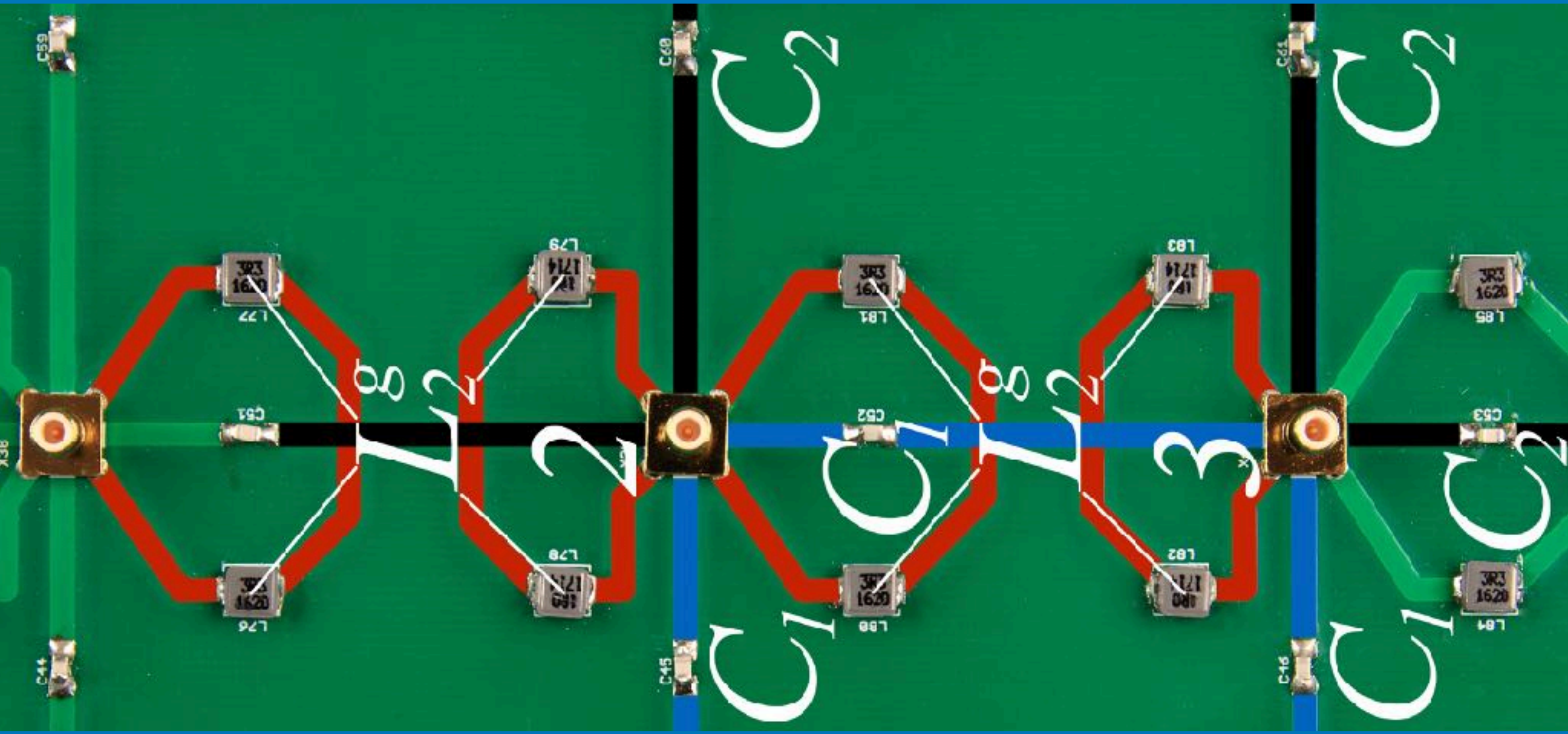
Higher-Order Topolectrical Circuits



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Cologne, 28 November 2017

Topolectrical Circuit Realization of Topological Corner Modes

arXiv:1708.03647

Outline

1) Higher-order topological phases
2) Corner states as vortex bound states
3) Topolectrical circuits
4) Corner circuit architecture and symmetries
5) Experimental realization

Collaborators

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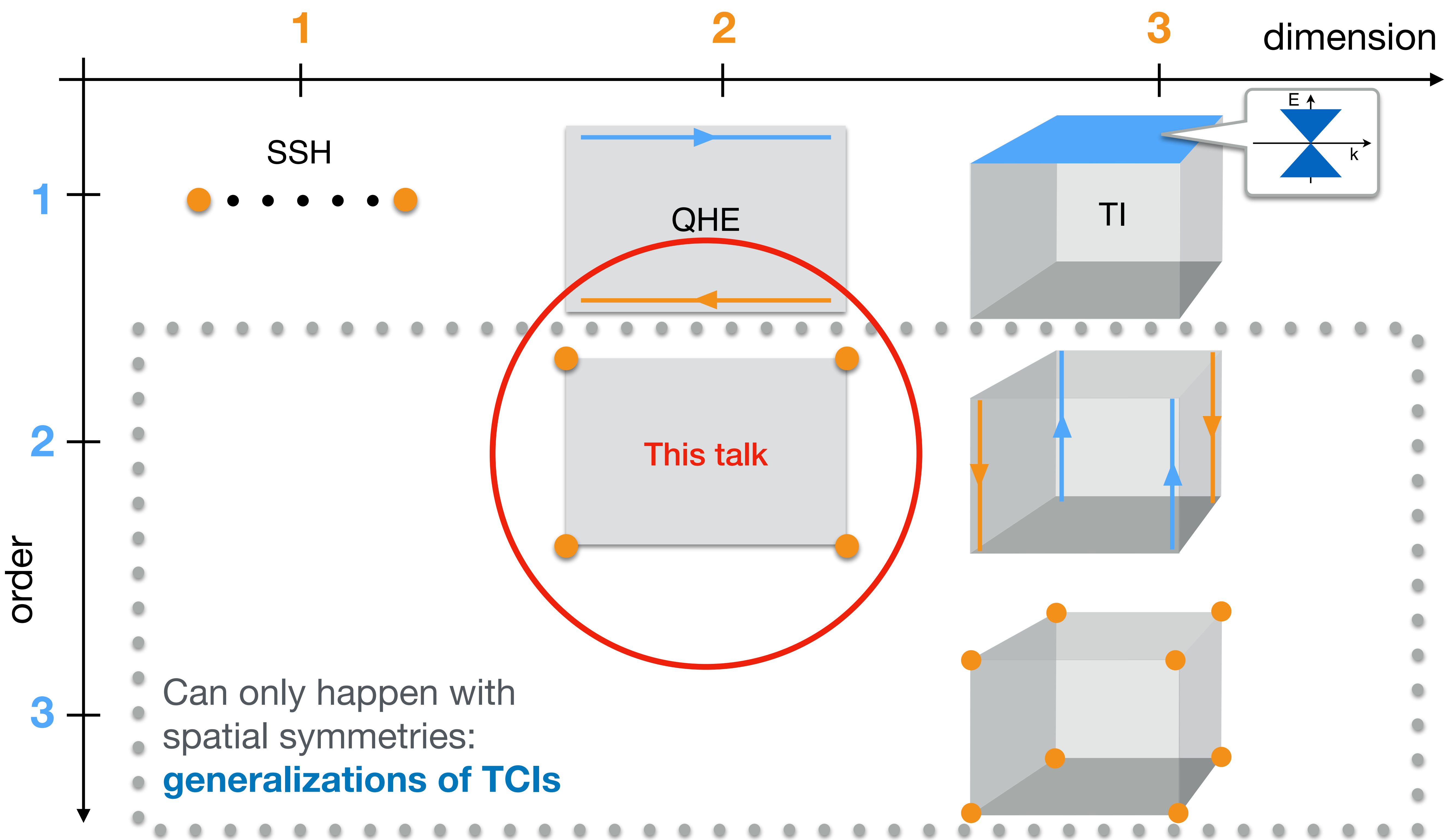
(Dated: September 27, 2017)

