Pairing Obstructions in Topological Superconductors

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Real-space picture of topology

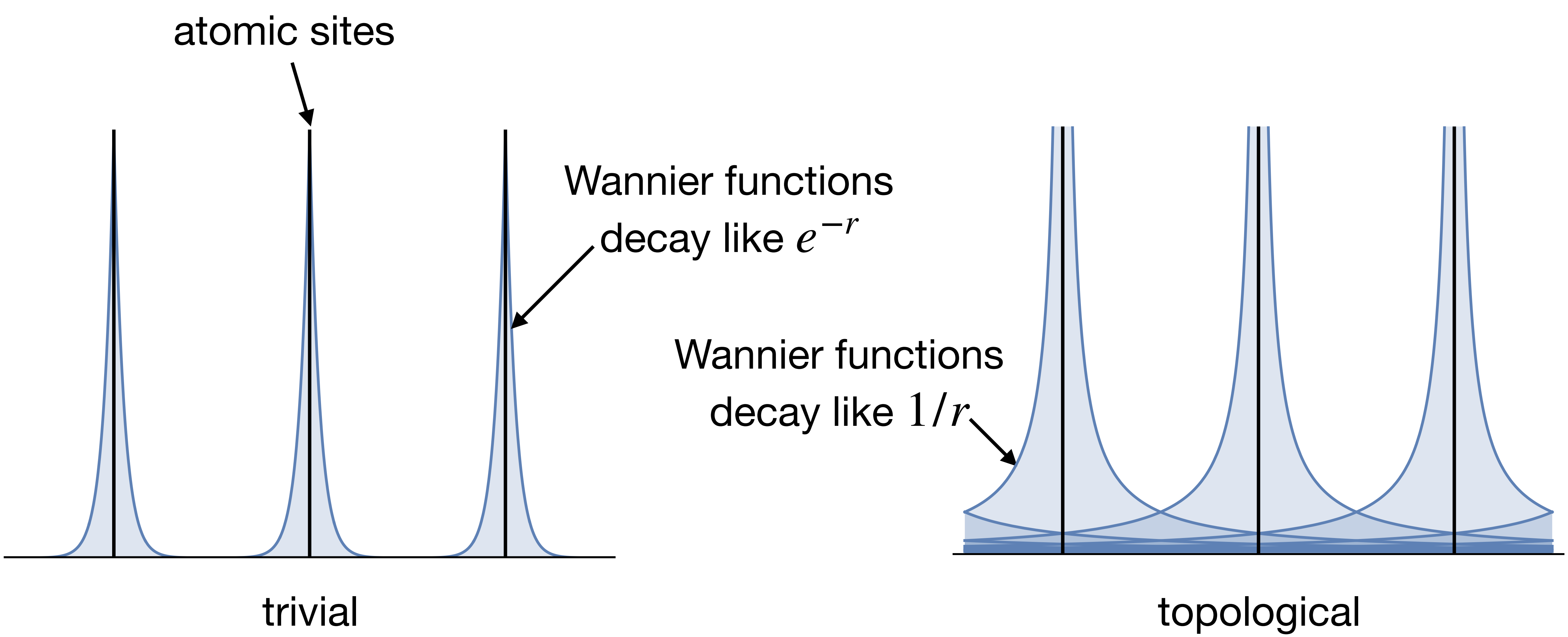
Insulators have **momentum** space topological invariants,

e.g. the Chern number $C = \int \mathrm{d}^2 k\, F$

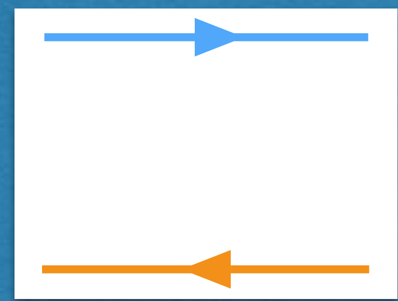
Uncertainty principle:

global properties in **momentum** space $\Longleftrightarrow$ local properties in **real** space

Wannier function $W_R = \int \mathrm{d}k\, e^{\mathrm{i}kR}\, \Psi_k$ ← Bloch function



