Chain Rule:

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

Bayes' Rule:

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Discussion 5A: tinyurl.com/frank-discussion 7/20/2020

1 (a) $P(\underset{W}{\text{windy}} \text{ and } \underset{R}{\text{rain}}) = ?$

$= P(\text{windy}) \cdot P(\text{rain}|\text{windy})$

$= (0.2) \cdot (0.3) = \boxed{6\%}$

(b) $P(\text{rain}) = ?$ Law of Total Probability

$= \underbrace{P(\text{rain AND windy})}_{0.06} + \underbrace{P(\text{rain AND not windy})}$

$= 0.06 + P(\text{not windy}) * P(\text{rain}|\text{not windy})$

$= 0.06 + (0.8) * (0.8)$

$= 0.06 + 0.64 = \boxed{0.7}$

(c) $P(\text{exactly 1 out of 2 days is rainy}) = ?$

R = rain S = sun RS or SR $0.3(0.7)^4$

~~$P(RS) = P(\text{first day is } R) * P(2^{nd} \text{ day is } S) = (0.7) * (0.3)$~~

$= 0.21$

Probability of weather today is independent of weather on all other days

$P(SR) = 0.21$ $P(RS) + P(SR) = 2(0.21) = \boxed{0.42}$

2 (a) $P(\text{Smartie is not poisonous})$

$= P(NP \cap BK) + P(NP \cap SF) + P(NP \cap USC)$

$= P(BK) \cdot P(NP|BK) + P(SF)P(NP|SF) + P(USC) \cdot P(NP|USC)$

$= .45(.99) + .2(.98) + .35(.985)$

$= \boxed{98.625\%}$

(b) $P(\text{poison}|\neg BK)$

$= \frac{P(\text{poison} \cap \neg BK)}{P(\neg BK)}$

$= \frac{P(\text{poison} \cap USC) + P(\text{poison} \cap SF)}{P(USC) + P(SF)}$

$= \frac{P(USC)P(\text{poison}|USC) + P(SF)P(\text{poison}|SF)}{0.35 + 0.2}$

$= \frac{.35(.015) + .2(.02)}{.35 + .2}$

$= \boxed{1.68\%}$

2(c) $P(SF \mid \text{poison}) = ?$

$$= \frac{P(\text{poison} \mid SF) \cdot P(SF)}{P(\text{poison})}$$

$$= \frac{0.02\ (0.2)}{\underbrace{1 - P(\cancel{\text{not poisonous}})}_{0.98625}} = \boxed{29.1\%}$$

3 (a) $P(FFB) = ?$

$$= \boxed{\left(\frac{n-k}{n}\right)\left(\frac{n-k-1}{n-1}\right)\left(\frac{k}{n-2}\right)}$$

(b) $P(3^{rd} \text{ coin biased}) = ?$ _ _ B

$\{FF, FB, BF, BB\}$

$P(3^{rd} \text{ is } B) = P(FFB) + P(FBB) + P(BFB) + P(BBB)$

Symmetry: $P(3^{rd} \text{ coin is } B) = \underbrace{P(1^{st} \text{ coin is } B)}_{\boxed{\frac{k}{n}}}$

(c) $P(\text{pick at least 2 fair coins}) = ?$

$= P(\text{pick exactly } \underset{F}{2} \text{ coins}) + P(\text{pick exactly } \underset{F}{3} \text{ coins})$

$\underbrace{\qquad}_{BFF,\ FBF,\ FFB} \qquad \underbrace{\qquad}_{FFF}$

$$= \boxed{3\left(\frac{n-k}{n}\right)\left(\frac{n-k-1}{n-1}\right)\left(\frac{k}{n-2}\right) + \left(\frac{n-k}{n}\right)\left(\frac{n-k-1}{n-1}\right)\left(\frac{n-k-2}{n-2}\right)}$$

(d) $P(2^{nd} \text{ coin is } B \mid 2^{nd} \text{ coin is } H)$

$$= \frac{P(2^{nd} \text{ is } H \mid 2^{nd} \text{ is } B) \cdot P(2^{nd} \text{ is } B)}{P(2^{nd} \text{ is } H)}$$

$$= \frac{P\left(\frac{k}{n}\right)}{P\left(\frac{k}{n}\right) + \left(\frac{1}{2}\right)\left(\frac{n-k}{n}\right)}$$

2nd coin is F

$$= \boxed{\frac{pk}{pk + \frac{n-k}{2}}}$$

2nd coin is B