Higher-order topological insulators

For an N-th order TI:

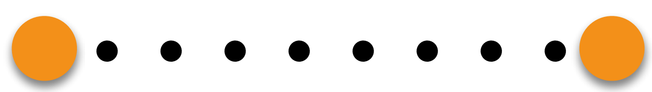
(d-m)-dimensional boundary components of a d-dimensional system are **gapless for m = N**, and are generically **gapped for m < N**

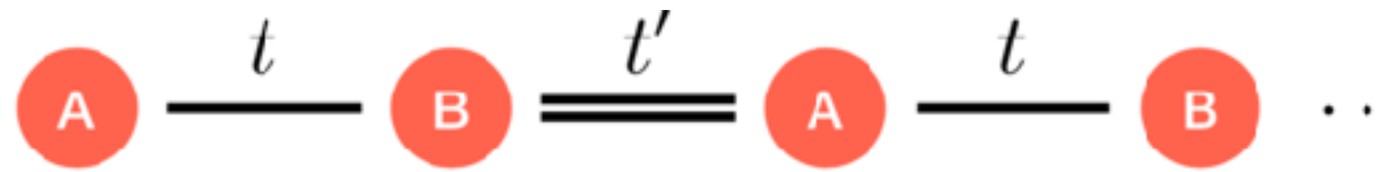


Second-order 2D topological insulator

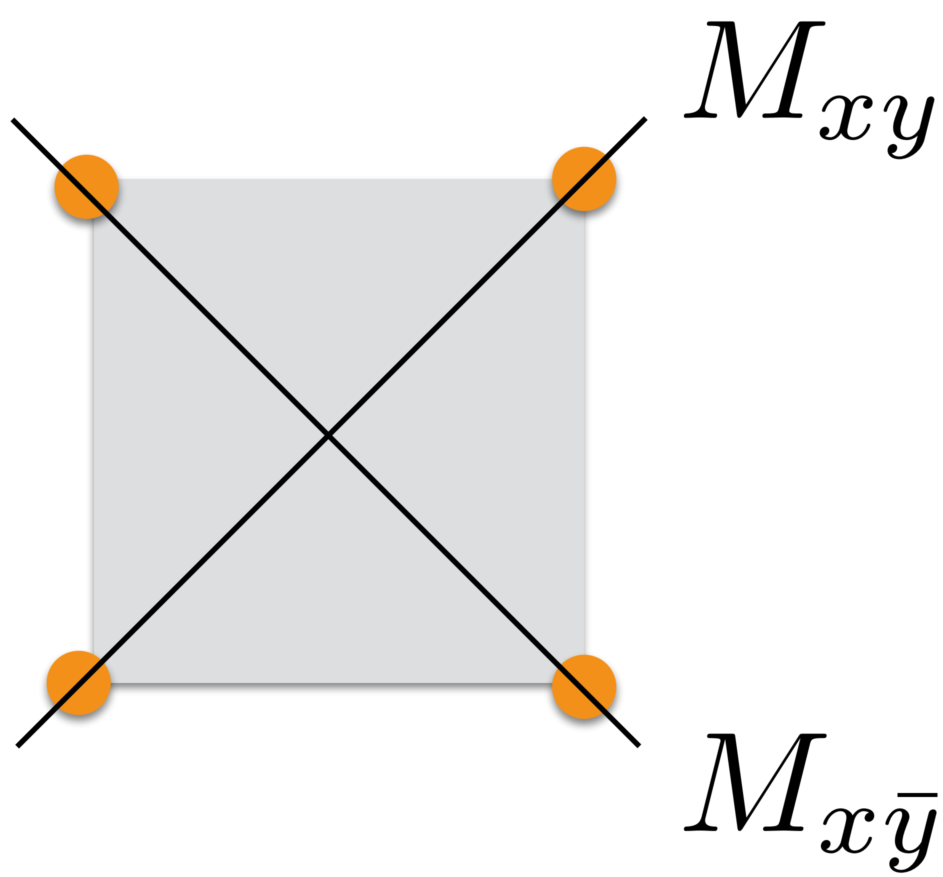
= quadrupole insulator *[Benalcazar, Bernevig and Hughes, Science 357, 61 (2017)]*

