CS 70 Discussion 2A: tinyurl.com/frank-discussion 6/29/2020

1 (a) $A \cap B$, A countable, B uncountable

countable

$A \cap B$ returns a subset of A

4 min since A is countable, $A \cap B$ is countable.

4 min

(b) $A \cup B$, A countable, B uncountable

uncountable

$A \cup B$ contains at the minimum all elements of B

B is uncountable, thus $A \cup B$ is uncountable.

(c) $\bigcap_{i \in A} S_i$ where A is countable set of indices but each S_i is uncountable

S_1 S_2 S_3 ...

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$A = \{1, 2, \quad \}$

Sometimes countable:

$S_i = \{\forall r \in \mathbb{R} \mid i < r < i+1\}$

all elements of S_i are different from each other

For example, $S_2 = \{\forall r \in \mathbb{R} \mid 2 < r < 3\}$

$A = \mathbb{Z}$ $\bigcap_{i \in A} S_i = \boxed{\emptyset}$

Sometimes uncountable:

$A = \mathbb{Z}$ $S_i = \{\forall r \in \mathbb{R} \mid 1 < r < 2\}$

all sets S_i are the same as each other

$\bigcap_{i \in A} S_i = S_i$

therefore uncountable

5:20

2 (a) Given countable A and B, prove that $A \times B$ is countable.

A is countable. $\exists$ bijection btwn A and a countable subset of $\mathbb{N}$.

B is countable, so similar bijection exists.

6 min We know that $\mathbb{N} \times \mathbb{N}$ is countable,

24 min proof is from the notes.

We build a bijection between a subset of $\mathbb{N} \times \mathbb{N}$ and $A \times B$ as follows:

$\forall i, j \in \mathbb{N}$ $f(i,j) = (A_i, B_j)$

Since we have built a bijection btwn $A \times B$ and $\mathbb{N} \times \mathbb{N}$, and $\mathbb{N} \times \mathbb{N}$ is countable, $A \times B$ is countable as well.

(b) Given finite n, prove that $A_1 \times \ldots \times A_n$ is countable (where each A_i is countable).

Induction:

Base case: $n = 2$ $A_1 \times A_2$ is countable ✓

Inductive Hypothesis: $\exists k \in \mathbb{N}$ $(k > 2)$ such that $A_1 \times \ldots \times A_k$ is countable

Inductive Step: $\underbrace{\underbrace{A_1 \times \ldots \times A_k}_{} \times A_{k+1}}_{\text{also countable}}$ $(n = k+1)$

$C = A_1 \times \ldots \times A_k$ C is countable

$C \times A_{k+1} =$ countable

countable

■

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5:50 $\boxed{3}$ (a) A common strategy to show that a program P is uncomputable is to reduce TestHalt to P.

3 min

7 min Problem A reduces to a Problem B if B can be used as a subroutine to solve A.

$$\int_0^1 x^2\,dx$$

A = integration B = addition, ⊖, ×

$\boxed{\text{Hint: TestHalt reduces to PrintHW}}$