

- 11 (a)  $A \cap B$ ,  $A$  countable,  $B$  uncountable  
countable

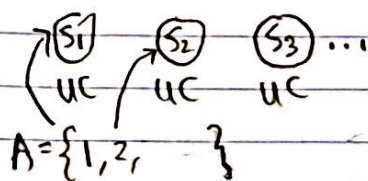
$A \cap B$  returns a subset of  $A$   
Since  $A$  is countable,  $A \cap B$  is countable.

uncountable

$A \cup B$  contains at the minimum  
all elements of  $B$

$B$  is uncountable, thus  $A \cup B$   
is uncountable.

- (c)  $\bigcap_{i \in A} S_i$  where  $A$  is countable set of  
indices but each  $S_i$  is uncountable



Sometimes countable:

$$S_i = \{\forall r \in \mathbb{R} \mid i < r < i+1\}$$

For example,

$$S_2 = \{\forall r \in \mathbb{R} \mid 2 < r < 3\}$$

all elements of  
 $S_i$  are  
different from  
each other

Sometimes uncountable:

$$A = \mathbb{Z} \quad S_i = \{\forall r \in \mathbb{R} \mid 1 < r < 2\}$$

all sets  $S_i$  are the same as  
each other

$$\bigcap_{i \in A} S_i = S_i$$

therefore uncountable

$$A = \mathbb{Z}$$

$$\bigcap_{i \in A} S_i = \emptyset$$

- 12 (a) Given countable  $A$  and  $B$ , prove that  $A \times B$  is countable.

$A$  is countable.  $\exists$  bijection btwn  
 $A$  and a countable subset of  $\mathbb{N}$ .

$B$  is countable, so similar bijection  
exists.

We know that  $\mathbb{N} \times \mathbb{N}$  is countable,  
proof is from the notes.

We build a bijection between a  
subset of  $\mathbb{N} \times \mathbb{N}$  and  $A \times B$  as follows:

$$\forall i, j \in \mathbb{N} \quad f(i, j) = (A_i, B_j)$$

Since we have built a bijection  
btwn  $A \times B$  and  $\mathbb{N} \times \mathbb{N}$ , and  
 $\mathbb{N} \times \mathbb{N}$  is countable,  $A \times B$  is  
countable as well.

Induction:

Base case:  $n=2$   $A_1 \times A_2$  is countable  $\checkmark$

Inductive Hypothesis:  $\exists k \in \mathbb{N} (k \geq 2)$  such that  
 $A_1 \times \dots \times A_k$  is countable

Inductive Step:  $A_1 \times \dots \times A_k \times A_{k+1}$  ( $n=k+1$ )  
also countable

$$C = A_1 \times \dots \times A_k \quad C \text{ is countable}$$

$$C \times A_{k+1} = \text{countable}$$

countable

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(a) A common strategy to show that a program  $P$  is uncomputable is to reduce TestHalt to  $P$ .

3 min

7 min

Problem A reduces to a Problem B if B can be used as a subroutine to solve A.

$$\int_c^1 x^2 dx$$

A = integration    B = addition,  $\ominus$ ,  $\times$

Hint = TestHalt reduces to PrintHW