X = Binomial RV
$10 = n, p = \frac{1}{2}$

$$\boxed{P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}}$$

Discussion 5C: tinyurl.com/frank-discussion 7/22/2020

1 (a) name of distribution of X and its parameters:

$$\boxed{X = \text{Binomial}(n=20, p=0.4)}$$

(b) $P(X=7) = ?$

$$\boxed{P(X=7) = \binom{20}{7}(0.4)^7(0.6)^{13}}$$

(c) $P(X \geq 1) = ?$ RV so it can take on values btwn 0-20 inclusive

$1 - P(X=0)$

$= 1 - \cancel{\binom{20}{0}}\cancel{(0.4)^0}(0.6)^{20}$

$= \boxed{1 - \left(\frac{3}{5}\right)^{20}}$

(d) $P(12 \leq X \leq 14) = ?$

$$\boxed{\sum_{i=12}^{14} \binom{20}{i}(0.4)^i(0.6)^{20-i}}$$

3 (a) $P(X=0)=?$ $P(X=1)=?$ $P(X=2)=?$ $P(X=3)=?$

$P(X=0) = \boxed{\dfrac{\binom{48}{3}}{\binom{52}{3}}}$

sampling w/o replacement

$P(X=1) = \boxed{\dfrac{\binom{48}{2}\binom{4}{1}}{\binom{52}{3}}}$

$P(X=2) = \boxed{\dfrac{\binom{48}{1}\binom{4}{2}}{\binom{52}{3}}}$

$P(X=3) = \boxed{\dfrac{\binom{4}{3}}{\binom{52}{3}}}$

(b) Answers from part (a) add up to what?

$\boxed{1}$ You can either get 0, 1, 2, or 3 queens.

(c) Compute $\mathbb{E}[X]$ from definition.

$\mathbb{E}[X] = \sum_x x \cdot P(X=x)$

$= \cancel{0 \cdot P(X=0)} + 1 \cdot P(X=1) + \ldots + 3 \cdot P(X=3)$

$$\boxed{\frac{\binom{48}{2}\binom{4}{1} + 2 \cdot \binom{48}{1}\binom{4}{2} + 3\binom{4}{3}}{\binom{52}{3}}} = \frac{3}{13}$$

(d) Compute $\mathbb{E}[X]$ using LOE.

$$\boxed{\begin{gathered}\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y] \\ \text{works regardless of } X, Y \text{ being} \\ \text{dependent of one another}\end{gathered}}$$

(e) Are indicators X_i independent?

$P(X_1 = 1 \cap X_2 = 1) \neq P(X_1 = 1) \cdot P(X_2 = 1)$

$\left(\frac{4}{52}\right)\left(\frac{3}{51}\right) \neq \left(\frac{4}{52}\right)\left(\frac{4}{52}\right)$

$\boxed{\text{dependent}}$

Bernoulli X_1, X_2, X_3 where each X_i is a random variable that = 1 if we draw a queen and 0 otherwise

$X_1 = \begin{cases} 1 & \text{if 1st draw is a Q} \\ 0 & \text{if 1st draw is } \neg Q \end{cases}$

all 3 are equal

$\mathbb{E}[X] = \mathbb{E}[X_1 + X_2 + X_3] = \mathbb{E}[X_1] + \mathbb{E}[X_2] + \mathbb{E}[X_3]$

$= \left[\left(\frac{1}{13}\right)(1) + \left(\frac{12}{13}\right)(0)\right] \cdot 3 = \boxed{\frac{3}{13}}$

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4 (a) $A_1 A_2 \dots A_{10}$

$B_1 B_2 \dots B_{20}$

$$\mathbb{E}[A_1] = \underbrace{\left(\frac{1}{3}\right)(3)}_{\text{win}} + \underbrace{\left(\frac{2}{3}\right)(0)}_{\text{lose}} = 1$$

$$\mathbb{E}[B_1] = \left(\frac{1}{5}\right)(4) + \left(\frac{4}{5}\right)(0) = \frac{4}{5}$$

$$\mathbb{E}\left[\sum_{i=1}^{10} A_i\right] = \sum_{i=1}^{10} \mathbb{E}[A_i] = \sum_{i=1}^{10} 1 = 10$$

$$\mathbb{E}\left[\sum_{j=1}^{20} B_j\right] = \sum_{j=1}^{20} \mathbb{E}[B_j] = \sum_{j=1}^{20} \frac{4}{5} = 16$$

$10 + 16 = \boxed{26}$

(b) 10^6 letters

$\mathbb{E}$[occurrences of "book"]

million letters ⇒ 999,997 places where "book" could show up

$A_1 \; A_2 \dots A_{999,997}$

= 1 if = "book"

= 0 otherwise

$$\mathbb{E}\left[\sum_{i=1}^{999,997} A_i\right] = \sum_{i=1}^{999,997} \mathbb{E}[A_i]$$

$$\mathbb{E}[A_i] = \left(\frac{1}{26^4}\right)(1) \cancel{+ (0)} = \frac{1}{26^4}$$

$$= \boxed{\frac{999,997}{26^4}}$$

≈ 2.19

joint probability:

$P(A \cap B)$ 2+ events occurring simultaneously

marginal probability:

$P(A)$ $P(B)$ just one event occurring

$$P(A) = \sum_{b} P(A \cap B = b)$$

b any possible value that the RV B can take on