CALIFORNIA STATE UNIVERSITY FULLERTON™

Analyzing Process Behaviors with Control Charts

Paulo de Oliveira
paulohgo@csu.fullerton.edu

Reading Assignment

- Chap 4
- Chap 5
- Appendix A

Note that you need to read these chapters several times

Analyzing Process Behavior

- *A phenomenon will be said to be controlled when, through the use of past experience, we can predict, at least within limits, how the phenomenon may be expected to vary in the future.*

  - Walter A. Shewhart 1931

- Culturally, this is a movement to statistically based decision making

Analyzing Process Behavior

- Data are generally collected as basis for action.
- Variation exists in all data and consists of both noise (common cause variation) and signal (assignable cause variation).

Total variation = common cause variation + assignable cause variation

- We wish to draw inferences that can be used to guide decisions and actions.
- One needs simple and effective ways to detect assignable causes surrounded by common causes.

Write the letter "a" on a piece of paper. Now make another a just like the first one; then another and another until you have a series of a's, a, a, a,... You try to make all the a's alike but you don't; you can't. You are willing to accept this as an empirically established fact. But what of it? Let us see just what this means in respect to control. Why can we not do a simple thing like making all the a's exactly alike? Your answer leads to a generalization which all of us are perhaps willing to accept. It is that there are many causes of variability among the a's: The paper was not smooth, the lead in the pencil was not uniform, and the unavoidable variability in your external surroundings reacted upon you to introduce variation in the a's. But are these the only causes of variability in the a's? Probably not.

We accept our human limitations and say that likely there are many other factors. If we could but name all the reasons why we cannot make the little a's alike, we would most assuredly have a better understanding of a certain part of nature than we now have. Of course, this conception of what it means to be able to do what we want to do is not new; it does not belong exclusively to any one field of human thought; it is commonly accepted.

The point to be made in this simple illustration is that we are limited in doing what we want to do; that to do what we set out to do, even in so simple a thing as making a's that are alike, requires almost infinite knowledge compared with that which we now possess. It follows, therefore, since we are thus willing to accept as axiomatic that we cannot do what we want to do and cannot hope to understand why we cannot, that we must also accept as axiomatic that a controlled quality will not be a constant quality. Instead, a controlled quality must be a ***variable*** *quality. This is the first characteristic.*

But let us go back to the results of the experiment on the a's and we shall find out something more about control. Your a's are different from my a's; there is something about your a's that makes them yours and something about my a's that makes them mine.

True, not all of your a's are alike. Neither are all of my a's alike. Each group of a's varies within a certain range and yet each group is distinguishable from the others. This distinguishable and, as it were, constant variability ***within limits*** *is the second characteristic of control.*

Walter A. Shewhart 1931

Understanding Variation

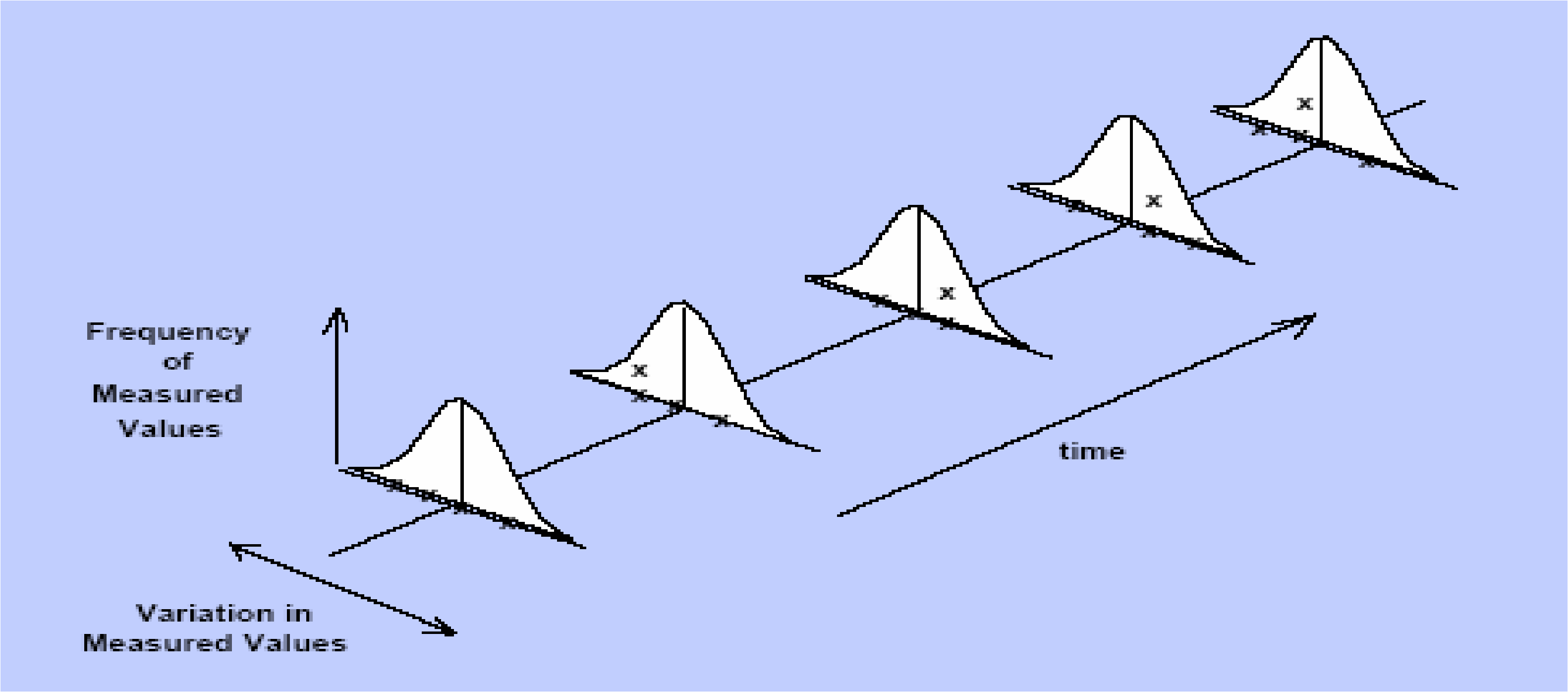
- When a process is stable, the variation in process performance is predictable and is attributable to common cause. *Predictable* being synonymous with *in control.*
- Assignable causes of variation arise from events that are not part of the normal process.

