Multivariate Statistical Analysis Final Project
Dynamic Hedging for Taiwan Stock Index and Put Warrants

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1 Introduction

1.1 Motivation

Market risk is a very import component in almost every investment portfolios. Every stock contribute to the overall rise and fall of the market, and also affected by the overall market environment. There are various scenarios we would like to hedge away market risks: an investor may hold a market portfolio and don't want to sell them in bear market, or a particular stock out-performs the market and we want to earn the excess return only. So we would like to use the popular warrant market to hedge Taiwan stock index or ETF position.

There are two major problem we want to answer in this work: Does dynamic hedge reduce the risk of the portfolio? Do warrants issued by different issuers have different effectiveness on hedging?

1.2 Background

The warrant market in Taiwan have some properties: First, sell short is not allowed for individual investors. So we can only long a put to hedge a long stock index position. Also the issuer of a warrant need to be the market maker, which means it has to place certain amount of buy/sell order even if there is no other order placed by other parties. So it is very important to check if different issuers has different market making power.

There are two index that can represent the market risk: the TWSE weighted index, and the exchange-traded fund(ETF) tracking this index, called Polaris Taiwan Top 50 Tracker Fund. Theoretically the ETF will have tracking error to the TWSE weighted index, but since the ETF is more accessible to individual

In the traditional hedging scheme, hedge ratios are fixed because the volatility of spot and warrant position are treated as time-invariant, which is easier to process but terribly wrong in real world observations. Therefore we would like to model the time-varying volatility for dynamic optimal hedge ratio estimation. The final goal is to employ the multivariate autoregressive conditional heteroscedasticity (MGARCH) model proposed in [4] to model volatility of spot and warrants, and derived hedge ratios.

2 Theoretical Background

2.1 Optimal Hedge Ratio

Consider a portfolio consisting of a spot position (stock index or its ETF) and some warrants position. The return at period t will be

$$r_{p,t} = r_{s,t} + \beta_{t-1} r_{w,t}$$

where $r_{p,t}$, $r_{s,t}$ and $r_{w,t}$ is the return of portfolio, spot and warrant respectively, from period $t-1$ to t. We can than calculate the conditional variance of the portfolio as:

$$\begin{aligned} var(r_{p,t}|\Omega_{t-1}) =& var(r_{s,t}|\Omega_{t-1}) + \beta_{t-1}^2 var(r_{w,t}|\Omega_{t-1}) \\ &+ 2\beta_{t-1} cov(r_{s,t}, r_{w,t}|\Omega_{t-1}) \end{aligned}$$

An optimal hedge is to reduce the overall variance of the portfolio, by adjusting the percentage of the two elements, i.e. changing the β. So the optimal hedge ratio β_{t-1}^* is defined as:

$$\beta_{t-1}^* = \arg\min_{\beta} var(r_{p,t}|\Omega_{t-1})$$

We can do a partial derivative on β_{t-1} and set the result to zero, and we can get:

$$\beta^*_{t-1} = -\frac{cov(r_{s,t}, r_{w,t}|\Omega_{t-1})}{var(r_{w,t}|\Omega_{t-1})}$$

A simple way is to use fixed historical variance/ covariance to calculate the hedge ratio. But the variance/covariance is time-varying, so we need a more complex model for estimation.

2.2 MGARCH-BEKK Model

To model the time-varying volatility, we follow the methodology proposed by [4]. Using the multivariate autoregressive conditional heteroscedasticity (MGARCH) model, we can create a time-varying model for conditional variances and covariances for calculating optimal hedge ratio.

Under the assumptions of no transaction costs, no market microstructure effects, efficient markets hypothesis and no arbitrage opportunities, we can assume the spot and warrant markets react identically to new information (shock/innovation).

For the return series

$$r_t = \mu_t + \Xi_t$$

where $\mu_t = E(r_t|\Omega_{t-1})$, the conditional expectation of r_t. We can apply a vector ARMA(p,q) model,

$$\mu_t = \Upsilon x_t + \sum_{i=1}^{p} \Phi_i r_{t-1} - \sum_{j=1}^{q} \Theta_j \Xi_{t-1}$$

where x_t are exogenous (explanatory) variables vector, and Ξ_t are shock/innovation matrix. This is the mean equation of r_t.

But what we really want is the conditional covariance model.There are more than one way to solve a MGARCH model, we choose BEKK parameterization proposed by Bollerslev, Engel, Kroner, and Kraft to solve it. [3]

$$H_t = W'W + A'H_{t-1}A + B'\Xi_{t-1}\Xi'_{t-1}B$$

In our bi-variate model, A and B are 2×2 matrices of parameters, W is an upper-triangular matrix of parameters, and Ξ is the innovation (disturbance) vector.

