

Exact Quantum Scars in the Chiral Non-Linear Luttinger Liquid

$$H = \sum_p \epsilon(p) c_p^\dagger c_p + \sum_{p>0} \left[V(p) \sum_{qk} c_{q+p}^\dagger c_q c_{k-p}^\dagger c_k \right]$$

$\epsilon(p) = vp$: integrable

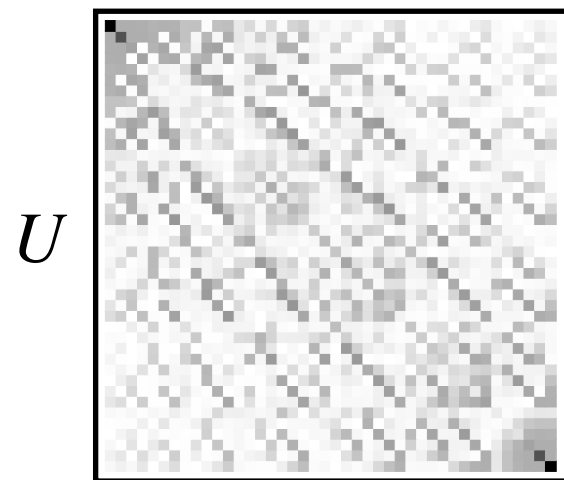
$\epsilon(p) = vp + ap^2 + \dots$: no exact solution

Bosonization is a unitary transformation:

$$\sum_{p>0} V(p) \sum_{qk} c_{q+p}^\dagger c_q c_{k-p}^\dagger c_k$$

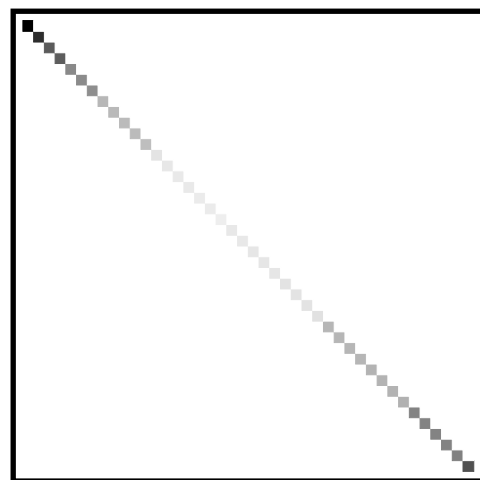
$$\sum_p p^2 c_p^\dagger c_p$$

breaks
integrability



Fermi

$U^\dagger =$



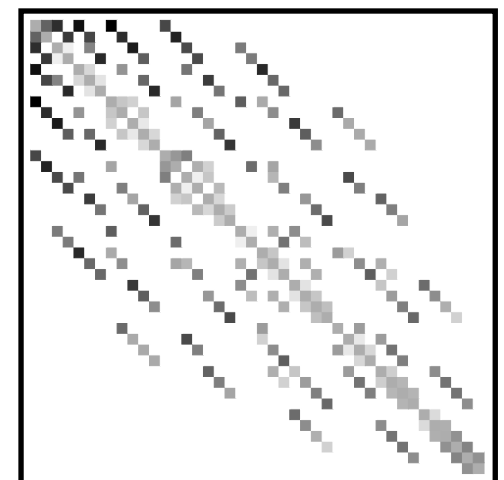
Bose

U



Fermi

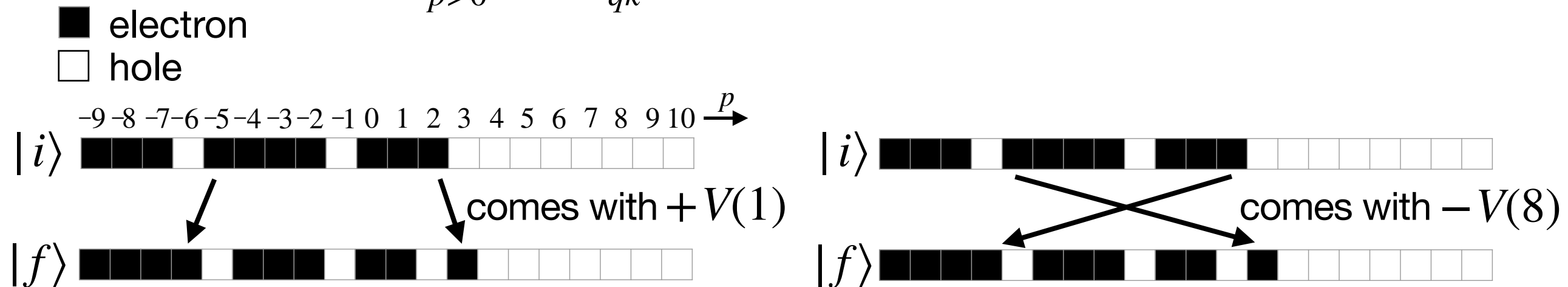
$U^\dagger =$



Bose

Exact Slater-determinant eigenstates

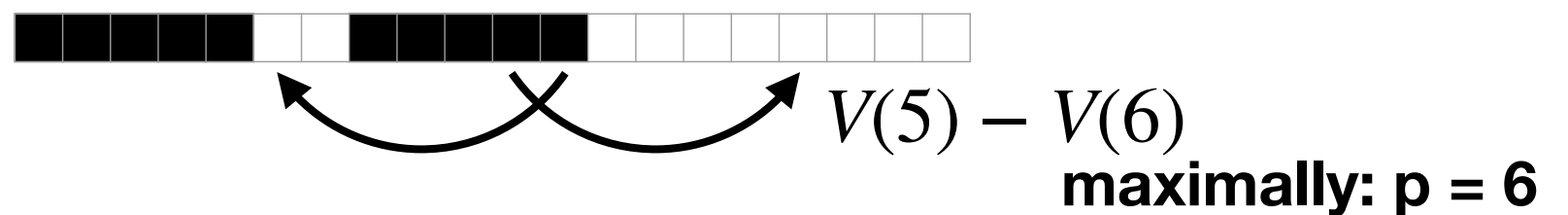
The Hamiltonian $H_{\text{int}} = \sum_{p>0} V(p) \sum_{qk} c_{q+p}^\dagger c_q c_{k-p}^\dagger c_k$ has a special structure:



$$\langle f | H_{\text{int}} | i \rangle = V(1) - V(8)$$

Always have **destructive interference** between **exactly two** scattering processes

Some states do not scatter with all $V(p)$!