Understanding Variation

- One of the techniques that is often used to establish operational limits for acceptable variation is the use of **control charts**.

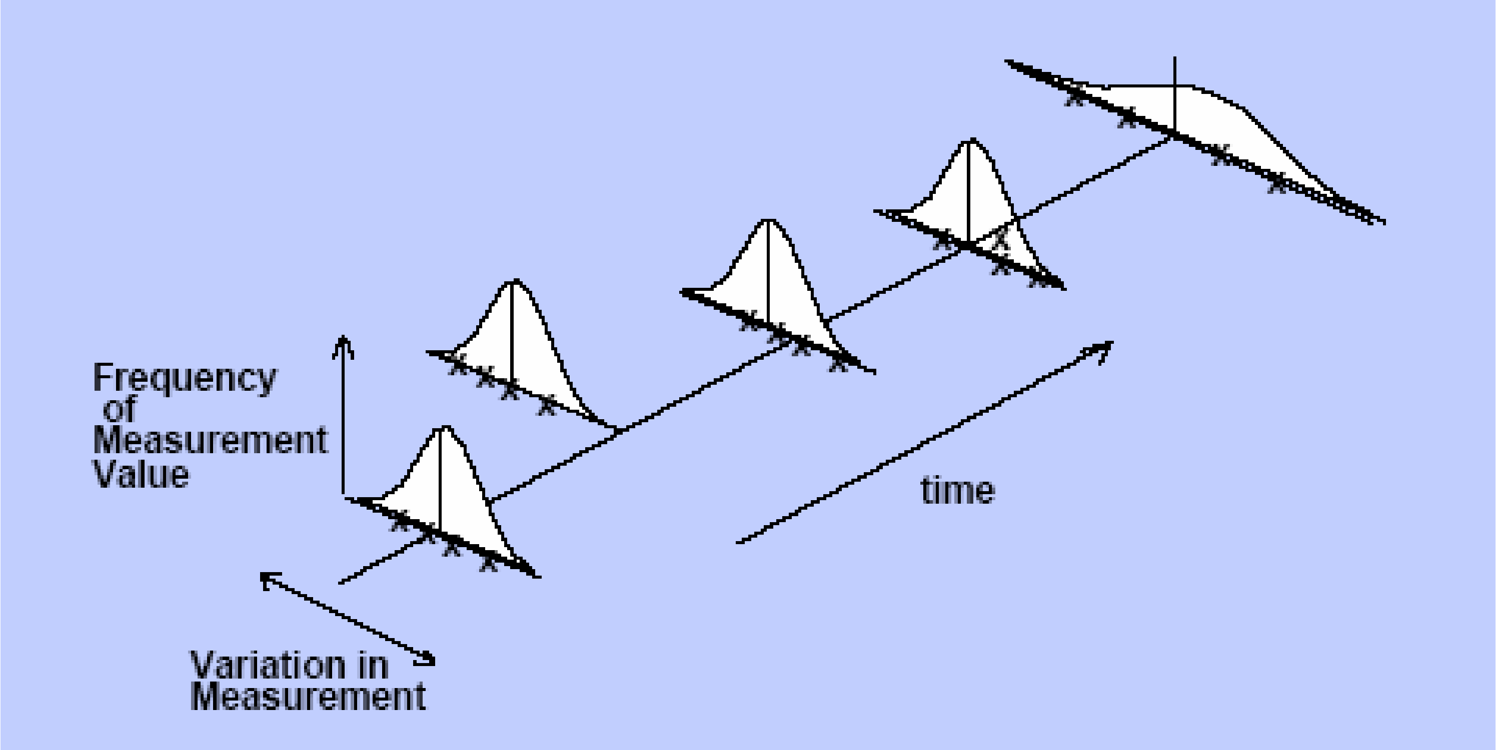
- Control charts are techniques for quantifying process behavior.

For your review



- Is there variation?
- Is this process under control?

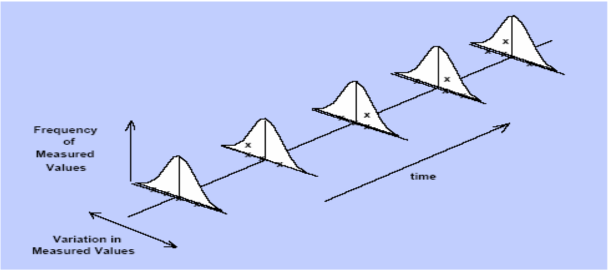
For your review

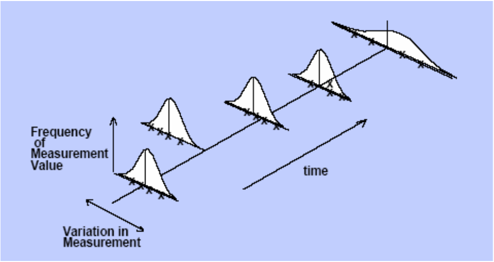


- Is there variation?
- Is this process under control?

Fantasy

- As we know, all data in our work environment is presented to us in this wonderful format.





- Not true.
- These are nice graphics, but we still must “feel” whether or not the process in or out of control, or do we…..

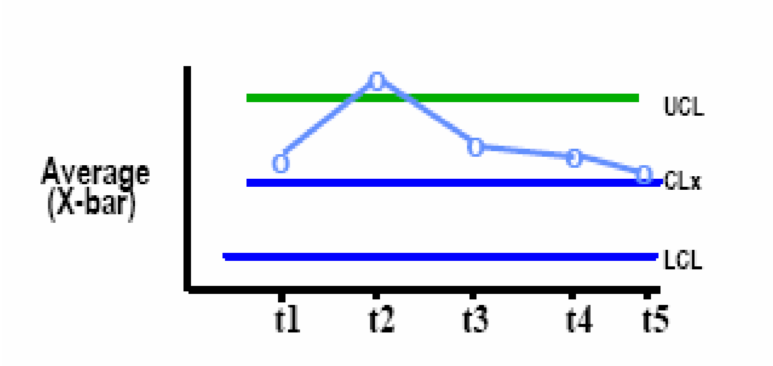
Reality

- As we know, all data in our work environment is presented to us in this format.

