CS 70 Discussion 7A

Markov Inequality:

$$P(X \geq a) \leq \frac{\mathbb{E}[X]}{a}$$

a: some constant

X must be a nonnegative RV

Chebyshev's Inequality:

$$P(|X-\mu| \geq a) \leq \frac{\text{Var}(X)}{a^2}$$

LLN:

As you take increasingly more samples from a population, the sample mean converges to the population mean.

$\boxed{1}$ (a) $\mathbb{E}[X^2] = 13$

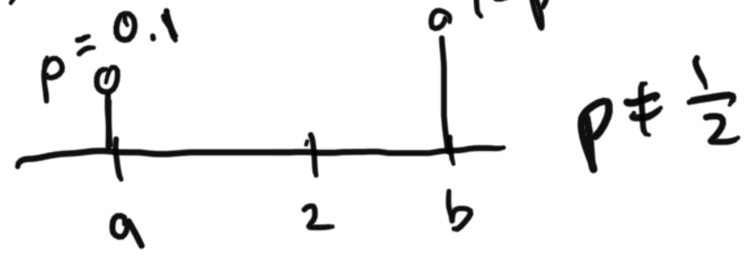
$\text{Var}(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$

$9 = \mathbb{E}[X^2] - 4$

$\boxed{\mathbb{E}[X^2] = 13}$

$\boxed{\text{True}}$

X



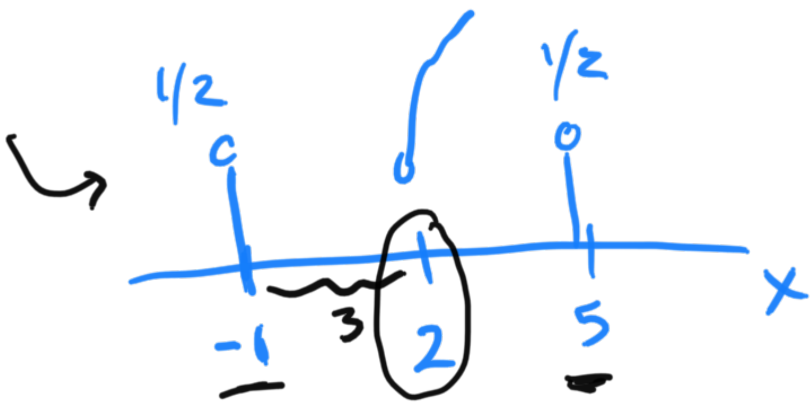
$p \neq \frac{1}{2}$

$$\begin{cases} \mathbb{E}[X] = 0.1a + 0.9b = 2 \\ \text{Var}(X) = \underline{0.1a^2 + 0.9b^2} = 13 \end{cases}$$

(b) $P[X=2] > 0$

X: PMF

$P(X=2) = 0$



$\mathbb{E}[X] = \frac{1}{2}(-1) + \frac{1}{2}(5) = \underline{2}$

$\underline{\text{Var}(X)} = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$

$= \frac{1}{2}(-1)^2 + \frac{1}{2}(5^2) - 2^2$

$= 13 - 4 = \underline{9}$ $\boxed{\text{False}}$

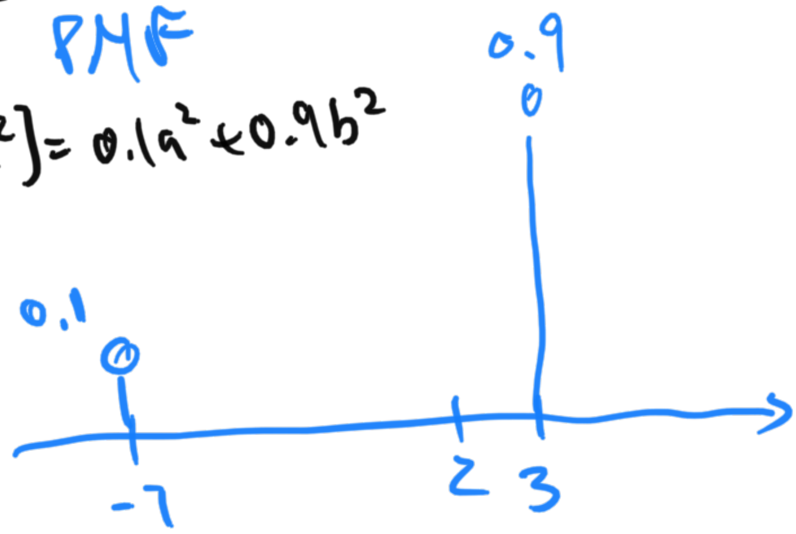
(c) $\overbrace{\underbrace{P[X \geq 2]}_{0.9}} = \overbrace{\underbrace{P[X \leq 2]}_{0.1}}$

X is a discrete RV:

