Discussion 4C: tinyurl.com/frank-discussion 7/16/2020

5-6 min alone
10 min go over

1 (a) Flip fair coin 3 times, list out my sample space:

$\boxed{\Omega = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}}$

(b) A = first flip is heads

$A = \{HHH, HHT, HTH, HTT\}$

$\boxed{4}$

(c) B = third flip is heads

$B = \{HTH, THH, HHH, HHT\}$

$\boxed{4}$

(d) C = 1st and 3rd flip are heads

$C = \{HHH, HTH\}$

$\boxed{2}$

(e) 1st or 3rd flip are heads

$D = \{HHH, HHT, HTH, THH, HTT, TTH\}$

$\boxed{6}$

(f) Express C, D in terms of A, B

1st 3rd

$\boxed{C = A \cap B \quad D = A \cup B}$

A, B are not disjoint b/c they have element(s) in common.

(g) Coin flipped $n \geq 3$ times, compute $|\Omega|, |A|, |B|, |C|, |D|$

$|\Omega| = \underbrace{2 \times 2 \times 2 \dots 2}_{n \text{ times}} = 2^n$

$|A| = \underbrace{2 \times 2 \times 2 \dots 2}_{n-1 \text{ times}} = 2^{n-1}$

$|B| = |A| = 2^{n-1}$

$|C| = |A \cap B| = \underbrace{2 \times 2 \times 2 \dots 2}_{n-2 \text{ times}} = 2^{n-2}$

$|D| = |A \cup B| = |A| + |B| - |A \cap B|$

$= 2^{n-1} + 2^{n-1} - 2^{n-2}$

$= 2^n - 2^{n-2} = \boxed{\frac{3}{4}(2^n)}$

(h) Coin could be HH, TH, or TT. Given that it lands H, what is probability that other side is H?

HH front	~~HT front~~	TT front
HH back	HT back	TT back

Ω

We account for the fact that we saw a heads when placing onto the table.

there are only 3 outcomes in Ω that are consistent w/ our observation

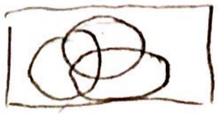
~~$\frac{3}{6} = \frac{1}{2}$~~

$\frac{2}{3}$ ← highlighted possible outcome

$\boxed{\frac{2}{3}}$

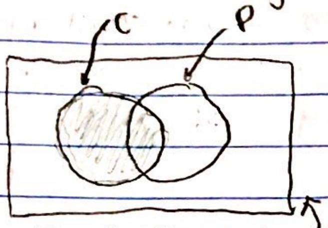
5:21

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C)$
$- P(B \cap C) + P(A \cap B \cap C)$



$P(A \cup B \cup C \cup D) = +P(\text{one-way} \cap) - P(\text{2-way} \cap) + P(\text{3-way} \cap) - P(\text{4 way} \cap)$

$\boxed{2}$ (a) Draw Venn Diagram (b). P(student $\in$ C)? (c) P(student $\in$ P)?



Ω = all CS students

$|C| = 400$ $|P| = 500$

$|C \cap P| = 50$

(b) $\boxed{\dfrac{400}{1000}}$ (c) $\boxed{\dfrac{500}{1000}}$

(d) P(student $\in$ C, P) (e) P(student $\in$ C or student $\in$ P)

$= P(s \in C) + P(s \in P) - P(s \in C, P)$

$\boxed{\dfrac{50}{1000}}$

$= \frac{2}{5} + \frac{1}{2} - \frac{1}{20} = \boxed{\frac{17}{20}}$

$\boxed{3}$ $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 70$

$x_1 + x_2 = 6$ $(x_1, x_2 \geq 0)$

$\underbrace{1+1+1+1+}_{x_1}|\underbrace{1+1}_{x_2} = 6$

$\Downarrow$

how many ways can I arrange six ones, and one divider? $\binom{7}{1}$

$\underbrace{1}_{x_1}|\underbrace{1}_{x_2}|\underbrace{1}_{x_3}|...|1 = 70$

70 ones

70 ones
5 dividers

(a) size of sample space?

| ← vertical dividers are the "bar"

1 ← the number one is the stars

$\boxed{\binom{75}{5}}$ arranging 75 objects in a particular order, where the ones and the dividers are identical

$\dfrac{75!}{70!\,5!}$

(b) $P(x_1 \geq 30 \text{ and } x_2 \geq 30)$

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 70$

$\uparrow$ 30 $\uparrow$ 30 $x_1' + x_2' + x_3 + x_4 + x_5 + x_6 = \underline{10}$

$\boxed{\dfrac{\binom{15}{5}}{\binom{75}{5}}}$

10 balls, 5 dividers

$\boxed{\binom{15}{5} / \binom{75}{5}}$ the total # of ways to assign values to $x_1 - x_6$ st they sum to 70

(c) $P(x_1 \geq 30 \text{ or } x_2 \geq 30)$

$= \underline{P(x_1 \geq 30)} + \underline{P(x_2 \geq 30)} - \underline{P(x_1, x_2 \geq 30)}$

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 70$

$\uparrow$ 30 $\binom{45}{5}$

$= \boxed{\dfrac{2\binom{45}{5} - \binom{15}{5}}{\binom{75}{5}}}$