	Sample1	Sample2	Sample3	Sample4
T1	13	11	9	7
T2	12	12	11	8
T3	12	9	8	8
T4	12	9	9	7
T5	13	11	11	8

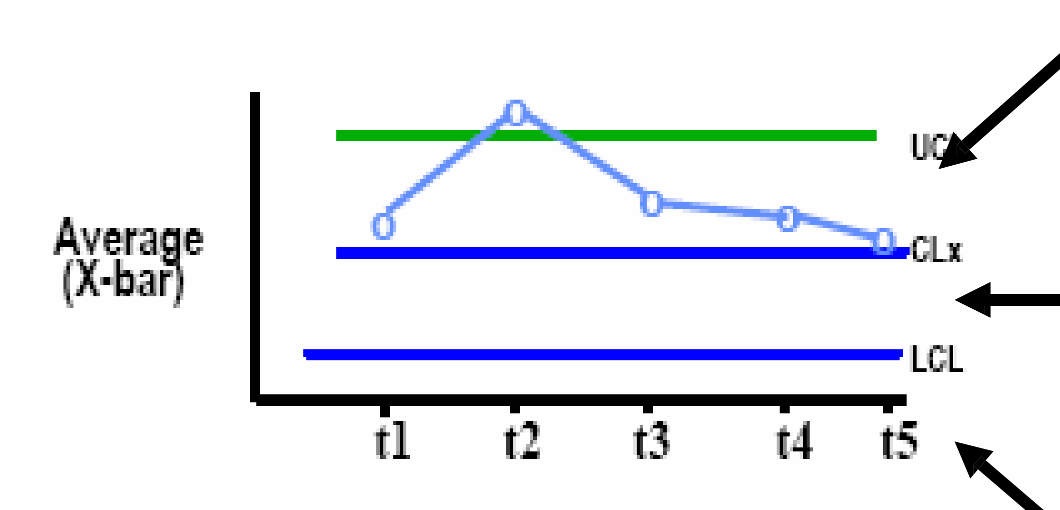
Control Charts to the Rescue!

Control charts separate signal from noise, so that you can recognize a process change when it occurs.



Control charts identify unusual events. They point you to fixable problems (assignable causes) and to potential process improvements.

Anatomy of a Control Chart



Upper Control Limit
+ 3 sigma

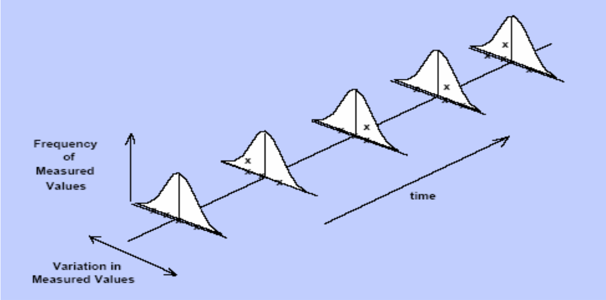
Centerline

Upper Control Limit
- 3 sigma

Building Xbar and R Charts

- Let's build an X-bar and R chart for our earlier data.
- Is this process stable, or in control?

	Sample1	Sample2	Sample3	Sample4
T1	13	11	9	7
T2	12	12	11	8
T3	12	9	8	8
T4	12	9	9	7
T5	13	11	11	8



The tools

	Sample1	Sample2	Sample3	Sample4
T1	13	11	9	7
T2	12	12	11	8
T3	12	9	8	8
T4	12	9	9	7
T5	13	11	11	8

1. Compute the average $\overline{X}$ and range R for each subgroup of size n, for each of the k subgroups:

$$\overline{X}_k = \frac{X_1 + X_2 + \dots + X_n}{n}$$

$$R_k = |X_{MAX} - X_{MIN}|$$

2. Compute the grand average $\overline{\overline{X}}$ by averaging each of the k subgroup averages:

$$\overline{\overline{X}} = \frac{\overline{X}_1 + \overline{X}_2 + \dots + \overline{X}_k}{k}$$

3. Compute the average range $\overline{R}$ by averaging each of the k subgroup ranges:

$$\overline{R} = \frac{R_1 + R_2 + \dots + R_k}{k}$$

The tools

Average X-bar Limits

$$UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R} = \text{ Grand Average } + \ A_2 \text{ times the Average Range}$$

$$CL_{\overline{X}} = \overline{\overline{X}} \ = \text{ the Grand Average}$$

$$LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R} = \text{ Grand Average } - \ A_2 \text{ times the Average Range}$$

Average Range Limits

$$UCL_{\overline{R}} = D_4\overline{R} = D_4 \text{ times the Average Range}$$

$$CL_{\overline{R}} = \overline{R} \ = \text{ the Average Range}$$

$$LCL_{\overline{R}} = D_3\overline{R} = D_3 \text{ times the Average Range}$$

Exercise 3 – The tools

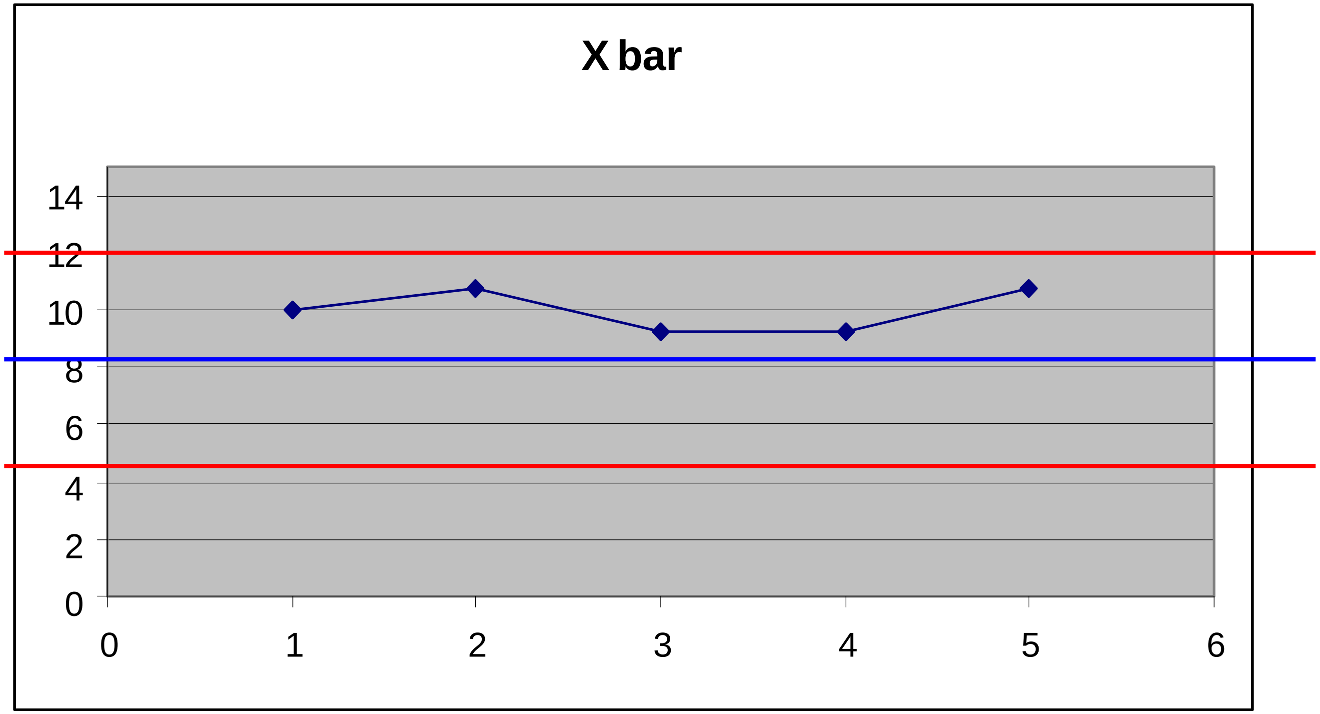
n	d_2	A_2	D_3	D_4
2	1.128	1.880	–	3.268
3	1.693	1.023	–	2.574
4	2.059	0.729	–	2.282
5	2.326	0.577	–	2.114
6	2.534	0.483	–	2.004
7	2.704	0.419	0.076	1.924
8	2.847	0.373	0.136	1.864
9	2.970	0.337	0.184	1.816
10	3.078	0.308	0.223	1.777