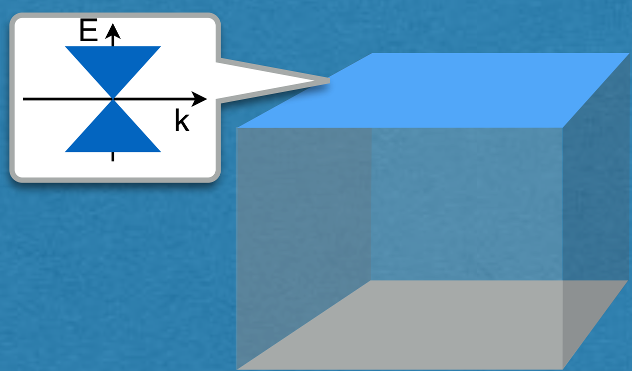
2D Chern Insulator



No exponentially localizable Wannier functions

Brouder, C., et al. (2007). Physical review letters, 98(4), 046402

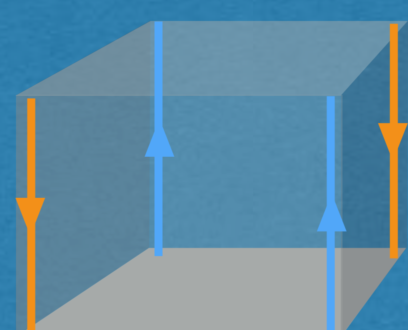
3D Topological Insulator



No exponentially localizable Wannier functions that satisfy time-reversal symmetry

Soluyanov, A. A., & Vanderbilt, D. (2011). Physical Review B, 83, 035108

3D Higher-Order TI



No exponentially localizable Wannier functions that satisfy crystalline symmetry

Topology as a Wannier Obstruction

unified picture of topological insulators, obstructed atomic limits, fragile phases, …

How does this generalize to superconductors?

Topological superconductors exist in 1D,
but 1D Wannier functions are *always* exponentially localized!

Fundamental difference between 1D and 2D

Wannier function $W_R = \int \mathrm{d}k\, e^{ikR}\, \Psi_k$

Bloch function: can always be made smooth in 1D by parallel transport

in 2D, not necessarily! "smoothing out" does not commute

Chern # = [parallel transport in x, parallel transport in y]

Periodic Table of Topological Superconductors

No Wannier obstruction in 1D

Wannier obstruction due to Chern #

q	$\pi_0(R_q)$	$d=1$	$d=2$	$d=3$
0	$\mathbb{Z}$		no symmetry (p_x+ip_y, e.g., SrRu)	T only (^{3}He-B)
1	$\mathbb{Z}_2$	no symmetry (Majorana chain)	T only ($(p_x+ip_y)\uparrow+(p_x-ip_y)\downarrow$)	T and Q (BiSb)
2	$\mathbb{Z}_2$	T only ($(\mathrm{TMTSF})_2\mathrm{X}$)	T and Q (HgTe)	
3	0	T and Q		

(Kitaev, 2009)

BdG Mean-Field Theory

$$H = \sum_k \left[h_k c_k^\dagger c_k + \frac{1}{2}\left(\Delta_k c_k^\dagger c_{-k}^\dagger + \Delta_k^\dagger c_{-k} c_k \right) \right]$$

We discuss two choices of basis:

$$H = \sum_k \Psi_k^\dagger \mathscr{H}_k \Psi_k, \quad \Psi_k = \frac{1}{\sqrt{2}} \begin{pmatrix} c_k \\ c_{-k}^\dagger \end{pmatrix}, \quad \mathscr{H}_k = \begin{pmatrix} \epsilon_k & \Delta_k \\ \bar{\Delta}_k & -\epsilon_{-k} \end{pmatrix}.$$

(1) Nambu basis

$$\begin{pmatrix} c_k \\ c_{-k}^\dagger \end{pmatrix} = \begin{pmatrix} 1 & +\mathrm{i} \\ 1 & -\mathrm{i} \end{pmatrix} \left[\frac{1}{2} \begin{pmatrix} a_k \\ b_k \end{pmatrix} \right]$$

(2) Majorana basis

They provide complementary physical interpretations

Fundamental origin of difference: $c_k^2 = (c_{-k}^\dagger)^2 = 0$, **whereas** $a_k^2 = b_k^2 = 1$.

