Exercise 1. Answer each of the following questions, and explain your reasoning.

(a) Can a Turing Machine ever write the blank symbol $\square$ on its tape?

Solution:

(b) Can the tape alphabet Γ be the same as the input alphabet?

Solution:

(c) Can a Turing Machine's head ever stay in the same location for two steps back to back?

Solution:

(d) Can a Turing Machine contain just a single state?

Solution:

Exercise 2. Show that the set of **decidable** languages is closed under:

(a) union

Solution:

(b) intersection

Solution:

(c) complement

Solution:

Exercise 3. Consider the following TM M that decides the language $L = \{0^{2^n} | n \geq 0\}$

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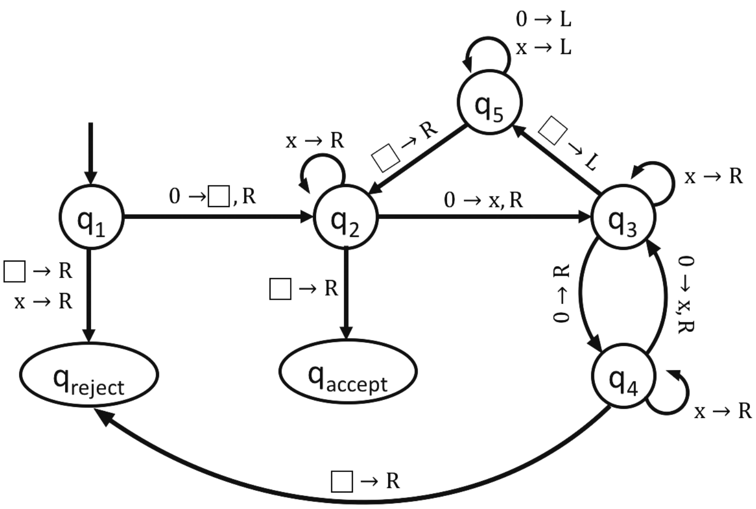
M = "On input string w:
    1. Sweep left to right across the tape, crossing off every other 0.
    2. If in stage 1 the tape contained a single 0, ACCEPT.
    3. If in stage 1 the tape contained more than a single 0
       and the number of 0s was odd, REJECT.
    4. Return the head to the left-land end of the tape.
    5. Go to stage 1."
```

We can describe this machine formally as:

$$\begin{aligned} Q &= \{q_1, q_2, q_3, q_4, q_5, q_{accept}, q_{reject}\} \\ \Sigma &= \{0\} \\ \Gamma &= \{0, x, \square\} \end{aligned}$$

with start state q_1, accepting state q_{accept} and rejecting state q_{reject}. We describe δ with the following diagram (Sipser, p. 144):



Give the sequence of configurations that M enters when started on each of the following strings:

(a) 0

...continued on next page.

Solution:

(b) 00

Solution:

(c) 000

Solution:

(d) 000000

Solution:

Exercise 4. Show that the following language is **decidable**:

$$\{\langle A \rangle \mid A \texttt{ is a DFA and } L(A) \texttt{ is infinite}\}$$

Hint: consider what you know about DFAs and their languages...

Solution:

Exercise 5 OPTIONAL. A Turing machine with STAY PUT instead of LEFT is similar to an ordinary Turing machine, but the transition function has the form:

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{R, S\}$$

At each point the machine can move its head right or let it stay in the same position on the tape. Show that this Turing machine variant is **not equivalent** to the usual version. What class of languages do these machines recognize?

Solution:

References