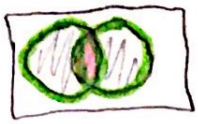


1 min

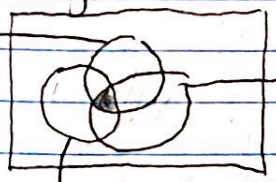
Discussion 4B: tinyurl.com/frank-discussion 7/15/2020

1 (a) How many of first 100 positive integers are divisible by 2, 3, or 5?

C: numbers div by 5

5-6 min

15 min



$|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$= \left\lfloor \frac{100}{2} \right\rfloor + \left\lfloor \frac{100}{3} \right\rfloor + \left\lfloor \frac{100}{5} \right\rfloor - \left\lfloor \frac{100}{6} \right\rfloor - \left\lfloor \frac{100}{10} \right\rfloor - \left\lfloor \frac{100}{15} \right\rfloor + \left\lfloor \frac{100}{30} \right\rfloor$

$= \boxed{74}$

(b) How many non-decreasing phone numbers having 7 digits?

stars and bars

_ _ _ _ _ _ _

dividers: partition the 7-digit phone number into distinct regions for numerical values

16 objects: 9 dividers + 7 digit locations

$\boxed{\binom{16}{9} = \binom{16}{7}}$

(c) How many strictly decreasing phone numbers having 7 digits?

Key insight: given 7 digits out of 10, (unique)

there is only one ordering of these 7 digits such that it is strictly decreasing $\boxed{\binom{10}{7}}$

5:32 2 (a) $\binom{2n}{2} = 2\binom{n}{2} + n^2$ (b) $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$ (c) $\sum_{k=1}^{n} k\binom{n}{k} = n2^{n-1}$

$2n$ choose 2

n ppl choose k of them

Let k = the number of actors Edward wants

6 min alone rest time go over

① choose 2 dirs. from the experienced group

② choose 2 dirs. from the inexperienced group

③ choosing 1 dir. from exp, 1 dir. from inexperienced

$k\binom{n}{k}$ — amongst the k actors hired, pick one of them to be lead actor

I have n applicants, choose k of them to hire

$\binom{2n}{2} = \binom{n}{2} + \binom{n}{2} + n(n)$

① ② ③

$\boxed{\binom{2n}{2} = 2\binom{n}{2} + n^2}$

(d) $\sum_{k=j}^{n} \binom{n}{k}\binom{k}{j} = 2^{n-j}\binom{n}{j}$

Story: exactly the same as part 2c, except we hire j lead actors instead of only 1

LHS:

$\sum_{k=j}^{n} \binom{n}{k}\binom{k}{j}$ ← choosing j lead actors out of k actors hired

