Network Analysis and the Economics of Influence in The 109th U.S. Congress

EC331 Research in Applied Economics

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Abstract

This paper aims to analyse the allocation of campaign contributions to legislators in the U.S. 109th Congress, as well as their Katz-Bonacich centrality measure. I adjust parameters to reflect the importance that legislators place on their connections and the uncertainty lobbyist groups have about the legislators' preferences. This is done within the framework initially presented by Battaglini & Patacchini (2018). I will use data on the adjacency matrix that represents the connections found in Battaglini & Patacchini (2018)'s online appendix. I employ Monte Carlo simulations to test the robustness of the model under various scenarios such as the changes I described above. My analysis aims to understand the complexities of legislative behaviour and assess whether any feasible policy recommendations arise from my analysis.

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Abbreviations

Katz Bonacich	KB
Modified Katz Bonacich	MKB
Optimal Vector of Transfers	OVT
Nash Equilibrium	NE

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1 Project Outline

1.1 Introduction

Legislators, recognised as utility-maximising individuals (Mattero 2015, Heim 2022), operate within frameworks designed to prevent corruption, such as prohibitions on direct monetary transfers contingent upon votes[1]. These constraints render the analysis of interactions between interest groups and legislators complex, as all interactions must comply with lawful behaviours, which means that unlawful behaviours go unobserved. Additionally, legislators are influenced by networks connecting them with peers, which may affect their decisions Battaglini & Patacchini (2018). This study aims to explore how external factors like the significance of connections, and legislators' preferences, influence the interactions between interest groups and legislators.

A model by Battaglini & Patacchini (2018) investigates how campaign contributions are allocated among legislators linked by social ties, establishing criteria for a unique Nash equilibrium in pure strategies[2] and characterising the resource allocation by interest groups. The model finds that campaign contributions correlate with the Katz-Bonacich centrality measure. The KB, which we will use to refer to the Katz-Bonacich centrality henceforth, extends traditional eigenvalue centrality by incorporating decay factors to account for the attenuation of influence through network paths; thus, modifications such as adding or removing nodes can significantly alter the centrality measure, reflecting its comprehensive consideration of both direct and indirect connections[3]

To advance this area of research I am going to use a slightly simplified version of the original model found in Battaglini & Patacchini (2018). I will use their model to analyse the network dynamics in the 109th U.S. Congress. The simplification of the model is done with the purpose of focusing on the dynamics that happen in the network as legislators place different scaling factors on the importance of their connections or as lobbyists have less certainty about the legislator's preferences. In order to explore the dynamics I will employ Monte Carlo simulations[4] which are aimed at rigorously looking at these arbitrary changes in parameters and its potential results. Moreover, this strategy not only tests the validity of the model proposed by Battaglini & Patacchini (2018) but also explores the broader economic and political repercussions of network influences in a legislative setting.

1.2 Literature Review

My research is primarily focused on decision-making in the context of Networks. This is an area of Economics that has been growing recently due to the many achievements within the last 30 years. Carrell et al. (2009) demonstrate the significant role of peer effects in educational outcomes, illustrating how individual achievements are influenced by the broader network of student interactions. This concept of peer effects has parallels in political economics, especially in understanding how legislators'

[1]For evidence from Brazil, please refer to Brollo (2011).

[2]Please see the appendix for the formal definition

[3]For more on centrality measure sensitivities, see Dequiedt & Zenou (2014).

[4]Monte Carlo simulations, a method pioneered by John Von Neumann and Stanislaw Ulam, are particularly effective for assessing models with complex dynamics or those that lack analytical solutions Metropolis & Ulam (1949).

decisions are influenced by their peers within the legislative network. Similar to how students are impacted by their peers, legislators are influenced by the preferences and actions of other legislators with whom they share connections.

This analogy is important to my research, as it supports the hypothesis that network effects in a Congressional setting extend beyond simply shared interest or party affiliations. It is important to point out that the definition of a link between two nodes which I borrowed from Battaglini & Patacchini (2018) might not be the most appropriate but it is a good proxy, more on this in the conclusion and extensions section. Carrell et al. (2009)'s findings support the use of Monte Carlo simulations in this context, I use it to explore how varying the scaling factor on social connections and the uncertainty of the lobbyist groups on the preferences of the legislators affect legislative outcomes, my research seeks to quantify the extent to which these peer effects can alter policy-making, offering a novel contribution to the understanding of network effects in legislative environments.

Finally, Battaglini & Patacchini (2018) applies this analysis to the U.S. congress to observe how Network dynamics affect the contributions given to Political Action Committees. I use their work as a stepping stone to explore their model and test their assumptions and the parameters exogenous to the model such as the uncertainty of the interest groups and the preferences of the legislators. I use the 109th U.S. Congress to achieve this.

Chaney (2014) analysis of international trade networks offers insights into how economic relationships (which can be thought of as networks) are structured around the globe. His research highlights the importance of central firms (central nodes in the network), which act as hubs in the trade network, greatly influencing trade patterns and outcomes due to their strategic position. Due to the nature of Battaglini & Patacchini (2018) we do not have central nodes in the same sense in our network of the 109th Congress, this makes it easier to analyse as we do not have congressmen which curve the whole congress and this is key to my research because if this were the case then lobbyists would simply target this central nodes within the congress.

The application of network analysis to understand political influence and decision-making can be traced back to the foundational work by Mark Granovetter on the strength of weak ties. His seminal paper posits that weak ties—relationships characterised by infrequent interactions and lower emotional closeness, such as acquaintanceship's in a workplace-are crucial for disseminating information and bridging disparate parts of a social network. This theory underscores the importance of these connections in facilitating information flow across different social groups, which is particularly relevant in the context of political networking and influence within the field of economics. Both Battaglini & Patacchini (2018) and Bramoullé & Genicot (2018) explore network centrality measures, yet they apply these concepts to different aspects of political influence and decision-making. Battaglini & Patacchini (2018) utilise the Katz-Bonacich centrality to model how political contributions are influenced by legislators' network positions, emphasising the static structure of these networks. In contrast, Bramoullé & Genicot (2018) introduces diffusion centrality, focusing on the dynamic process of information spreading within networks, which is crucial in scenarios where policy information and influence disseminate rapidly and decay just as quickly. This dynamic approach contrasts with the more static analysis by Battaglini & Patacchini (2018), suggesting that while Katz-Bonacich centrality is effective for long-term structural influence, diffusion centrality might better capture short-term and rapid changes in political behaviour.