What replaces Wannier functions?

For non-interacting insulators, Wannier functions show up in the ground state:

$$|\Omega\rangle = \prod_k \left(u_k c_k^\dagger\right)|0\rangle = \prod_R \left(\sum_r W_R(r) c_r^\dagger\right)|0\rangle$$

Read & Green (2000) studied the ground state of a **2D** p-wave superconductor:

$$\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_N) = \frac{1}{2^{N/2}(N/2)!}\sum_P \operatorname{sgn} P \times \prod_{i=1}^{N/2} g(\mathbf{r}_{P(2i-1)} - \mathbf{r}_{P(2i)})$$

$g(\mathbf{r}) \sim e^{-r/r_0}$ trivial, C = 0

$g(\mathbf{r}) \propto 1/z$ topological, C = 1

Compare this with the IQHE:
Chern # as an obstruction
to finding a smooth gauge
(Wannier obstruction)

(1) Is this property adiabatically stable, i.e., in one-to-one correspondence with topology?

(2) How is this "pairing obstruction" related to Wannier obstructions, can we apply it to 1D?

BdG Mean-Field Theory $H = \sum_k \left[h_k c_k^\dagger c_k + \frac{1}{2} \left(\Delta_k c_k^\dagger c_{-k}^\dagger + \Delta_k^\dagger c_{-k} c_k \right) \right]$

ground state: $|\Omega\rangle \propto \exp\left(\sum_k \frac{v_k}{u_k} c_k^\dagger c_{-k}^\dagger \right) |0\rangle \propto \exp\left(\sum_{xy} g_{xy} c_x^\dagger c_y^\dagger \right) |0\rangle$

$$g_{xy} = \int \frac{\mathrm{d}k}{2\pi} e^{\mathrm{i}k(x-y)} \frac{v_k}{u_k} .$$

decays exponentially $\longleftrightarrow$ smooth in k space

Topological invariant: Berry phase $\gamma = \mathrm{i} \int \mathrm{d}k \; \left(\bar{u}_k \partial_k u_k + \bar{v}_k \partial_k v_k \right) \quad \mathrm{mod}\ 2\pi$

PHS implies $\gamma = 0, \pi$.