The data

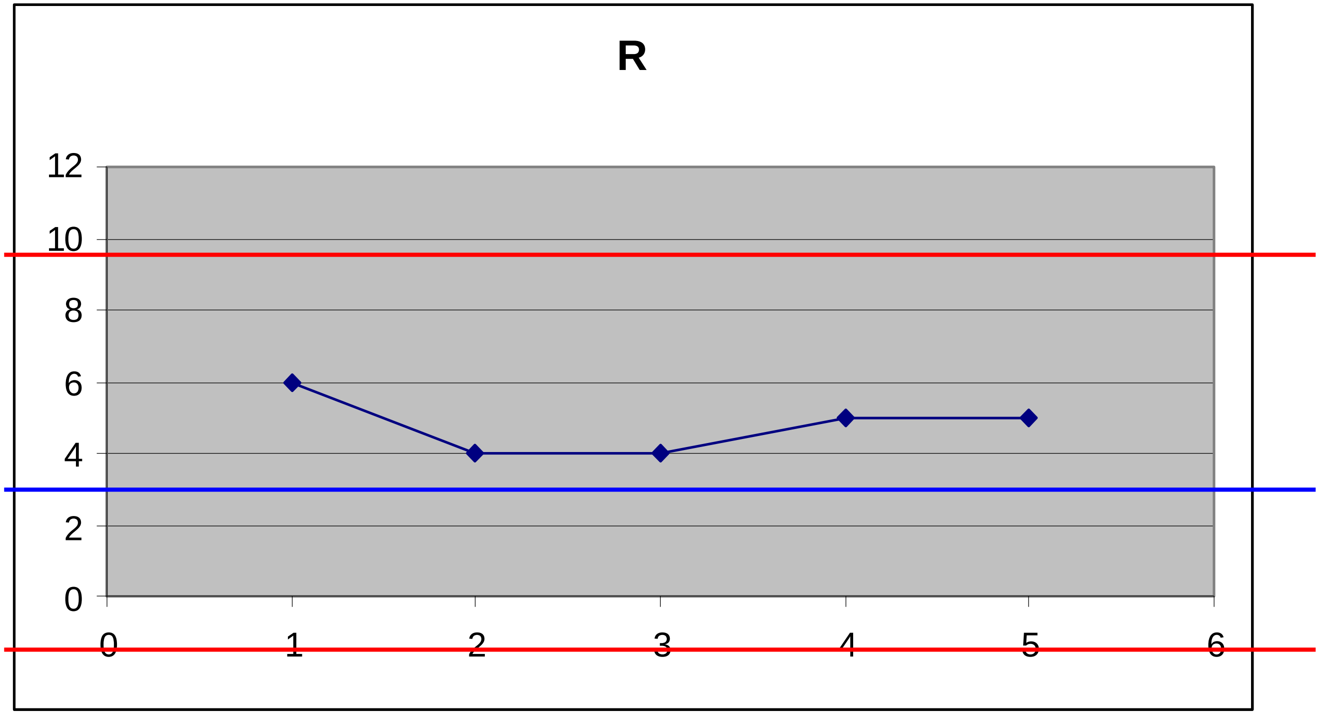
	Solutions					
	Sample1	Sample2	Sample3	Sample4	Xbar	R
T1	13	11	9	7	10	6
T2	12	12	11	8	10.75	4
T3	12	9	8	8	9.25	4
T4	12	9	9	7	9.25	5
T5	13	11	11	8	10.75	5

Xdblbar	10			
Rbar	4.8			
Xbar limits			R limits	
UCL	13.4992		UCL	10.9536
CL	10		CL	4.8
LCL	6.5008		LCL	0

The charts

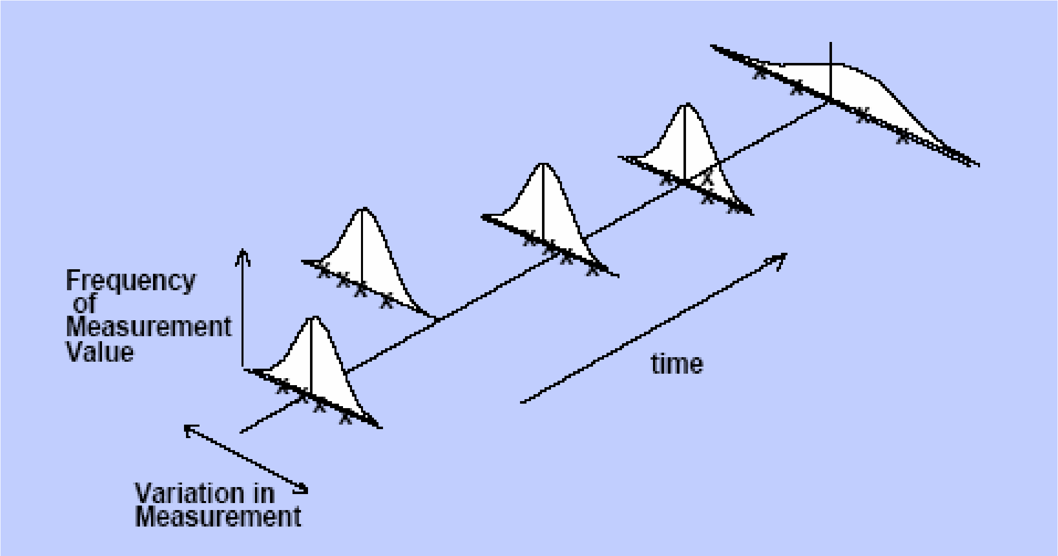


The charts



Exercise 3 – The Challenge

- Let's build an X-bar and R chart for our earlier data, the picture we "feel" is out of control.
- Is this process stable, or in control?