This distinction is critical for my dissertation as it highlights the importance of selecting appropriate centrality measures based on the specific temporal and structural characteristics of the network being studied. My approach builds on this foundation by employing Monte Carlo simulations to test these different centrality measures under various hypothetical conditions in the 109th Congress, assessing not only static but also dynamic network effects on legislative decision-making.

In recent years, the model introduced by Battaglini & Patacchini (2018) has gained significant attention for its innovative approach to capturing the dynamics of how interest groups interact with legislators. While the initial findings suggest that the Katz-Bonacich centrality measure is a strong predictor of political action committees contributions, the robustness of this model under varying conditions remains an open question. This gap is particularly relevant given the critical role that such models play in informing policy decisions.

Using simulations for analysing networks is very common. For example, Ruppert et al. (2014) discusses decision making in policy-making, where conflicting objectives and the need to consider economic, social and environmental impacts are significant. They proceed to run simulations as a method to measure the effectiveness of different policies. Additionally, Fowler (2006) provides strong evidence for the importance of social ties in legislative influence, underscoring the importance of network analysis in political economics. Chaudhury et al. (2023) builds upon Battaglini & Patacchini (2018) and they add an extra layer of complexity to the analysis and study how the dynamics change once you introduce polarisation in the political process. This is very interesting given the fact that even if not first perceived there is political polarisation going on than might be manifested in the network dynamics but not in the data.

1.3 Research Hypotheses

Due to the inherent complexity of networks, changes in exogenous parameters—such as the uncertainty faced by lobbyists and the significance legislators ascribe to their connections—yield unpredictable effects on network dynamics. Consequently, Monte Carlo simulations emerge as a natural and crucial method to explore these effects, particularly in assessing how variations in connection weights influence the Nash equilibrium of the model. Additionally, I am working with a large 429×429 matrix, which is inherently difficult to analyse. Moreover, I have to consider that the U.S. Congress was, in a way, designed with the intent of constraining the power of the congressmen within it. Therefore, it is not surprising that there is no central node in the U.S. Congress, which adds an extra layer of complexity. However, I will limit my discussion of the historical aspects of this design, as it is not relevant beyond what has already been stated.

Furthermore, I hypothesise that the network position of a node i exhibits robustness against moderate alterations in the model's primitives, provided these changes do not violate the foundational assumptions required to maintain equilibrium (Assumptions 1-3)[5]. For instance, a highly connected legislator (node i) is likely to remain a focal point for lobbying efforts unless changes to the model's parameters are substantial enough to disrupt the game's equilibrium. Additionally, because congressmen in this model cannot alter their connections it makes it easy to find a stable equilibrium which was the one found in Battaglini & Patacchini (2018).

[5]These assumptions are there to ensure the existence of a unique NE in pure strategies

2 Optimising Lobbying Efforts

2.1 Context

2.2 Set-up: Legislators

In the refined model based on Battaglini & Patacchini (2018), I consider a legislature with a set of $N = \{1, \ldots, 429\}$ legislators, each making a binary decision: to enact a new law, A, or maintain the status quo, B, with a simple majority rule required for policy adoption. The network of legislators, represented as a adjacency matrix $\hat{G}$ of size 429×429. I take this network from `'G_party.dta'` which is found in Battaglini & Patacchini (2018)'s online appendix.

We define the link $g_{j,i}$, as in (Battaglini & Patacchini 2018), in the alternative network definitions section. I define the link between congressmen i and j reflecting their shared academic background and party alignment:

$$g_{j,i} = \begin{cases} -0.5 & \text{If non-alumni with differing parties,} \\ 0.5 & \text{If alumni with differing parties,} \\ 0 & \text{If of the same party but non-alumni,} \\ 1 & \text{If alumni of the same school and party.} \end{cases}$$

v^i denotes the legislator's utility from approving the policy A over the status quo, B, which has a normalised utility of zero[6]. A positive v^i indicates a preference for A, whilst a negative v^i suggests a disfavour towards B, I assume v^i cannot be zero, explicitly excluding legislators motivated solely by private benefits such as power or status. Decision-making is influenced by both direct utilities from casting votes and social dynamics within the legislative body. Legislators are monitored by lobbyists who may encourage certain votes with informational contributions and are affected by peers' voting patterns, aligning their votes to resonate with the collective behaviour of other legislators[7]. We simulate each v^i and save our results in a diagonal matrix V where each diagonal entry corresponds to a legislator. I randomly assigned a value of v^i to each node in the network such that it can take the discrete values of 1, 0.5, -0.5, and -1. The results presented in this paper hold even if we pick v^i from $[-1, 1] \setminus \{0\}$. For more on this, this can be checked if the simulation is adjusted slightly. The key here is that the absolute maximum value of v^i remains such that Assumption 1 is satisfied.

2.3 Set-up: Lobbyists

There are two lobbyists[8] as in Battaglini & Patacchini (2018). Interest group A aims to maximise the number of legislators voting for policy A, and interest group B aims to minimise it, both acting exogenously to $\hat{G}$. For instance, during the George W. Bush administration, specifically within the 109th Congress (2005-2007), significant lobbying efforts surrounded the Social Security reform proposal. Although the reform did not pass due to substantial opposition and insufficient support Jaenicke & Waddan (2006), this period illustrates the dynamics described in the model.

[6]This normalisation sets a benchmark, evaluating the utility of policy A against policy B's baseline utility.

[7]Studies such as Arnold et al. (2000), Cohen & Malloy (2014), and Battaglini & Patacchini (2018) support the significant influence of social connections on voting behaviours.

[8]'Interest groups' and 'Lobbyists' are used interchangeably in this context.

Group A (proponents of the reform) and Group B (opponents) allocated fixed budgets X_A and X_B, respectively, and offered transfer vectors $\mathbf{s_A} = (s_A^1, \ldots, s_A^n)$ and $\mathbf{s_B} = (s_B^1, \ldots, s_B^n)$ to legislators. As posited by Battaglini & Patacchini (2018), these vectors reach an equilibrium $\mathbf{s_A} = \mathbf{s_B} = \mathbf{s}$, reflecting a balanced exertion of influence. For analytical purposes, we assume $X_B = X_A = X$ denotes the equal and substantial resource both groups could mobilise to sway legislative support.

