

Exercise 1. Consider the following language:

$$COMPOSITE_n = \{n \mid n = ab \text{ for some integers } a, b\}$$

What is the smallest class that contains this language (finite, regular, context-free, decidable, recognizable, or unrecognizable)? Prove it.

Solution: Your solution here

Exercise 2. Consider the following language:

$$COMPOSITE_{TM} = \{ \langle M, w \rangle \mid M \text{ is a TM and } M \text{ halts on } w \text{ in} \\ n = ab \text{ steps for some integers } a, b \}$$

What is the smallest class that contains this language (finite, regular, context-free, decidable, recognizable, or unrecognizable)? Prove it.

Solution: Your solution here

Exercise 3. Consider the following language:

$$COMPOSITE_{RE} = \{n \mid n = ab \text{ for some regular expressions } a, b\}$$

What is the smallest class that contains this language (finite, regular, context-free, decidable, recognizable, or unrecognizable)? Prove it.

Solution: Your solution here

Exercise 4. Describe the primary differences between a Turing reduction (\leq_T) and a Mapping reduction (\leq_m).

Solution: Your solution here

Exercise 5. Show that the following language is in P :

$$\text{RELATIVELY-PRIME} = \{\langle x, y \rangle \mid x \text{ and } y \text{ are integers, } \gcd(x, y) = 1\}$$

Exercise 6. A Caesar cipher is a simplified encryption protocol in which all letters are shifted $0 < k < 26$ positions *mod* 26, e.g. when $k = 3$:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

To use this encryption method, look up the substitution for each letter, like this:

SMITH COLLEGE \rightarrow VPLWK FROOHJH

Show that this encryption scheme can be broken in $O(n)$ where n is the length of the message.

Exercise 7. Consider the language:

$$VERTEX - COVER = \{\langle G, k \rangle \mid G \text{ is a graph that has a} \\ \text{vertex cover of size } k\}$$

where a **vertex cover** is a set of k vertices such that every edge in the graph touches at least one of the vertices.

- (a) Draw a diagram of a graph on 10 vertices with an **vertex cover** of size 5.
- (b) Prove that $VERTEX - COVER$ is NP -complete.

Exercise 8. Consider the language:

$SET - COVER = \{ \langle U, S, k \rangle \mid U \text{ is a set of elements } \{1, 2, \dots, n\} \text{ (the "universe")},$
 $S \text{ is a set of } m \text{ subsets where } \bigcup S = U,$
 $\text{and } S \text{ contains a set cover of size } k \}$

where a **set cover** is a set of k subsets $\in S$ such that every element in U is contained in at least one of the selected subsets.

- (a) Draw a diagram of a universe with 10 elements, partitioned into 5 subsets with a **set cover** of size 3.
- (b) Prove that $SET - COVER$ is NP -complete.

References

- [1] Sipser, Michael. *Introduction to the Theory of Computation*. Course Technology, 2005. ISBN: 9780534950972
- [2] Critchlow, Carol and Eck, David *Foundation of computation.*, Critchlow Carol, 2011