

Statement to check :

$$\sum_1^n i^3 = \frac{n^2(n+1)^2}{4}$$

Base Case : $n = 1$

$$\sum_1^1 i^3 = \frac{1^2(1+1)^2}{4}$$

$$1^3 = 1 = \frac{1^2(1+1)^2}{4} = 1 \quad \checkmark$$

Induction Hypothesis : $n = k$

$$\sum_1^k i^3 = \frac{k^2(k+1)^2}{4} \quad \checkmark$$

Induction Step : $n = k + 1$

$$\sum_1^{k+1} i^3 = \frac{(k+1)^2((k+1)+1)^2}{4}$$

$$\begin{aligned} \sum_1^{k+1} i^3 &= \sum_1^k i^3 + (k+1)^3 \\ &= \frac{k^2(k+1)^2}{4} + (k+1)^3 \\ &= \frac{k^2(k+1)^2}{4} + \frac{4(k+1)^3}{4} \\ &= \frac{k^2(k+1)^2}{4} + \frac{4(k+1)(k+1)^2}{4} \\ &= \frac{k^2(k+1)^2 + 4(k+1)(k+1)^2}{4} \\ &= \frac{(k+1)^2(k^2 + 4(k+1))}{4} \\ &= \frac{(k+1)^2(k^2 + 4k + 4)}{4} \\ &= \frac{(k+1)^2(k+2)^2}{4} \\ &= \frac{(k+1)^2((k+1)+1)^2}{4} \quad \square \end{aligned}$$