2.4 The Model

Legislator i's direct utility from voting for policy $p \in \{A, B\}$ with a quasi-linear utility function as in Battaglini & Patacchini (2018):

$$U^i(p) = \ln(s^i(p)) + \phi \sum_{\substack{j=1 \\ j \neq i}}^{n} g_{j,i} \cdot \mathbb{1}\{j \text{ votes for p}\} + \varepsilon_p^i \tag{2.1}$$

The utility of legislator i from supporting policy p, $\ln(s^i(p))$, derives from total contributions $s^i(p)$ and shows quasi-concave diminishing returns. Social interaction effects, sourced from Ballester et al. (2006), are included in the adjacency matrix $\hat{G}$. The error term ε_p^i accounts for external decision influences, with $\varepsilon_A^i = \varepsilon^i$ reflecting both impacts and ε_B^i set to zero. The constant ϕ across legislators scales social interaction in utility calculations. As noted by Battaglini & Patacchini (2018), interest groups, lacking perfect information on legislators' preferences, cannot precisely predict the impact of their contributions. ε^i is assumed independently uniformly distributed, $\varepsilon^i \sim U\left[-\frac{1}{2\boldsymbol{\Psi}}, \frac{1}{2\boldsymbol{\Psi}}\right]$, known only to legislator i.

I will give a formal definition of the primitives.

Definition 1. Scaling Factor ϕ: is a scaling factor in the utility function of legislators, quantifies the influence of social connections. Specifically, ϕ scales the effect of the influence matrix G on the utility derived from others' votes.

Definition 2. Uncertainty of the Lobbyists $\boldsymbol{\Psi}$: Represents the distribution parameter associated with the uncertainty that lobbyists have regarding legislators' preferences. It influences the variability of ϵ, the error term in the utility function of legislators, which reflects the lobbyists' uncertainty about how each legislator will vote.

Let $P_A(i)$ be the probability that legislator i votes for option A, and $\boldsymbol{P_A} = (P_A(i))_{i=1}^n$ is the associated vector of probabilities. Moreover, let $q^i(\boldsymbol{P_A})$ be legislator i's pivot probability. That is, this is the probability that legislator i voting for policy A changes the outcome from option B to option A. Please note that the calculation of pivot probabilities gets more complex as n gets large. For my simulation, I used logarithmic computations to help manage the very small probabilities as n increases. given $\boldsymbol{P_A}$. Legislator i is willing to vote for option A if and only if:

$$\mathbb{E}[U^i(B) - U^i(A)] \leq v^i q^i(\boldsymbol{P_A}) \tag{2.2}$$

This can be rewritten[9] as:

$$\varepsilon^i \geq \ln(s_B^i) - \ln(s_A^i) - v^i q^i(\boldsymbol{P_A}) + \phi \sum_{\substack{j=1 \\ j \neq i}}^{n} g_{j,i}(1 - 2P_A(j)) \tag{2.3}$$

Let $\bar{v}$ be the highest valuation in absolute value: $\bar{v} = \max_i |v^i|$. Additionally, throughout the paper we will hold Assumptions 1-3 which can be found in the appendix and were taken from Battaglini & Patacchini (2018).

Definition 3. Katz-Bonacich Centrality For a node i in a network with adjacency matrix $\mathbf{G}$ and attenuation factor ϕ^*, the Katz-Bonacich centrality, which we will refer to as KB from now on, is defined mathematically as:

$$b_i(\phi^*, \mathbf{G}) = [(\mathbf{I} - \phi^* \mathbf{G})^{-1} \mathbf{1}]_i$$

This definition is taken from Battaglini & Patacchini (2018)
Additionally, we need to borrow a definition from Battaglini & Patacchini (2018) which would be a modified version of the Katz Bonacich centrality measure, MKB from now on.

Definition 4. Modified Bonacich Centrality Measure Let $D\boldsymbol{q}_*$ be the Jacobian matrix of the function $\boldsymbol{q}(\boldsymbol{P_A}) = (q^1(\boldsymbol{P_A}), \ldots, q^n(\boldsymbol{P_A}))^T$ evaluated at equilibrium. Suppose V is a diagonal matrix where the i-th diagonal entry is v^i. The *Modified Bonacich Centrality* b^M at ϕ^* with respect to V, the transpose of the matrix G (denoted G^T), and a coefficients matrix $\mathbf{\Psi}$, is defined by the equation. This definition is taken from Battaglini & Patacchini (2018)

$$b^M(\phi^*, V, G^T) = [I - (\phi^* G^T + \mathbf{\Psi} D\boldsymbol{q}_*^T \cdot V)]^{-1} \cdot \mathbf{1},$$

I will rely on the following proposition which can also be found in Battaglini & Patacchini (2018). This will allow us to compute the unique equilibrium in pure strategies using optimisation algorithms.

Proposition 1. There exists a unique equilibrium in which lobbyists choose the same vector of transfers $\mathbf{s}^*$. The vector $\mathbf{s}^*$ solves the problem

$$\max_{s \in S} \left\{ \sum_j b_j^M(\phi^*, V, G^T) \cdot \ln_j(s_j) \right\}, \tag{2.4}$$

where $b_j^M(\phi^*, V, G^T)$ is the modified Katz-Bonacich centrality of j in V, G^T with coefficient $\phi^* = 2\mathbf{\Psi}\phi$. This is a modified version of the one presented in Battaglini & Patacchini (2018).

This proposition is particularly important for my purposes, it ensures that the vector of transfers that I find is unique thus making the problem a simple nonlinear optimisation problem. Please refer to Battaglini & Patacchini (2018) for the proof.

[9] See Appendix

2.5 Monte Carlo Simulation

I aim to test the robustness of findings from Battaglini & Patacchini (2018), so first let's solve the maximisation problem using numerical methods, compute the MKB measure and solve for the optimal vector of transfers for each n. Then we will test the hypothesis of whether the MKB is proportional to the optimal vector of transfers.I regress OVT on the MKB using OLS, and plot a scatter plot to see the results when we use the 109th congress. We get the following graph:

