

Lab 1

By  Alejandra ,  Lily  , and  Franklin 

MATH0154 Computational Statistics with Prof.  Gabe Chandler 

```
In [22]: h <- function(x) {
  #
  # function with many local maxima, but only one global maximum
  #
  # Args:
  #   x: value at which to evaluate the function
  # Returns:
  #   y: value of the function evaluated at x, f(x)

  return(sin(10*x) - x^(2))
}

Explore <- function(x, tau, h, n_iter, eps){
  #
  # function that explores local neighborhood
  # Args:
  #   @param x: initial starting value
  #   @param tau: the value of tau for the short random walk
  #   @param h: function that we want to maximize
  #   @param n_iter: number of iterations to perform for this set v
  alue of tau.
  # Returns:
  #   y: resting location after a short random walk

  i <- 1

  while(i <= n_iter){

    y <- x + eps*runif(1,-1,1)
    val <- min(1, exp((h(y)-h(x))/tau)) #probability of moving

    if(runif(1)<val){ #update according to the probability
      x <- y
    }

    i <- i + 1 #update i
  }
}
```

```

    }

    return(x)
}

SimAnnealing <-function(x, h, tau_range, n_iter, tau_length, eps){

    # function that finds the global maximum of a function using simulated annealing
    #
    # Args:
    #   @param x: initial starting value
    #   @param h: function that we want to maximize
    #   @param n_iter: number of iterations
    #   @param tau_length: length of the cooling schedule
    #   @param eps: maximum length of one random walk step
    # Returns:
    #   path: list of locations that the simulated annealing algorithm has visited

    path <- c(x) #initialize the path as a vector of the seed value.
    tau <- exp(seq(tau_range[1], tau_range[2], length.out=tau_length)) #
    create a tau_schedule

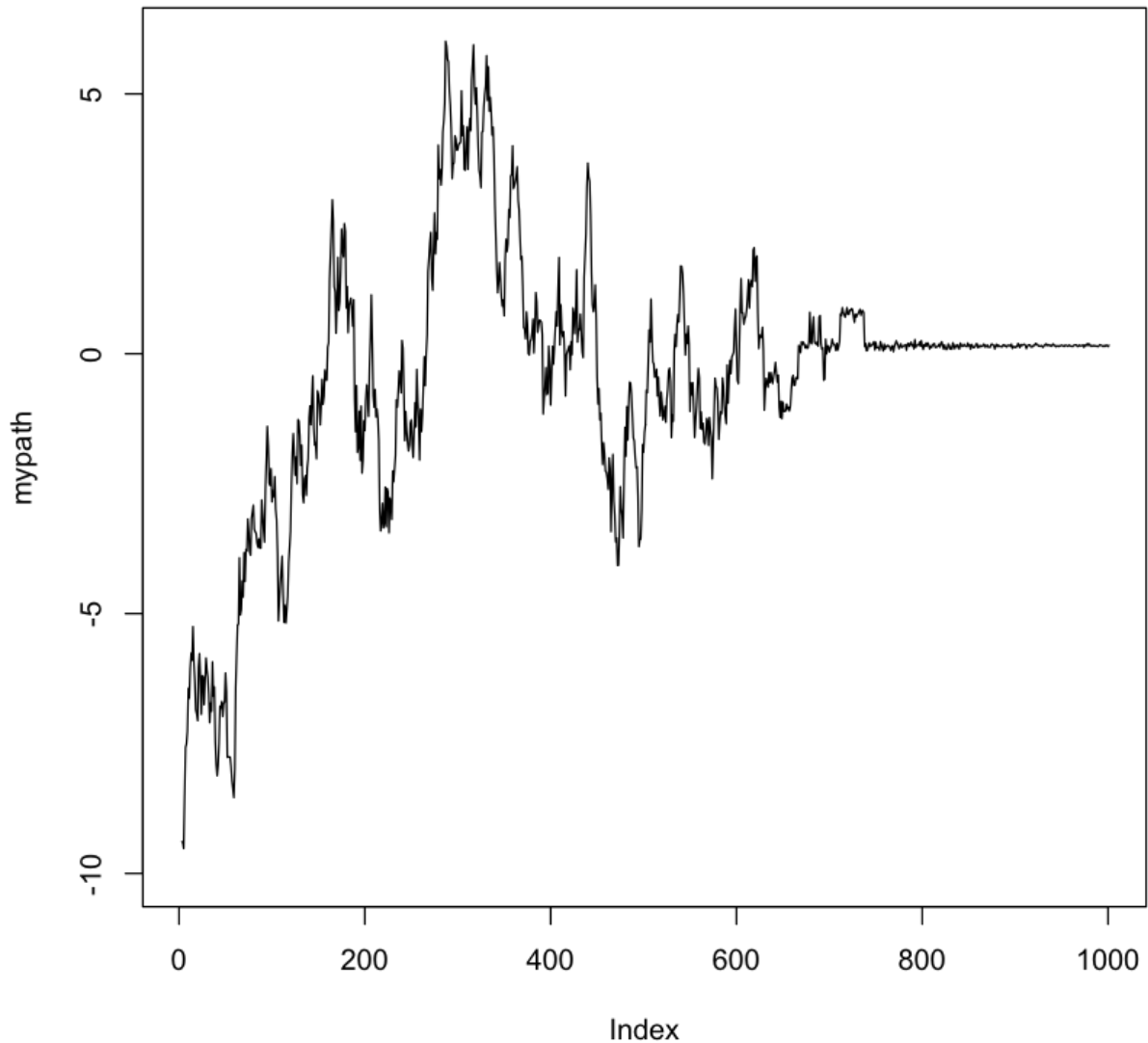
    for(i in (2:length(tau) + 1)){ #for a tau in the tau schedule
        x <- explore(x, tau[i - 1], h, n_iter, eps) #run a short random walk
        path[i] <- x # store the value of x into the "path" of the walker
    }
    return(path)
}

```

Now that we have wrote our simulated annealing function, we will run it to find the global maximum of $h(x)$. We will start the walker at $x = -10$. Our temperature will start at 10^7 and drop to 10^{-4} in 1000 linearly-spaced steps. We will walk 100 times between each drop of the temperature. Our maximum step size will be 0.1 units.