The data

	Sample1	Sample2	Sample3	Sample4	Xbar	R
T1	12	11	9	8		
T2	17	16	15	14		
T3	12	11	10	8		
T4	12	8	8	6		
T5	18	13	8	3		

Stability and Capability

Stability

- *Is the process that we are managing behaving predictably?*
  - business value: foundation for estimating (predicting) and making commitments

Operating within natural boundaries UCL and LCL

Capability

- *Is the process capable of delivering products that meet requirements?*
- *Does the performance of the process meet the business needs of the organization?*
  - business value: foundation for making commitments

Operating within business goals or objectives

Detection Rules

Test 1: A single point falls outside the 3-sigma control limits.

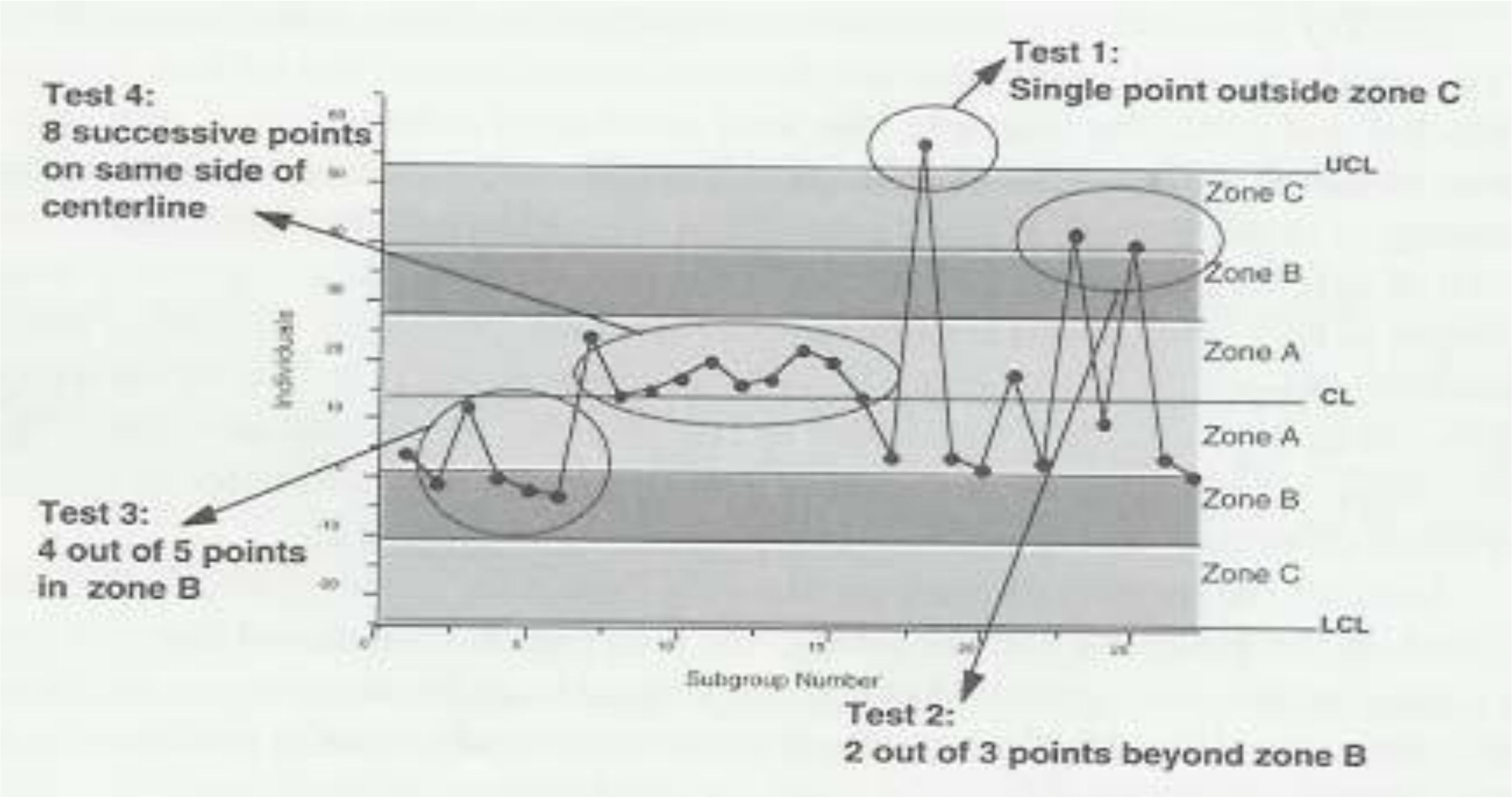
Test 2: At least two of three successive values fall on the same side of, and more than two sigma units away from, the center line.

Test 3: At least four out of five successive values fall on the same side of, and more than one sigma unit away from, the center line.

Test 4: At least eight successive values fall on the same side of the center line.

Source: Western Electric Handbook

Detection Rules Example



The Western Electric Handbook 1958

Detection Rules Example

Warning: These rules apply to XbarR charts

However, only rule 1 applies to XmR charts.

Detection Rules Example

Wheeler: XbarR and XmR charts should satisfy 99% or more of your control charting needs.

X_{bar} is average of rational (homogenous) subgroups, usually with 4-6 items sampled.

For most software processes, individual data are preferred, leading to the other basic control chart, the XmR chart.

