

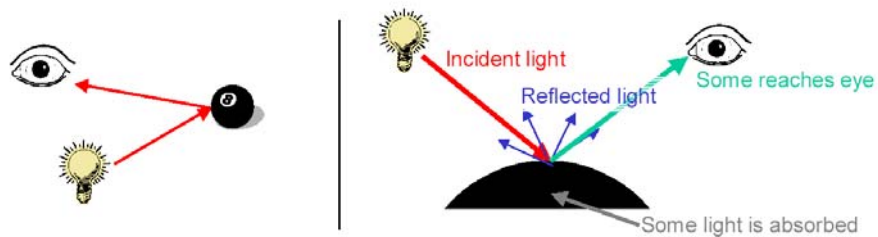


## Determining and Object's Appearance

Ultimately, we're interested in modeling **light transport** in scene

- Light is emitted from light sources and interacts with surfaces
- on impact with an object, some is reflected and some is absorbed
- distribution of reflected light determines "finish" (matte, glossy, ...)
- composition of light arriving at eye determines what we see

Let's focus on the local interaction of light with single surface point



## Modeling Light Sources

In general, light sources have a very complex structure

- incandescent light bulbs, the sun, CRT monitors, ...

To simplify things, we'll focus on **point light sources** for now

- light source is a single infinitesimal point
- emits light equally in all directions (**isotropic** illumination)
- outgoing light is set of rays originating at light point

Creating lights in OpenGL

- `glEnable(GL_LIGHTING)` — turn on lighting of objects
- `glEnable(GL_LIGHT0)` — turn on specific light
- `glLight(...)` — specify position, emitted light intensity, ...

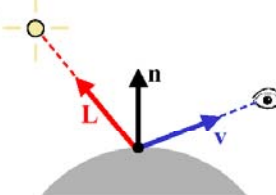
## Basic Local Illumination Model

We're only interested in light that finally arrives at view point

- a function of the light & viewing positions
- and local surface reflectance

Characterize light using RGB triples

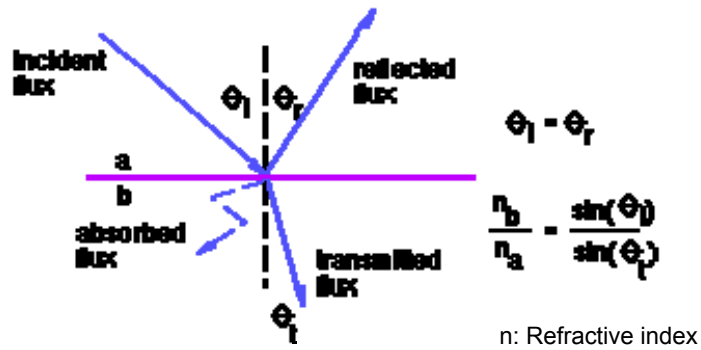
- can operate on each channel separately



Given a point, compute intensity of reflected light

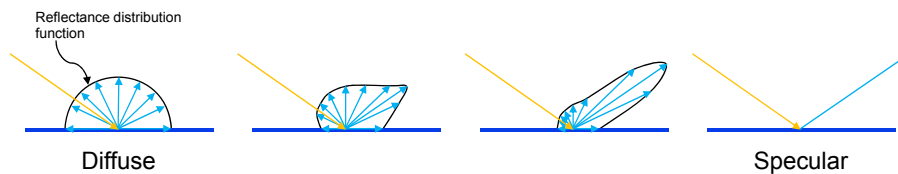
## Local Illumination Physics

### *Law of reflection and Snell's law of refraction*



## What Are We Trying to Model ?

### *From diffuse to specular reflectance*



## Diffuse Reflection

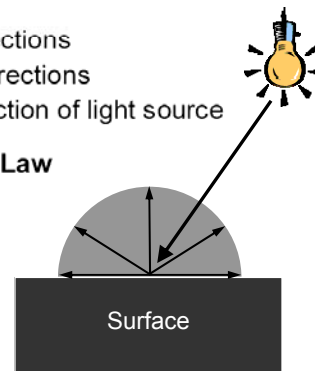
This is the simplest kind of reflection

- also called **Lambertian** reflection
- models dull, matte surfaces — materials like chalk