Theorem

$$\gamma = \pi \longleftrightarrow u_0\, u_\pi = 0$$

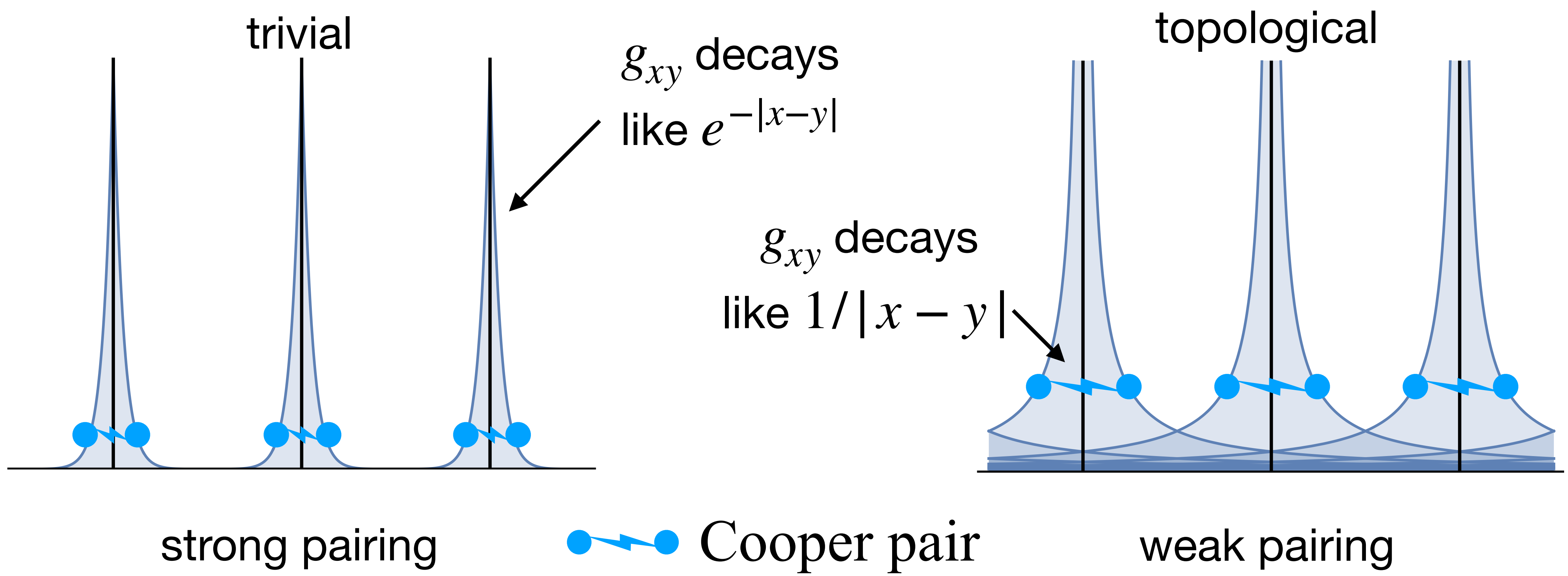
Pairing obstruction

$$|\Omega\rangle \propto \exp\left(\sum_{xy} g_{xy} c_x^\dagger c_y^\dagger\right)|0\rangle, \quad g_{xy} = \int \frac{\mathrm{d}k}{2\pi} e^{\mathrm{i}k(x-y)} \frac{v_k}{u_k}$$

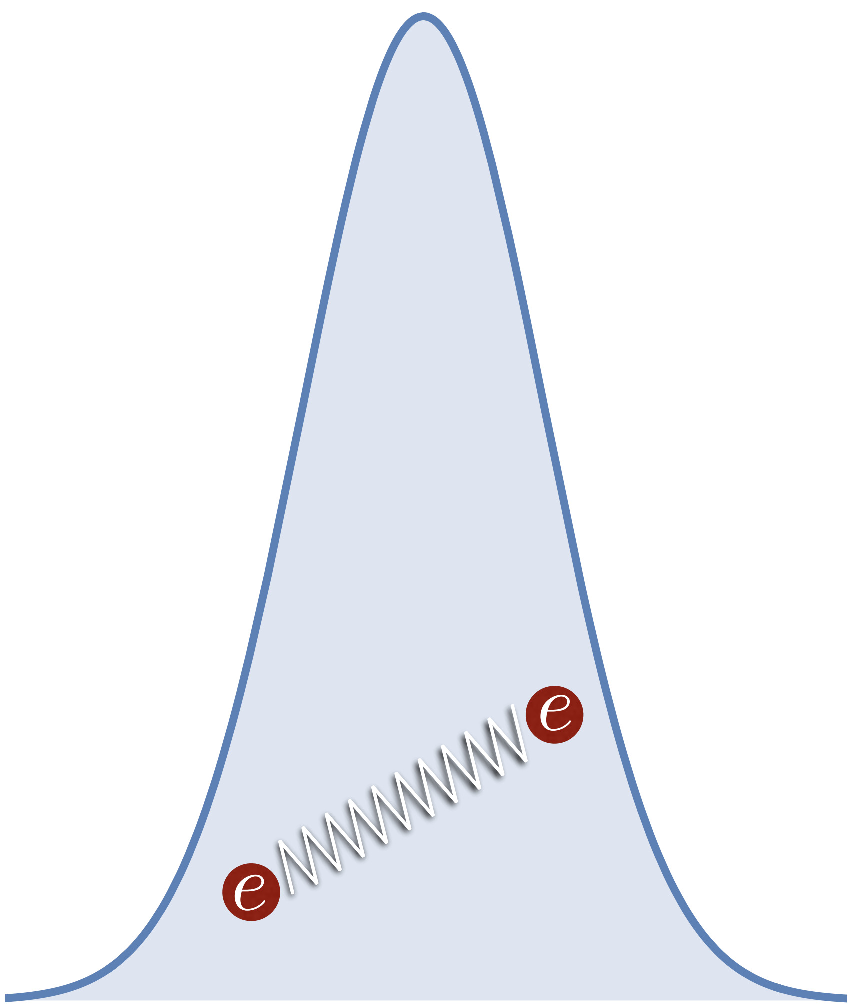
$$\boxed{\gamma = \pi \Longleftrightarrow u_0 u_\pi = 0}$$

