strongly interacting fermions [11,12]. However, quantum effects of the light were widely neglected to a large extent. In both complex experimental setups and theoretical models, laser light is commonly treated as a classical auxiliary tool to prepare, probe and manipulate intriguing atomic states. Atom-field interaction can be well captured by the minimal-coupling Hamiltonian at the dipole approximation level. In the far red detuned limit, coherent scattering of photon dominate and the resulting dipole light force can be derived from the optical potential proportional to laser intensities due to Stark-shift.

The level playing field changes when atom is confined in high finesse optical cavity [13–15], where atom and the light field will mutually affect each other. This is simply because intra-cavity photon and atoms highly frequently scatter with each other due to the geometric confinement, the light field no longer serves as a conservative potential. Dipole force gets strongly enhanced and and atom's back-action onto light gets significant. In general, this demands a self-consistent solution for both light and atom by treating them on equal footing. On the one hand, the quantum properties of atom will manifest themselves in the scattered light, which leads to novel technical advancements in light measurement of atomic states. On the other hand, the quantized light field imprints nontrivial marks on atomic many-body dynamics as well as equilibrium states. In this regard, cavity quantum electrodynamics (CQED) further enriches the picture of using quantum simulator to understand real world materials, [16–18].

Following the cornerstone of traditional CQED, where only internal dynamics of the atom is relevant in typical setups, we have witnessed important experimental progresses in putting ultracold atoms inside optical cavities. The center-of-mass (COM) motion of the atom gets non-negligible in this "ultracold atom + optical cavity" system, which has been extensively explored both experimentally [19–23] and theoretically [24,25].

Another branch of series breakthroughs in cold atom research originate in the realization of artificial (synthetic) gauge potentials for neutral atoms, first in bosonic systems [26,27] and later in fermionic counterparts [28,29]. Laser fields are properly aligned and designed in such a way that trapped atoms may mimic charged particles in a magnetic field with emergence of Lorentz-like force. The synthesis is achieved by inducing two-photon Raman transition between two hyperfine ground state. Using a group of degenerate (or quasi-degenerate) pseudospin eigenstates, non-abelian dynamics of cold atoms in light fields is generated, which effectively leads to the spin-orbit coupling (SOC) for cold atoms, simulating the one appearing for electrons in condensed matter. Synthetic SOC refers to the coupling between pseudospins (a.k.a. hyperfine states) and atom's COM, rather than the generic interaction between electron's spin (or magnetic moment) and angular momentum operator in quantum mechanics. SOC is essential in understanding numerous underlying condensed matter phenomena and nuclear particle physics [30], including *inter alia* topological insulators, Majorana and Weyl fermions, spin-Hall effects, etc [31–35].

Thus far, all the experimental realizations of synthetic SOC in quantum gases, utilize classical laser fields to assist Raman transition, which are not affected by atom's COM reciprocally. In this article of special issue, we first of all briefly review our previous work [36], and theoretically explore the full quantum treatment beyond semi-classical mean field formalism, then investigate the correspondences in quantum and semi-classical regions. We consider a single atom (or an ensemble of non-interacting BEC) being confined by a single-mode unidirectional ring cavity, whose cavity mode together with an additional coherent laser beam form a pair of Raman beams that flips atomic transition between $|\uparrow\rangle$ and $|\downarrow\rangle$ while transferring recoil momentum of $\pm 2\hbar q_r \hat{z}$ from and/or to photon field. Hence, the so-realized effective coupling between atom's external and internal degrees of freedom is generated by the quantized light field, which is affected by atomic dynamics in return. In this sense, the *cavity-assisted* SOC becomes *dynamic*. We show that, at mean field level, the dynamic SOC dramatically modifies the atomic dispersion relation, in particular, with emergence of a loop structure under certain circumstances. We systematically characterize dispersion relation of atomic state and photon number, both as a function of atom's quasi-momentum. For given cavity parameters, we found with increasing Raman coupling strength Ω, dispersion curve changes from double minima to gapped single minimum, looped structure, and gapless single minimum in sequence. Furthermore, we carry out the full quantum mechanical treatment by solving master equations of density operators, and find excellent agreement by comparing averaged photon number with mean field results. The two distinctively different approaches give us an unified understanding of the atom-light effective non-linearity and induced dynamical instability in this system.

