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CS 194-10 Machine Learning
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Assignment 5

1. Conjugate priors

   (a)

   (b) Given the geometric distribution

   $$P(X_i = k|\theta) = (1-\theta)^{k-1} \cdot \theta$$

   and the beta distribution

   $$\beta(\theta|a,b) = \alpha\theta^{a-1}(1-\theta)^{b-1}$$

   we prove that the beta distribution is the conjugate prior for a likelihood with a geometric distribution.

   $$\begin{aligned} P(\theta|X) &= P(\theta) \cdot P(X|\theta) \\ &= \alpha \cdot \theta^{a-1} \cdot (1-\theta)^{b-1} \cdot (1-\theta)^{k-1} \cdot \theta \\ &= \alpha \cdot \theta^{a} \cdot (1-\theta)^{b+k-2} \\ &= \beta(\theta|a+1, b+k-1) \end{aligned}$$

   The posterior has the form of a beta distribution so therefore the beta distribution is the conjugate prior for the geometric distribution.
   The update procedure for a beta posterior simply involves updating the a and b parameters

   $$\begin{aligned} a_{N+1} &\leftarrow a_N + 1 \\ b_{N+1} &\leftarrow b_N + k - 1 \end{aligned}$$

2. Bayesian Naive Bayes

3. Logistic regression for credit scoring