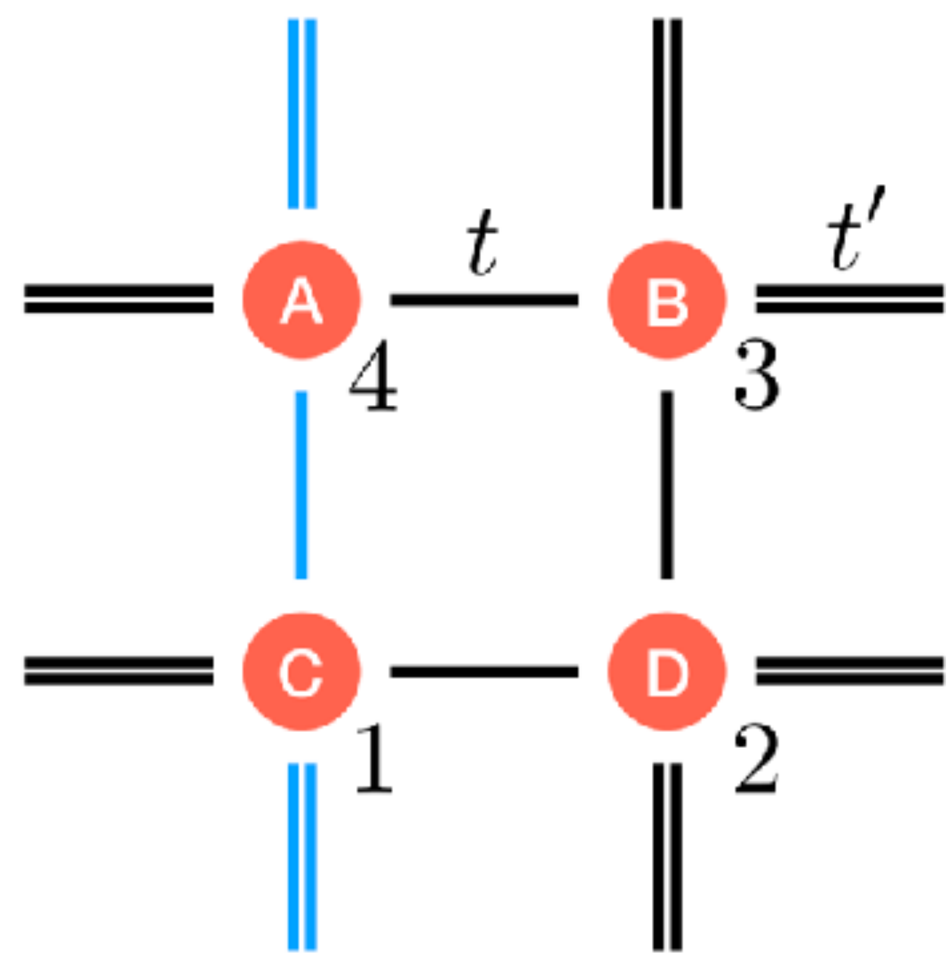
Start with SSH model… (protected by chiral symmetry)





…and generalize to 2D:





Protected in addition by mirror symmetries (otherwise can annihilate corner modes with each other without closing the bulk gap).

→ Cannot glue together 1D models to get same phase

Corner states as vortex bound states

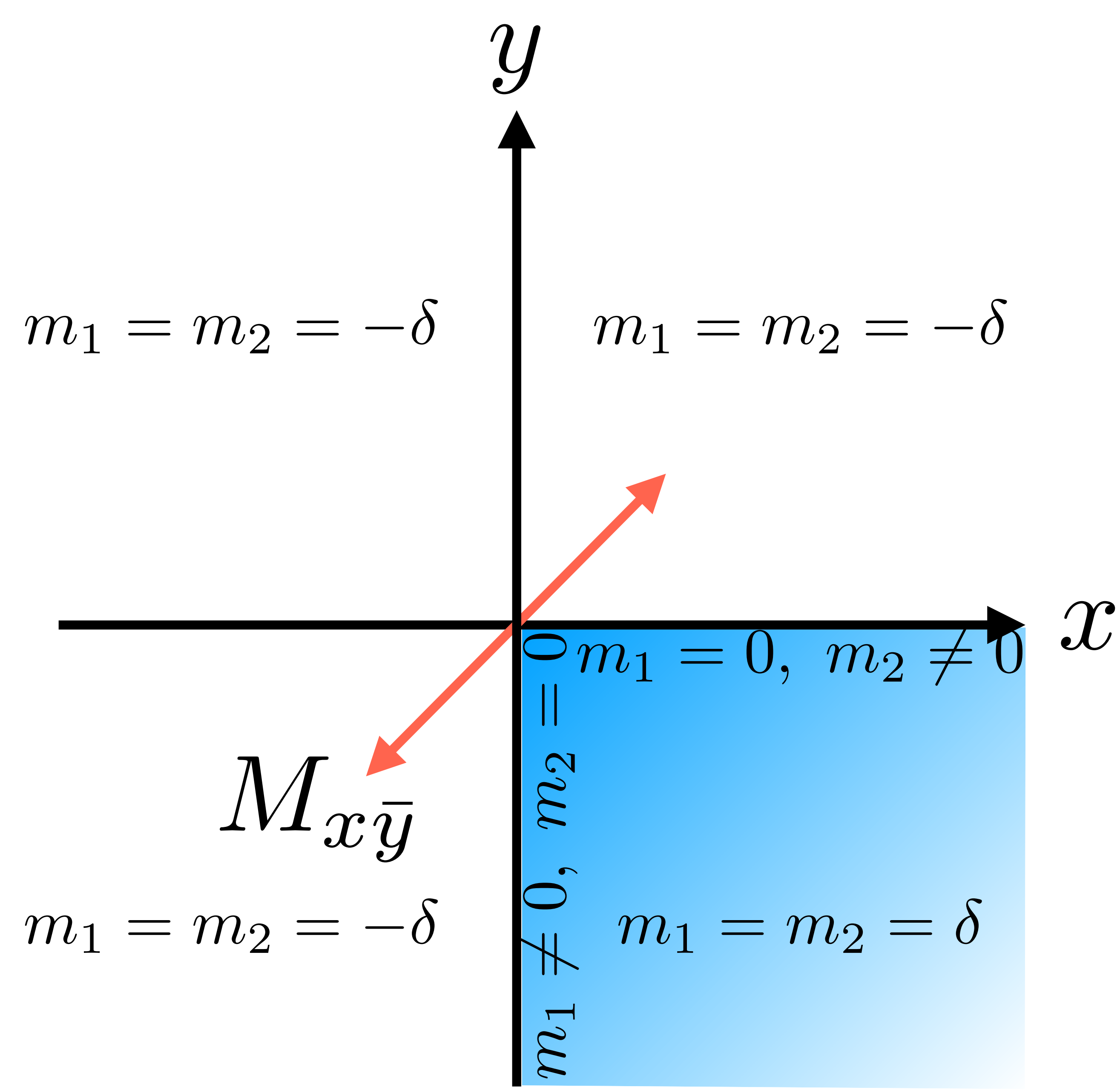
Expand Hamiltonian around gap closing point in momentum space

$$\mathcal{H}(\boldsymbol{k}) \approx \delta\tau_0\sigma_x + \delta\tau_y\sigma_y + k_x\tau_z\sigma_y - k_y\tau_x\sigma_y$$

$M_{x\bar{y}}$

Open boundary conditions in real space are modeled by domain wall of Dirac mass terms

