CS 70 Discussion 7C

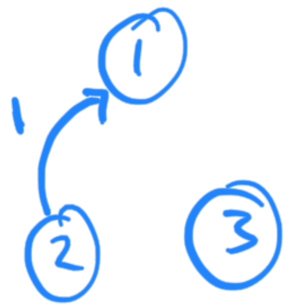
1 (a) X: set of all states inside the MC

P: transition probability matrix
(prob. of transitioning from every state to every other state)

π_0: gives the initial distribution across states
- the sum of entries in π_0 must equal 1
- each entry of π_0 must be ≥ 0

$$P = \begin{bmatrix} .2 & .2 & .6 \\ 1 & 0 & 0 \\ & & \end{bmatrix}$$

each row adds to 1 to signify that we move somewhere, w/ prob. 1, after each step



$$\pi_0 = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

(b) This is true b/c of the Markov property.
Prob. of being in future states is independent of where we were in the past given knowledge of our current state.

row vector

$\pi_1 = \pi_0 P$, $\pi_2 = \pi_1 P$, ...

$\pi_1 = P\pi_0$, $\pi_2 = P\pi_1$

Once we have the initial distribution π_0 and P, we can find distributions at all future timesteps by repeatedly multiplying P

$P = 3\times 3 \quad \pi_0 = 1\times 3$

$\pi_1 = \underset{1\times 3}{\overset{\pi_0}{[\quad]}} \cdot \underset{3\times 3}{\begin{bmatrix} P \end{bmatrix}} \qquad \pi_0 = [\,\_\ \_\ \_\,]$

$\underset{1\times 3}{[\quad]}$

(c) $P(X_1 = j)$ - we can transition from any state in the MC to the state j

$$\boxed{= \sum_{x \in X} \underline{\pi_0(x)}\, \underline{P(x,j)}}$$

$\pi_1 = \pi_0 P$

$\vdots$

$\underline{\pi_n} = \underline{\pi_0 P^n}$

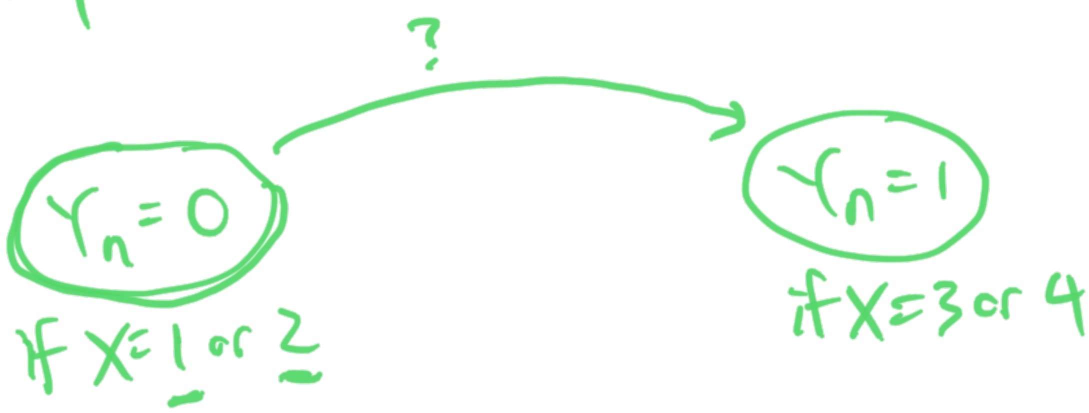
10 states in our MC

$\sum_{i=1}^{10} P(\text{of being @ state } i) \cdot P(\text{moving from } i \to j \mid \text{@ state } i)$

2

(a)

