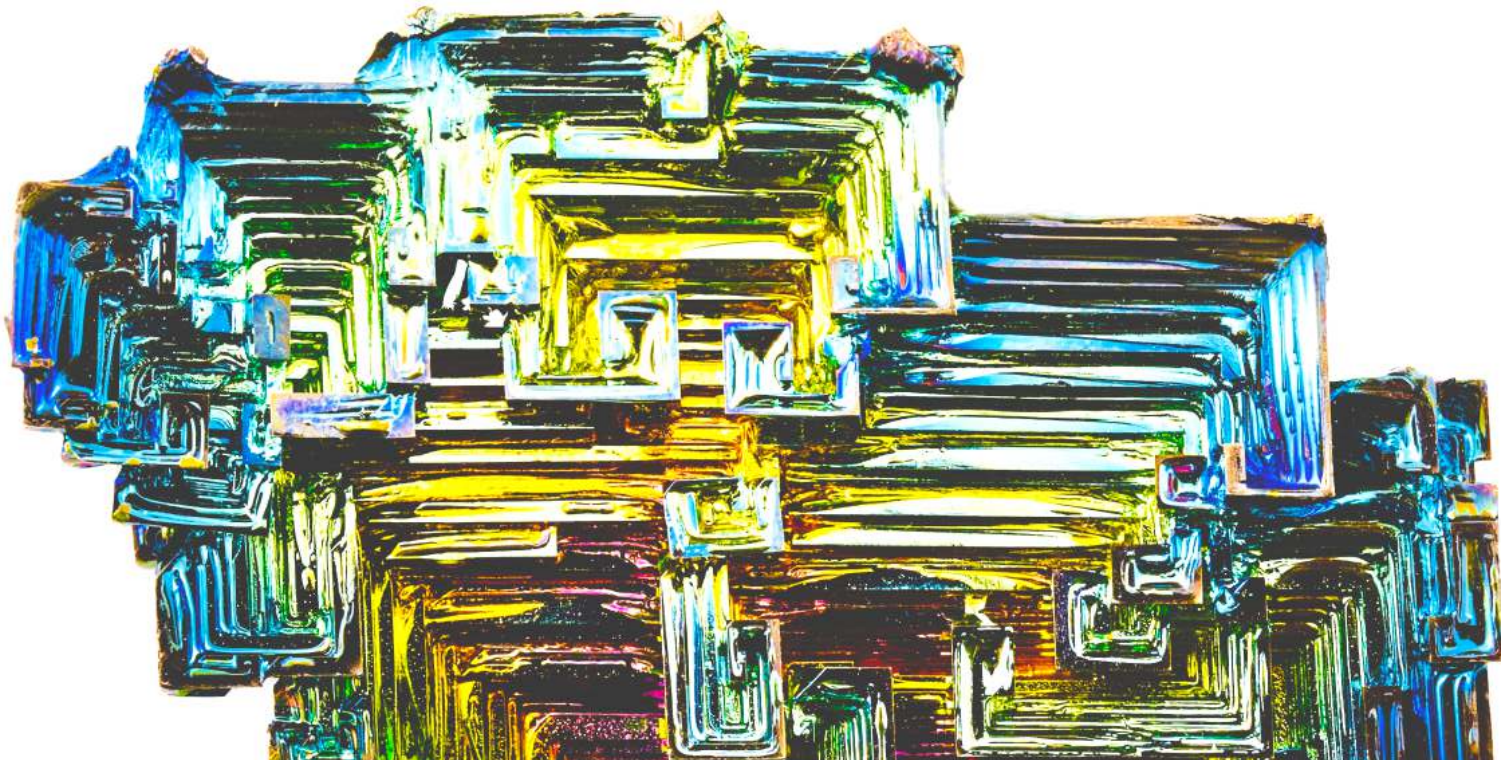


# Higher-Order Topological Insulators

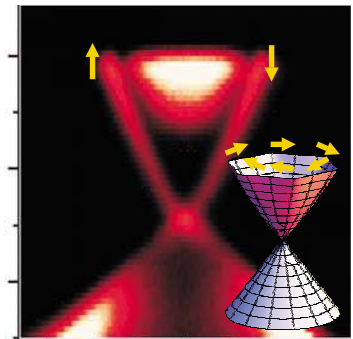
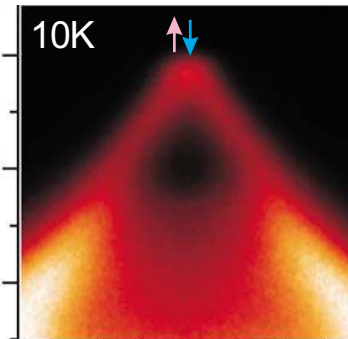
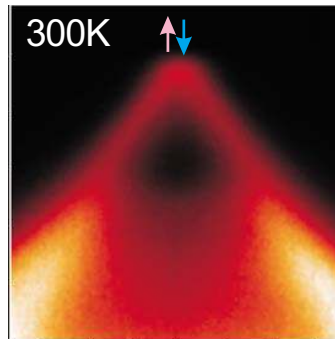
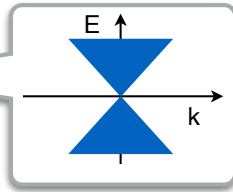
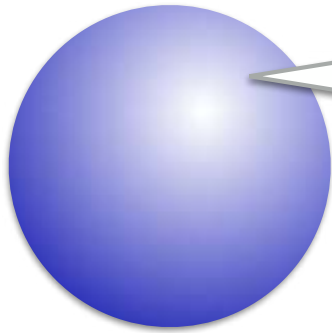
Frank Schindler, University of Zurich  
Solid State Seminar 03/10/2018