Xbar and XmR Charts

- K= # of sequential measurements
- n=2 the subgroup size for 2-point ranges
- If n=2, r=K-1, D4=3.268, d2=1.128
- mRi=|Xsubi+1 – Xsubi| mRbar= avg of mRi’s

Xbar and XmR Charts

Individuals Chart Limits

$$\text{Upper Natural Process Limit} = UNPL_X = \overline{X} + \frac{3\overline{mR}}{d_2} = \overline{X} + 2.660\overline{mR}$$

$$\text{Center line} = CL_x = \overline{X} = \frac{1}{k}\sum_{i=1}^{i=k} X_i$$

$$\text{Lower Natural Process Limit} = LNPL_X = \overline{X} - \frac{3\overline{mR}}{d_2} = \overline{X} - 2.660\overline{mR}$$

Moving Range Chart Limits

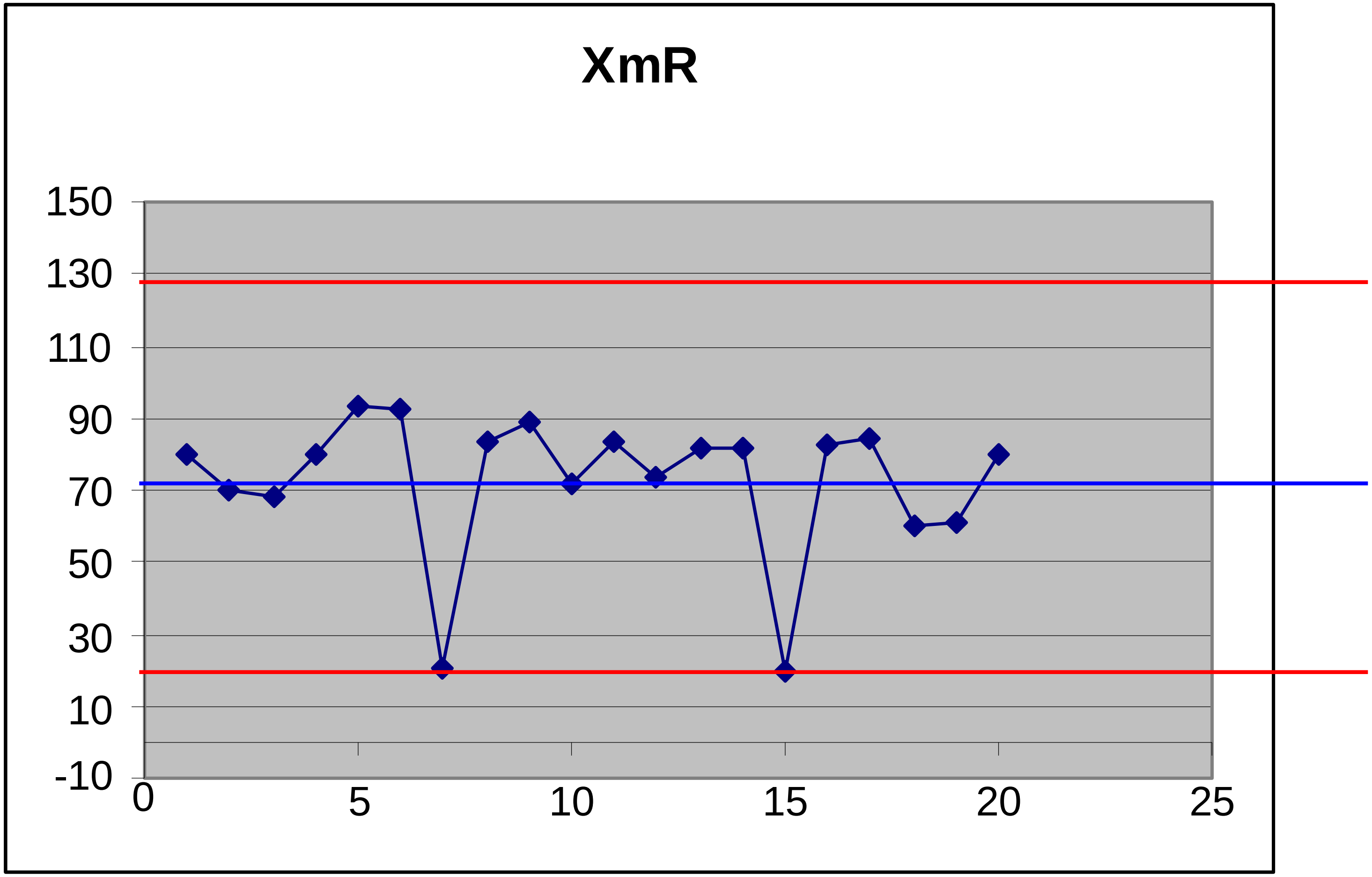
$$\text{Upper control Limit for moving range} = UCL_R = D_4\overline{mR} = 3.268\overline{mR}$$