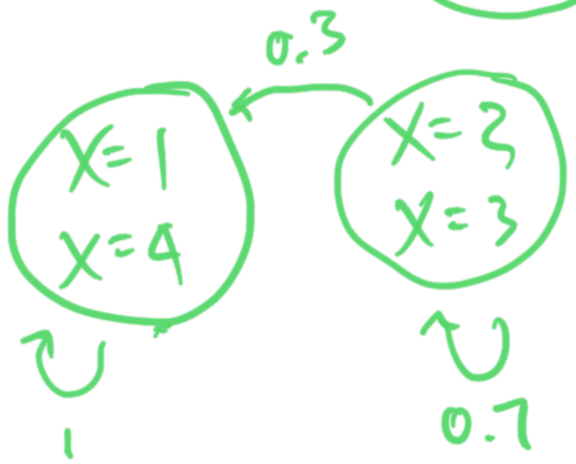
(b) $m=4$



b/c if $Y_n=0$ and $X=1$, $?=0$

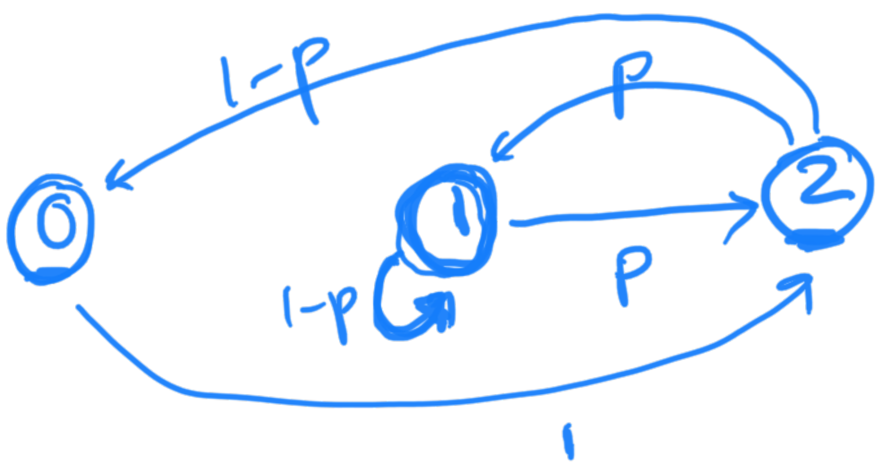
but if $Y_n=0$ and $X=2$, $?=0.3$

$\boxed{\text{This process is not a MC}}$



$X=2$	0.3
$X=3$	0.3

$\boxed{3}$ (a)



irreducible?

aperiodic?

$\mathcal{X}=\{0,1,2\}$

0, 1, 2 represent the number of umbrellas that Alan has at his current location

(b) π_0 $\qquad$ $\pi_1 = \pi_0 P$ $\qquad$ $\pi_2 = \pi_0 P^2$ $\qquad$ $\pi_n = \pi_0 P^n$

① As long as MCs are <u>irreducible</u>, they will have a unique invariant/stationary distribution.

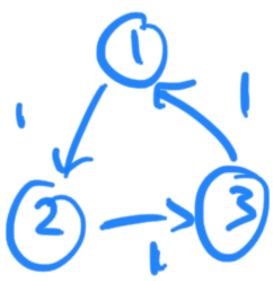
② <u>Moreover</u>, if the MC is additionally aperiodic, as $n \to \infty$ X_n will converge to the Stationary distribution.

period = 1 means aperiodic

period 2

$\to$ 2, 4, 6, ...

gcd(2, 4, 6 ...) = 2



period 3

3, 6, 9, ...

gcd(3, 6, 9 ...) = 3

1, 2, 3, 4, ...

gcd = 1

$$\boxed{\text{If a MC has a self-loop, then it is immediately aperiodic.}}$$

Let $\pi(0)$, $\pi(1)$, $\pi(2)$ denote long-term probabilities of being in each state.

$\pi(0) + \pi(1) + \pi(2) = 1$

$$\begin{cases} \pi(0) = (1-p)\pi(2) \\ \pi(1) = (1-p)\pi(1) + p\pi(2) \end{cases}$$

$\cancel{\pi(2) = \pi(0) + p\pi(1)}$

one of them is redundant

$$\boxed{\pi(0) = \frac{1-p}{3-p} \qquad \pi(1) = \pi(2) = \frac{1}{3-p}}$$

long term fraction Alan walks thru rain w/ no umbrella:

$$\underline{\pi(0) \cdot p} = \boxed{\left(\frac{1-p}{3-p}\right)p}$$