Toolkit: Direct proof, contradiction, contraposition

What we're going to cover today: proof techniques

2 min

CS 70 Discussion 1B: tinyurl.com/frank-discussion 6/23/2020

1 (a) $\forall n \in \mathbb{N}$ if n is odd then n^2+4n is odd.

$n = 2k+1$ $(k \in \mathbb{Z})$ (odd)

$n = 5 = 2(2)+1$

3 min think
3 min break
6 min disc

$n^2+4n = (2k+1)^2 + 4(2k+1)$

$= 4k^2+4k+1+8k+4$ $(q \in \mathbb{Z})$

$= 2(2k^2+2k+4k+2)+1 = \boxed{2q+1}$

(b) $(\forall a, b \in \mathbb{R})$ if $a+b \leq 15$ then $a \leq 11$ or $b \leq 4$

If $a > 11$ and $b > 4$, then $a+b > 15$

$\begin{cases} a > 11 \\ b > 4 \end{cases}$

$a+b > 15$ ■

(c) $\forall r \in \mathbb{R}$ if r^2 is irrational, then r is irrational.

If r is rational, then r^2 is rational.

$r = \frac{p}{q}$ $(p, q \in \mathbb{Z})$

$r^2 = \frac{p^2}{q^2}$ ← integers

r^2 is also rational

(d) $(\forall n \in \mathbb{Z}^+)$ $5n^3 > n!$

$n = 7$ $5n^3 = 5(343) = 1715$

$7! = 5040$

$n! > 5n^3$

Counterexample

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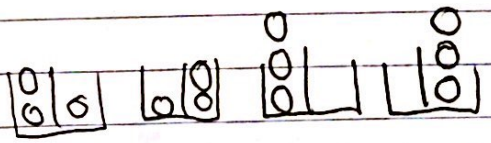
2 Pidgeonhole Principle: If I have $n+1$ balls (objects), and I want to put them inside n bins, then at least one bin will have ≥ 2 balls.

(give numeric example)

$n = 2$ 3 balls, 2 bins



Proof by contradiction:

"$\exists$ some bin w/ ≥ 2 balls"

↓ negate

"$\forall$ bins have < 2 balls"

(all bins have 0 or 1 balls)

2 min think
3 min break
7 min disc

If all bins have ≤ 1 ball, then the max number of balls I can have is n. But we started w/ $n+1$ balls in the statement we're trying to prove. Hence, contradiction.

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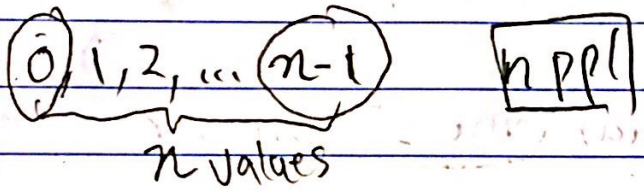
[5] Prove that if there are $n \geq 2$ people at a party, then at least 2 of them have the same number of friends.

4 min think
4 min break
discuss

Contradiction:

We assume there are $n \geq 2$ people at a party, but every one of them has a unique number of friends.



We have a person w/ 0 friends, and we also have a person w/ n-1 friends (friends w/ everybody else). This is a contradiction b/c you can't have someone who's friends w/ everybody and someone who is friends with nobody. ■