Bernoulli, Binomial, Poisson, Geometric, Exponential,

Normal (Gaussian), Uniform

① What the dist describes
② PMF / PDF / CDF
③ $\mathbb{E}[X]$ ④ $Var(X)$
⑤ Special Properties

① Bernoulli(p) (indicator RV)

$$X = \begin{cases} 0 & \text{with prob. } 1-p \\ 1 & \text{with prob. } p \end{cases}$$

0 = Failure
1 = success

Flipping a coin. success = heads

PMF: $X = \begin{cases} 0 & \text{with prob } 1/2 \\ 1 & \text{with prob } 1/2 \end{cases}$

$\mathbb{E}[X] = 0(1-p) + 1(p) = p$

$Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]$
$= p(1) - p^2 = p - p^2$

② Binomial(n, p)

n = num trials p = prob. of success per trial

what's the probability of k successes in n trials

PMF:
$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$\mathbb{E}[X] = np$
$Var(X) = np(1-p)$

You can think of a Binomial RV as the sum of n Bernoulli RV.

$$P(X=7) = \binom{10}{7}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^3$$

The outcome of each trial is

$$P(X \geq 7) = \sum_{i=7}^{10} \binom{10}{i}\left(\frac{1}{2}\right)^{10}$$

an iid Bernoulli RV

$X > 7 \quad i = 8$

iid = independent and identically distributed

③ Geometric(p): p is the probability of success on each trial

counting how many trials occur until 1st success

PMF:

$$P(X=k) = (1-p)^{k-1} p$$

$$\mathbb{E}[X] = \frac{1}{p}$$

$$Var(X) = \frac{1-p}{p^2}$$

$\boxed{4}$ Exponential(λ) λ: "rate" of exponential decay $\mathbb{E}[X] = 50$

continuous-time analog of the geometric RV $\frac{1}{\lambda} = 50$

$\lambda = \frac{1}{50}$

PDF: $f_X(k) = \lambda e^{-\lambda k}$ CDF: $F_X(k) = 1 - e^{-\lambda k}$

$P(X \leq k)$

cCDF: $e^{-\lambda k} \leftarrow P(X \geq k)$

Memoryless:

$$\boxed{P(X > a+b \mid X > a) = P(X > b)}$$

$a, b > 0$

$$\mathbb{E}[X] = \frac{1}{\lambda} \qquad Var(X) = \frac{1}{\lambda^2}$$

$\boxed{5}$ Poisson(λ) ← rate

models an event that occurs w/ rlly low probability, but over a long period of time $\lambda \approx n \cdot p$

PMF:

$$P(X=k) = \frac{e^{-k}\lambda^k}{k!}$$

an approximation of the binomial

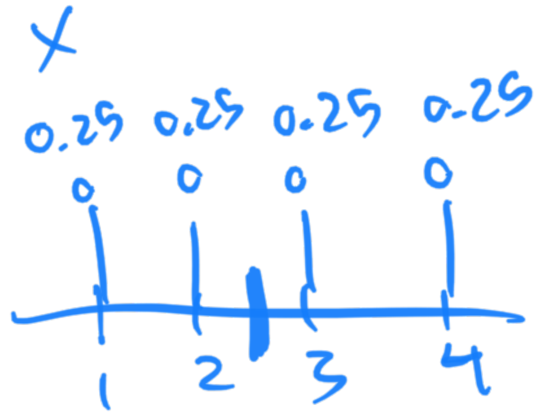
$\mathbb{E}[X] = \lambda$

$Var(X) = \lambda$

RV when we take the limit as $n \to \infty$, $p \to 0$

6 Uniform (Discrete)

Uniform [



$$\mathbb{E}[X] = \frac{a+b}{2} = 2.5$$

(a = 1, b = 4)

7 Uniform (Continuous)

$[a, b]$ $\quad a \le X \le b \quad$ PDF: $\frac{1}{b-a}$

$\mathbb{E}[X] = \frac{a+b}{2}$ $\quad Var(X) = \frac{(b-a)^2}{12}$

$= \mathbb{E}[X^2] - (\mathbb{E}[X])^2$

8 Normal (Gaussian)

$N(\mu, \sigma^2)$ $\quad \mu$ = mean $\quad \sigma^2$ = variance

PDF: $f_X(k) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$\mathbb{E}[X] = \mu$

$Var(X) = \sigma^2$

$X \sim N(\mu, \sigma^2)$ $\quad aX + b \sim N(a\mu + b, a^2\sigma^2)$