Econ 441 Reading Notes

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1 Introduction & Basic Tools

1.1 Choice under uncertainty, expected utility

1.1.1 Reading: van Zandt Ch. 1 & 2

Overview

- Weak Axiom of Revealed Preference (WARP)
- Independence Axiom
- First Order Stochastic Dominance
- Continuity Axiom
- Expected Utility Maximization
- von Neumann-Morgenstern Theorem

1.1.2 Chapter 1: Introduction to decision theory

Let X be the finite set of all consumption bundles. Let A be a non-empty subset of X of a potential feasible set. Let $C(A)$ be the elements of A that the agent might choose. $C(A)$ may contain more than one item because of indifference.

Definition 1.1. (Preference Maximization): Let $\succeq$ be the relation defined for a choice rule $C(\cdot)$.

$$x \in C(A) \iff x \succeq y \qquad \forall y \in A$$

Definition 1.2 (Preference Relation is Rational)**.** The preference relation $\succeq$ is rational if it satisfies the following two axioms:

1. Completeness: $\forall x, y \in X$, either $x \succeq y$ or $y \succeq x$ or both.
2. Transitivity: $\forall x, y, z \in X$, if $x \succeq y$ and $y \succeq z$, then $x \succeq z$.

Proposition 1.3. *The choice rule satisfies WARP if and only if satisfies preference maximization and the preference relation is rational.*

Definition 1.4. (Weak Axiom of Revealed Preference): Let $x, y \in X$. Let A and B be subsets of X containing both x and y. If $x \in C(A)$ and $y \in C(B)$, then $x \in C(B)$. In other words, if x is revealed weakly preferred to y, then y is not revealed preferred to x.

Proposition 1.5. *2. If the preference relation $\succeq$ is rational, then there is a utility function $U : X \to \mathbb{R}$ such that $\forall x, y \in X$,*

$$x \succeq y \iff U(x) \geq U(y)$$

- Note: the utility representation is not unique. Consider monotonic transformations on U. - Utility functions are useful to present an example of preferences when X is large. (e.g. $U(x) = \log(x_1) + \log(x_2)$)

1.1.3 Chapter 2: Lotteries and objected expected utility

Definition 1.6. (Probability measure): $P : X \to [0, 1]$ is a probability measure and $P(x)$ is the probability of outcome x if and only if
1. $P(x) \geq 0$ for all $x \in X$ 2. $\sum_{x \in X} P(x) = 1$

Consequentialism: The decision maker is indifferent between a compound lottery and its reduced lottery (the lottery of the outcomes of the compound lottery).

Definition 1.7. (Independence Axiom) For all lotteries $P, Q, R \in \mathcal{L}$ and $\alpha \in [0, 1]$

$$P \succeq Q \iff \alpha P + (1 - \alpha)R \sim\succeq Q + (1 - \alpha)R$$

- We can rewrite simple lotteries as compound lotteries to invoke the independence axiom to find inconsistencies in choices. (Exercise 2.1)

Definition 1.8. First Order Stochastic Dominance Let P and Q be two lotteries with outcomes $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$, and $P(x_i) = Q(y_i) = \alpha_i$ for all i. We allow that $x_i = x_j$ or $y_i = y_j$ for some $i \neq j$. P weakly first order stochastically dominates Q if and only if

$$x_i \succeq y_i \qquad \forall i \in \{1, \ldots, n\}$$

P (strictly) first order stochastically dominates Q (P f.o.s.d. Q) if and only if

$$x_i \succ y_i \qquad \forall i \in \{1, \ldots, n\}$$

Proposition 1.9. *1. Suppose that $\succeq$ satisfies the independence axiom. If P weakly (resp. strictly) first order stochastically dominates Q, then $P \succeq Q$ (resp. $P \succ Q$). - Proof with the independence axiom and the compound lottery representation of simple lotteries. - We can use this proposition to show conclude that lotteries are preferred to others by rewriting the lotteries into simple lotteries with the same probability distribution. (p. 22)*

Proposition 1.10. *2. Let $X = \{x_1, \ldots, x_n\}$ and WLOG assume that $x_1 \succeq \cdots \succeq x_n$. P weakly f.o.s.d Q if and only if $\forall k \in \{1, \ldots, n\}$*

$$\sum_{i=1}^{k} P(x_i) \geq \sum_{i=1}^{k} Q(x_i)$$

Because the probabilities sum to 1, this can be rewritten as

$$\sum_{i=k}^{n} P(x_i) \leq \sum_{i=k}^{n} Q(x_i)$$

In other words, we can either compare the cumulative probabilities of the worst outcomes or the best outcomes to determine first order stochastic dominance.

Definition 1.11. (Continuity Axiom) If $P, Q \in \mathcal{L}$ and $P \succ Q$, then $\forall R \in \mathcal{L}$, there exists $\alpha \in (0, 1)$ such that

$$P \succ (1 - \alpha)Q + \alpha R$$

and there exists $\beta \in (0, 1)$ such that

$$(1 - \beta)P + \beta R \succ Q.$$

In words, there is nothing so good (or so bad) that it does not become insignificant if it occurs with small enough probability. This is also called the Archimedean axiom. - Note: this is still unintuitive for me. There is an example on page 25.

Definition 1.12. (Expected utility maximization) Preference $\succeq$ over lotteries $\mathcal{L}$ satisfies expected utility maximization if there is a function $u\colon Z \to \mathbb{R}$ such that $\forall P, Q \in \mathcal{L}$ and $P \succeq Q$ if and only if

$$\sum_{z \in Z} u(z)P(z) \geq \sum_{z \in Z} u(z)Q(z)$$

- A decision maker is an expected utility maximizer if for some function u, she always prefers the lottery with the highest expected utility.

Theorem 1.13. *1 (von Neumann-Morgenstern) If $\succeq$ satisfies the independence axiom and the continuity axiom, then $\succeq$ satisfies expected utility maximization. - Because VNM utility functions measure strength of preferences over outcomes, they are sometimes called cardinal utility functions. - Only positive affine transformations preserve the ranking of lotteries. If $u\colon X \to \mathbb{R}$ is a VNM utility function, then $v\colon X \to \mathbb{R}$ is a positive affine transformation of u if and only if there exists $a > 0$ and b such that $v(x) = a + bu(x)$ for all $x \in X$.*

1.2 Bayesian inference

1.2.1 Reading: van Zandt Ch. 4

Chapter 4: Choosing when there is new information

Conditional beliefs: - Let $\pi\colon S \to \mathbb{R}$ (a probability measure) be the prior belief. - Let $\pi(\cdot \mid E)\colon S \to \mathbb{R}$ be the posterior belief after observing E. - The general rule is (to rescale the probabilities of possible states after observing E, some states may be impossible after observing E)

$$\pi(s \mid E) = \begin{cases} \frac{\pi(s)}{\pi(E)} & \text{if } s \in E \\ 0 & \text{if } s \notin E \end{cases}$$

Bayes' Rule

$$\pi(A \mid E) = \frac{\pi(A \cap E)}{\pi(E)}$$