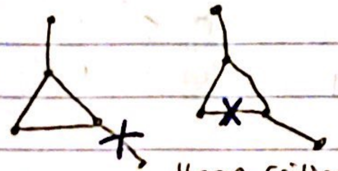
CS 70 Discussion 2C: tinyurl.com/frank-discussion 7/1/2020

$\boxed{1}$ (a) remove degree 3 node from tree; how many connected components left?

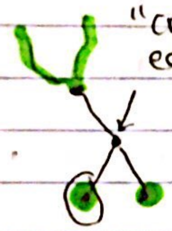
3-4 min

6 min



"critical edge" "non-critical edge"

$\boxed{3}$



(b) Bob adds 10 edges to tree, Alice removes 5 edges, and 3 connected components left over. How many more edges to remove to eliminate all cycles?

- tree: a graph w/ no cycles
- Bob: adds 10 edges, Bob creates extra cycles inside the graph
- Alice: removes 5 edges, 3 connected components left

Key insight:

3 CC leftover = 2 critical edges removed

2 critical edges removes

↓

3 non-critical edges Alice removed

All 10 of Bob's edges were non-critical!

$10 - 3 = \boxed{7}$ edges left to remove in the graph

5:21 $\boxed{2}$ Given planar graph G s.t. $e = 3v - 6$, prove that every face in this graph must be bounded by exactly three edges.

1 min alone

3 min group

10 min

5:35

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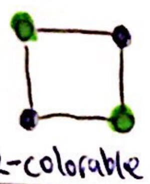


A graph is (k+1)-colorable if you can use a minimum of (k+1) colors to assign a color to each vertex such that no two adjacent vertices have the same color.

common misconception: induction on max degree inside the graph.

3 Prove graph w/ max degree k is (k+1)-colorable.

Hint: induction on num. vertices inside graph

Proof: We prove by induction on the number of vertices inside the graph, n.

1 min

4 min

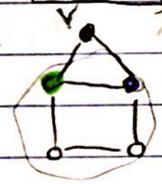
10 min

Base case (n=1): degree 0 maximum $0+1=1$ 1-colorable ✓

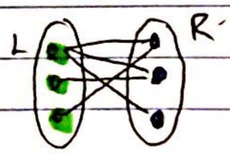
Inductive Hypothesis: $\forall n \in \mathbb{N}$ $(n \geq 1)$ A graph w/ n vertices having max. degree k is (k+1)-colorable.

Inductive step: Consider any graph G having (n+1) vertices and max degree k. I pick a vertex v arbitrarily to remove. The resulting graph has n vertices, and therefore, by our inductive hypothesis, the resulting graph is still (k+1)-colorable. Now, I add vertex v back into the graph and notice that v has a maximum of k neighbors. However, since we have (k+1) colors to choose from, there will thus exist a unique color left over for us to use to color the last vertex v.

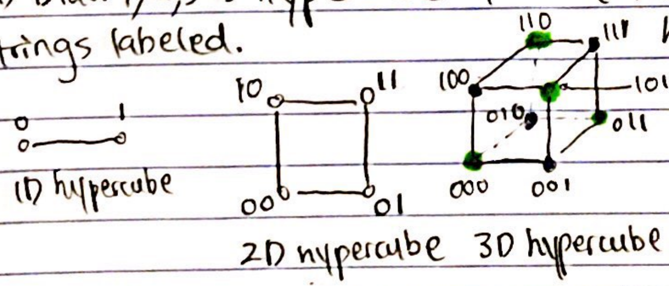
~~Why can't I: start w/ any graph having n vertices and max degree k, and just add a vertex to the graph in order to show the inductive step?~~ Issue: it only proves to be the case for one particular graph having (n+1) vertices.



max degree 2 ← this is 3-colorable



5:50 **4** (a) Draw 1,2,3-D hypercubes w/ bit strings labeled.



By definition, you have an edge between 2 vertices in a hypercube if their bit strings differ by one bit

(b) Show that for any $n \geq 1$, n-dimensional hypercube is bipartite.

Key Idea: in a bipartite graph, every edge goes in between a vertex $\in L$ and a vertex $\in R$. Every other vertex in a hypercube connected by an edge must switch off from being in L or being in R.

~~L = all vertices w/ odd num of zeros inside bit string~~

~~R = all vertices w/ even num of zeros inside bit string~~