Model Predictive Control for a Novel Wave Energy Converter

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Abstract—**Wave energy converters can harvest power from ocean waves, but to do so effectively requires a sophisticated control force on the power take off mechanism that is able to maximize the power while satisfying a set of constraints. While other control strategies have been explored, they have some fundamental disadvantages. Model predictive control is proposed for the optimal control of a novel wave energy converter because of it ease of implementation, its ability of handle constraints, and the its ability to maximize power without perfect knowledge of the sea state.**

I. Introduction

The world is experiencing a pivot towards sustainable energy, away from fossil fuels. In this transition, energy sources such as solar, wind, geothermal, hydroelectric are all going to play a role in the transition. Wave energy converters (WECs) could provide a source of renewable energy in certain environments where other sources are not as effective. Wave power is currently not widely employed, but the technology is still in its early stages and has room to grow. Many designs for WECs are still being explored today. Not only is the physical design of WECs important, but also the control strategies that accompany them. By including a controllable force in the power take off mechanism (PTO) the power harvest can be greatly increased.

A novel design of a WEC is being investigated by the author along with a couple potential control strategies. The goal of this project is to explore a different control strategy and compare its performance with previously used methods. This control strategy is model predictive control with optimization methods (MPC), which was chosen because of it can take into account the time varying excitation force over a finite horizon while satisfying a state constraints.

The paper will be organized into these sections

1) Description of the System: Introduces basic concepts in wave body interaction. Describes the formulation of the frequency and time domain equations of motion.
2) Previous Work: Talks about previously explored control methods, including resistive control, conjugate control and optimal control via Pontryagin's Minimum Principle (PMP). Describes the limitations of each approach
3) Methods: Describes optimal control of the WEC using Model Predictive Control using optimization methods. Sets up optimization problem and the constraints
4) Results: Shows data from simulating the WEC with MPC control and compares results with those from previous methods
5) Conclusions and Future Work: Summarizes findings and proposes future work

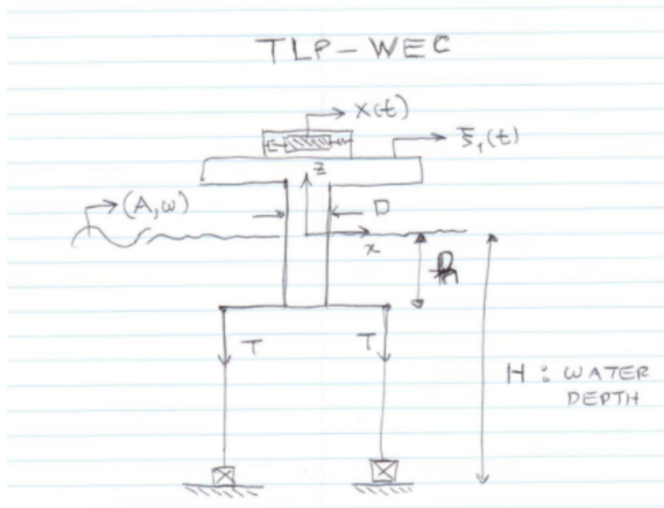

Figure 1. The WEC consists of a TLP than contains a oscillating proof mass

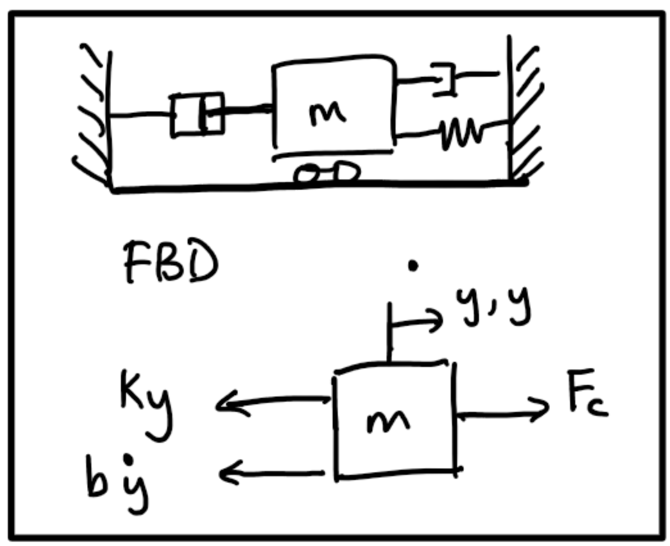

Figure 2. The proof mass is coupled with the TLP via spring, dashpot, and a PTO. The forces acting on the proof mass is shown in the free body diagram (FBD)