The article is organized as the following: After briefly reviewing key ideas of our previous work and semi-classical mean field approach in Sec. 2, we develop the full quantum mechanical formalism to the physical system of interest in Sec. 3 and discuss about the intimate correspondence between the two in Sec. 4, and finally conclude in Sec. 5.

2. Model Setup and Semi-classical Mean Field Formalism

We follow the model Hamiltonian proposed in [36],

$$\begin{aligned} H &= \sum_\sigma \int d\mathbf{r} \left[\Psi_\sigma^\dagger(\mathbf{r}) \left(\frac{\hat{\mathbf{k}}^2}{2m} + \epsilon_\sigma^0\right) \Psi_\sigma(\mathbf{r})\right] + \frac{\Omega}{2} \int d\mathbf{r} \left[e^{+2iq_r z} \Psi_\uparrow^\dagger(\mathbf{r}) \Psi_\downarrow(\mathbf{r}) \tilde{c} e^{+i\omega_R t} + c.c\right] \\ &+ i\varepsilon_p(\tilde{c}^\dagger e^{-i\omega_p t} - \tilde{c} e^{+i\omega_p t}) + \omega_c \tilde{c}^\dagger \tilde{c} - i\kappa \tilde{c}^\dagger \tilde{c}, \end{aligned} \tag{1}$$

where $\Psi_\sigma(\mathbf{r})$ ($\sigma = \uparrow, \downarrow$) is the atomic annihilation operator, ϵ_σ^0 is the corresponding bare atomic energy, and $\tilde{c}$ represents the photon annihilation operator. Ω describes the atom-photon coupling strength, q_r denotes recoil momentum, ω_R refers to the frequency of the coherent laser beam. The cavity is pumped by a coherent laser field with frequency ω_p and pumping rate ε_p. It supports a single mode traveling wave and has an intrinsic angular frequency ω_c. In the semi-classical approach, we have treated the leakage of cavity photon phenomenologically by introducing a cavity decay rate κ.

3. Master Equation Approach: Full Quantum Mechanical Treatment

4. Results and Discussions

5. Conclusions

Acknowledgments

Author Contributions

H.P. conceived the idea of the project, L.D. and C. Z. explored the theoretical and numerical aspects of the physics. All authors contributed to writing and revising the manuscript and participated in the discussions about this work.

Conflicts of Interest

The authors declare no conflict of interest.

References

1. Anderson, M. H., J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, *Science* **1995**, *269*, 198.
2. Bradley, C. C., C. A. Sackett, J. J. Tollett, and R. G. Hulet, *Phys. Rev. Lett.* **1995**, *75*, 1687.
3. Davis, K. B., M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, *Phys. Rev. Lett.* **1995**, *75*, 3969.
4. DeMarco, B., and D. D. Jin, *Science* **1999**, *285*, 1703.
5. Truscott, A., K. Strecker, W. McAlexander, G. Partridge, and R. G. Hulet, *Science* **2001**, *291*, 2570.
6. Schreck, F., L. Khaykovich, K. L. Corwin, G. Ferrari, T. Bourdel, J. Cubizolles, and C. Salomon, *Phys. Rev. Lett.* **2001** *87*, 080403.
7. Bloch, I., *Nature Physics* **2005**, *1* 23.
8. Bloch I. and M. Greiner, *Adv. At. Mol. Opt. Phys.* **2005**, *52* 1.
9. Jaksch, D., C. Bruder, J. I. Cirac, C. W. Gardiner, and P. Zoller, *Phys. Rev. Lett.* **1998**, *81*, 3108.
10. W. Zwerger, *J. Opt. B* **2003**, *5*, 9.
11. Chevy, F.;Salomon, C. Thermodynamics of Fermi Gases. In *The BCS-BEC Crossover and the Unitary Fermi Gas*; Zwerger, W.; Springer: Lecture Notes in Physics, Vol. 836, 2012; pp. 407-446.
12. Ketterle, W.; Zwierlein, M. W., Making, probing and understanding ultracold Fermi gases. In *Ultra-cold Fermi Gases*; Inguscio, M., Ketterle, W., Salomon, C.; IOP Press:Proceedings of the International School of Physics “Enrico Fermi”, 2007; pp.95-287.
13. Brennecke, F., Donner, T., Ritter, S., Bourdel, T., Kohl, M., and Esslinger, T., *Nature* **2007**, *450* 268.
14. Colombe, Y., Steinmetz, T., Dubois, G., Linke, F., Hunger, D. and Reichel, J. *Nature* **2007**, *450* 272.
15. Slama, S., Bux, S., Krenz, G., Zimmermann, C. and Courteille, Ph. W. *Phys. Rev. Lett.* **2007**, *98* 053603.
16. J. M. Raimond, M. Brune, and S. Haroche, *Rev. Mod. Phys.* **2001**, *73*, 565.
17. R. Miller, T. E. Northup, K. M. Birnbaum, A. Boca, A. D. Boozer, and H. J. Kimble, *J. Phys. B* **2005**, *38*, S551.
18. H. Walther, B. T. H. Varcoe, B.-G. Englert, and T. Becker, *Rep. Prog. Phys.* **2006**, *69*, 1325.
19. F. Brennecke, T. Donner, S. Ritter, T. Bourdel, M. KÃűhl, and T. Esslinger, *Nature* (London) **2007**, *450*, 268.