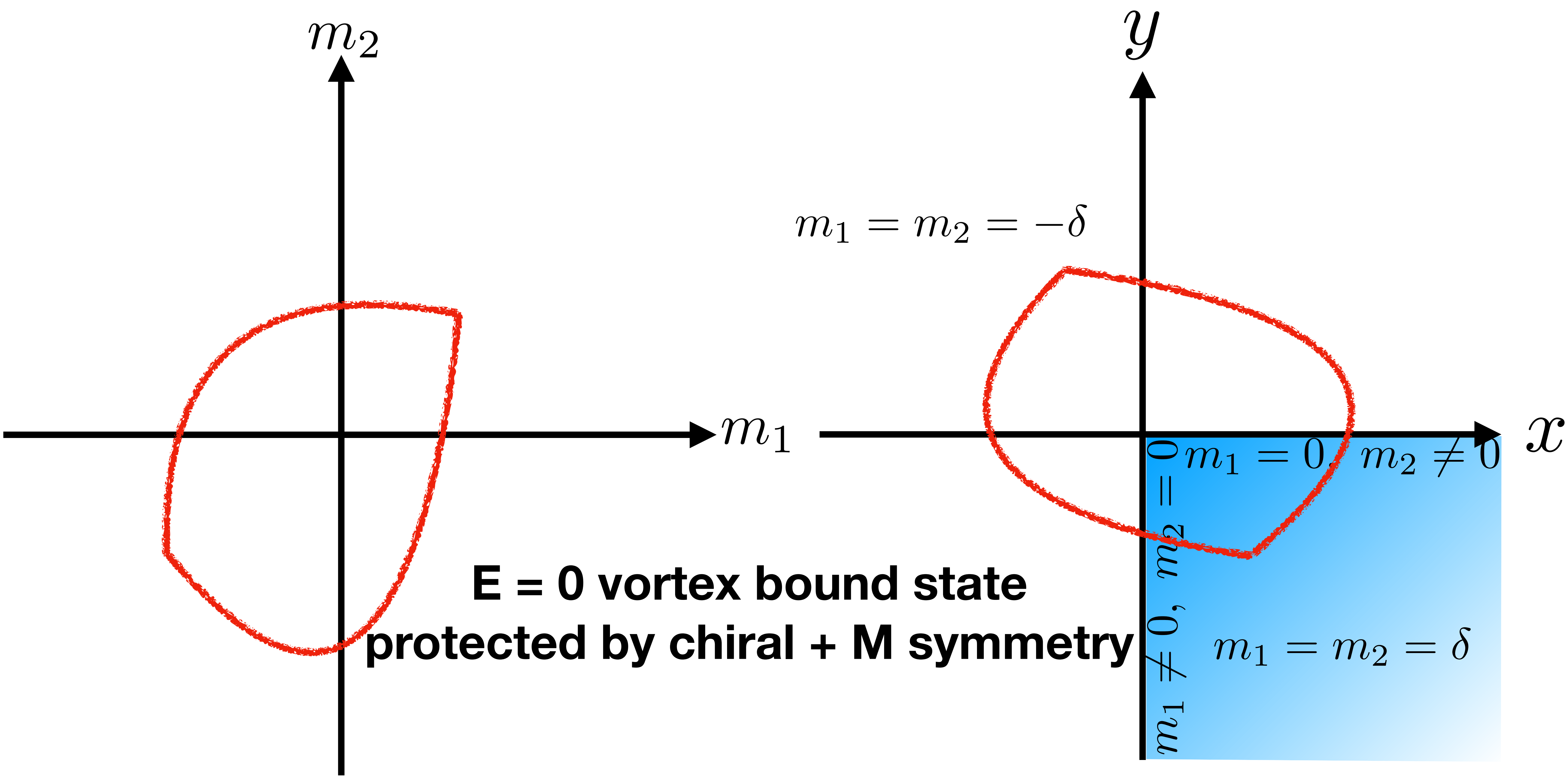
Mirror Symmetry relates different boundaries with each other and so imposes constraints on mass terms



Corner states as vortex bound states

Bulk mass switches from inside to outside, surface mass switches from one surface to the other

→ Corner-localized vortex
in „superconducting order parameter“ $m_1 + \mathrm{i}m_2$)



E = 0 vortex bound state protected by chiral + M symmetry

Mathematically equivalent to vortex on 3D TI surface → Binds a zeromode!

Topological invariant

Can define a topological invariant in presence of chiral symmetry C and mirror symmetry M

M^2 = +1 or -1 → **H block-diagonal in mirror subspaces in the BZ part invariant under M**

[M, C] = 0 →

M=+ subspace

$$\mathcal{H}(k,k) = \begin{pmatrix} 0 & q_+(k) & 0 & 0 \\ q_+(k)^\dagger & 0 & 0 & 0 \\ 0 & 0 & 0 & q_-(k) \\ 0 & 0 & q_-(k)^\dagger & 0 \end{pmatrix}$$

M=- subspace

$$\nu_\pm := \frac{\mathrm{i}}{2\pi}\int_0^{2\pi} \mathrm{d}k \, \mathrm{tr}\, \tilde{q}_\pm^\dagger(k) \partial_k \tilde{q}_\pm(k),$$

→ **spectrally flattened q**

$$\nu := \frac{\nu_+ - \nu_-}{2} \in \mathbb{Z}$$

„Mirror winding number“

(compare with mirror Chern #)

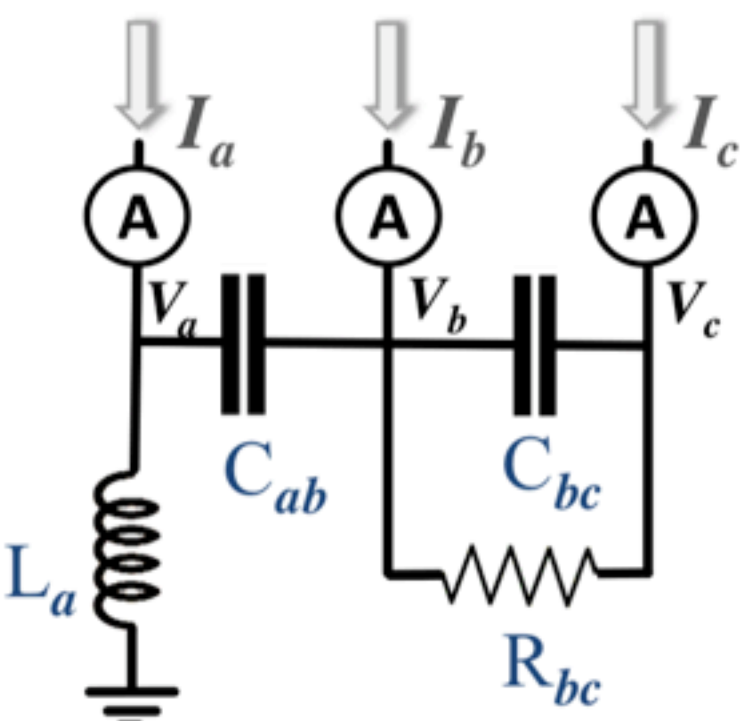
Electrical Circuits 101

Kirchhoff's Law $$I_a(\omega) = \sum_{b=1,2,\cdots} J_{ab}(\omega)\, V_b(\omega)$$

where $$J_{ab}(\omega) = \mathrm{i}\omega\, C_{ab} - \frac{\mathrm{i}}{\omega} W_{ab}.$$

$$C_{aa} = -C_{a0} - \sum_{b=1,2,\cdots} C_{ab}$$ **Capacitances**

$$W_{aa} = -L_{a0}^{-1} - \sum_{b=1,2,\cdots} L_{ab}^{-1}$$ **Inductivities**



Circuit Ground

$$W = \begin{pmatrix} (i\omega \mathrm{L}_a)^{-1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Adjacency matrix

$$C = \begin{pmatrix} 0 & i\omega \mathrm{C}_{ab} & 0 \\ i\omega \mathrm{C}_{ab} & 0 & i\omega \mathrm{C}_{bc} + \mathrm{R}_{bc}^{-1} \\ 0 & i\omega \mathrm{C}_{bc} + \mathrm{R}_{bc}^{-1} & 0 \end{pmatrix}$$