**Ideal diffuse reflection**

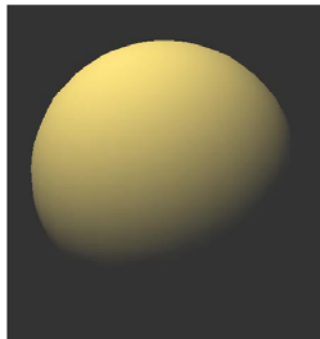
- scatters incoming light equally in all directions
- identical appearance from all viewing directions
- reflected intensity depends only on direction of light source

Light is reflected according to Lambert's Law

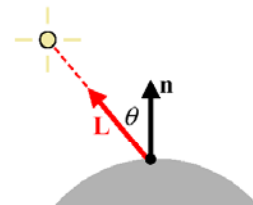


## Lambert's Law for Diffuse Reflection

*Purely diffuse object*



$$I = I_L k_d \cos \theta$$
$$= I_L k_d (\mathbf{n} \cdot \mathbf{L})$$



$I$  : resulting intensity

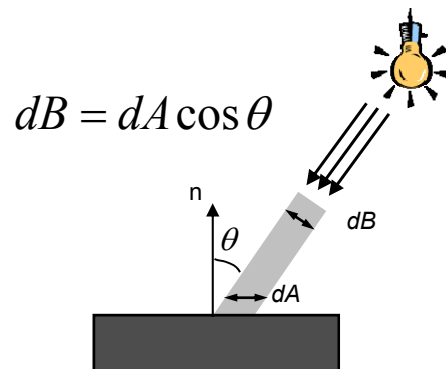
$I_L$  : light source intensity

$k_d$  : (diffuse) surface reflectance coefficient

$$k_d \in [0, 1]$$

$\theta$  : angle between normal & light direction

## Proof of Lambert's Cosine Law



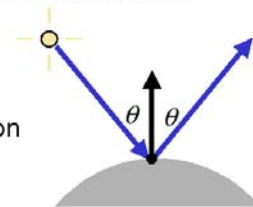
## Specular Reflection

Diffuse reflection is nice, but many surfaces are shiny

- their appearance changes as the viewpoint moves
- they have glossy **specular highlights** (or specularities)
- because they reflect light coherently, in a preferred direction

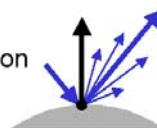
A mirror is a perfect specular reflector

- incoming ray reflected about normal direction
- nothing reflected in any other direction



Most surfaces are imperfect specular reflectors

- reflect rays in cone about perfect reflection direction



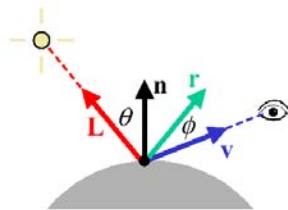
## Phong Illumination Model

$$I = I_L k_d \cos \theta + I_L k_s \cos^n \phi$$

$$= I_L k_d (\mathbf{n} \cdot \mathbf{L}) + I_L k_s (\mathbf{r} \cdot \mathbf{v})^n$$

One particular specular reflection model

- quite common in practice
- it is purely empirical
- there's *no physical basis* for it