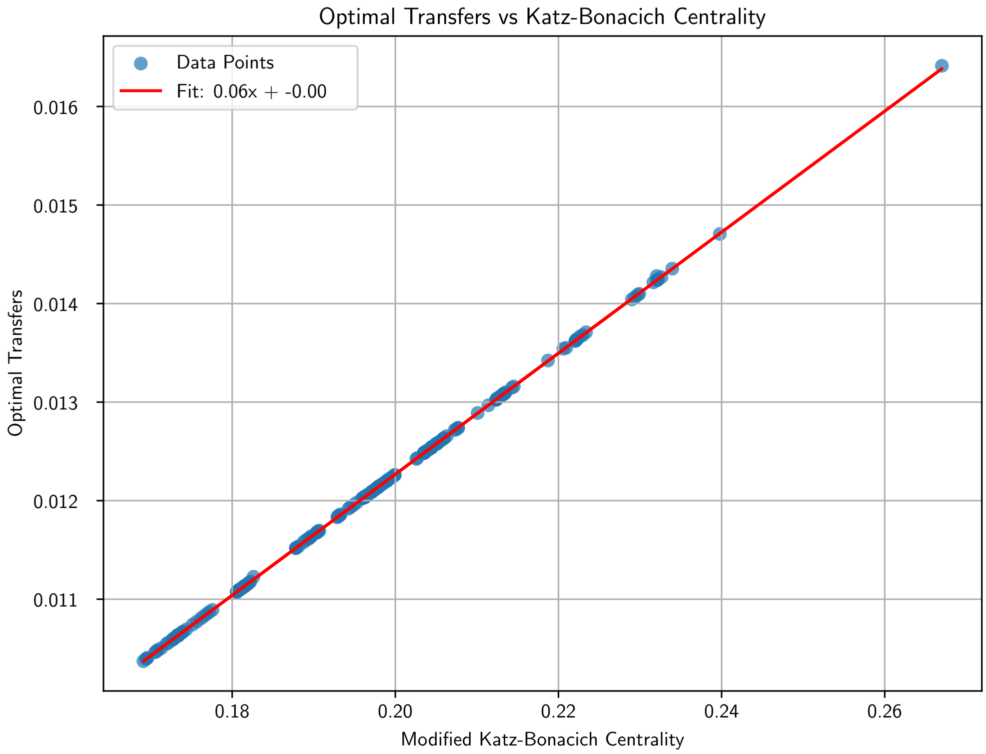

Figure 2.1: Scatter plot of Optimal Vector of Transfers on the Modified Katz-Bonacich Measure

Coefficient	Estimate	Std. Error	t value	$Pr > \|t\|$
β_0	1.94e-06	8.28e-06	-8.384	0.000
MKB	0.0614	1.02e-05	6022.471	0.000

Table 2.1: OLS Regression: Optimal Vector of Transfers on the Modified Katz-Bonacich Measure

The coefficient of variation is 0.0002 The p-value being close to zero shows that results from Battaglini & Patacchini (2018) hold even when you simulate V in fact, this supports the importance of the MKB in networks, it is also important to notice that these results might be robust against eigenvector centrality measures given the fact that they are highly correlated Valente et al. (2008). Moreover, I also argue that this result is very likely to hold for other congresses as well not just the 109th congress which is the one I used. It would be interesting to see the dynamics of other types of congresses like the Catalan congress where you have representatives from more than two parties and the system is based on proportional representation. It is important to remember that my results are subject to the primitives of the model proposed by Battaglini & Patacchini (2018) e.g., I assume majority voting and this is crucial as it renders individual votes increasingly unimportant as n grows.

3 Monte Carlo Simulations: Importance of Connections

3.1 Set-up

Primitive	Value
ϕ	[0.1, 0.98]
$\boldsymbol{\Psi}$	0.10
X	5
q	0.5
$\bar{v}$	1

Table 3.1: Values of the primitives: Simulating with the Scaling Factor

A Note on Methods

All parameters adhere to Assumptions 1-3. ϕ is defined as per Definition 1. Prior to executing these simulations, predicting the specific impact on the network is challenging because each legislator i occupies a nearly unique position, which significantly influences their MKB measure. Additionally, the choice of parameters is guided by the theoretical model as presented in Battaglini & Patacchini (2018) to ensure everything is well behaved, I suspect that the analysis will be similar for any set of parameters that remain far from the boundary as defined by Assumptions 1-3. As discussed in section 2, one might expect that an increase in a legislator's MKB would correspond to an increase in the optimal transfers they receive. However, this may not always occur. For instance, if another legislator's MKB increases more substantially, the transfers might instead be diverted to them. This unpredictability underscores the necessity of employing Monte Carlo methods to model these complex scenarios effectively. It is also important to notice that due to the complexity of the problem at hand I have had to use nonlinear constrained optimisation algorithms[10].

3.2 MKB vs. Scaling Factor

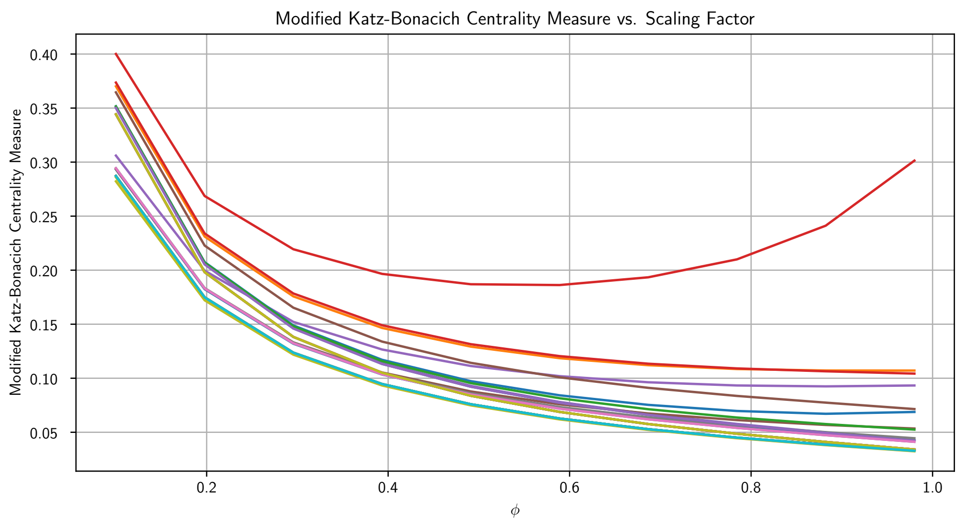

Figure 3.1: Modified Katz-Bonacich vs Scaling Factor

[10]More on this can be found on my replication file

Figure 3.1 graphs the MKB measures of 30 legislators against the scaling factor. Intuitively, the graph shows what happens to the MKB measure for each legislator as the scaling factor increases, i.e., as they place more importance on the connections they possess. The results might seem a bit counter intuitive at first as every single legislator ends up with an MKB score lower than at the beginning of the simulation. It is challenging to determine how much of this result is due to the choice of parameters from Table 3.1. One could conclude that these legislators were not as well connected as defined by $g_{j,i}$, and thus increasing the scaling factor decreases the MKB measure because of this factor. It is difficult to clearly observe the dynamics when we are not representing the entire Congress; however, including the whole would have made the image too convoluted. Additionally, one could question whether our definition of $g_{j,i}$ is an accurate measure of the connections between the legislators. A compelling argument could be made that the true measure of $g_{j,i}$ is immeasurable and likely not consciously realised by the legislators. Nevertheless, for the purposes of this paper, the definition provided by Battaglini & Patacchini (2018) serves as a good proxy.