(d) $P[X \leq 1] \leq \frac{8}{9}$

PMF

$E[X^2] = 0.1a^2 + 0.9b^2$



$\underline{E[X]} = 0.1(-7) + .9(3) = \underline{2}$

$\underline{Var(X)} = E[X^2] - (E[X])^2$

$= .1(-7)^2 + .9(3^2) - 4$

$= \underbrace{4.9 + 8.1}_{13} - 4 = \underline{9}$

(e) $P[\underline{X \geq 6}] \leq \frac{9}{16}$

$P[|X - \mu| \geq a] \leq \frac{Var(X)}{a^2}$

$P[\underbrace{|X - 2| \geq 4}_{X \leq -2}] \leq \frac{Var(X)}{4^2}$

$\boxed{\leq \frac{9}{16}}$

$\mu = E[X] = 2$

Let $Y = 10 - X$ ← $X \leq 10$ implies that $Y \geq 0$

$P[X \leq 1]$

$P[-X \geq -1]$

$P[\underbrace{10 - X}_{Y} \geq \underbrace{10 - 1}_{9}]$

$P[Y \geq 9]$

$E[Y] = E[10 - X]$

$= \underbrace{\cancel{E[10]}}_{10} - \underbrace{\cancel{E[X]}}_{2} = 8$

$P[Y \geq a] \leq \frac{E[Y]}{a}$

$\boxed{P[Y \geq 9] \leq \frac{8}{9}}$ $a = 9$

$\boxed{3}$ (a) more than 60% heads

$\boxed{\text{10 tosses}}$ b/c w/ 100 tosses you're more likely to get around 50% heads

(b) more than 40% heads

$\boxed{\text{100 tosses}}$

again, w/ 100 tosses you're more likely to get around 50% heads

(c) btwn 40% to 60% heads

$\boxed{\text{100 tosses}}$

w/ 100 tosses you're more likely to get around 50% heads

(d) exactly 50% heads

$\boxed{\text{10 tosses}}$

according to binomial distribution, we're more likely to get exactly 5/10 heads, than we are to get 50/100 heads

$\binom{10}{5}\left(\frac{1}{2}\right)^5 \approx 0.246$ $\qquad$ $\binom{100}{50}\left(\frac{1}{2}\right)^{100} \approx 0.0796$

49% to 51% heads

10 toss: 40%, 50%, 60%

100 toss: 49%, 50%, 51%

$\boxed{2}$ Y_1 = be the score that the GSI picks for HW #1

$\mathbb{E}[Y_1] = 5 \quad Var(Y_1) = 1$

Y_2: be the score that GSI picks for HW #2

$\mathbb{E}[Y_2] = 5 \quad Var(Y_2) = 1$

X_1, X_2: scores on 1st and 2nd HWs:

$X_1 = 3Y_1 \qquad X_2 = 4Y_2$

$\mathbb{E}[X_1 + X_2] = \mathbb{E}[3Y_1 + 4Y_2] = 3\mathbb{E}[Y_1] + 4\mathbb{E}[Y_2]$

$= 3(5) + 4(5) = 35$

$Var(X_1 + X_2) = Var(X_1) + Var(X_2) = Var(3Y_1) + Var(4Y_2)$

$Var(aX) = a^2 Var(X)$

$= 9\,\underbrace{Var(Y_1)}_{1} + 16\,\underbrace{Var(Y_2)}_{1}$

$= 25$

$P(|X - \mu| \geq a) \leq \dfrac{Var(X)}{a^2}$

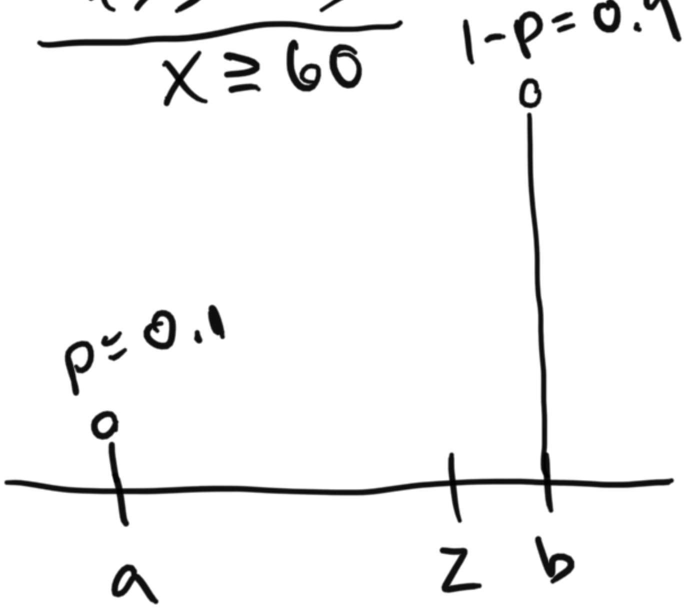
$\dfrac{P(X \geq 60)}{\mu = 35}$

$\underbrace{P(|X - 35| \geq 25)}_{P(X \geq 60)} \leq \dfrac{25}{25^2}$

$\leq \dfrac{1}{25} \approx 4\%$

$\boxed{\text{prob. that you get an A is } \leq 4\%}$

$X - 35 \geq 25$
$+35 \quad +35$
$X \geq 60$



$Var(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$

$\mathbb{E}[X^2] = (0.1)a^2 + (0.9)b^2$

$= 0.1a^2 + 0.9b^2$

$\mathbb{E}[Y] = 2$

$$(\mathbb{E}[X])^2 = 4$$

$$\mathrm{Var}(X) = 0.1a^2 + 0.9b^2 - 4$$