BEKK model can guarantee that H_t is positive-definite and allows dynamic dependence between series. But the number of parameters increase rapidly with dimension, make it very time-consuming to run a estimations. [15]

To estimate the above parameters, we try to maximize the log-likelihood function

$$l(\theta) = -\frac{TN}{2}log2\pi - \frac{1}{2}\sum_{t=1}^{T}(log|H_t| + \Xi'_t H_t^{-1} \Xi_t)$$

, in which θ means all the parameters, T is the number of observations and N is the number of time series (in our case $N = 2$). We will not go into the detail of this estimation procedure further.

3 Hedging Strategy

We will use long put with daily rebalance as our hedge strategy. For every day t, we will use a time windows of the last 30 trading days (i.e. $t-30$ to t) to estimate H. The estimated H will be used to determine the hedge ratio of tomorrow $(t+1)$ using the procedure explained in section 2.1. So on every trading day we will adjust the proportion of the warrants position in our hedge portfolio, we will round the hedge ratio to make the size of both position integer. If the data available in the time window is less than 30. We will still use the available data to estimate. So in the first few days the number of data points will be too few to achieve statistical significance.

4 Experiments

4.1 Data Source

We use the price quotes of TWSE weighted index (symbol: TSEA), Polaris Top 50 Tracker Fund (symbol: 0050), three put warrants on TSEA (symbols: 06156P, 06246P, and 06487P) and three on 0050 (symbols: 05936P, 05449P, and 06233P). The warrants are issued by 3 different issuers: FuBon, KGI and Capital. These data are fetched from the Captial Group Stock Trading Program, which is a real-world trading platform. Because the warrants starts and ends on different dates, we take only the intersection of the trading dates, results in a 220 day sequence.

We will test 0050 with each of its three warrants, and do the same to TSEA. Table 1 lists all the test pairs.

Spot	Hedge with
0050	06156P
0050	06246P
0050	06487P
TSEA	05936P
TSEA	05449P
TSEA	06233P

Table 1: Hedging Pairs

4.2 Experiment design

All the spot-warrant pairs are test for three kinds of hedging strategy: no hedge, simple hedge, and symmetric dynamic hedge. No hedge means we only buy and hold one spot position. Simple hedge means we buy one spot with a fixed amount of spot position, the ratio is determined by the historical variance of the first 30 days of return data. The position is held until the end of test without rebalance. Symmetric dynamic hedge is described in the previous section (section 3).

For each hedge method, we evaluate the net value in a 30 days sliding window to get the hedge result. The window starts from the day 30 to 60, and shift by 5 days. So we will get 10 samples for each subject under each hedge method. There will be a total of 180 samples (6 subjects, 3 hedge methods, 10 samples each). For each hedge result, we calculate the mean and variance of the net value sequence.

4.3 Results

The daily hedge ratio is shown in Figure 1 and 2.

We can see that the six daily hedge ratios have very similar trends (except the outliers in the first 30 days). This is not very surprising because the 0050 are suppose to be tracking the index. And the warrants are suppose to track the spots in the same way.

The hedge result is shown in the following table: Table 2 is the mean of the returns of the 10 samples from each subject (Note there are problems calculating the hedge ratio for 0050–06156P.); Table 3 is the mean of the variance of the 10 samples from each subject. We can see that symmetric dynamic hedge yields the lowest variances, and returns are more close to 0. Simple hedge gives worse return and higher variance, which is not desirable in hedging.

We would expect the variance of portfolio net value to drop after hedging. We will use repeated-measure ANOVA to evaluate the following hypothesis:

1. No interaction between issuer and hedge method effects.
2. There is no difference in net value variance between different issuers.
3. There is no difference in net value variance between different hedge methods.

We use ANOVA (analysis of variance) to check these assumptions. We have 6 subjects (spot-warrant pair), one between-subject factor – issuer (3 level) and one within-subject factor – hedge method (3 level). The ANOVA table and Wilk's lambda results are shown in Table 4:

We can tell from the p-value that the first and second null hypothesis are not rejected at 5% significance level. However the third one is rejected at 5% significance level. So we can conclude that:

1. Who is the issuer does not affect the hedge performance of the results.
2. Hedge method do have effect on the variance of the hedge result.

Combining the information from Table 3 and Table 4, we can see dynamic hedge actually reduce the variance of the portfolio.

5 Conclusion and Outlook

We used MGARCH-BEKK model to estimate the dynamic hedge ratio between Taiwan Stock Index/0050 ETF and their put warrants. The dynamic hedge method is then tested on 6 subjects, with no hedge and simple hedge as control group. By applying ANOVA on the hedge results, we can conclude that the issuer of the warrants don't affect the hedging performance. And The hedge method has statistically significant effect on reducing the overall risk of portfolios.