$I$ : resulting intensity

$I_L$ : light source intensity

$k_s$ : (specular) surface reflectance coefficient

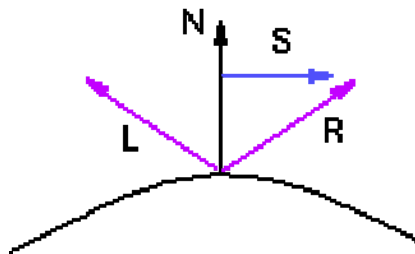
$k_s \in [0, 1]$

$\phi$ : angle between viewing & reflection direction

$n$ : "shininess" factor

## Computing R

**All vectors are unit length!**

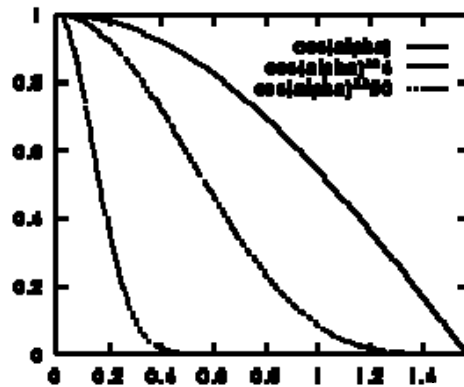


$$\mathbf{R} = (\mathbf{N} \cdot \mathbf{L}) \mathbf{N} + \mathbf{S}$$

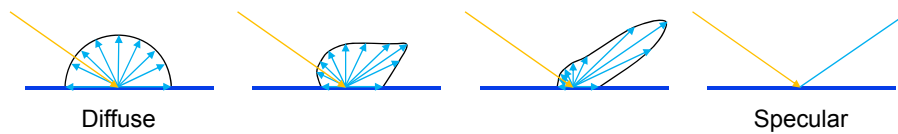
$$\mathbf{S} = (\mathbf{N} \cdot \mathbf{L}) \mathbf{N} - \mathbf{L}$$

$$\mathbf{R} = 2\mathbf{N} (\mathbf{N} \cdot \mathbf{L}) - \mathbf{L}$$

## The Effect of the Exponent $n$

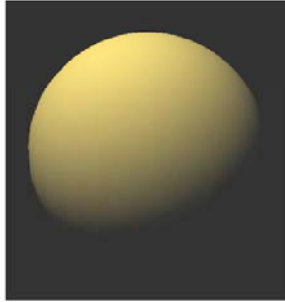


## Comparison



## Examples of Phong Specular Model

*Diffuse only*



*Diffuse + Specular  
(shininess 5)*



*Diffuse + Specular  
(shininess 50)*



## The Blinn-Torrance Specular Model

**Agrees better with experimental results**

$$I_s = I_L K_s (H \cdot V)^n$$

Halfway vector H

$$H = \frac{L + V}{\|L + V\|}$$





## Advantages of the Blinn-Torrance Specular Model

- Theoretical basis
- $N \cdot H$  cannot be negative if  $N \cdot L > 0$  and  $N \cdot V > 0$
- If the light is directional and we have orthographic projection then  $N \cdot H$  is constant

$$H = \frac{L + V}{\|L + V\|}$$



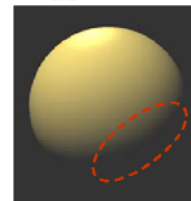
## The Ambient Glow

So far, areas not directly illuminated by any light appear black

- this tends to look rather unnatural
- in the real world, there's lots of ambient light

To compensate, we invent new light source

- assume there is a constant ambient "glow"
- this ambient glow is *purely fictitious*



Just add in another term to our illumination equation

$$I = I_L k_d \cos \theta + I_L k_s \cos^n \phi + I_a k_a$$

$I_a$  : ambient light intensity

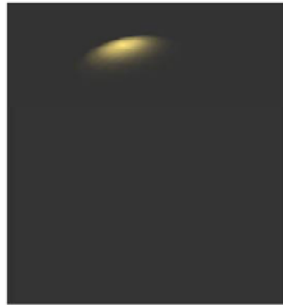
$k_a$  : (ambient) surface reflectance coefficient

## Our Three Basic Components of Illumination

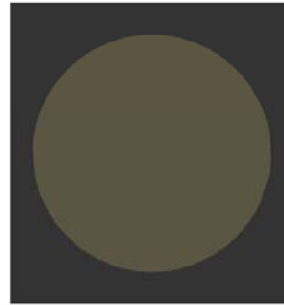
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Diffuse



