CS 70 Discussion 1D: tinyurl.com/frank-discussion 6/25/2020

$\boxed{1}$ $\mathbb{R}$ - real numbers
$\mathbb{Q}$ - rational numbers $\frac{p}{q}$ $(p, q \in \mathbb{Z})$
$\mathbb{Z}$ = integers $\{-\infty, -2, -1, 0, 1, 2, .. +\infty\}$
$\mathbb{N} = \{0, 1, 2, \dots +\infty\}$

(d) $\mathbb{R} \cap \mathbb{Z} = \mathbb{Z}$
(e) $\mathbb{N} \cup \mathbb{Q} = \mathbb{Q}$
(f) $\mathbb{R} \setminus \mathbb{Q}$
= irrational number

4 min alone
6 min discuss

(a) $A = \{1, 2, 3, 4\}$

$P(A) = \{\{\}, \{1\}, \{2\}, \{3\}, \{4\},$
$\{1,2\}, \{1,3\}, \{1,4\}$
$\{2,3\}, \{2,4\}, \{3,4\}$
$\{1,2,3\}, \{1,2,4\}, \{1,3,4\}$
$\{2,3,4\}, \{1,2,3,4\}\}$

$A \setminus B$
= elems in set A absent from set B

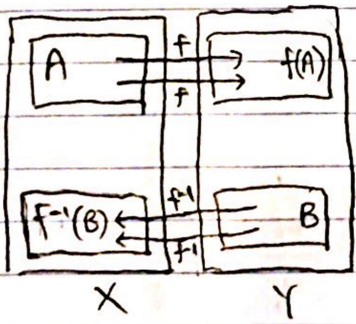
(g) $S \subseteq T$, what $S \setminus T$?
$= \boxed{\emptyset}$

(b) $P(B) = \{T \mid T \subseteq B\}$

(c) $\mathbb{R} \cap P(A) = \boxed{\emptyset}$
$\boxed{A}$

$\mathbb{R} = \{1, 1.1, 1.2, 1.3\}$
$\mathbb{Z} = \{\dots -2, -1, 0, 1, 2, \dots\}$

5:23 $\boxed{2}$



$f(A) = \{f(x) \mid x \in A\}$
$f^{-1}(B) = \{x \mid f(x) \in B\}$

5 min break
15 min disc

(a) $B \subseteq f(X)$ Prove that $f(f^{-1}(B)) = B$

$f(f^{-1}(B)) \subseteq B$

$f^{-1}(B) \subseteq X$
Consider generic $x \in f^{-1}(B)$
By definition, $\boxed{f(x) \in B}$
over all x inside $f^{-1}(B)$

5:43

$\boxed{f(f^{-1}(B)) \subseteq B}$

$B \subseteq f(f^{-1}(B))$

generic $y \in B$
Since $B \subseteq f(X)$, given
$\exists\, x \in f^{-1}(B)$ such that $f(x) = y$
If we consider this over all generic x inside $f^{-1}(B)$, then
$y \in f(f^{-1}(B))$ hence
$\boxed{B \subseteq f(f^{-1}(B))}$

Scanned with CamScanner

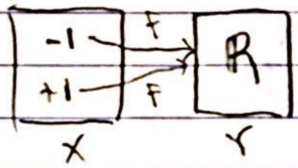
4 min break

b) Let $A \subseteq X$. Show $A \subseteq f^{-1}(f(A))$

Consider generic $x \in A$. Then $f(x) \in f(A)$.

$x \in f^{-1}(f(A))$ $\boxed{A \subseteq f^{-1}(f(A))}$

c) example where $A \neq f^{-1}(f(A))$



$f(x) = x^2$

$\boxed{A = \{1\}}$

$f(A) = \{1\}$

$\boxed{f^{-1}(f(A)) = \{-1, 1\}}$

injective function: where if $f(x) = f(y)$, then $x = y$

Distinct elements map to distinct outputs.

d) suppose f is injective. Is true that $A = f^{-1}(f(A))$

$A \subseteq f^{-1}(f(A))$ proved ✓ from part b

$f^{-1}(f(A)) \subseteq A$.

Consider generic $x \in f^{-1}(f(A))$ (x is a preimage)

$\exists y \in f(A)$ such that $f(x) = y$ (y is that image)

Since $y \in f(A)$, $\exists x' \in A$ st $f(x') = y$

However f is injective! $x = x'$

$x \in A$

Thus, $\boxed{f^{-1}(f(A)) \subseteq A}$

3 surjective: every element in the range (onto) has a preimage

bijective: both injective and surjective

(a) injective, surjective, bijective

(b) No for all 3

(c) not injective, yes surjective, not bijective