Total node conductance

$$D = \begin{pmatrix} i\omega \mathrm{C}_{ab} & 0 & 0 \\ 0 & i\omega(\mathrm{C}_{ab} + \mathrm{C}_{bc}) + \mathrm{R}_{bc}^{-1} & 0 \\ 0 & 0 & i\omega \mathrm{C}_{bc} + \mathrm{R}_{bc}^{-1} \end{pmatrix}$$

Linear Response

$G(\omega) = J^{-1}(\omega)$ **Circuit Green's Function**

determines the impedance between two sites a and b

$$Z_{ab}(\omega) = G_{aa}(\omega) + G_{bb}(\omega) - G_{ab}(\omega) - G_{ba}(\omega)$$

→ **Impedance dominated by smallest eigenvalues of** $J_{ab}(\omega)$

EoM $$\sum_{b=1,2,\cdots} C_{ab} \frac{\mathrm{d}^2}{\mathrm{d}t^2} \phi_b(t) + \sum_{b=1,2,\cdots} W_{ab} \phi_b(t) = 0.$$

determines the eigenfrequencies of the circuit for which $J_{ab}(\omega)$ **has a zeromode.**

→ **Spectrum** ω^2 **of eigenmodes determined by eigenvalues of**

$$D = C^{-1/2} W C^{-1/2}$$

Topology in Circuits

Spectrally isolated zeromodes of $J_{ab}(\omega)$ dominate the linear response.

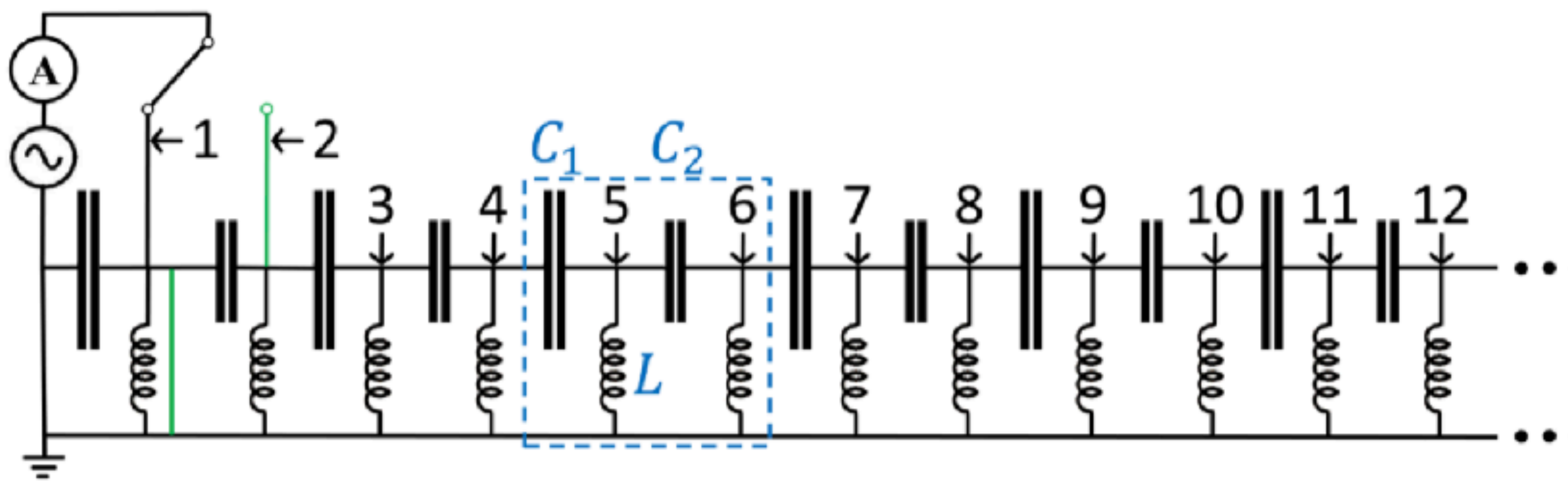
Such modes can be of topological origin

(just as in the topological band theory of non-interacting, quantum mechanical electrons)

Quantum Mechanics	Electrical Circuits
Locality of interactions	Circuit connectivity determined by lattice
Energy gap (allows to define adiabaticity)	Gap in spectrum of $J(\omega_0)$ about 0 (= gap in spectrum of D around ω_0)
Gapless boundary modes contributing to macroscopic observables	Gapless zeromodes of $J(\omega_0)$, implying peaks in the impedance profile

