Exercise 1. Consider the following language:

$$COMPOSITE_n = \{n \mid n = ab \texttt{ for some integers } a, b\}$$

What is the smallest class that contains this language (finite, regular, context-free, decidable, recognizable, or unrecognizable)? Prove it.

Solution: Your solution here

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Exercise 2. Consider the following language:

$$\begin{aligned} COMPOSITE_{TM} &= \{\langle M, w\rangle \mid M \texttt{ is a TM and } M \texttt{ halts on } w \texttt{ in} \\ &\qquad n = ab \texttt{ steps for some integers } a, b\} \end{aligned}$$

What is the smallest class that contains this language (finite, regular, context-free, decidable, recognizable, or unrecognizable)? Prove it.

Solution: Your solution here

Exercise 3. Consider the following language:

$$COMPOSITE_{RE} = \{n \mid n = ab \texttt{ for some regular expressions } a, b\}$$

What is the smallest class that contains this language (finite, regular, context-free, decidable, recognizable, or unrecognizable)? Prove it.

Solution: Your solution here

Exercise 4. Describe the primary differences between a Turing reduction ($\leq_T$) and a Mapping reduction ($\leq_m$).

Solution: Your solution here

Exercise 5. Show that the following language is in P:

$$\texttt{RELATIVELY-PRIME} = \{\langle x, y\rangle \mid x \texttt{ and } y \texttt{ are integers, } gcd(x, y) = 1\}$$

Exercise 6. A Caesar cipher is a simplified encryption protocol in which all letters are shifted $0 < k < 26$ positions *mod* 26, e.g. when $k = 3$:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

To use this encryption method, look up the substitution for each letter, like this:

$$\text{SMITH COLLEGE} \rightarrow \text{VPLWK FROOHJH}$$

Show that this encryption scheme can be broken in $O(n)$ where n is the length of the message.

Exercise 7. Consider the language:

$$VERTEX-COVER \;=\; \{\langle G,k\rangle \mid G \texttt{ is a graph that has a vertex cover of size } k\}$$

where a **vertex cover** is a set of k vertices such that every edge in the graph touches at least one of the vertices.

(a) Draw a diagram of a graph on 10 vertices with an **vertex cover** of size 5.

(b) Prove that $VERTEX-COVER$ is NP-complete.

Exercise 8. Consider the language:

$$\begin{aligned} SET-COVER \; = \; \{\langle U, S, k\rangle \mid & \; U \texttt{ is a set of elements } \{1, 2, ..., n\} \texttt{ (the "universe"),} \\ & S \texttt{ is a set of } m \texttt{ subsets where } \bigcup S = U, \\ & \texttt{and } S \texttt{ contains a set cover of size } k\} \end{aligned}$$

where a **set cover** is a set of k subsets $\in S$ such that every element in U is contained in at least one of the selected subsets.

(a) Draw a diagram of a universe with 10 elements, partitioned into 5 subsets with a **set cover** of size 3.

(b) Prove that $SET-COVER$ is NP-complete.

References

[1] Sipser, Michael. *Introduction to the Theory of Computation.* Course Technology, 2005. ISBN: 9780534950972

[2] Critchlow, Carol and Eck, David *Foundation of computation.*, Critchlow Carol, 2011