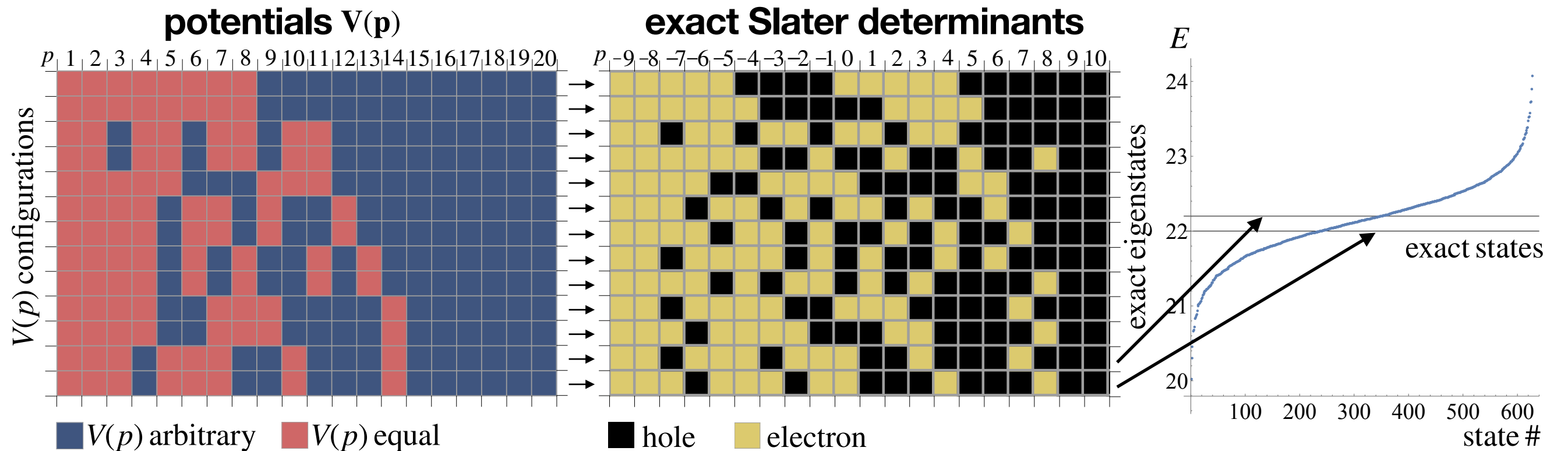
If we set $V(1) = V(2) = \dots = V(6)$,
 then this state and similar ones will be eigenstates of H_{int} .

These states are eigenstates **irrespective of the kinetic term**.

More intricate tuning patterns

The choice of potentials $V(1) = V(2) = V(3) = \dots = V(p_{\max})$ is not the only type of configuration giving exact states!

For total momentum $P = 20$, the frugal potentials (minimal amount of tuning) and associated exact states are:



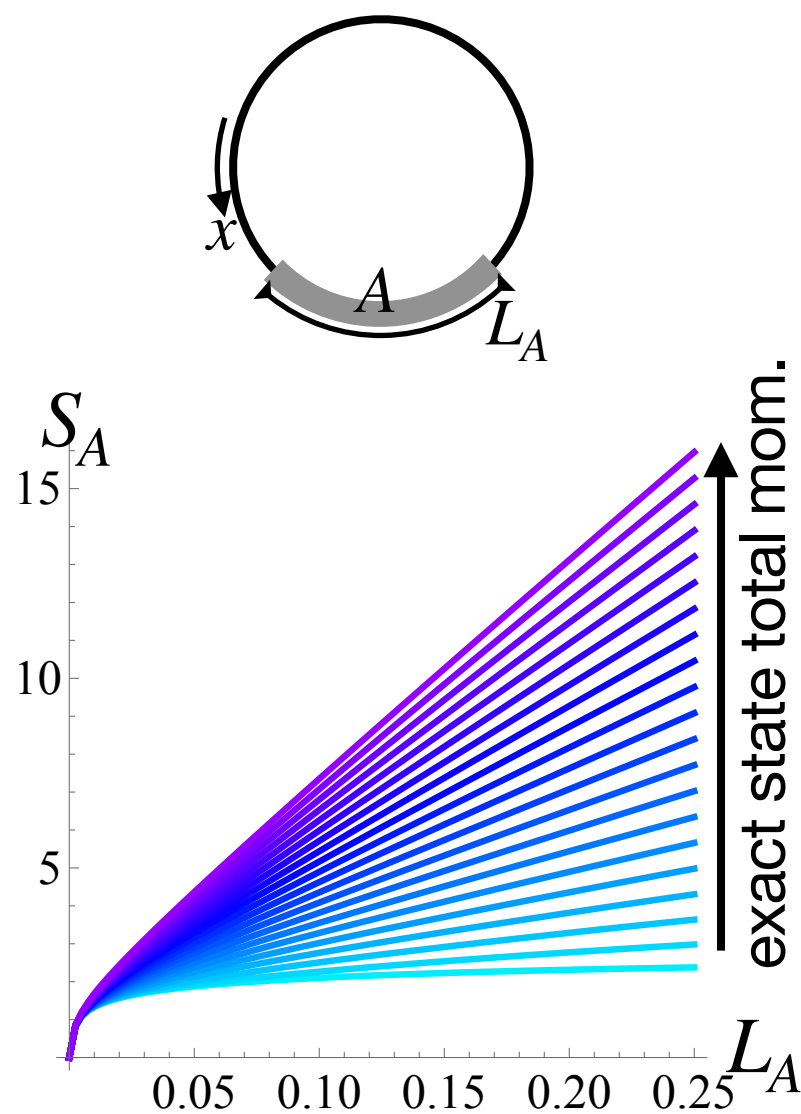
tuning less potentials
gives rise to no exact states