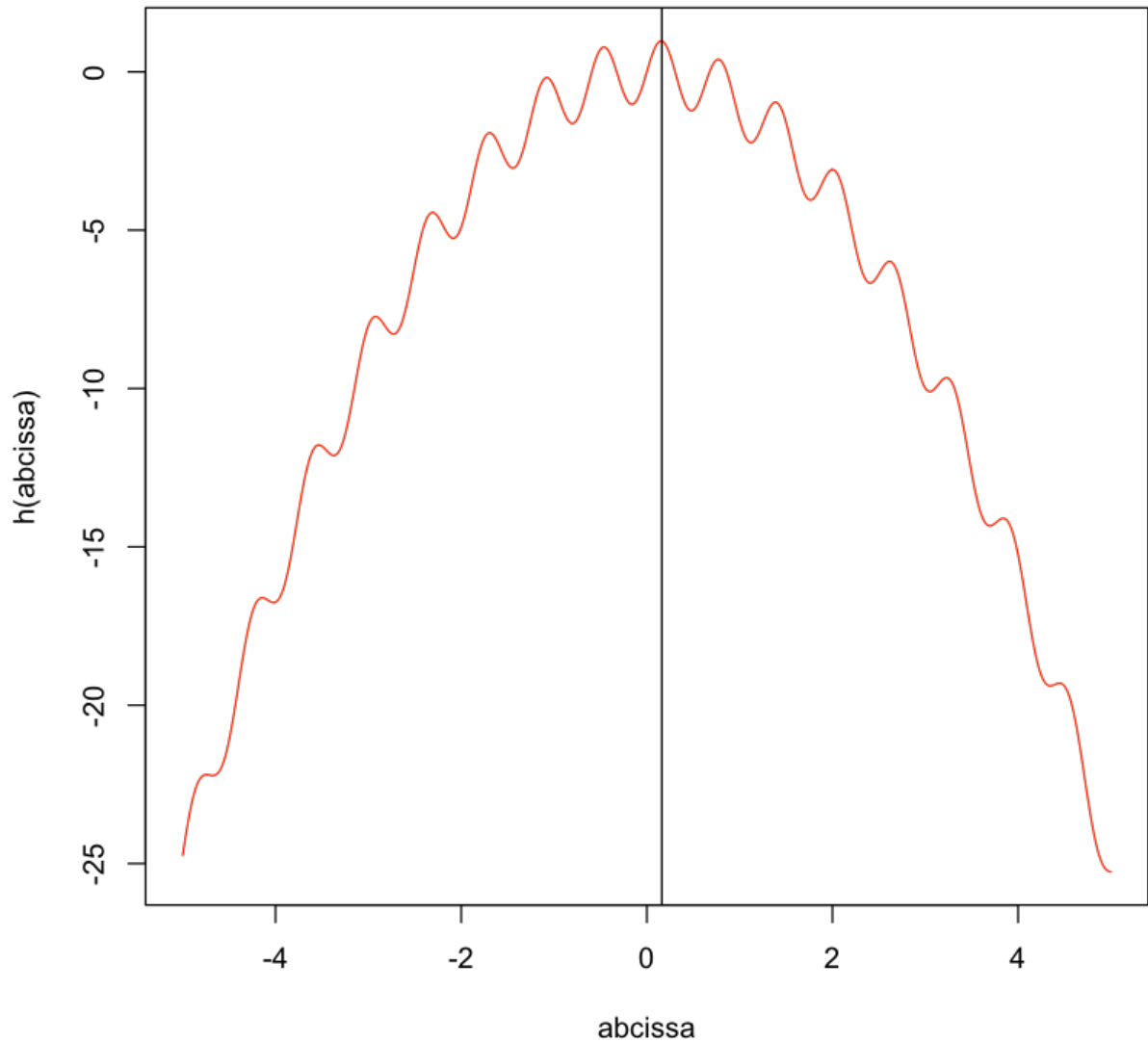
```
In [30]: mypath <- SimAnnealing(-10,h,c(7,-4), 100, 1000, 0.1)
plot(mypath, type = 'l')
print(mypath[length(mypath)])

[1] 0.1610027
```



We find that the walker explores a wide range of values when the temperature is high, ranging from -10 to 5. After about 800 drops of the temperature, it eventually settles to a value of 0.0161. In the next plot, we will plot the solution as a vertical line, and the function (as a red line) and see if the global maximum.

```
In [32]: abciissa <- seq(-5,5,0.01)
plot(abciissa, h(abciissa), type = 'l', col = 'red')
abline(v = mypath[length(mypath)] )
```



Indeed, we have found the global maximum! 🙌🙌🙌🍻