# Motivation

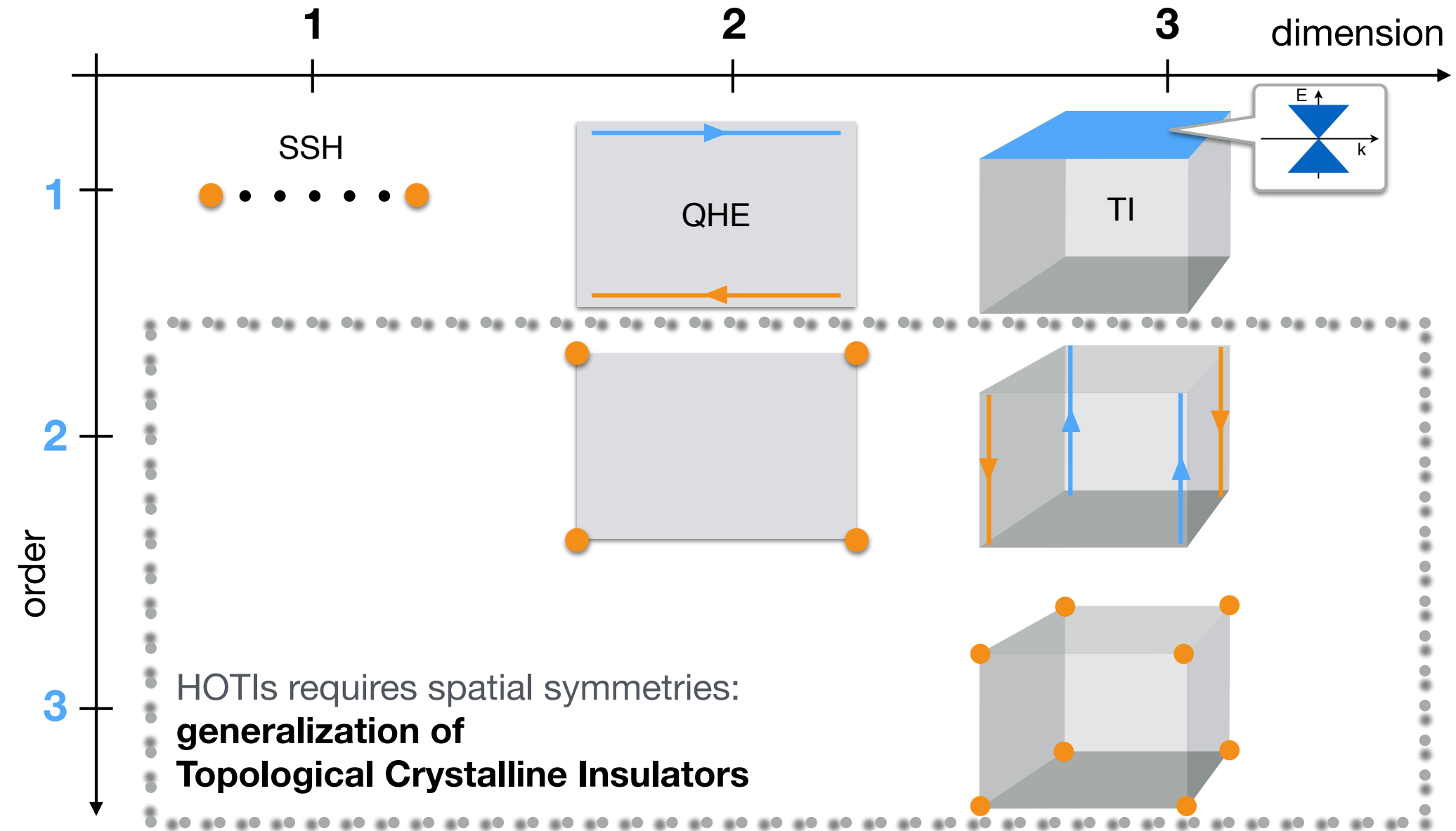
A 3D topological insulator has gapless states on its **2D surfaces**

