This is the first of two examinations for
CSC250: Theory of Computation
as taught by R. Jordan Crouser and Pablo Frank Bolton in Spring 2024.

This exam is **open book and open note**.
The following materials are **permitted** while taking this examination:

- your own notes
- lecture slides / videos
- scribe notes
- homework solutions
- any of the recommended textbooks

Honor code: no other resources are permitted during this exam.
This includes (but is not limited to): online materials, tutors, teaching assistants, and other students.

YOUR NAME: _____

Question 0. Getting in the Groove (0 points)

Note: This question is optional, but strongly recommended.

Educational research studies[1,2] have suggested that people perform better on tests when they spend a few minutes thinking about things they're good at before they begin.

In the space below, briefly tell us about a time when you were **really successful** at doing something challenging (it doesn't have to be related to this course). If you prefer, you can draw a picture instead of writing.

[1] Lang, Jonas WB, and Jessica Lang. "Priming competence diminishes the link between cognitive test anxiety and test performance: Implications for the interpretation of test scores." *Psychological Science* 21.6 (2010): 811-819.

[2] Barrows, Jennifer, Samantha Dunn, and Carrie A. Lloyd. "Anxiety, self-efficacy, and college exam grades." *Universal Journal of Educational Research* 1.3 (2013): 204-208.

Question 1. Valid or Invalid Reasoning (6 points)

For each of the following English arguments, express the argument in terms of propositional logic and briefly justify whether the argument is valid or invalid. Be sure to clearly label your propositions.

(a) *When the weather is nice, Max either rides their bike on the rail trail or goes for a walk through the gardens (but never both on the same day). The weather is nice, and Max is going for a walk through the gardens. Therefore, Max will not ride their bike on the rail trail today.*

(b) *When students go to Neilson Library, they want to check out a book. No students went to Neilson Library today. This means that no one wanted to check out a book.*

Question 2. Interpreting regular expressions (3 points)

Describe the language matched by the following regular expression:

$$0(10)^*1 + 1(01)^*0$$

Question 3. Writing regular expressions (3 points)

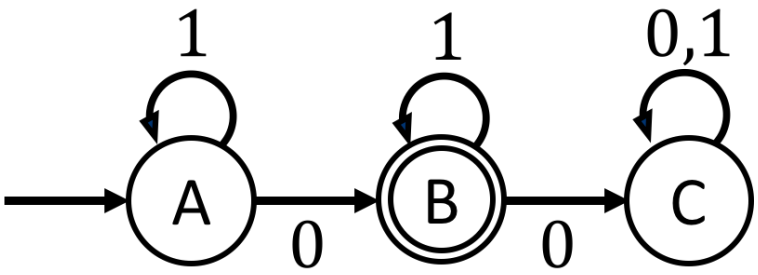
Consider the following language on the alphabet $\Sigma = \{0, 1\}$:

$$L = \{w \mid w \texttt{ has even length, and starts and ends with the same symbol}\}$$

Write a regular expression for L.

Question 4. Interpreting Finite Automata (4 points)

Consider the following finite automaton:



(a) What is the start state?

(b) What is the set of accepting states?

(c) Is this a DFA, an NFA, neither, or both?

(d) What is the language accepted by this FA?

Question 5. Building Finite Automata (6 points)

Draw the transition diagram for a finite automaton that recognizes each of the following languages. In all cases, the alphabet is $\Sigma = \{0, 1\}$.

(a) $\{w \in \Sigma^* \mid w$ `begins with` 1 `and ends with` $0\}$.

(b) $\{w \in \Sigma^* \mid w$ `contains an even number of` $1s\}$.

(c) $\{w \in \Sigma^* \mid w$ `does not contain the substring` $10\}$.

Question 6. Short proofs (8 points)

Determine whether each of the following statements is `true` or `false`.
If it is `true`, provide a short proof. If it is `false`, give a counterexample.

(a) All regular languages are finite.

(b) All finite languages are regular.

(c) All finite automata accept the empty string ε.

(d) There exists some finite automaton that accepts the empty string ε.

Question 7. Non-Regular Languages (6 points)

Prove that the language:

$$DOUBLEZERO = \{w \mid w \texttt{ contains twice as many } 0s \texttt{ as 1s}\}$$

is NOT a regular language.

Question 8. Context-Free Languages (4 points)

Prove that the language:

$$DOUBLEZERO = \{w \mid w \texttt{ contains twice as many } 0s \texttt{ as 1s}\}$$

is context-free.

Question 9. Decidable Languages (4 points)

Prove that the language:

$$DOUBLEZERO = \{w \mid w \texttt{ contains twice as many } 0s \texttt{ as 1s}\}$$

is decidable.

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