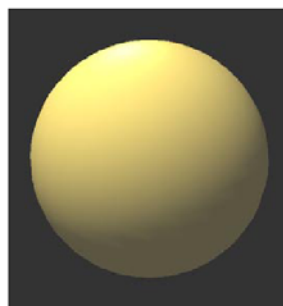
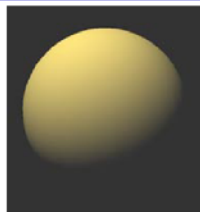
Specular



Ambient

## Combined for the Final Result

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# Lights and Materials

## Light properties

$$I_{d(iffuse)}, \underbrace{I_{s(pecular)}}_{\text{Add Specular Light}}, I_{a(ambient)}$$

## Material properties:

$$k_{d(iffuse)}, k_{s(pecular)}, k_{a(ambient)}$$

$$I_r = I_{d_r} k_{d_r} (N \cdot L) + I_{s_r} k_{s_r} (R \cdot V)^n + I_{a_r} k_{a_r}$$

$$I_g = I_{d_g} k_{d_g} (N \cdot L) + I_{s_g} k_{s_g} (R \cdot V)^n + I_{a_g} k_{a_g}$$

$$I_b = I_{d_b} k_{d_b} (N \cdot L) + I_{s_b} k_{s_b} (R \cdot V)^n + I_{a_b} k_{a_b}$$

## Questions

*If you shine red light (1,0,0) on a white object (1,1,1) what color does the object appear to have?*

*What if you shine red light (1,0,0) on a green object (0,1,0) ?*

*If the object is shiny, what is the color of the highlight?*

## Special cases

$$I_r = I_{d_r} k_{d_r} (N \cdot L) + I_{s_r} k_{s_r} (R \cdot V)^n + I_{a_r} k_{a_r}$$

$$I_g = I_{d_g} k_{d_g} (N \cdot L) + I_{s_g} k_{s_g} (R \cdot V)^n + I_{a_g} k_{a_g}$$

$$I_b = I_{d_b} k_{d_b} (N \cdot L) + I_{s_b} k_{s_b} (R \cdot V)^n + I_{a_b} k_{a_b}$$

- What should be done if  $I > 1$ ?  
Clamp the value of  $I$  to 1
- What should be done if  $N \cdot L < 0$ ?  
Clamp the value of  $I$  to 0 or flip the normal
- How can we handle multiple light sources?  
Sum the intensity of the individual contributions

## Shading Polygons: Flat Shading

Illumination equations are evaluated at surface locations

- so where do we apply them?

We could just do it once per polygon

- fill every pixel covered by polygon with the resulting color

OpenGL — `glShadeModel(GL_FLAT)`

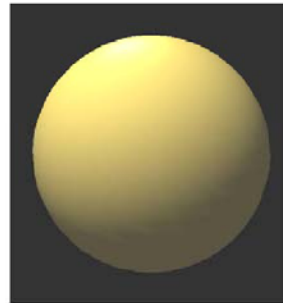
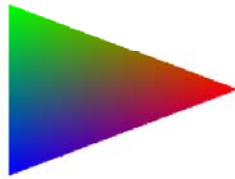


## Shading Polygons: Gouraud Shading

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Alternatively, we could evaluate at every vertex

- compute color for each covered pixel
- linearly interpolate colors over polygon



Misses details that don't fall on vertex

- specular highlights, for instance

OpenGL — `glShadeModel(GL_SMOOTH)`

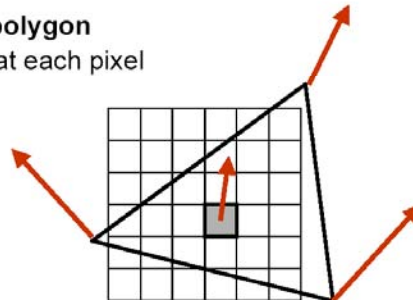
## Shading Polygons: Phong Shading

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Don't just interpolate colors over polygons

Interpolate surface normal over polygon

- evaluate illumination equation at each pixel



## Summarizing the Shading Model

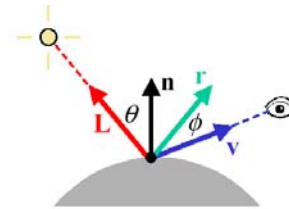
We describe local appearance with illumination equations