3D TI



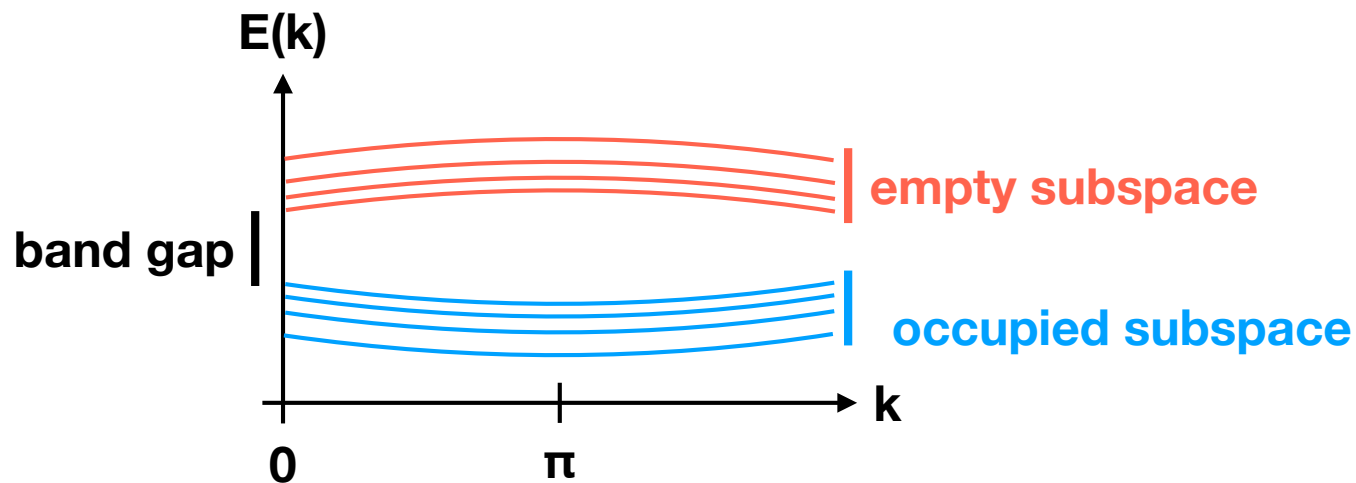
crystals also have **1D edges**!

# Overview



# Topological Crystalline Insulators

insulating, non-interacting, translationally invariant electron systems are described by a gapped Bloch Hamiltonian  $H(k)$

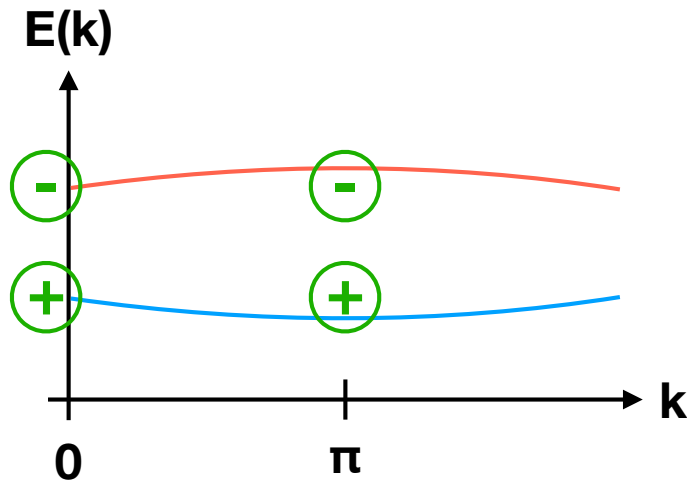


$H(k)$  of a TCI cannot be continuously deformed to that of a trivial insulator without closing the band gap or breaking crystalline symmetries

# 1D Topology from Inversion

$|H(k)|^{-1} = H(-k)$ ,  $I^2 = 1$ , so have **eigenvalues**  $\pm 1$

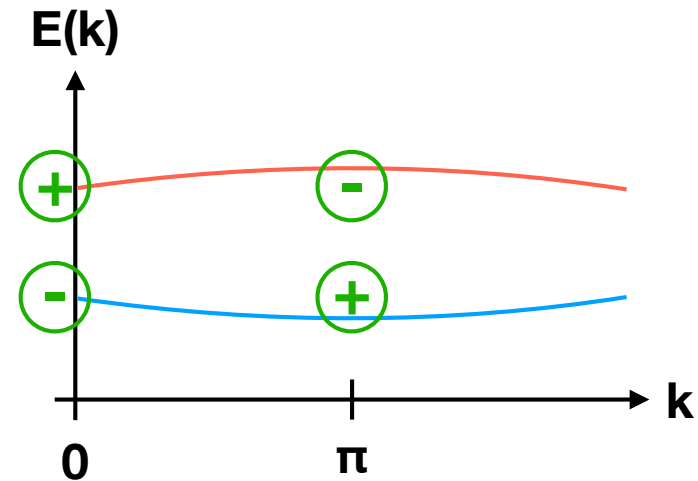
for a 1D system, have two possibilities:



trivial

• • • • •

gapped bulk  
trivial boundary



topological

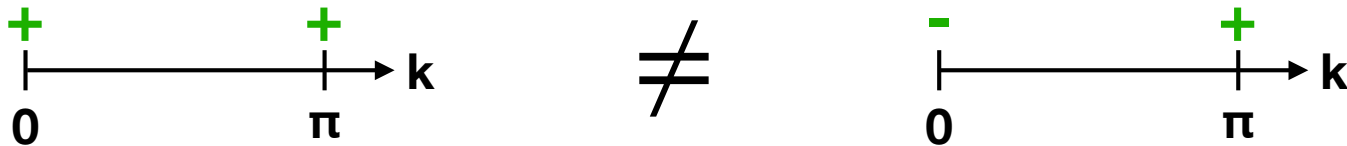
● • • • • ●

gapped bulk  
quantized 1/2 end charges

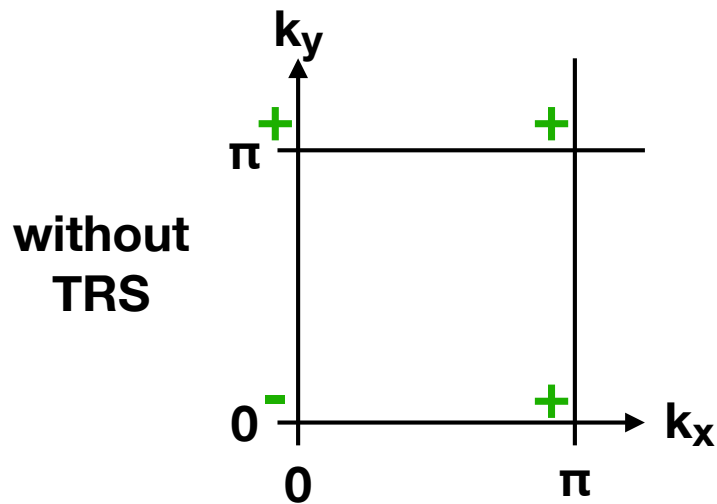
# 2D Topology from Inversion

forget about energies, they are not topological

what's important is that



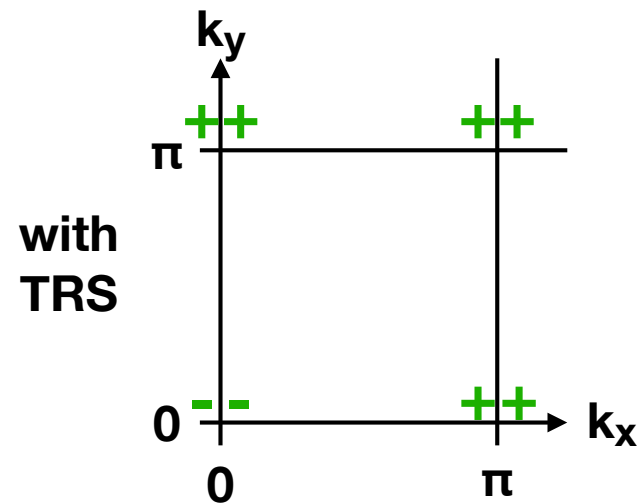
in 2D, famous examples of non-trivial inversion eigenvalues are



without  
TRS

**Chern insulator**

~ integer quantum Hall effect



with  
TRS

**2D topological insulator**

~ quantum spin Hall effect

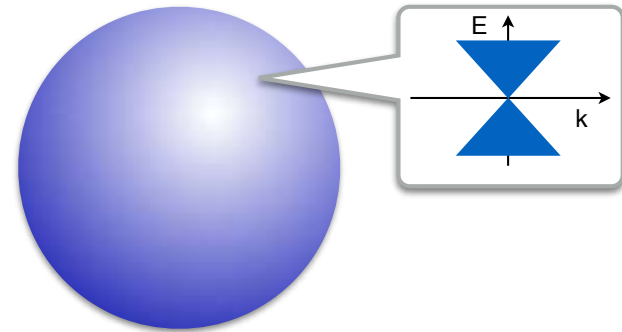
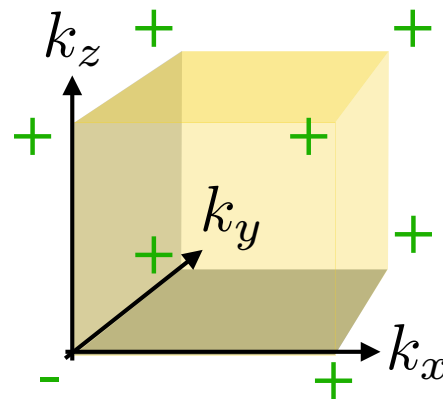
these 2D phases in fact don't even require inversion, but we can use it to diagnose them

# 3D Topology from Inversion

from now on, assume TRS:

$+$ ,  $-$  now denotes 2 Kramers paired bands with inversion eigenvalues  $\{\pm 1, \pm 1\}$