In this work we did not consider transaction costs. But daily rebalance will cause high transaction costs. So we may need to take that into account in model estimation. Another issue is that

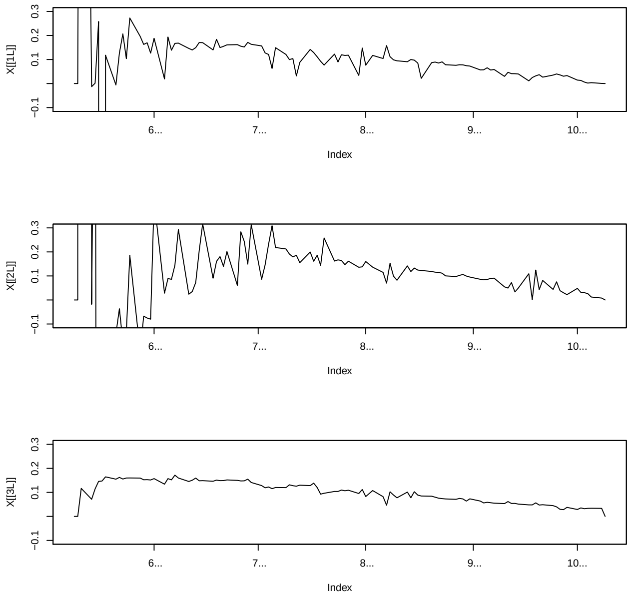

Figure 1: Daily Hedge Ratio for 0050 and 06156P, 06246P, 06487P (top to bottom).

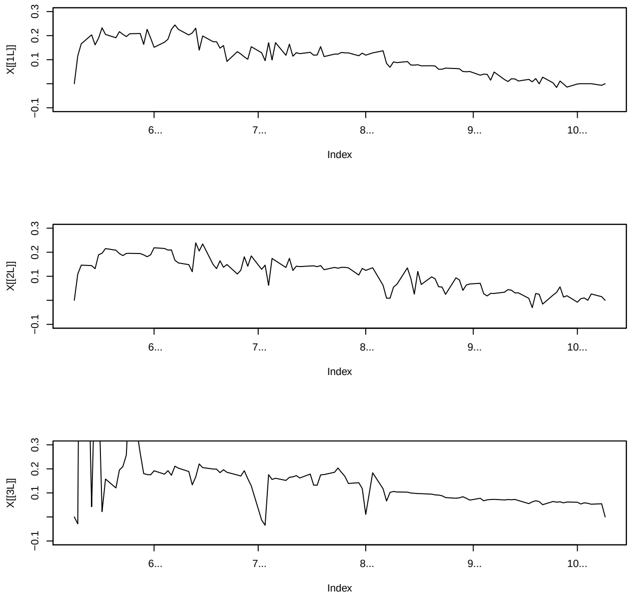

Figure 2: Daily Hedge Ratio for TSEA and 05449P, 05936P, 06233P (top to bottom).

	warrant	noHedge	simpleHedge	symmetricHedge
1	0050.06156P	0.04501327	-0.2721746	0.0000000000
2	0050.06246P	0.04501327	-0.2352882	-0.0003683609
3	0050.06487P	0.04501327	-0.2772216	0.0001241523
4	TSEA.05449P	0.02838937	-0.2820709	-0.0117454559
5	TSEA.05936P	0.02838937	-0.2808293	-0.0161438739
6	TSEA.06233P	0.02838937	-0.2483118	-0.0195794636

Table 2: Mean of Hedge Portfolio Returns

	warrant	noHedge	simpleHedge	symmetricHedge
1	0050.06156P	7.322145e-05	0.0008210118	NA
2	0050.06246P	7.322145e-05	0.0005637808	1.127919e-06
3	0050.06487P	7.322145e-05	0.0007705453	3.318314e-05
4	TSEA.05449P	6.362993e-05	0.0004856155	4.397624e-05
5	TSEA.05936P	6.362993e-05	0.0008118012	4.135964e-05
6	TSEA.06233P	6.362993e-05	0.0007452160	3.012211e-05

Table 3: Mean of Hedge Portfolio Variance

```
Error: warrant
          Df    Sum Sq   Mean Sq F value Pr(>F)
issuer     2 2.720e-08 1.360e-08   0.084  0.922
method     1 9.631e-07 9.631e-07   5.989  0.134
Residuals  2 3.216e-07 1.608e-07

Error: warrant:method
              Df   Sum Sq   Mean Sq F value   Pr(>F)
method         2 2.29e-05 1.145e-05  67.889 0.000238 ***
issuer:method  4 3.49e-07 8.700e-08   0.517 0.728722
Residuals      5 8.43e-07 1.690e-07
---
Signif. codes:  0 ‘’ *** 0.001 ‘’ ** 0.01 ‘’ * 0.05 ‘’ . 0.1 ‘’  1

Error: Within
           Df    Sum Sq  Mean Sq F value Pr(>F)
Residuals 153 6.074e-06 3.97e-08
```

Table 4: ANOVA Table

the testing data is too short in time. We may need to use a roll-over strategy to create a very long warrant position. Short before a warrant is about to expire, we will switch to a new warrant. But special care is need to avoid jumps when switching.

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