in the topological phase, v_k/u_k is ill defined at 0 or π

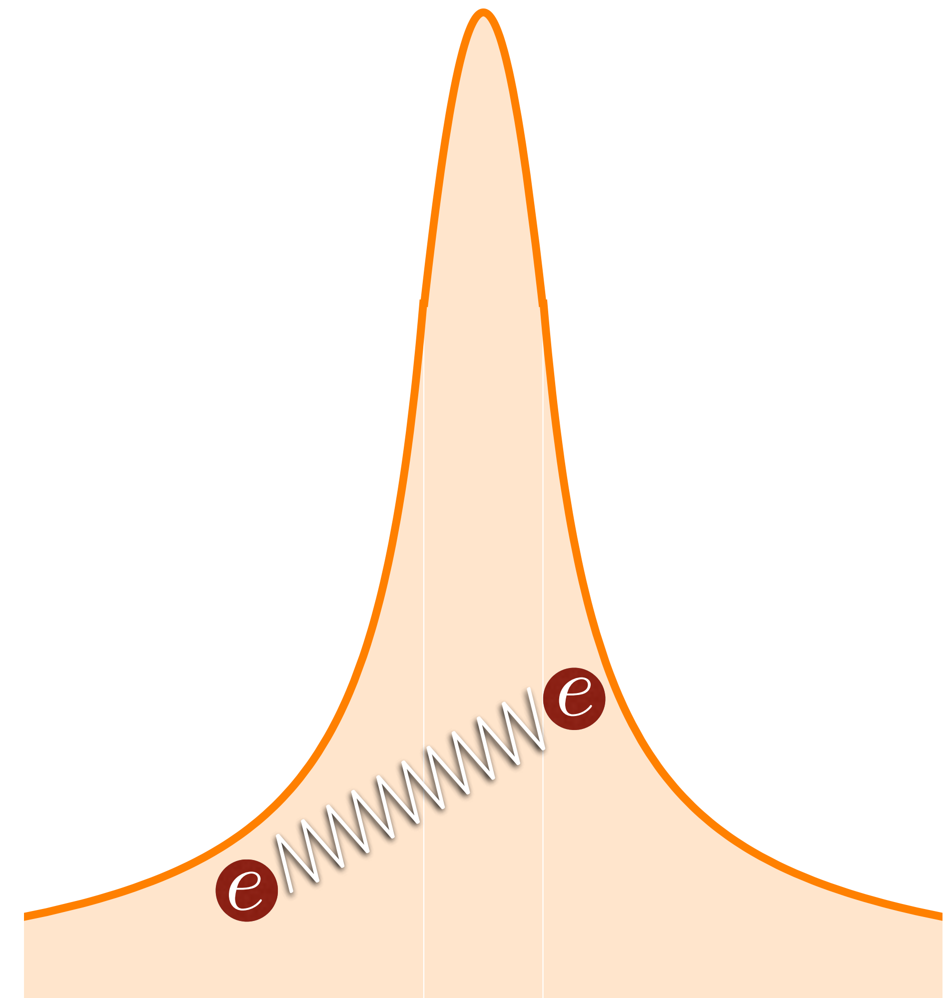
g_{xy} decays only polynomially



Thank you for your attention!



Cooper
pairs



trivial

(exponential decay)

topological

(polynomial decay)

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