**3D TI:**



Fu-Kane 3D TI index:

$$\prod_{k \in \text{TRIM}} \xi_k$$

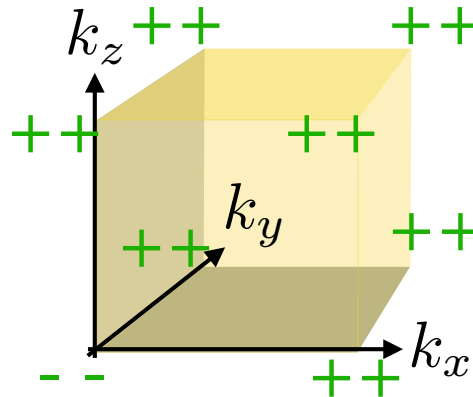
product over inversion eigenvalues  
per Kramers pair at time-reversal  
invariant momenta

evaluates to - 1



# 3D Higher-Order Topology

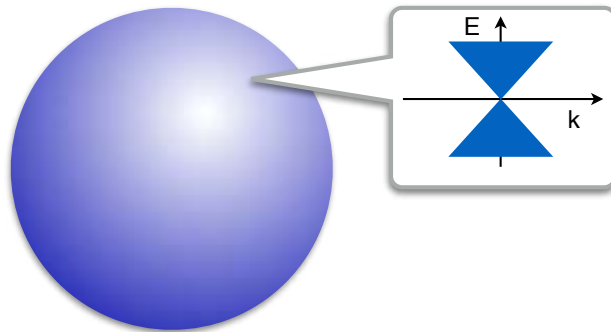
what about



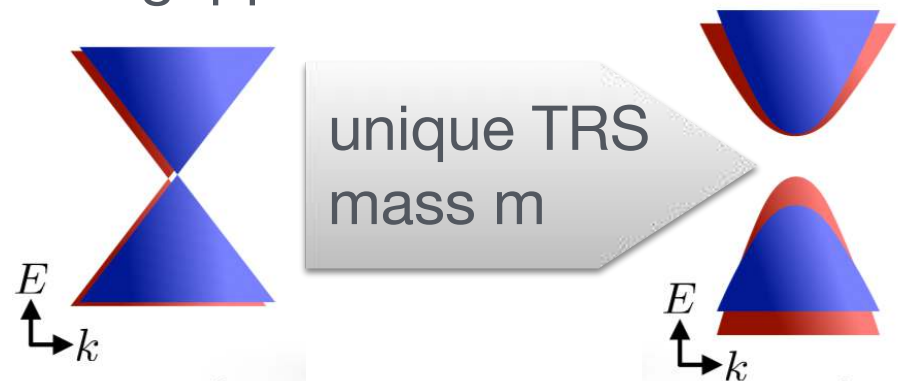
? Has Fu-Kane index +1.

= 2 x topological insulator

= 2 x



on surface:  
Two gappable Dirac cones

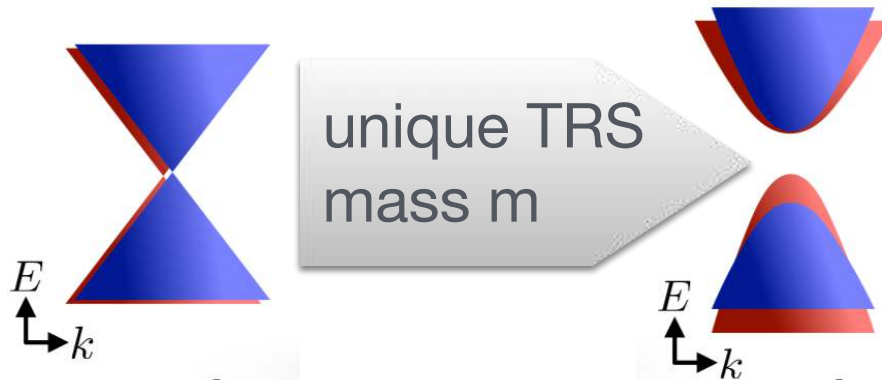


but this configuration of inversion eigenvalues is not quite trivial!