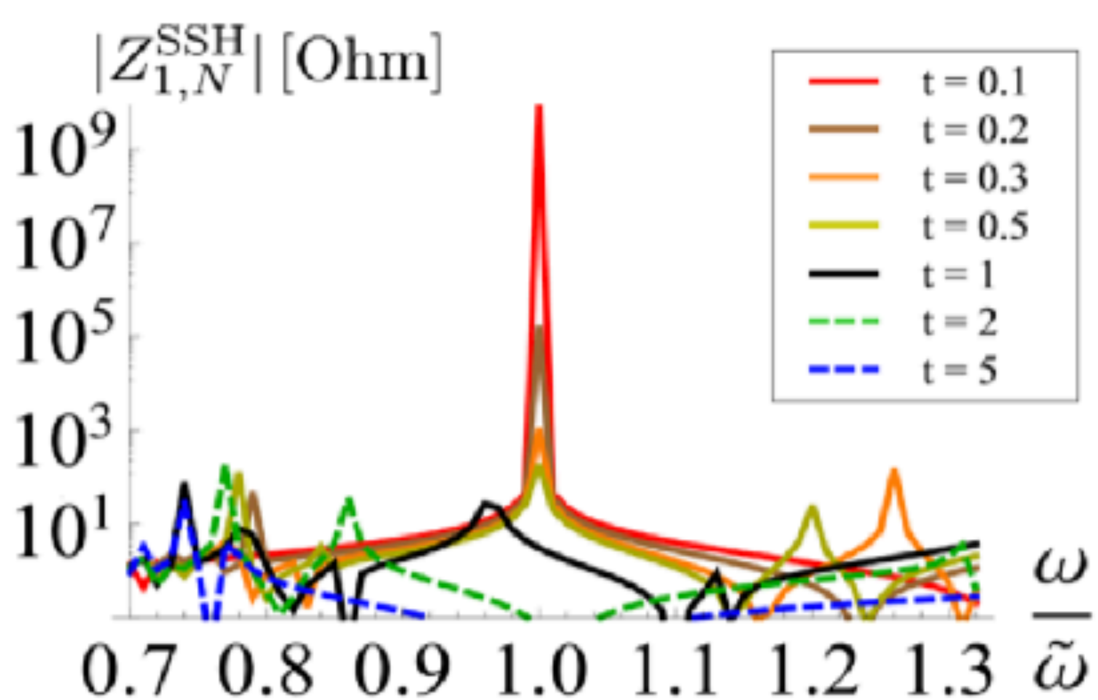
Example: SSH circuit

C. H. Lee, S. Imhof, C. Berger, F. Bayer, J. Brehm, L. W. Molenkamp, T. Kiessling, and R. Thomale, arXiv:1705.01077.



Alternating capacitances C1 > C2 with inductances to ground

Circuit Laplacian is equivalent to the tight-binding Hamiltonian of a a SSH chain



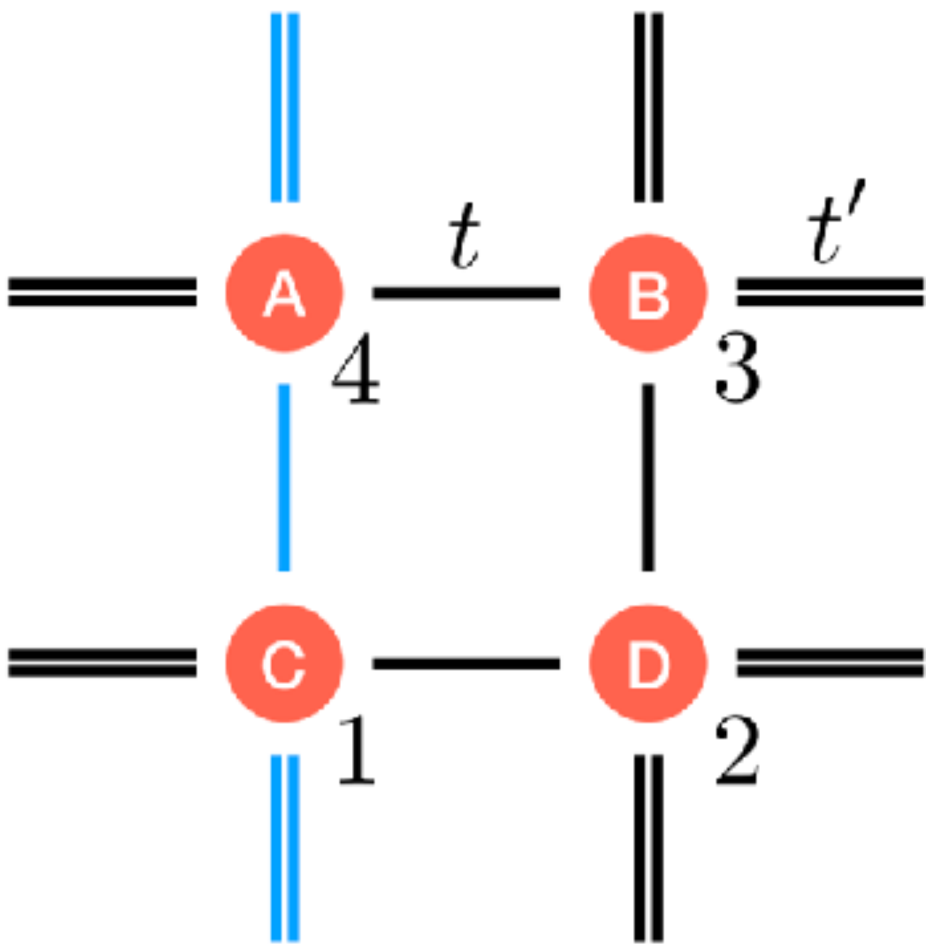
Impedance measurement:
End mode dominates response
(t = C1/C2)

Corner Circuit Design

Reminder: $J_{ab}(\omega) = \mathrm{i}\omega\, C_{ab} - \frac{\mathrm{i}}{\omega} W_{ab}.$

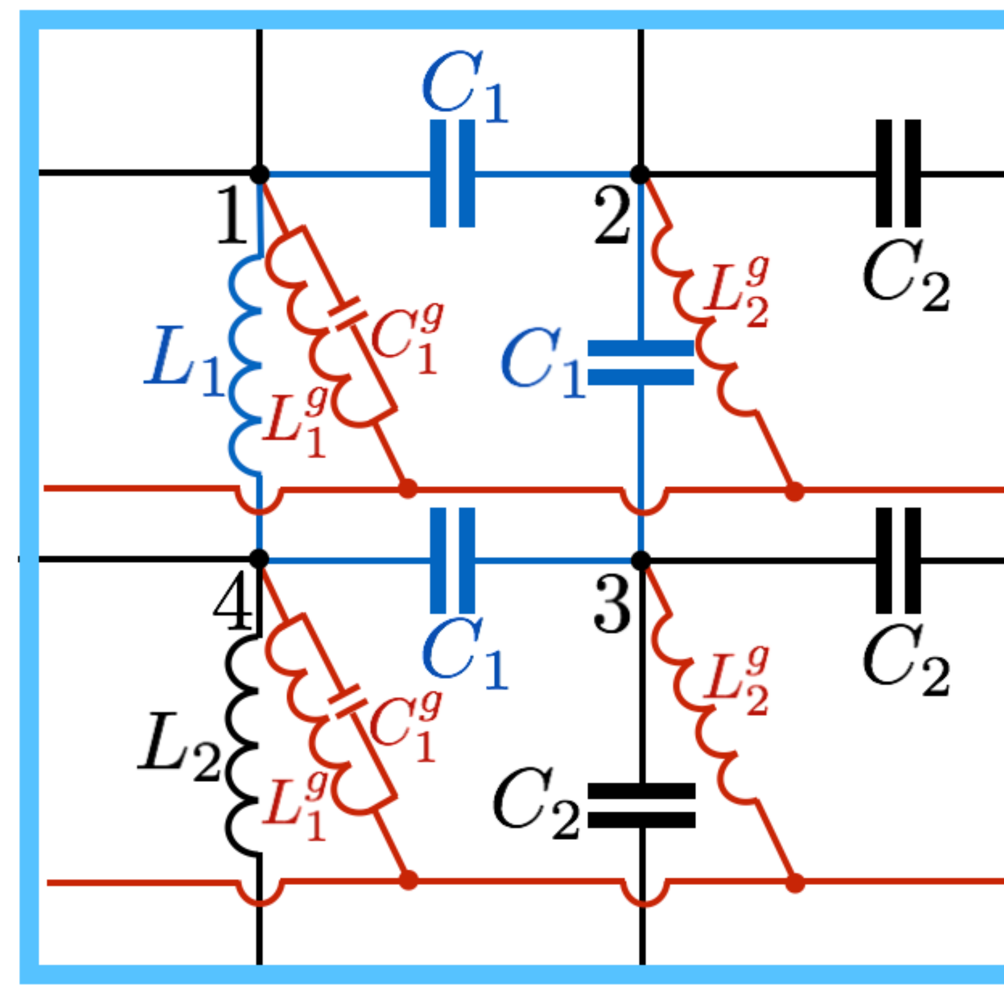
$$C_{aa} = -C_{a0} - \sum_{b=1,2,\cdots} C_{ab}$$

