Exercise 1. For each of the following English arguments, express the argument in terms of **propositional logic** and determine whether the argument is valid or invalid.

Recommended solution format: 1) Show the variables you will use to represent statements; 2) fill a table with the truth values of the parts of the propositional logic statements up to and including the stated conclusion; 3) add a clear answer of whether or not the stated conclusion is valid.

(a) If it is sunny and you have finished your homework, you always go for a run. Yesterday, you did not run. I conclude that you did not finish your homework. (Is that a valid conclusion?)

Solution: Your proof here □

(b) Every time you drink coffee after 5 PM, you lose sleep; When you lose sleep, you forget things; You forgot your wallet at home, so I can conclude for sure that yesterday you had a coffee after 5 PM. (Is that a valid conclusion?)

Solution: Your proof here □

Exercise 2. Determine whether each of the following is true or false. If an example or counterexample are sufficient, use that, if not, you must provide a general proof.

Recommended solution format: 1) indicate *true* or *false*; 2) add any required proof in concise form (math or a couple of statements).

(a) Every odd number has an even divisor

Solution: Your proof here □

(b) The multiplication of two even numbers is even

Solution: Your proof here □

(c) The multiplication of two odd numbers is odd

Solution: Your proof here □

(d) There is an even prime number

Solution: Your proof here □

Exercise 3. The **pigeonhole principle** is the following somewhat intuitive observation:

If you have n pigeons in k pigeonholes and if $n > k$,
then there is at least one pigeonhole that contains more than one pigeon.

Even though this observation seems obvious, it's a good idea to prove it. Prove the pigeonhole principle using a proof by contradiction.

Recommended solution format: 1) indicate clearly each of the steps for going from a statement to its contradiction; 2) conclude with what it means to have arrived at that contradiction. Note: a few sentences are more than enough.

Solution: Your proof here $\square$

Exercise 4. Use Induction to prove the next statements.

Recommended solution format: 1) Label each of the steps; 2) indicate clearly each of the mathematical manipulations taken to complete each step; 3) If needed, add small comments to clarify the meaning of each step.

(a) Prove that $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$

Solution: Your proof here

$$\begin{aligned} &\text{this is a nice way to align} \\ &\text{equations, in case} \\ &\text{you need to} \end{aligned}$$

$\square$

(b) Prove that $\sum_{i=1}^{n}(2i-1) = n^2$

Solution: Your proof here $\square$

References

[1] Sipser, Michael. *Introduction to the Theory of Computation.* Course Technology, 2005. ISBN: 9780534950972.