hiring k actors out of n applicants

we have to hire at least j actors

k = number of actors we hire

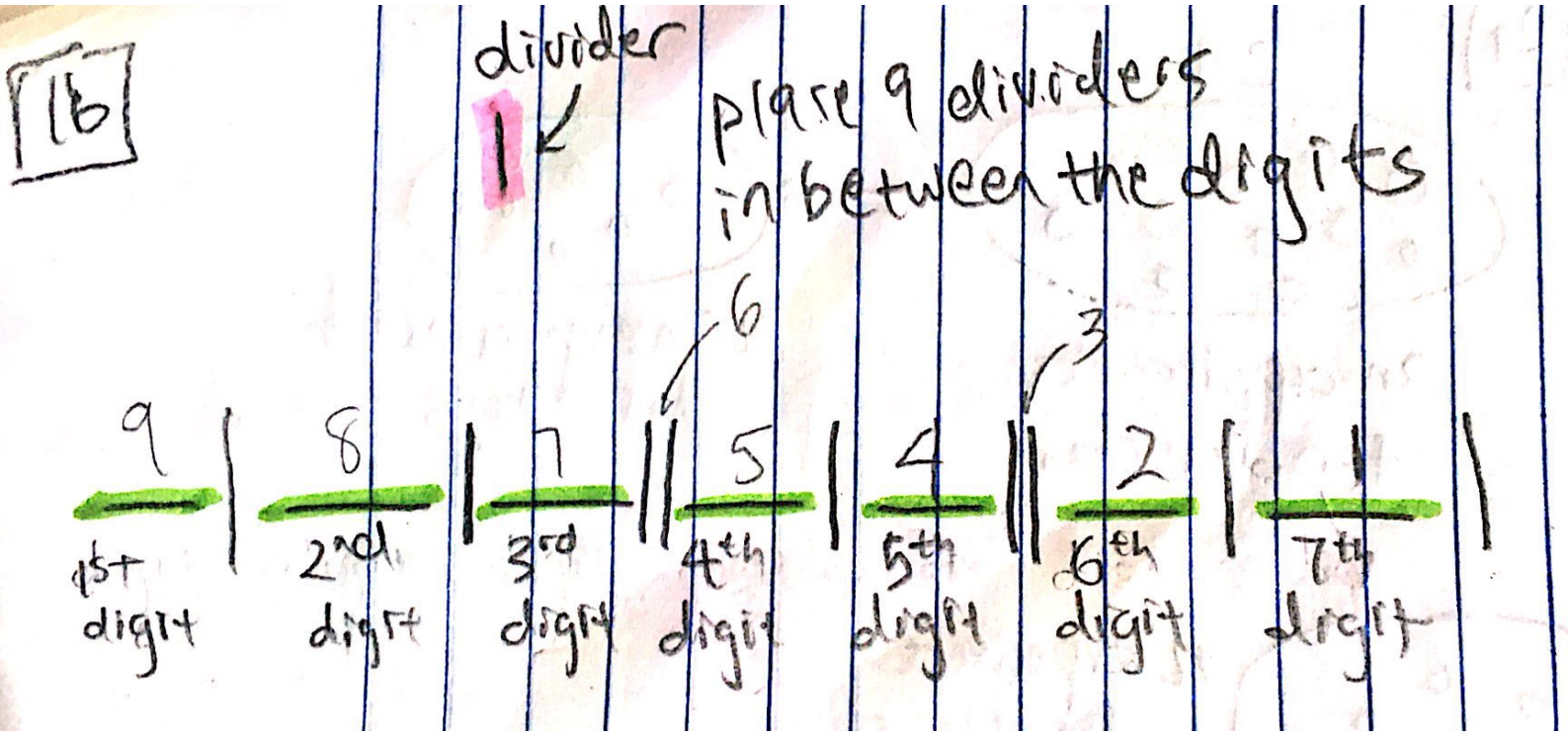
RHS: $2^{n-j}\binom{n}{j}$

① Of n applicants, choose j of them to be lead actors.
$\binom{n}{j}$ ways

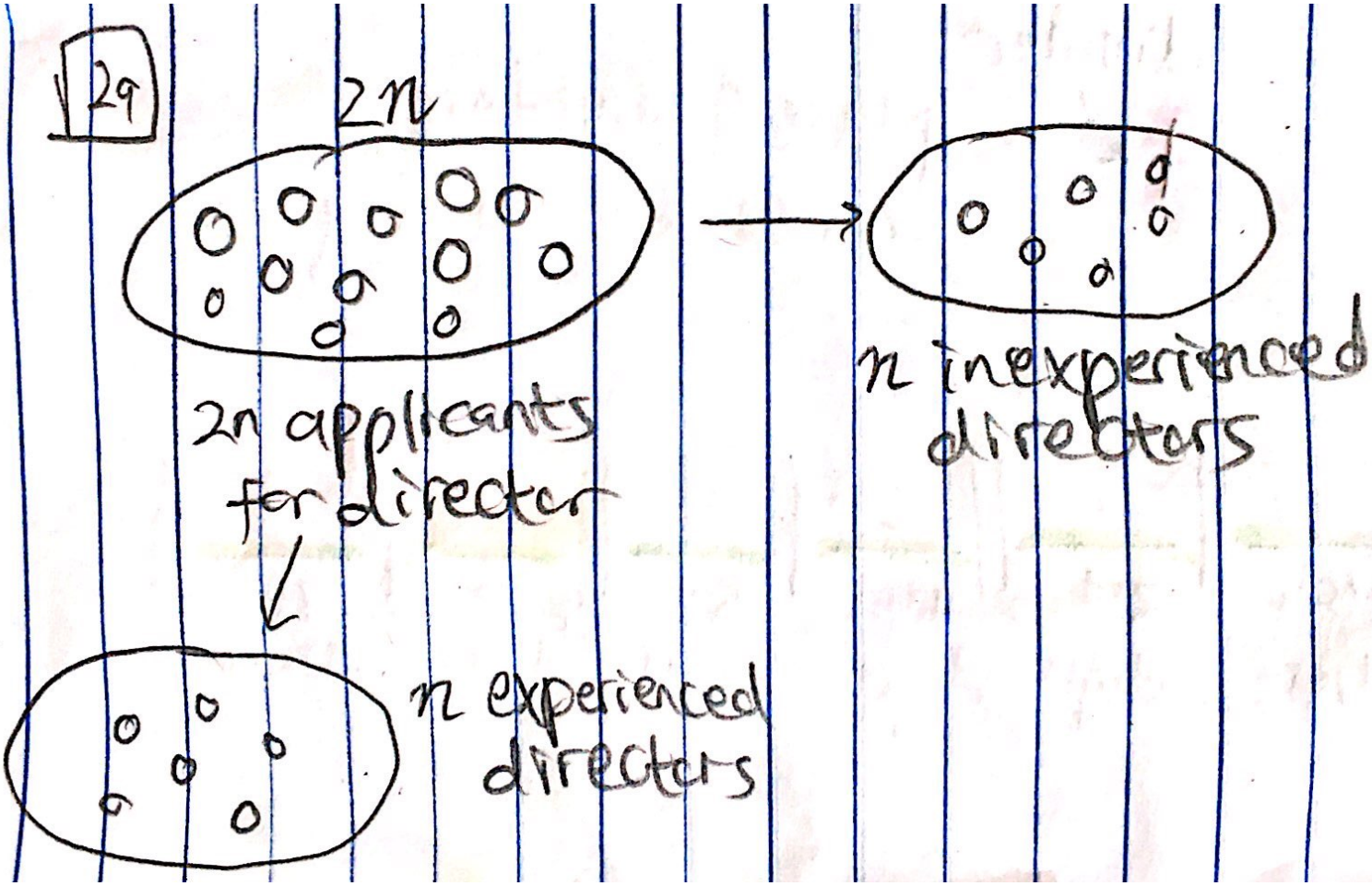
② For $n-j$ ppl left over, either hire them or don't hire
2^{n-j} ways $2^{n-j}\binom{n}{j}$

In both cases, I am selecting j lead actors to hire, along with some subordinate actors, out of n applicants

[3] How many bit strings of length 10 have at least 5 consecutive zeros?



29



26

n members

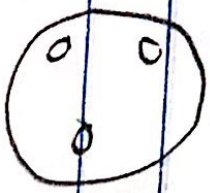
include him or exclude him?

include:

$\binom{n-1}{k-1}$

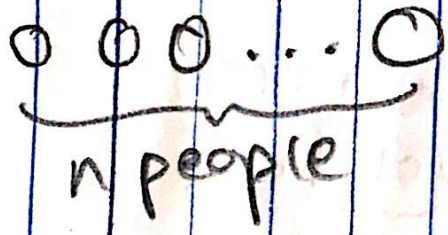
exclude:

$\binom{n-1}{k}$



select k to be a part of the crew

2c



k = possible number of actors hired, ranging from 1 to n

pick one lead actor too

① First, choose a lead actor
n ways

② Decide whether everyone else in line is hired or not.
$(n-1)\ 2^{n-1}$ options

$$n2^{n-1}$$