$$W_{aa} = -L_{a0}^{-1} - \sum_{b=1,2,\cdots} L_{ab}^{-1}$$



Mathematical Equivalence

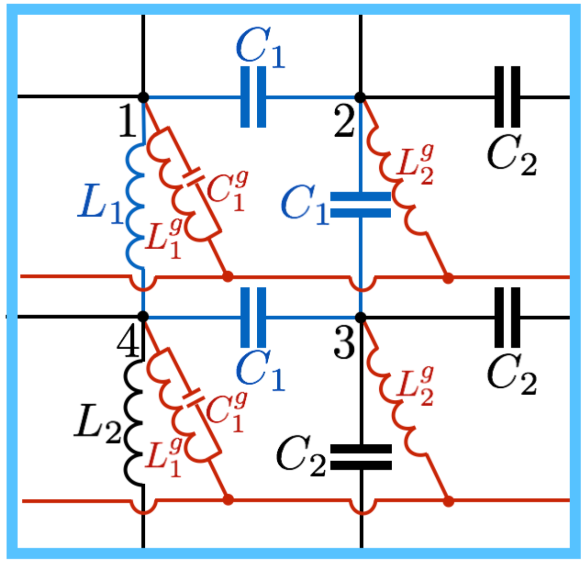
$=$



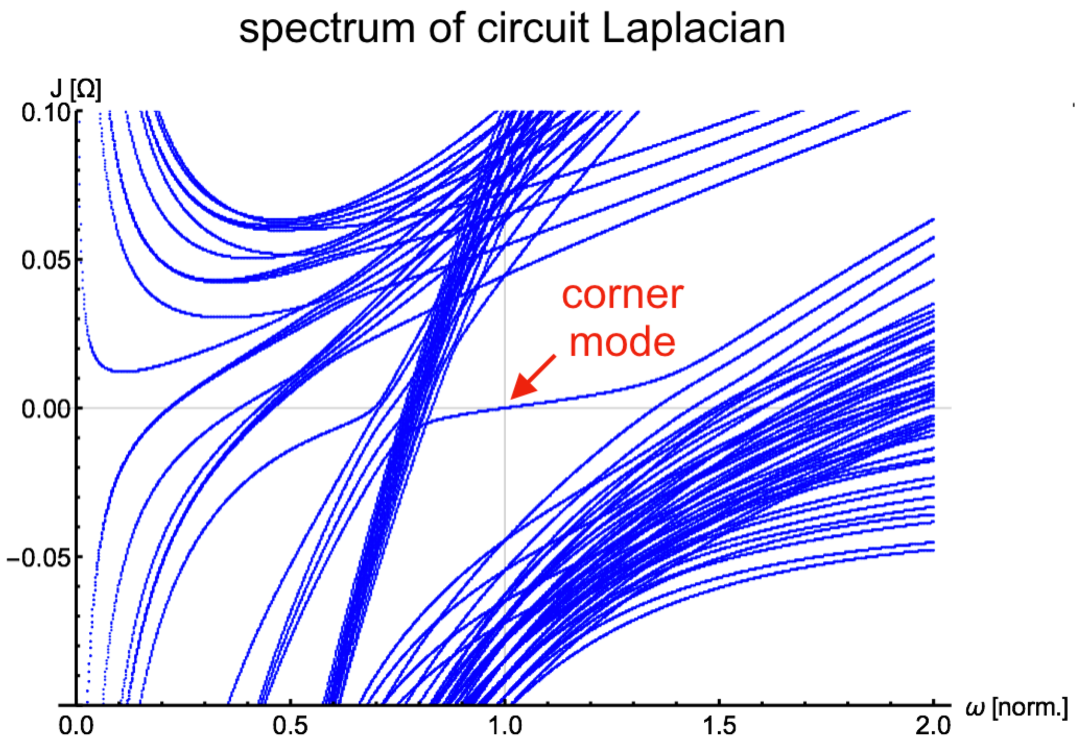
Corner Circuit Design

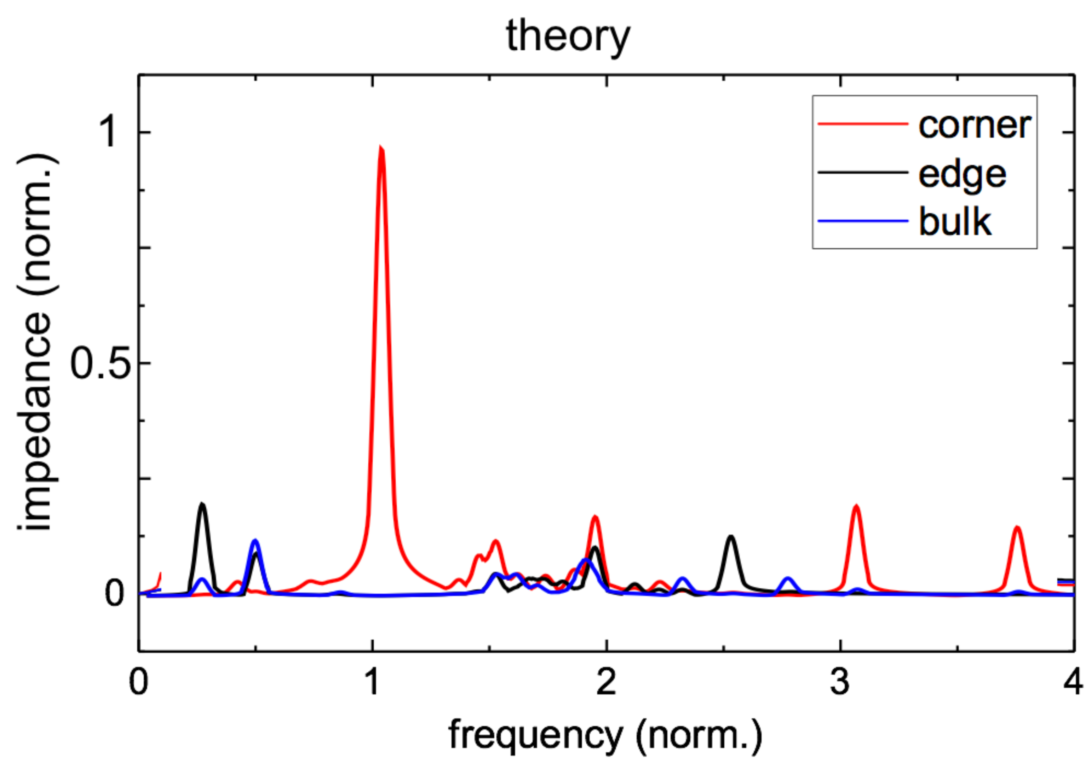
Choose a geometry with one trivial and one non-trivial corner termination (so that both can be measured in the same system)