As we can see there is a region where all of their MKB falls and then after 0.4 they start to converge in the case of 29 legislators and then we see the last one which starts increasing. This could be because other legislators in the network which are connected to him started placing more importance on their connections and thus further increasing its position in the network as defined by the MKB. For supporting evidence of our results you can see Bloch et al. (2023) which explain the theoretical framework in which this dynamic plays out.

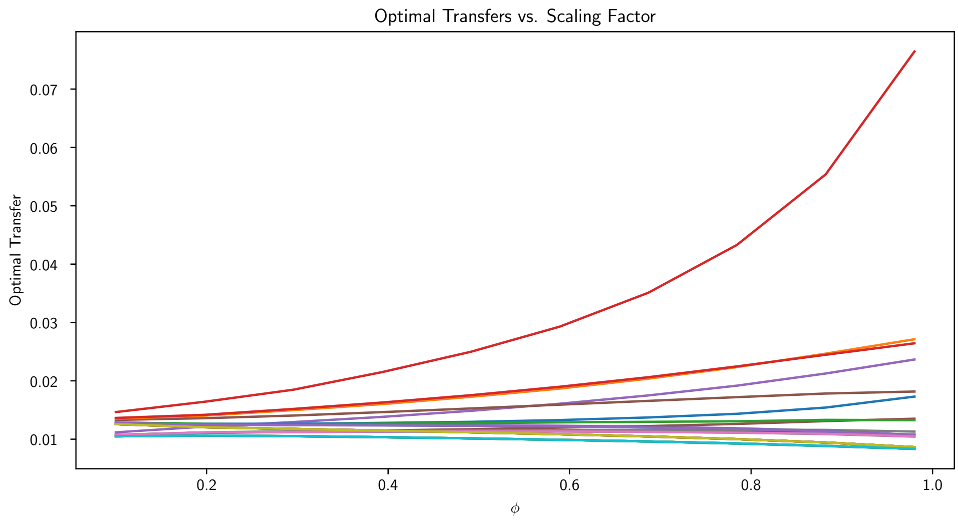

Figure 3.2: Optimal Transfers vs. Scaling Factor

In this analysis, we continue to explore the nuanced dynamics within the legislative network, maintaining consistent colour schemes for each legislator across all figures. Observations reveal that legislators who rank high according to the MKB centrality measure tend to see an increase in their optimal vector of transfers. This finding supports the results from the OLS regression analysis detailed in Section 2, highlighting that legislators with greater network centrality attract more substantial campaign contributions. This trend mirrors strategies observed in the commercial sector, such as marketing companies prioritising contracts with highly visible figures like Taylor Swift, who have extensive outreach and influence. Similarly, lobbyists target well-connected legislators for their ability to sway broader legislative outcomes, as quantified by the centrality measure $g_{j,i}$.

However, the influence of the scaling factor ϕ introduces complexity into the correlation between centrality and transfer volumes. Its impact varies depending on the specific characteristics of the network and the individual connectivity of legislators. For example, less connected legislators might experience a decline in targeted transfers as lobbyists reallocate resources towards more influential nodes within the network. This strategic behaviour is underscored by findings such as those presented in Ringe & Wilson (2016), which link covoting centrality directly to political influence within legislative networks. Further empirical support for the influence of network centrality on resource allocation can be found in studies like those by Valente (2012), who explores how network positions affect the effectiveness of targeted interventions, andBorgatti et al. (2006) who discusses the robustness of centrality measures and their implications for network strategy. These contributions significantly enhance our understanding of how network structure influences the distribution of resources and underscore the sophisticated strategies lobbyists employ to maximise their influence within legislative bodies.

Notice that as the network gets larger the MKB should approximate the MB given that the effect of your vote being pivotal goes to zero. To do this I will replace the same analysis as in Figure 4.1 but using the KB instead of the MKB

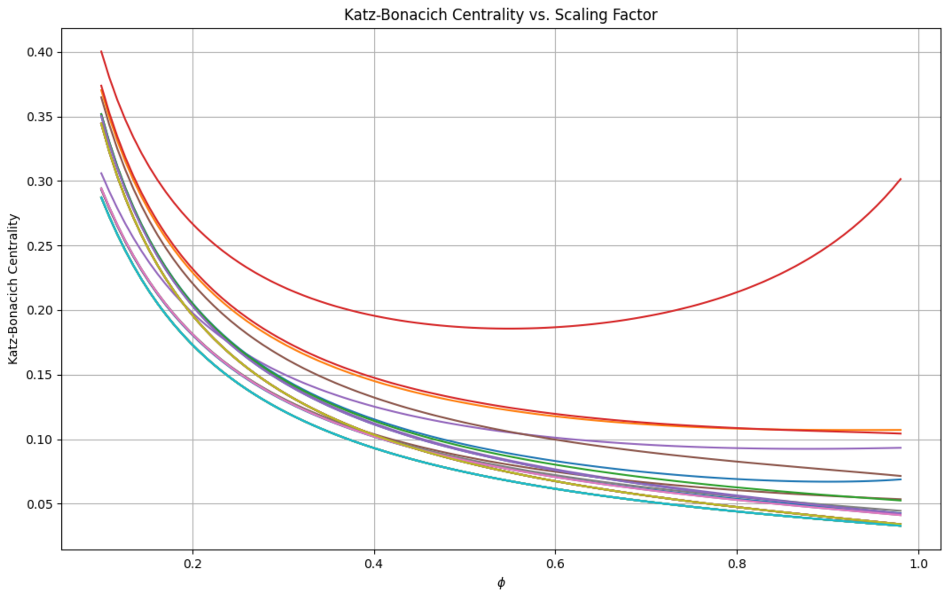

Figure 3.3: Katz-Bonacich vs. Scaling Factor

This graph is slightly different from the one in Figure 4.1 for one reason and it is because I used a greater number of intervals for ϕ which would naturally make the graph look smoother. However, the dynamics are the same. You can see the effect of the pivotal probabilities going to zero as n gets large if you look at Definition 3 and 4. Given this, it makes our analysis significantly easier as we could conclude the behaviour of the MKB by looking at the KB as ϕ increases given that we have a large network. Refer to the appendix to figure g to see the general trend of ϕ for the whole network.