$$\text{Center line or average moving range} = CL_R = \overline{mR}$$

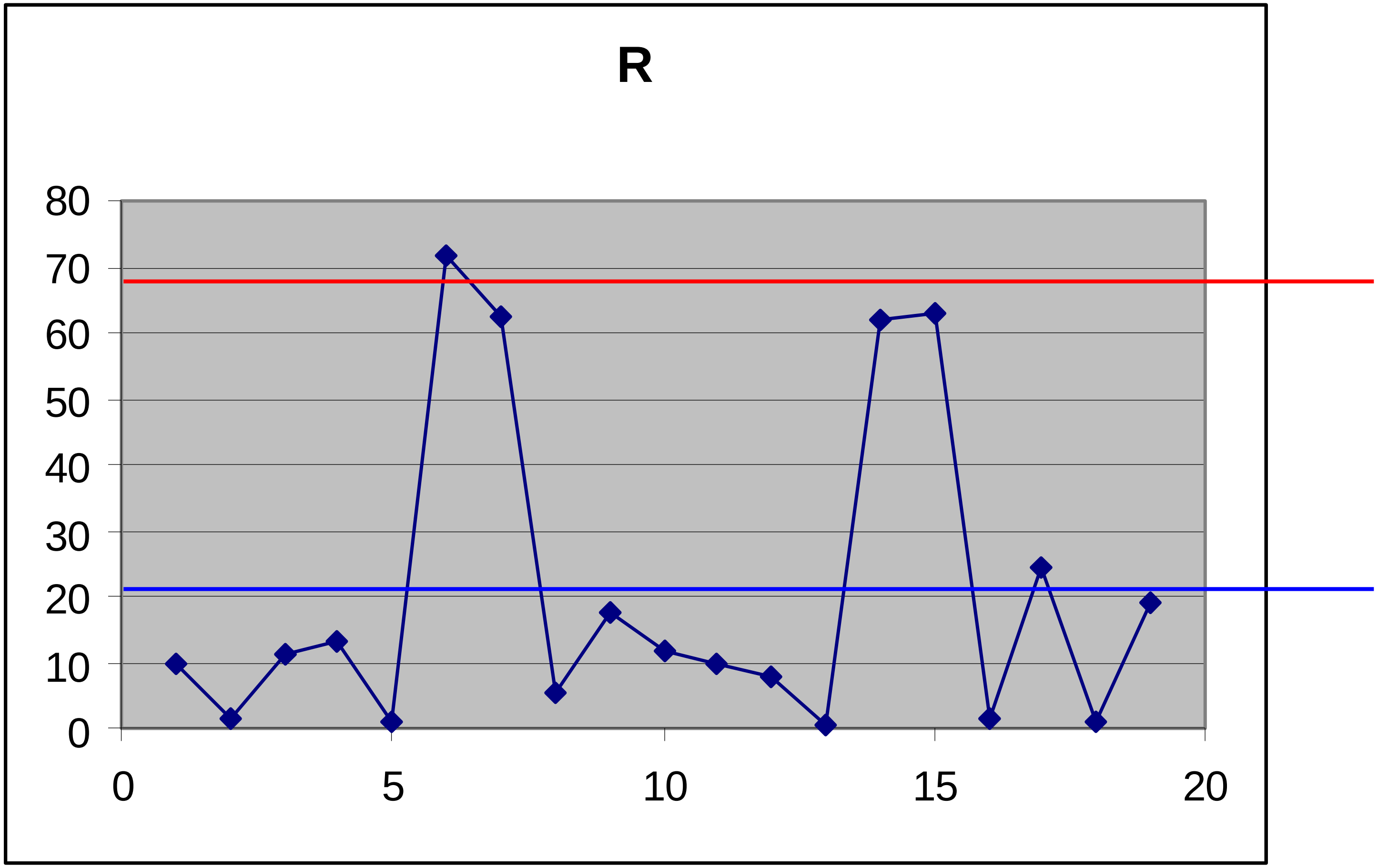
$$\text{Lower control Limit for moving range} = LCL_R = D_3\overline{mR} = 0 \text{ for } n = 2$$

$d_2 = 1.128$ for $n = 2$

XmR Chart Def Density



XmR Chart Def Density



Trial Limits for XmR Charts

IF limits are based on k < 20
OR
IF using trial limits established at 20 < k < 35

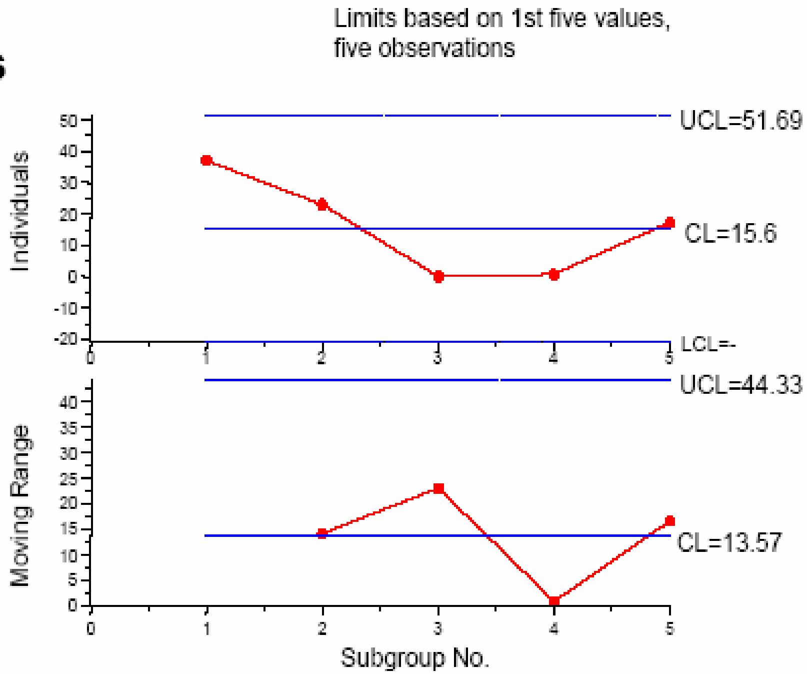
… AND potential assignable cause is encountered

… THEN recalculate limits before pursuing assignable cause

Trial Limits for XmR Charts

Using data from a real inspection process, plot the first five (of 40) values of defect density.

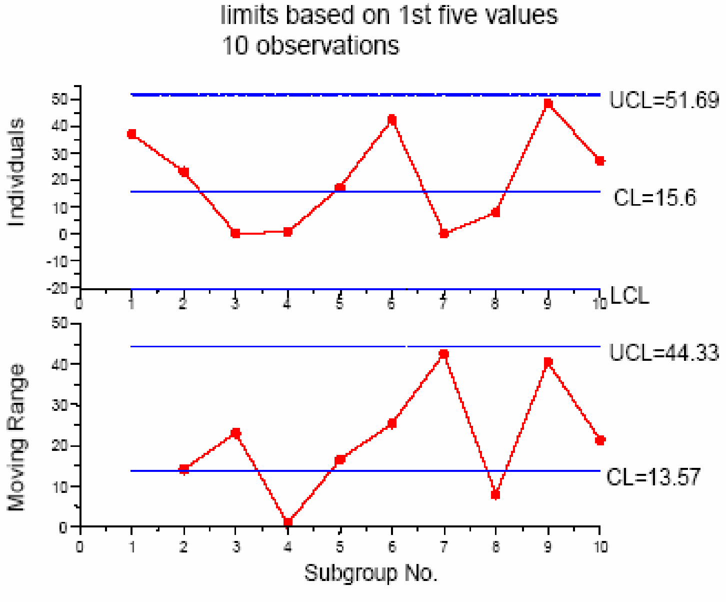
No apparent assignable causes.



Trial Limits for XmR Charts

Keeping same limits, plot next five observations.

No assignable causes with first ten observations.