II. Description of System

The WEC being considered is a tension leg platform (TLP), where the inside holds containers with oscillating proof masses. The WEC is categorized as an oscillating point absorber, which means that the dimensions are much smaller than the wavelength of the ocean waves and energy is absorbs through its movements near the water surface. The forces of the waves will cause the proof masses to oscillate relative to the TLP. The relative motion of the proof masses will be harvested as energy. A diagram of this WEC is shown in Figure 1. The proof mass container is shown in Figure 2. In this paper, WEC will refer to the entire oscillating system including both the TLP and the oscillating proof masses and the broader definition of a WEC. The TLP will refer to the WEC minus the proof masses. Ocean waves will excite the states of the WEC and induce relative motions between the proof mass and the TLP. A power take off (PTO) mechanism can be used to extract power from this relative motion. The PTO will exert a controllable force $F_c(t)$. The power harvested by the PTO is given by $P(t) = F_c(t)\dot{y}(t)$. The equation of motion of the system is given in state space form. To keep things simple, we will only model motion in surge, while in real world there will be many more degrees of freedom, some of which are coupled. The state space equations are given in equation 1. The variables inside the matrices were derived using system identification methods [1] and a tool called WAMIT. The details of that will be skipped over in this paper. This is a multiple input system, with inputs $F_e(t)$ and $F_c(t)$.

$$\frac{d}{dt}x(t) = Ax(t) + B_eF_e(t) + B_cF_c(t) \tag{1}$$

$$x(t) = \begin{bmatrix} \xi(t) \\ \dot{\xi}(t) \\ y(t) \\ \dot{y}(t) \\ \mathbf{x_r(t)} \end{bmatrix} \tag{2}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \mathbf{0} \\ \frac{-C_{11}}{M+A_\infty} & 0 & \frac{k}{M+A_\infty} & \frac{b}{M+A_\infty} & \frac{-\mathbf{C_r}}{M+A_\infty} \\ 0 & 0 & 0 & 1 & \mathbf{0} \\ \frac{C_{11}}{M+A_\infty} & 0 & \frac{-k}{M+A_\infty} - \frac{k}{m} & \frac{-b}{M+A_\infty} - \frac{b}{m} & \frac{\mathbf{C_r}}{M+A_\infty} \\ \mathbf{0} & \mathbf{B_r} & \mathbf{0} & \mathbf{0} & \mathbf{A_r} \end{bmatrix} \tag{3}$$

$$B_e = \begin{bmatrix} 0 \\ \frac{1}{M+A_\infty} \\ 0 \\ \frac{-1}{M+A_\infty} \\ \mathbf{0} \end{bmatrix} \tag{4}$$

$$B_c = \begin{bmatrix} 0 \\ \frac{-1}{M+A_\infty} \\ 0 \\ \frac{1}{M+A_\infty} + \frac{1}{m} \\ \mathbf{0} \end{bmatrix} \tag{5}$$

This continuous time state space model can be converted into a discrete time model, where $u_d[n] = [F_e[n], F_c[n]]^T$

$$x[n+1] = A_dx[n] + B_du_d[n] \tag{6}$$

With a discrete time formulation of the system, we can now carry out simulations of the WEC under different control strategies.

III. Previous Work

Previous work has explored using resistive control, conjugate control and PMP for the control force [2] [3].
Resistive control is simply a passive damping system, where the control force is $F_c(t) = -b_c\dot{y}(t)$. The damping term can be chosen so that the control force is maximized while the variance of the displacement is kept below a threshold. The limitations of resistive control is that it represents a lower bound on power harvested
Another strategy is conjugate control, which is obtained from the idea of impedance matching. This is represents the theoretical maximum power that can be harvested. The frequency domain expression for the complex conjugate control force is given by $F_c(\omega) = -Z_i(\omega)Y(\omega)i\omega$ This can be converted into the time domain expression for the control force in terms of the incident wave.

$$F_c(t) = A * |F_c(\omega)|cos(\omega t + \theta + phase(F_c(\omega))) \tag{7}$$

The limitations of conjugate control is that it requires perfect knowledge of the sea state and also often leads to the states of the system to exceed the maximum threshold.

