CS 70 Discussion 6A

1 (a) iid - independent and identically distributed

$P(\text{hit } 8^{th} \text{ throw}) = P(\text{missing on first } 7) \cdot P(\text{hit on } 8^{th} \text{ throw})$

Geometric RV $(p = 0.17)$

$$= \boxed{(1-0.17)^7 (0.17)} = (0.83)^7 (.17)$$

(b) $X \sim Geom(p = 0.2)$

$$\mathbb{E}[X] = \frac{1}{p} = \frac{1}{0.2} = \boxed{5} \quad Var(X) = \frac{1-p}{p^2} = \frac{0.8}{\frac{1}{25}} = \boxed{20}$$

(c) $\lambda = 3$ cars/week $X = Poisson(\lambda = 3)$

$$P(X \geq 1) = 1 - P(X=0) = 1 - \frac{e^{-\lambda}\lambda^k}{k!} = \boxed{1 - e^{-3}}$$

$(\lambda = 3, k = 0)$ $\quad 1 - \frac{e^{-3} \cancel{\lambda^0}^{1}}{\cancel{0!}_{1}}$

(d) $\lambda = 3.2$ particles/sec

$X \sim Poiss(\lambda = 3.2)$ $\quad \frac{e^{-\lambda}\lambda^k}{k!}$

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{e^{-3.2} 3.2^0}{0!} + \frac{e^{-3.2} 3.2^1}{1!} + \frac{e^{-3.2} (3.2)^2}{2!} = \boxed{0.382}$$

2 Hint: Let X_i be the number of visits we

need to make before collecting the i^{th} unique coupon

$$\underbrace{\text{num visits}} = \underline{X} = \sum_{i=1}^{n} X_i$$

a. Are each of the X_i independent? yes

b. What is the distribution of each X_i?

$$X_i \sim \text{Geom}\left(p = \frac{n-i+1}{n}\right)$$

prob. that I get a new coupon on my 1st draw $\quad 1$

prob. I get a new coupon on 2nd draw $\quad \frac{n-1}{n}$

prob. that a new draw gives 3rd unique coupon $\quad \frac{n-2}{n}$

$X_1 = \text{Geom}(p = 1)$

$X_2 = \text{Geom}\left(p = \frac{n-1}{n}\right)$

$X_3 = \text{Geom}\left(p = \frac{n-2}{n}\right)$

$\vdots$

$$\mathbb{E}[X] = \mathbb{E}\left[\sum_{i=1}^{n} X_n\right] = \sum_{i=1}^{n} \mathbb{E}[X_i] = \underline{\sum_{i=1}^{n} \frac{n}{n-i+1}} = \underline{\sum_{j=1}^{n} \frac{n}{j}}$$

Let $j = \underline{n-i+1}$

i goes from 1 to n

$$\begin{cases} \text{when } i=1, & j = n-1+1 = \underline{n} \\ \text{when } \underline{i=n}, & j = n-n+1 = \underline{1} \end{cases}$$

$\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y)$

only when X, Y are independent

$$\underline{\text{Var}(X)} = \underbrace{\text{Var}\left(\sum_{i=1}^{n} X_i\right)} = \sum_{i=1}^{n} \text{Var}(X_i) = \sum_{i=1}^{n} \frac{1-\left(\frac{n-i+1}{n}\right)}{\left(\frac{n-i+1}{n}\right)^2}$$

$\frac{1-p}{p^2}$

$$= \sum_{j=1}^{n} \frac{1-\frac{j}{n}}{\left(\frac{j}{n}\right)^2} \cdot \frac{n^2}{n^2} = \sum_{j=1}^{n} \frac{n^2 - nj}{j^2}$$

Let $j = n - i + 1$ / n

i goes from 1 to n,

j goes from $\frac{1}{n}$ to $\frac{n}{n}$

$j = n - i + 1$

$$= \sum_{j=1}^{n} \frac{n^2}{j^2} - \frac{n}{j} = n^2 \sum_{j=1}^{n} \frac{1}{j^2} - \sum_{j=1}^{n} \frac{n}{j}$$

$$= \boxed{n^2 \sum_{j=1}^{n} \frac{1}{j^2} - \mathbb{E}[X]}$$

3 Poisson splitting:

$X \sim \text{Poisson}(\lambda)$ number of email I get per hour

$\xrightarrow{p}$ email is spam

$\xrightarrow{1-p}$ email is not spam

Y = number of spam emails I get in 1 hour $\qquad Y \sim \text{Poisson}(\lambda p)$

Z = num of non-spam emails in 1 hour $\qquad Z \sim \text{Poisson}(\lambda(1-p))$