4 Monte Carlo Simulations: Uncertainty of the Lobbyists

4.1 Set-up

In this section, I shall examine the underlying principles outlined below. The aim is to investigate the conduct of certain legislators within the network as the distribution of the epsilon terms narrows around zero, thereby diminishing the uncertainty lobbyists face concerning the preferences of the legislators.

Primitive	Value
ϕ	0.20
$\mathbf{\Psi}$	[0.05,0.14]
X	5
q	0.5
$\bar{v}$	1

Table 4.1: Values of the primitives: Simulation with Uncertainty of the Lobbyists

A Note on Methods

The choice of the parameters presented in the table above was picked using the same reasoning for Table 3.1. When solving for the unique NE, any nonlinear constrained optimisation algorithm and it should arrive to the same solution as the NE is unique.

4.2 Uncertainty of the Lobbyists

Firstly, let's see what happens to the MKB as we have $\mathbf{\Psi}$ increasing within the range specified in Table 4.1

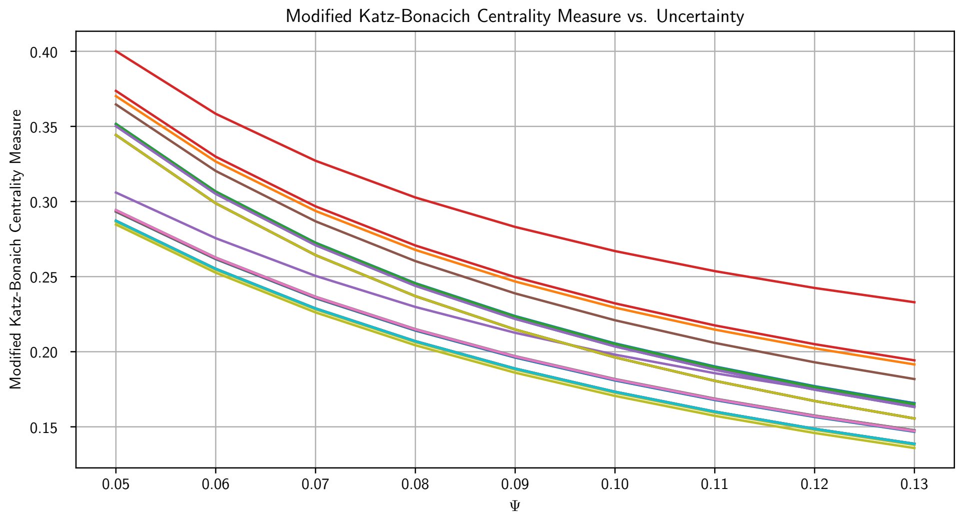

Figure 4.1: Modified Katz-Bonacich Measure vs Uncertainty of the Lobbyists.

This graph shows what happens to the modified Katz-Bonacich centrality measure as defined above as the uncertainty about the legislator's preferences drops. Notice that no matter where you are in the network or how many connections you have, the modified Katz-Bonacich measure decreases as the lobbyists learn more about the distribution of your preferences. This has one interpretation, if the modified Katz-Bonacich centrality measure is proportional to the transfer you get by the lobbyists as shown in Battaglini & Patacchini (2018) then by them reducing their uncertainty (and by this I mean the knowledge they have on $\mathbf{\Psi}$) then the optimal level of transfers should decrease if your modified Katz-Bonacich measure of centrality is decreasing.

Please notice two things from this graph. Because of the nature of this network it is hard to represent it graphically and because of its nature it is also hard to extract summary statistics given that there are four possible entries that define a connection between two legislators. Even though we showed in the last section that the MKB is proportional to the OVT this does not mean that there will be a one to one correspondence as here we show that even though the MKB falls for the 30 legislators the OVT has to rise for some others as we assume that the budgets X are spent.

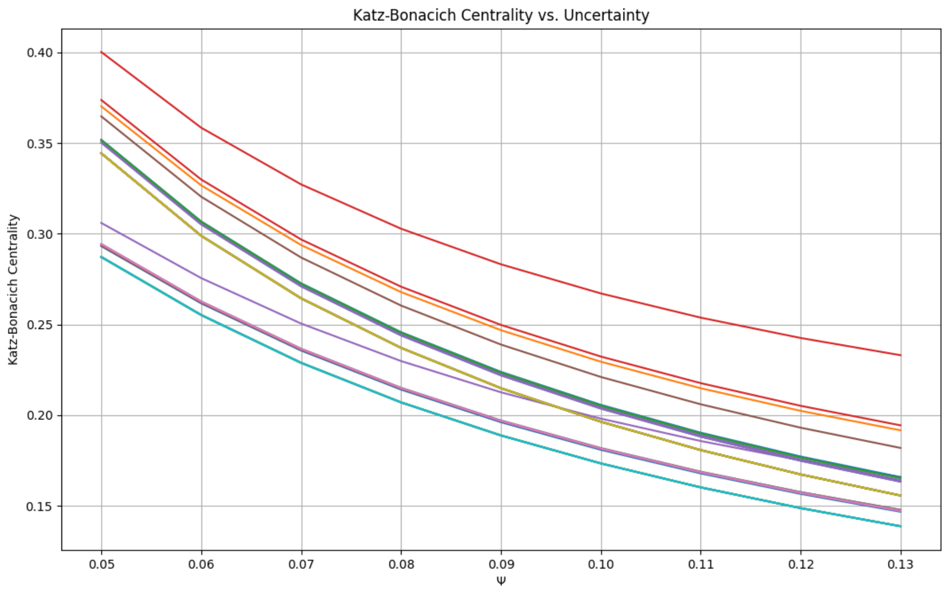

Figure 4.2: Katz-Bonacich Measure of Centrality vs Uncertainty of the Lobbyists

I will attempt to show that the MKB approaches the KB as n gets large as in section 3. The intuition for this is simple, as n increases the probability that your vote becomes pivotal gets closer to zero. Additionally, an interesting potential paradox arises, the more people in the system the less the probability that their vote becomes pivotal and thus the less impact their vote has. However, this does not mean that you as a legislator shouldn't vote because you will still get a possibly positive amount of transfers by the lobbyist groups and assuming that voting is costless your best response is always to vote. It would be interesting to see what happens when there is a cost of entry to voting and how the dynamics play out in this scenario. I proceed to plot the same graph plotted at the beginning of chapter 4 but I will use the KB instead of the MKB.