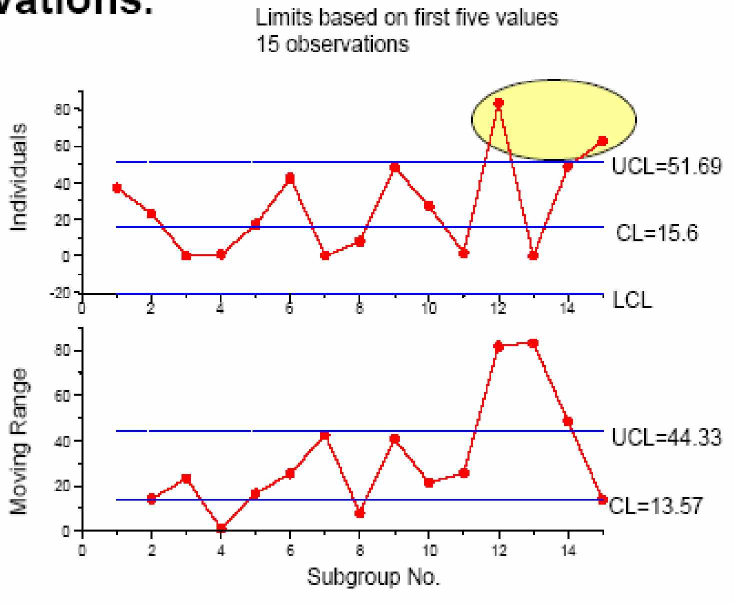
Trial Limits for XmR Charts

Keeping same limits, plot next five observations.

Chart indicates assignable causes with given limits.

Recalculate limits since k < 20.

Limits based on 15 observations are more reliable.

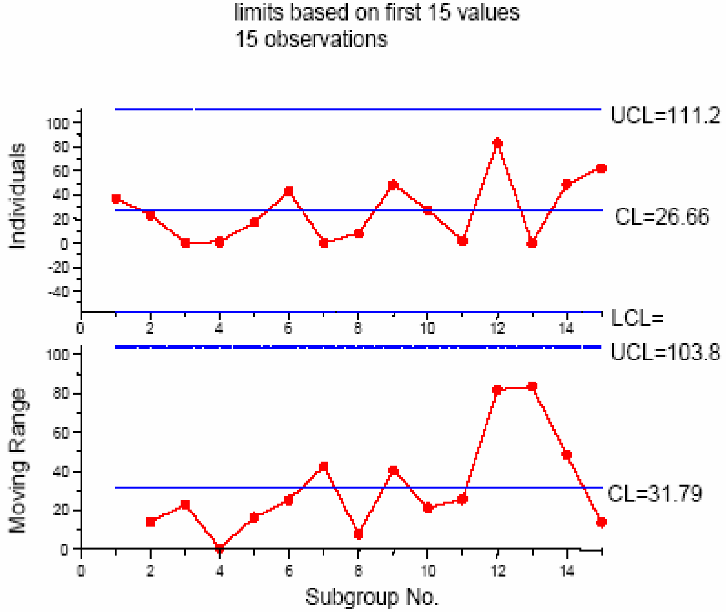


Trial Limits for XmR Charts

Recalculated limits with first 15 values.

Limits change, but they are more reliable.

No assignable causes.

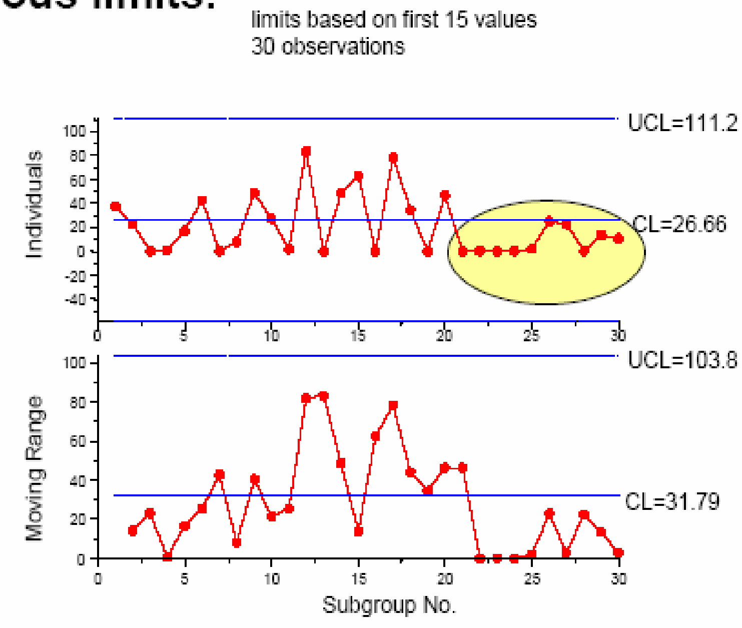


Trial Limits for XmR Charts

Continuing… plot 30 values using previous limits.

Assignable cause with limits based on k=15 (eight points on same side of central line).

Recalculate limits.



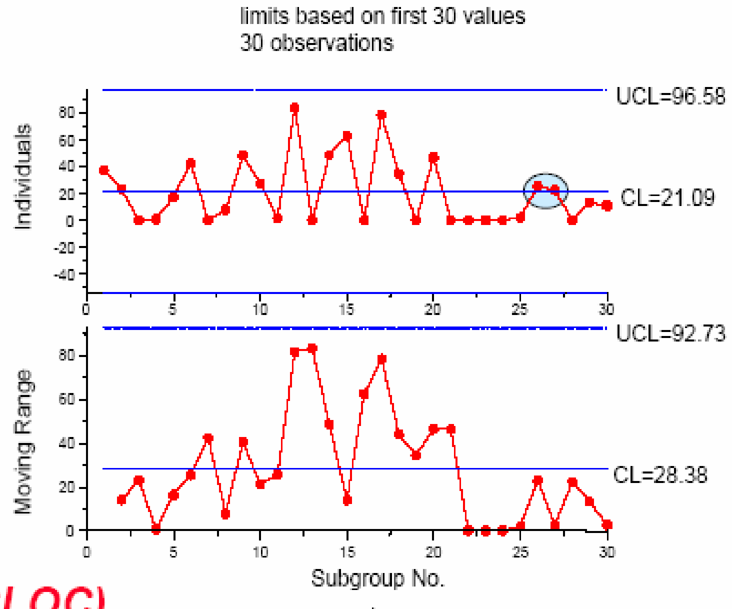
Trial Limits for XmR Charts

Recalculated limits for k =30.

No assignable causes.

Limits reduced in this case.

Remove point ee as an obvious outlier (650 defects/KSLOC).



Trial Limits for XmR Charts

Continuing… recalculate limits to see if k=39 results in any assignable causes.

Observation #12 is assignable cause in mR chart.

Removing #12 brings #17 out of limits.