2 min

Discussion 4A: tinyurl.com/frank-discussion 7/14/2020

1 (a) 4 categories of clothes; how many distinct outfits if pick one per category?

4 min

$\underline{10} \times \underline{10} \times \underline{10} \times \underline{10} = \boxed{10^4}$

(b) How many outfits if wear exactly two categories?

4 min

① Pick 2 categories } order doesn't matter

② Pick items from categories

$\boxed{\binom{4}{2} \cdot \underline{10} \cdot \underline{10}}$

(c) How many ways to hang 4/10 hats in a row on the wall?

$\boxed{10(9)(8)(7)} = \frac{10!}{6!}$

① Pick 4 hats to hang

② Choose an ordering for those 4 hats ← permutation

$\binom{10}{4} 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{\binom{10}{4} 4!}$

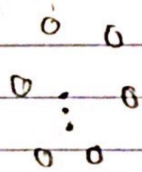
(d) How many ways to pick 4 hats?

$\boxed{\binom{10}{4}}$ choose notation b/c order of hats selected does not matter

$\binom{a}{b} = \frac{a!}{b!\,(a-b)!}$

$\frac{(c)}{4!} = (d)$

2 (a) How many undirected graphs have n vertices?



how many edges?

Where are the edges?

6 min

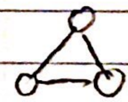
10 min

n vertices → $\binom{n}{2}$ edges

choosing two points inside the graph to be start/end vertices for an edge

$\boxed{2^{\binom{n}{2}}}$

(b) How many distinct cycles exist in complete graph K_n.



minimum length of a cycle in G: 3

max length of a cycle in G: n

Let $\underline{k}$ = number of vertices in a cycle

$\sum_{k=3}^{n} \frac{n!}{(n-k)!}$ ← $\underbrace{n(n-1)(n-2)\dots(n-k+1)}_{\text{number of cycles having } k \text{ vertices}} = \boxed{\frac{n!}{(n-k)!}}$

① Reflection

② Rotation

$\boxed{\sum_{k=3}^{n} \frac{n!}{(n-k)!\,2k}}$

2 (c) How many ways to color a bracelet with n beads if you have n colors?

$n! = n(n-1)(n-2)\ldots(1)$

① Rotation ② Reflection

$$\frac{n!}{2 \cdot n} = \boxed{\frac{(n-1)!}{2}}$$

(d) How many ways to color the faces of a cube with 6 colors, where each face has a unique color?

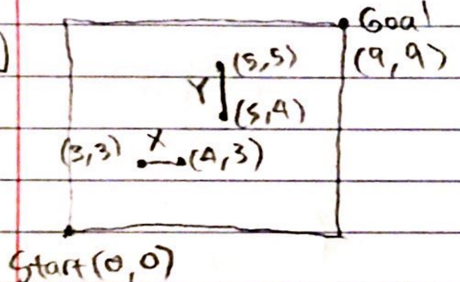
$6! = 6(5)(4)\ldots$

$$\frac{6!}{6 \cdot \underline{4}}$$

6: I choose which face of the cube is on top

4: this is the number of different ways that I can see the lateral faces (side faces) of the cube

3



(a) paths from start to goal:

9 steps up, 9 steps right

$\boxed{\binom{18}{9}}$ ← 18 steps

(b) pass thru X?

X (3,3) → (4,3)

① Start → X

② X → Goal

① (0,0) → (3,3)

3 steps up, 3 steps right

$\binom{6}{3}$

② (4,3) → (9,9)

6 steps up, 5 steps right

$\binom{11}{6}$ $\boxed{\binom{6}{3}\binom{11}{6}}$

(c) pass thru Y?

① start (0,0) → Y (5,4)

4 steps up, 5 steps →

$\binom{9}{4}$

② Y(5,5) → Goal (9,9)

4 steps up, 4 steps →

$\binom{8}{4}$ $\boxed{\binom{9}{4}\binom{8}{4}}$

(d) pass thru X, Y

① Start(0,0) → X(3,3)

$\binom{6}{3}$

② X(4,3) → Y(5,4)

$\binom{2}{1}$

③ Y(5,5) → G(9,9)

$\binom{8}{4}$ $\boxed{\binom{6}{3}\binom{2}{1}\binom{8}{4}}$