CS 70 Discussion 1A: tinyurl.com/frank-discussion 6/22/2020

$\forall, \exists, \Rightarrow$, how to prove $\Rightarrow$

$\forall$ = for all $\exists$ = there exists $\boxed{A \Rightarrow B}$

$\boxed{1}$ (a) $\forall x \forall y\, P(x,y) \Rightarrow \forall y \forall x\, P(x,y)$

x = boys y = girls

"All boys and all girls have heads" means same as "all girls and all boys have heads".

(b) $\exists x \exists y\, P(x,y) \Rightarrow \exists y \exists x\, P(x,y)$

"There exists an integer x and there exists an integer y such that $x+y=3$" means the same thing as "there exists an integer y and an integer x such that $x+y=3$".

(c) $\boxed{\forall x \exists y\, P(x,y)} \Rightarrow \exists y \forall x\, P(x,y)$

Let x, y be integers

Let $P(x,y)$ be "$x<y$"

$\boxed{\text{False}}$

(since there does not exist a largest integer)

(d) $\exists x \forall y\, P(x,y) \Rightarrow \forall y \exists x\, P(x,y)$

special $\boxed{x'}$ that can partner with any value of y to satisfy $P(x,y)$ $\boxed{x'}$

$\boxed{\text{True}}$

$\boxed{2}$ 1) A is F, B is T $\rightarrow \boxed{T}$

A is T, B is F $\rightarrow \boxed{T}$

hard-code the cases that work

$\boxed{(\neg A \land B) \lor (A \land \neg B)}$

2) $(A \oplus B) \rightarrow (A \lor B)$

A is F, B is T $\quad F \lor T \equiv T$

A is T, B is F $\quad T \lor F \equiv T$

$\boxed{\text{Valid}}$

3) $(A \lor B) \Rightarrow (A \oplus B)$

A is T
B is T $\quad \boxed{\text{False}}$

$A \lor B$ is T $\quad A \oplus B$ is F

3 (a) $\boxed{P \wedge (Q \vee P) \equiv P \wedge Q}$ (b) $(P \vee Q) \wedge R \equiv (P \wedge R) \vee (Q \wedge R)$

P	Q	$P \wedge (Q \vee P)$	$P \wedge Q$
T	T	T	T
T	F	T	F
F	T	F	F
F	F	F	F

Not equivalent due to discrepancy highlighted above

P	Q	R	$(P \vee Q) \wedge R$	$(P \wedge R) \vee (Q \wedge R)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

equivalent expressions

(c) $(P \wedge Q) \vee R \equiv (P \vee R) \wedge (Q \vee R)$

P	Q	R	$(P \wedge Q) \vee R$	$(P \vee R) \wedge (Q \vee R)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	F	F

equivalent expressions

4 (a) $(\forall x \in \mathbb{N})(4 \mid x \Rightarrow 2 \mid x)$

$x = 4k = 2(2k)$ True

(k is some integer) another integer

(b) $P \Rightarrow Q$ $\quad \neg P \Rightarrow \neg Q$

inverse: $4 \nmid x \Rightarrow 2 \nmid x$ False

counterexample: 2

(c) $P \Rightarrow Q$ $\quad Q \Rightarrow P$

converse: $2 \mid x \Rightarrow 4 \mid x$

counterexample: 6 False

(d) $P \Rightarrow Q$ $\quad \neg Q \Rightarrow \neg P$

contrapositive: $2 \nmid x \Rightarrow 4 \nmid x$ True

If x isn't divisible by 2, then x isn't divisible by 4, since dividing by 4 is same as dividing by 2 twice.