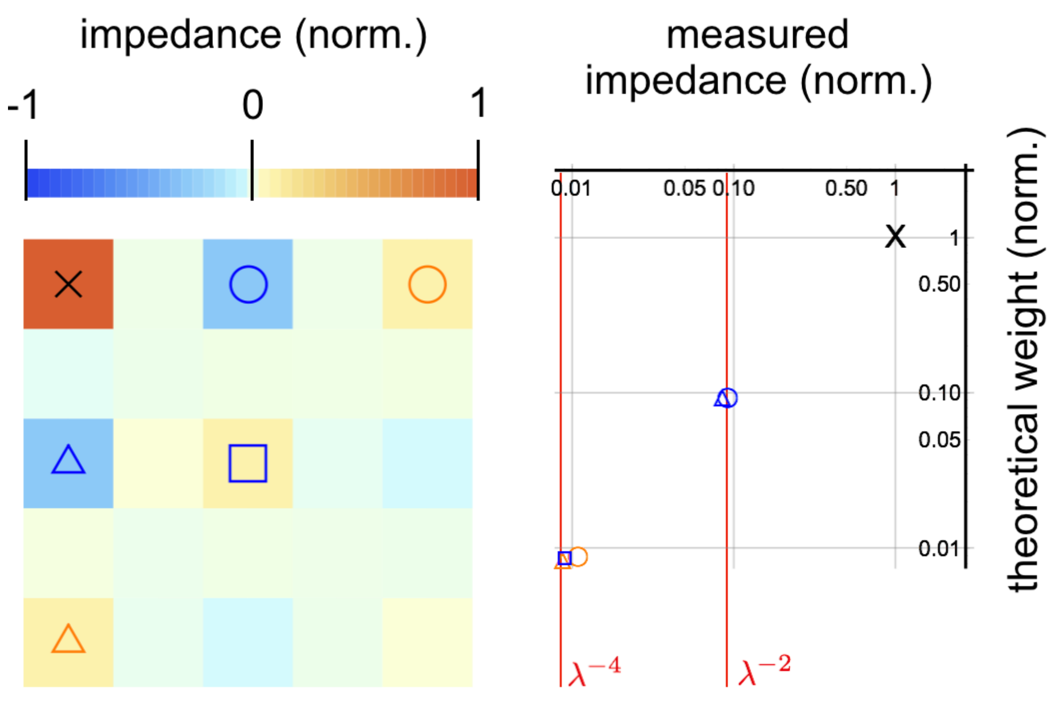
add disorder to make it more realistic!

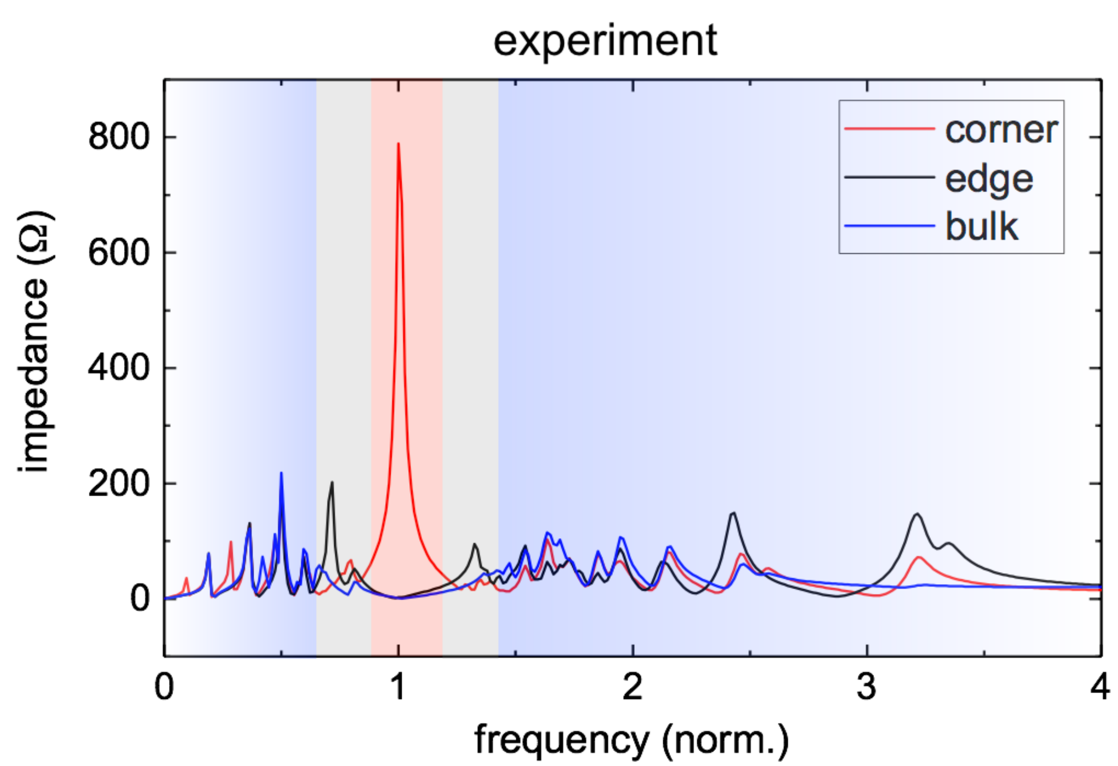


Impedance Measurement









Summary and Outlook

Topolectrical Circuits as a playground to explore concepts from band theory

… but not only those! Can add nonlinearities, domain walls, aperiodic lattices, and consider arbitrary lattice connectivities, and so, arbitrary dimensions

First-order topological phases usually require T^2 = -1

topology in classical analogues of these first-order Tis is not protected

… the second-order 2D TI realized here however only requires spatial symmetries, and its topology is thus protected even on the classical level

One other recent experimental realization of a 2nd order 2D TI in an analogue system:

Observation of a phononic quadrupole topological insulator
M. Serra-Garcia, V. Peri, R. Süsstrunk,
O. R. Bilal, T. Larsen, L. G. Villanueva, and S. D. Huber, arXiv:1708.05015 .