The graph above shows the Optimal vector of transfers for our set of 30 legislators. We have the opposite here as we do in the last section. For the line red which is

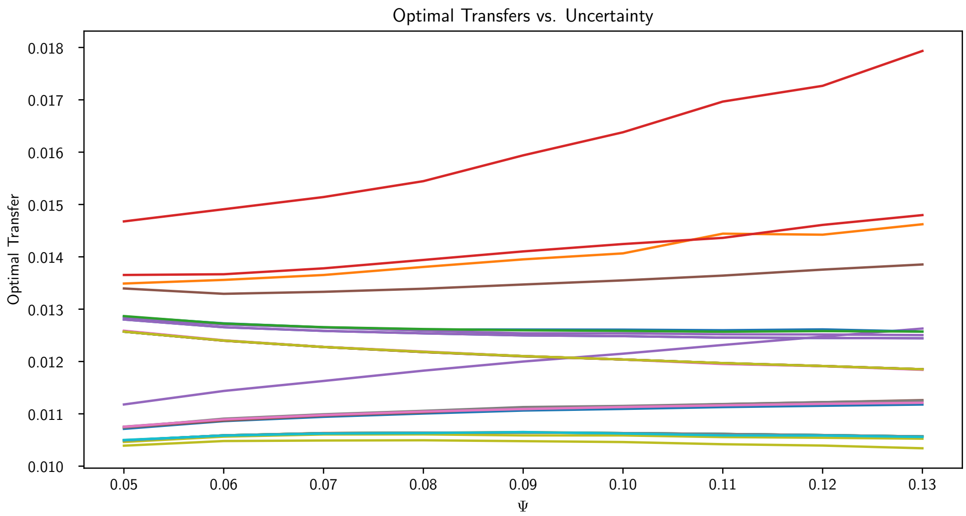

Figure 4.3: Optimal Transfers vs Uncertainty of the Lobbyists.

the same legislator for all the graphs we have that his optimal vector of transfers increases the less uncertain the lobbyists are about the preferences of the legislators (distribution of ε^i). This can be because of a few reasons, one of them is that the more certain they are about how legislators are likely to vote the more they are going to target those higher up in the network which in this case is defined by the MKB.

The graph above shows the optimal vector of transfers for our set of 30 legislators. We have the opposite result here as in the last section for some legislator,s, as the level of uncertainty among the lobbyists groups get smaller we have that the MKB measure gets increasingly important when calculating the OVT.

Given that we have shown that the KB is close to the MKB as n grows we can test out the hypothesis of whether the MKB falls or increases as $\mathbf{\Psi}$. Given that computing the KB is computationally less expensive than computing the MKB I can graph all the nodes in the network and see whether this is the case. I have included the graph in the appendix. Given these results we can safely conclude that the MKB falls as $\mathbf{\Psi}$

5 Conclusions and Extensions

As shown in Section 3 and 4, being highly connected in the network as defined by the MKB does indeed generally mean that the campaign contributions you get will be greater as shown by Battaglini & Patacchini (2018). Additionally, if n becomes large the probability of any legislator's vote being pivotal approaches zero. One of the implications of this is that as shown in Section 3 and 4 the MKB and the KB start to converge as n becomes large. Although not shown in my research, it is reasonable to assert that most eigenvalue centrality measures are correlated which means that similar results will be found if another eigenvalue centrality measure similar to the MKB is used. Moreover, the matrix V was generated randomly with the constraint that $\bar{v} = 1$ this is of course unrealistic but necessary as I did not have access to the real preferences of the legislators and I suspect that they would not know if asked.

I can also conclude that for this specific example, with the parameters from Table 3.1 and Table 4.1 both the MKB and the OVT tend to decrease as the parameters are increased

It would also be interesting to consider alternative ways of defining a connection between two legislators. I suppose that the definition we have used borrowed from Battaglini & Patacchini (2018) could be improved upon if we had information on whether the legislators followed each other on twitter or were friends outside of the congress. This adds another layer of complexity, but it could be interesting for future research.

We have a deeper insight into how it affects the MKB and the OVT but the assumption of the scaling factor being homogeneous for every legislator is a very strong assumption. We could benefit from relaxing this assumption and allowing a different ϕ_i for each i. However, then we have another challenge which is how do we measure ϕ_i for every legislator in the network. I suppose that a more flexible model will allow us to arrive at more conclusive results, for the moment, any policy analysis based on this research will be highly inappropriate as even if you had perfect information on how the dynamics of the congress worked you still cannot change the rules of the game as they have in some sense been set in stone.

It is also natural to assume that as $\mathbf{\Psi}$ increases then the random element of how each legislator is going to vote becomes smaller in the eyes of the lobbyists which in turn affects the MKB as defined in definition 4. This is in fact proven by our simulation but it might not hold for every legislator in the network. However, to be completely sure you can check the appendix where I run the same simulation using the KB centrality measure given the fact that it converges to the MKB as n becomes large to test this hypothesis and turns out that it still holds.

AI Content

I hereby declare that I have used ChatGPT version 4 during the production of this document. I have used it to help me find more efficient ways of writing my code which can be found on my GitHub page. Specifically, I have used the Python GPT by Nicholas Barker. Additionally, I have used it to solve questions I had at the time of writing the code.

I have also used it to find spelling mistakes and punctuation mistakes in my text. Additionally, I have used it to enhance my English.

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Appendix

From (2.2) we have:

$$\mathbb{E}[U^i(B) - U^i(A)] \leq v^i q^i(\boldsymbol{P_A})$$

$$U^i(A) = \ln(s_A^i) + \phi \sum_{\substack{j=1 \\ j\neq i}}^{n} g_{j,i} \mathbb{1}\{j \text{ votes for } A\} + \varepsilon_A^i$$

$$U^i(B) = \ln(s_B^i) + \phi \sum_{\substack{j=1 \\ j\neq i}}^{n} g_{j,i} \mathbb{1}\{j \text{ votes for } B\} + \varepsilon_B^i$$

$$\varepsilon_A^i \equiv \varepsilon^i, \quad \varepsilon_B^i \equiv 0$$

$$P_A(i) = \mathbb{E}[\mathbb{1}\{i \text{ votes for } A\}], \quad (1 - P_A(i)) = \mathbb{E}[\mathbb{1}\{i \text{ votes for } B\}]$$

$$\mathbb{E}[U^i(A)] = \ln(s_A^i) + \phi \sum_{\substack{j=1 \\ j\neq i}}^{n} g_{j,i} P_A(j) + \varepsilon^i$$

$$\mathbb{E}[U^i(B)] = \ln(s_B^i) + \phi \sum_{\substack{j=1 \\ j\neq i}}^{n} g_{j,i} (1 - P_A(j))$$