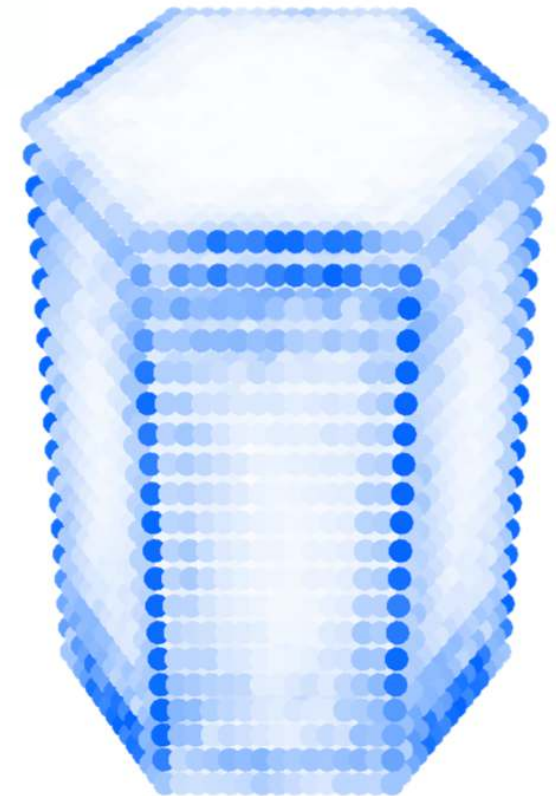
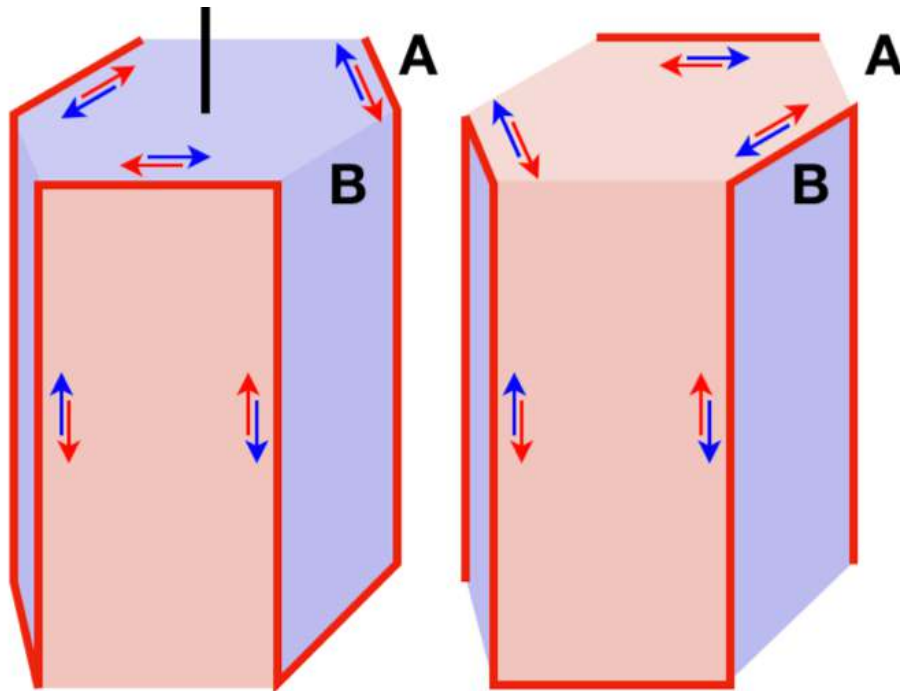


# Bulk-boundary correspondence for “double band inversion”

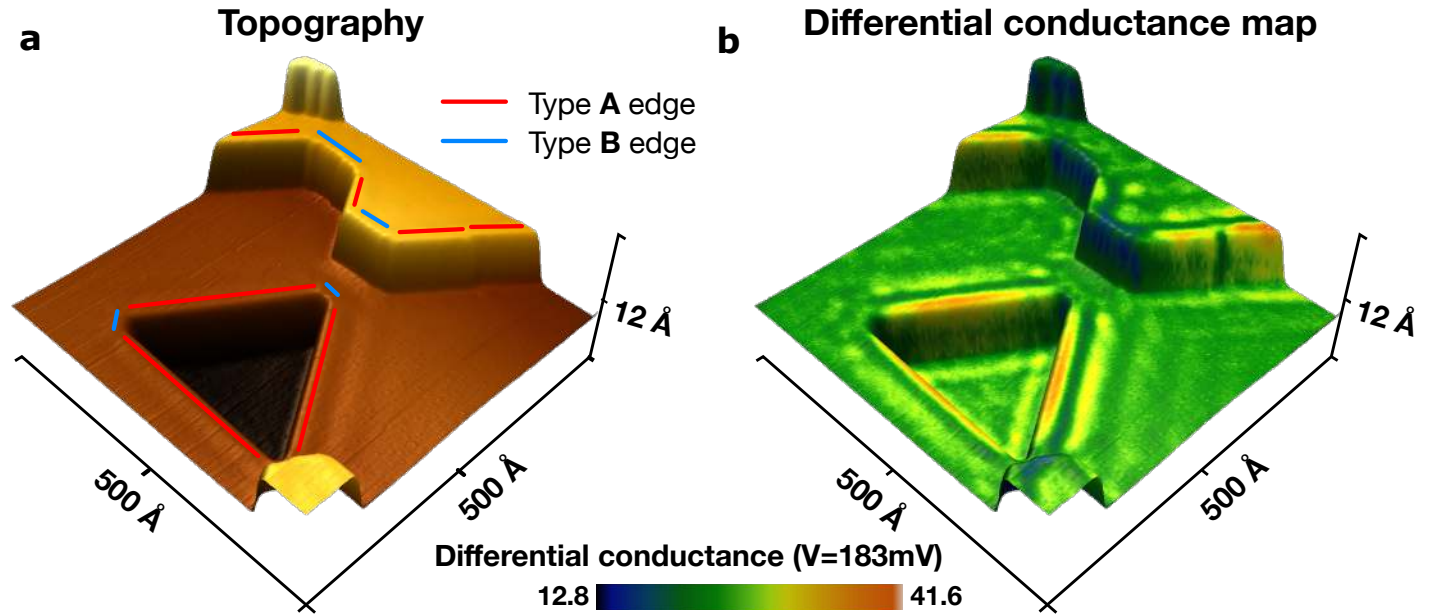
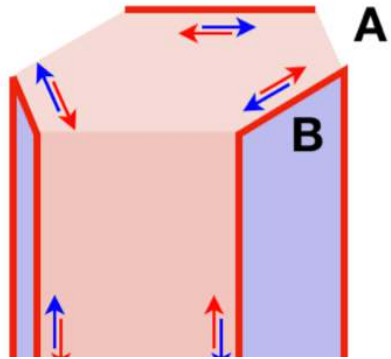
Decoupled subspaces:  
Two surface Dirac cones