Another approach that was explored was PMP. PMP is a necessary condition for optimal control. We can try to find a solution that satisfies this optimality condition. A guess of what the optimal states are computed using forward simulation where the control force is set to 0. This is assuming that the states won't be affected too much by a control force. Then a backwards iteration is used to solve for a control force that satisfies the optimality conditions. This control force is then used to drive the PTO. However, this method doesn't necessarily produce an optimal controller because PMP is a necessary condition but becomes sufficient only when the problem is convex (which it is not). Guess at optimal states isn't always correct and can cause the solution to not be optimal.

These control strategies have their limitations. Ideally we can find a controller that is able to effectively maximize power while satisfying constraints and without requiring perfect knowledge of the sea state.

IV. Methods

Model predictive control optimizes a cost function over a finite horizon, while satisfying a set of constraints. The optimal control is calculated using optimization methods [4]. The objective we want to minimize is given below, with state constraints on $x[n]$ and control constraints on $u[n]$. The state vector is given by $x = [\xi, \dot{\xi}, y, \dot{y}, x_{r1}, x_{r2}...]^T$. Exogenous input due to wave excitation is given by $F_e[n]$. The optimization was carried out for some finite horizon of size N.

$$\min_{x[.],u[.]} \sum_{n=0}^{N-1} \dot{y}[n]u[n] \tag{8}$$

$$subject\ to \quad x[n+1] = Ax[n] + B_eF_e[n] + B_cu[n] \tag{9}$$

$$|y[n]| <= y_{max} \tag{10}$$

$$|u[n]| <= u_{max} \tag{11}$$

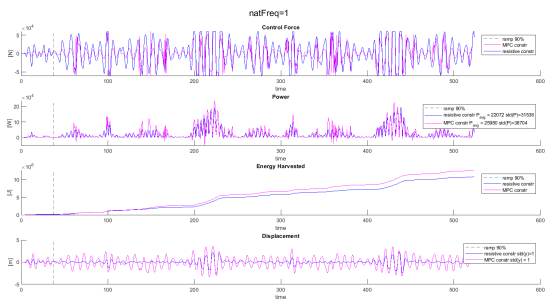

Figure 3. The simulation results with resistive control versus MPC. The subplots show (1) the constrained control force, (2) instantaneous Power being harvested, (3) the total energy harvested, (4) the relative position of the proof mass. From subplot 3, we can see that MPC is able to harvest more power than resistive control.

Using MATLAB's optimization functions, I was able to solve for an optimal control force. The optimization was solved using a quadratic program and was identified as a nonconvex problem.

The simulation was carried out in a specific sea state in over 500 seconds. A hyperbolic tangent function was used to ramp up the control force starting at time equals zero. Max control force was set to 60 kN. The finite horizon was chosen to be 5 seconds because this reflects the amount of time we can / need to forecast the excitation force into the future.

V. Results

The simulation results show an average power harvested of 22kJ for resistive control vs 26kJ for for MPC, which is an almost 20 percent increase. The drawbacks for MPC is that computing the optimal control takes slightly longer time than for resistive control. The simulation time for MPC is 56 seconds vs 0.06 seconds for resistive control.

VI. Conclusions and Future Work

MPC control was explored as a method for optimal control of a WEC to maximize power. MPC control is able to extract more power and at the same time consider constraints, while not requiring perfect knowledge of the sea state. To improve on this method, we can try to speed up the calculation of the optimal control force. In real life application, the control force is created using a set of pressurized hydraulic chambers that can only exert a discrete set of control forces. By limiting the control force to a discrete set, we can reduce the search space to optimize over. Another possible improvement is to turn the problem into a convex one. Currently the optimization solves a nonconvex quadratic program. Having a convex problem will make computation much easier.

References

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