$$\ln(s_B^i) + \phi \sum_{\substack{j=1 \\ j\neq i}}^{n} g_{j,i}(1 - P_A(j)) - \ln(s_A^i) - \phi \sum_{\substack{j=1 \\ j\neq i}}^{n} g_{j,i}(\varphi_j) - \varepsilon^i \leq v^i q^i(P_A(i))$$

$$\ln(s_B^i) - \ln(s_A^i) + \phi \sum_{\substack{j=1 \\ j\neq i}}^{n} g_{j,i}[1 - P_A(j) - P_A(j)] - \varepsilon^i \leq v^i q^i(P_A(i))$$

$$\ln(s_B^i) - \ln(s_A^i) + \phi \sum_{\substack{j=1 \\ j\neq i}}^{n} g_{j,i}[1 - 2\varphi_j] - \varepsilon^i \leq v^i q^i(P_A(i))$$

$$\ln(s_A^i) - \ln(s_B^i) + \phi \sum_{\substack{j=1 \\ j\neq i}}^{n} g_{j,i}[2\varphi_j - 1] + \varepsilon^i \geq -v^i q^i(P_A(i))$$

$$\varepsilon^i \geq \ln(s_B^i) - \ln(s_A^i) - v^i q^i(P_A(i)) + \phi \sum_{\substack{j=1 \\ j\neq i}}^{n} g_{j,i}(1 - 2P_A(j))$$

Thus, we arrive at (2.3)

Assumption 1. $\boldsymbol{\Psi}(\bar{v} + \phi + \ln(2X)) < \frac{1}{2}$.

Assumption 2. The matrix $I - 2\mathbf{\Psi G}$ is invertible and the associated KB vector is positive: $(I - 2\mathbf{\Psi G})^{-1} \cdot \mathbf{1} > 0$. This guarantees that the KB centrality measure exists.

Assumption 3: Legislators are more likely to vote for a policy if they receive more support from a group that favours it, and less likely if they receive more support from a group that opposes it. The effect of additional support diminishes over time, meaning early support influences voting decisions more strongly than later support. This assumption reflects how real-world incentives can sway legislators' votes based on the timing and amount of support they receive from different groups. I assume that for this setting assumption 3 always holds as the utility functions I have used and the parameters I have selected make it so. Please refer back to Battaglini & Patacchini (2018) for the more technical version.

A strategy for interest group $p \in \{A, B\}$ is a probability distribution over the set of feasible transfers S, that is:

$$S = \left\{ \mathbf{s} \in \mathbb{R}^n : \sum_{i=1}^{n} \mathbf{s}^i \leq X,\ \mathbf{s}^i \geq 0,\ \forall i \in \{1, \ldots, n\} \right\}.$$

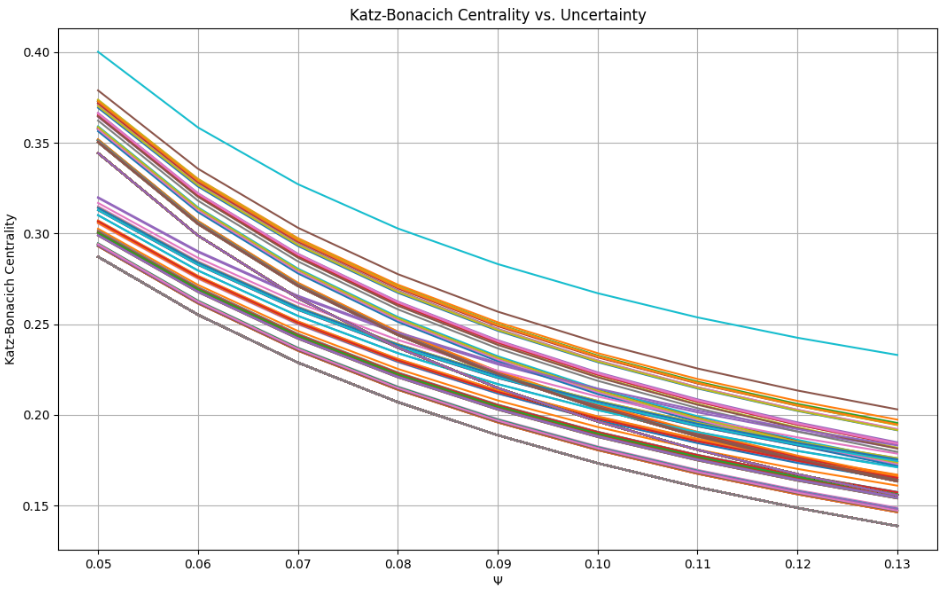

Figure A1: 429 Legislators Katz-Bonacich against Uncertainty of the Lobbyists.

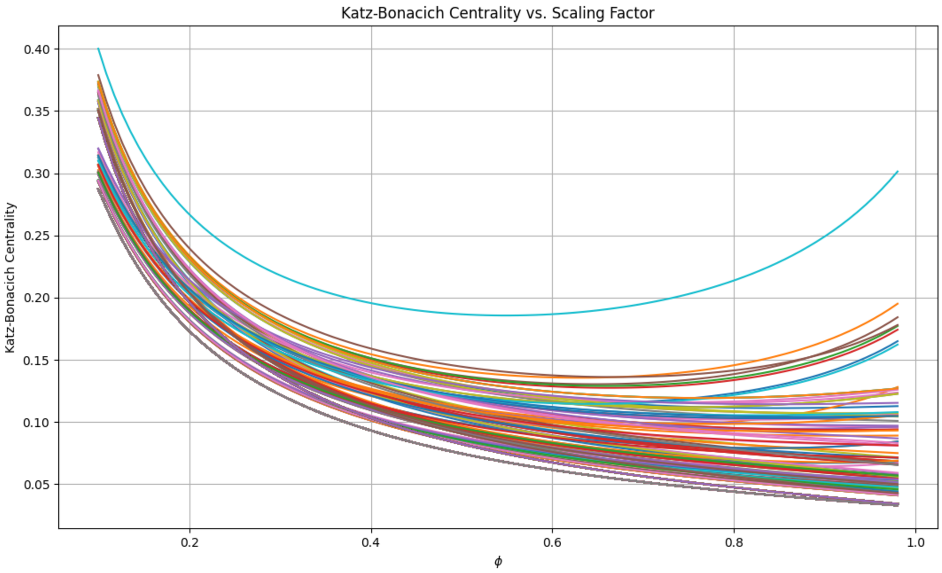

Figure A2: 429 Legislators Katz-Bonacich against the Scaling Factor.