$$I: \quad m \rightarrow -m$$

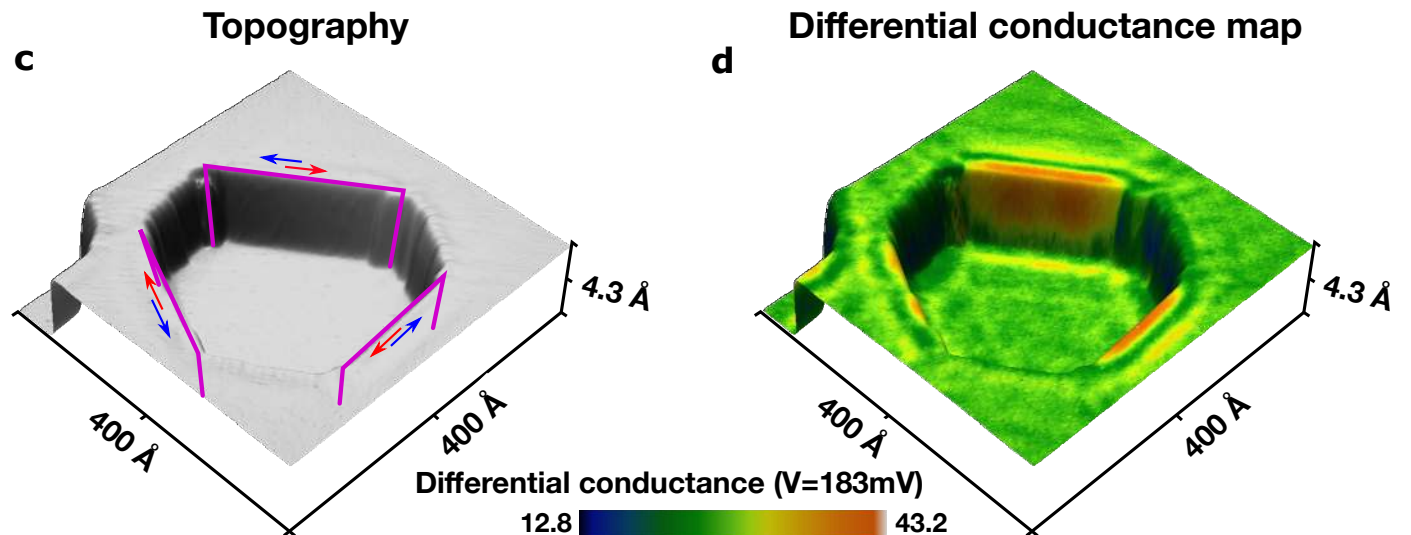


# Bismuth Experiment1) STM

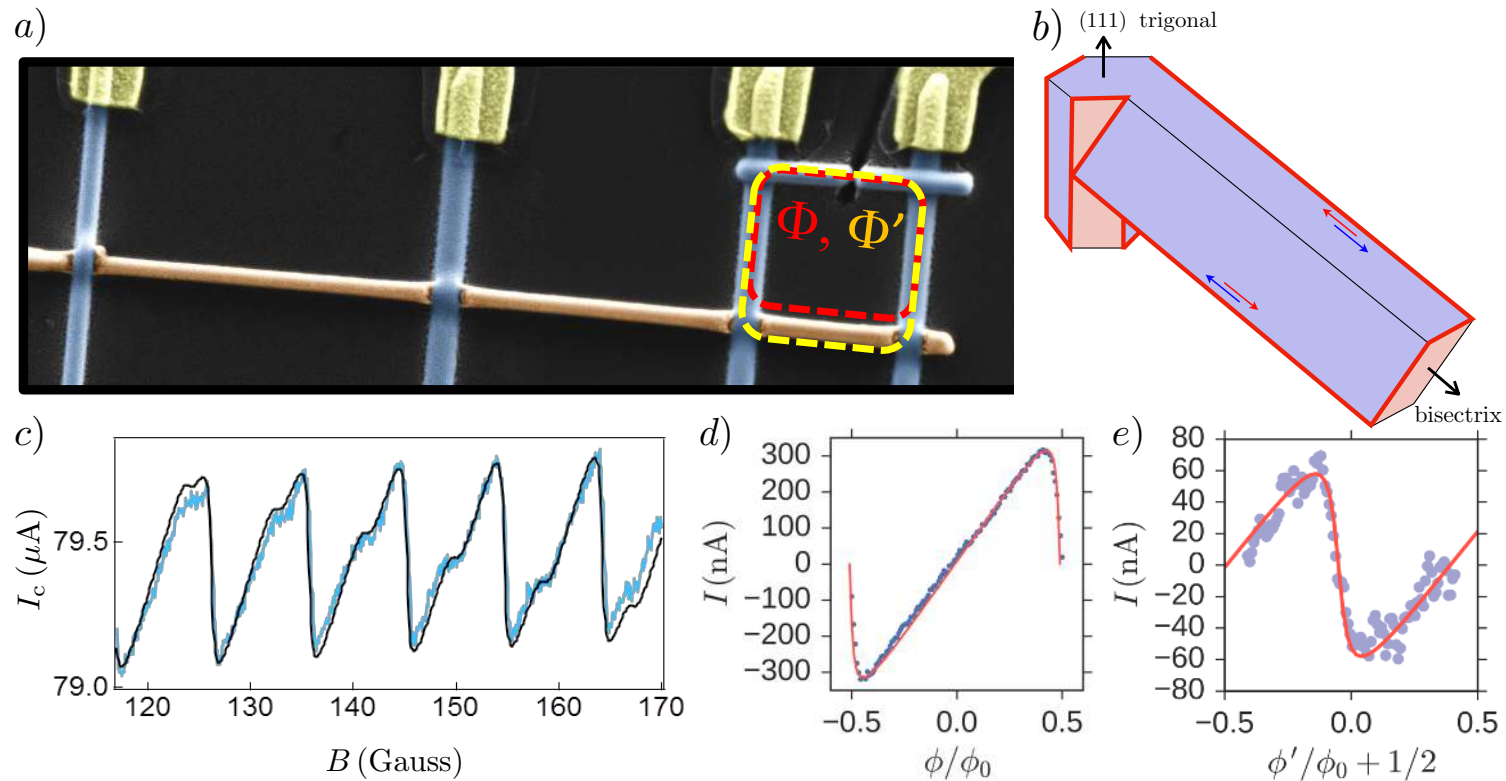


questions:

- i) thickness dependence
- ii) hybridization with lower hinge mode



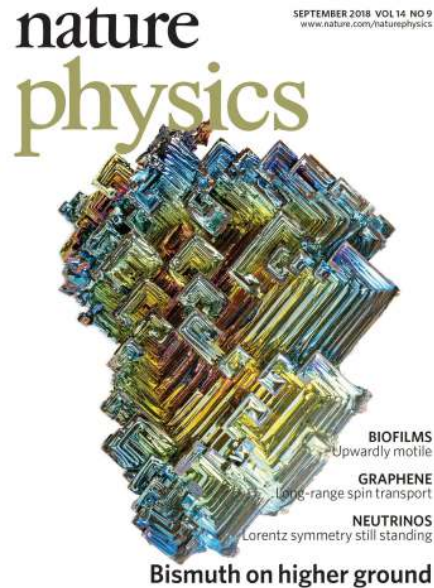
# Bismuth Experiment 2) Josephson interferometry



Sawtooth current-phase relation:

Mean free path required way longer than that of Bismuth bulk  
—> due to lossless edge states

# References



Higher-Order Topological Insulators,

F. Schindler, A. M. Cook, M. G. Vergniory, Z.J. Wang, S. S. P. Parkin, B. A. Bernevig, T. Neupert

**Science Advances, eaat0346 (2018)**

Higher-Order Topology in Bismuth,

F. Schindler, Z. J. Wang, M. G. Vergniory, A. M. Cook, A. Murani, S. Sengupta, A. Y. Kasumov, R. Deblock, S. Jeon, I. Drozdov, H. Bouchiat, S. Guéron, A. Yazdani, B. A. Bernevig, T. Neupert

**Nature Physics 14, 918 (2018)**

# Thanks to

## Theory

M. G. Vergniory,  
Z. J. Wang,  
S. S. P. Parkin,  
B. A. Bernevig,  
T. Neupert

## Experiment

A. Murani,  
S. Sengupta,  
A. Y. Kasumov,  
R. Deblock,  
S. Jeon,  
I. Drozdov,  
H. Bouchiat,  
S. Guéron,  
A. Yazdani