exact states show up
at intermediate energies

**In our paper, we describe the general method for finding
all exact state solutions in all Hilbert space sectors**

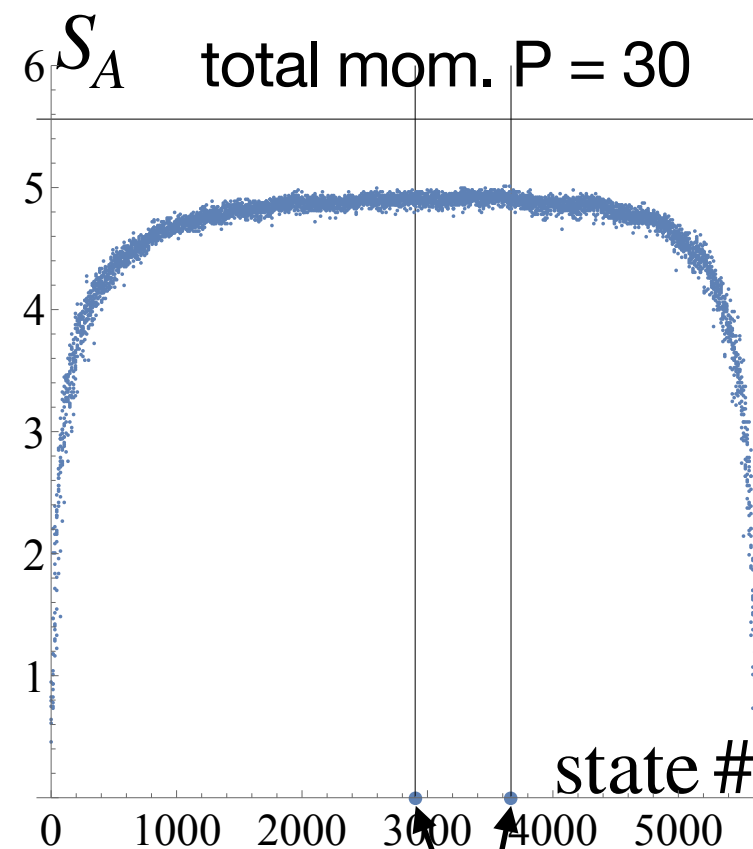
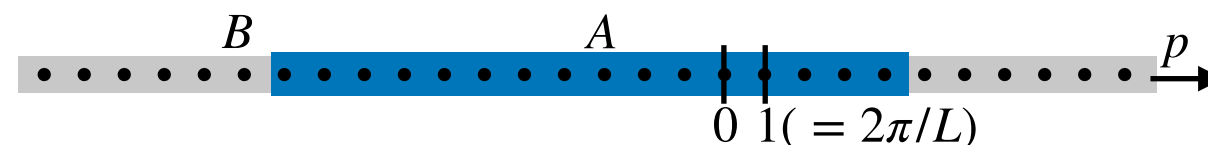
Exact state entanglement

Real-space cut



exact state entanglement can range from **sub-volume** to **volume-law**

Momentum-space cut



exact state entanglement entropy is **identically zero**

exact states are non-thermal, but with respect to momentum space entanglement!