20. Y. Colombe, T. Steinmetz, G. Dubois, F. Linke, D. Hunger, and J. Reichel, *Nature* (London) **2007**, *450*, 272.
21. S. Slama, S. Bux, G. Krenz, C. Zimmermann, and Ph. W. Courteille, *Phys. Rev. Lett.* **2007**, *98*, 053603.
22. D. Schmidt, H. Tomczyk, S. Slama, and C. Zimmermann, arXiv:1311.2156 (2013).
23. S. Gupta, K. L. Moore, K. W. Murch, and D. M. Stamper-Kurn, *Phys. Rev. Lett.* **2007**, *99*, 213601.
24. M. Lewenstein, A. Sanpera, V. Ahufinger, B. Damski, A. Sen De, and U. Sen, *Adv. Phys.* **2007**, *56*, 243.
25. I. B. Mekhov and H. Ritsch, *J. Phys. B* **2012**, *45*, 102001.
26. Y.-J. Lin, K. Jimenez-Garcia, and I. B. Spielman, *Nature* (London) **2011**, *471*, 83.
27. Y.-J. Lin, R. L. Compton, K. Jimenez-Garcia, W. D. Phillips, J. V. Porto, and I. B. Spielman, *Nat. Phys.* **2011**, *7*, 531.
28. P. Wang, Z.-Q. Yu, Z. Fu, J. Miao, L. Huang, S. Chai, H. Zhai, and J. Zhang, *Phys. Rev. Lett.* **2012**, *109*, 095301.
29. L. W. Cheuk, A. T. Sommer, Z. Hadzibabic, T. Yefsah, W. S. Bakr, and M. W. Zwierlein, *Phys. Rev. Lett.* **2012**, *109*, 095302.
30. V Galitski, IB Spielman, *Nature* **2013**, *494* 7435.
31. Hasan, M. Z., Kane, C. L. *Rev. Mod. Phys.* **2010**, *82* 3045âĂŞ3067.
32. Sau, J. D., Lutchyn, R. M., Tewari, S. and Sarma, Das, S. *Phys. Rev. Lett.* **2010**, *104* 040502.
33. Burkov, A. A. and Balents, L., *Phys. Rev. Lett.* **2011**, *107* 127205.
34. Sinova, J., Cilcer, D., Niu, Q., Sinitsyn, N., Jungwirth, T., and MacDonald, A., *Phys. Rev. Lett.* **2004**, *92* 126603.
35. Kato, Y. K., Myers, R. C., Gossard, A. C. and Awschalom, D. D., *Science* **2004**, *306* 1910âĂŞ1913.
36. Lin Dong, Lu Zhou, Biao Wu, B. Ramachandhran, and Han Pu, *Phys. Rev. A* **2014** *89*, 011602(R).