- consists of a sum of set of components — light is additive
- treat each wavelength independently
- currently: diffuse, specular, and ambient terms

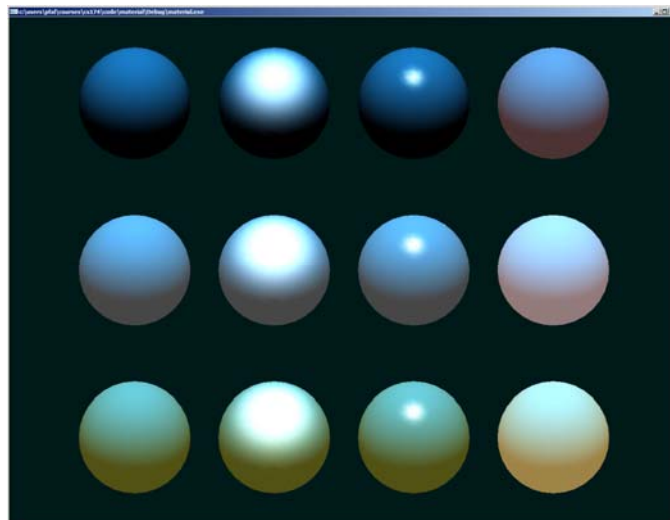
$$I = I_L k_d \cos \theta + I_L k_s \cos^n \phi + I_a k_a$$

Must shade every pixel covered by polygon

- flat shading: constant color
- Gouraud shading: interpolate corner colors
- Phong shading: interpolate corner normals



## Examples



## Problems with Shading Algorithms

*Orientation dependence*

*Silhouettes*

*Perspective distortion*

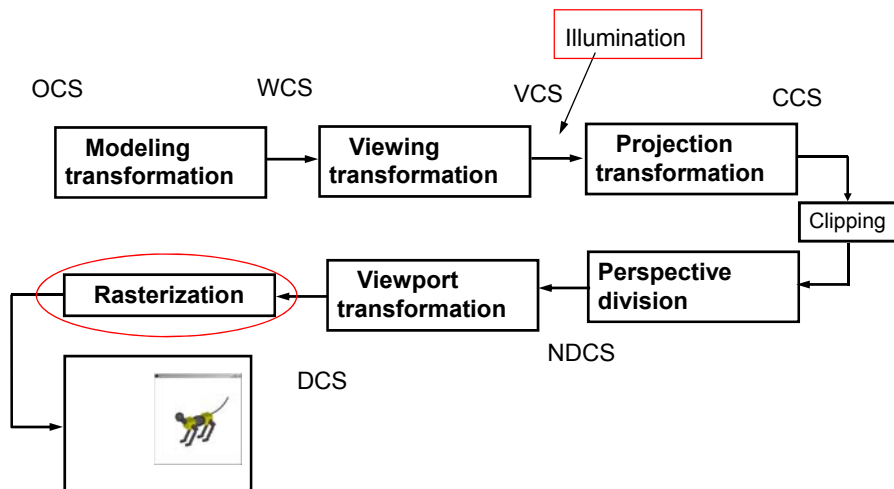
- It happens in screen space, so need to use hyperbolic interpolation

*T-vertices*

- If you do not have smooth normals, color changes if polygon order changes

*Generation of vertex normals*

## Illumination in the Graphics Pipeline



# Z-Buffer Algorithm

*for each polygon in model*

*project vertices of polygon onto image plane*

*for each pixel inside projected polygon*

*calculate pixel z-value*

*if z-value is smaller than pixel's z-value currently in z-buffer*

*calculate pixel color*

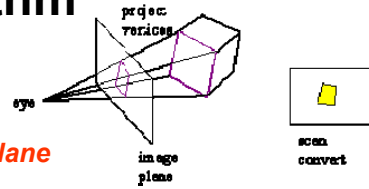
*draw pixel*

*update pixel z-value in z-buffer*

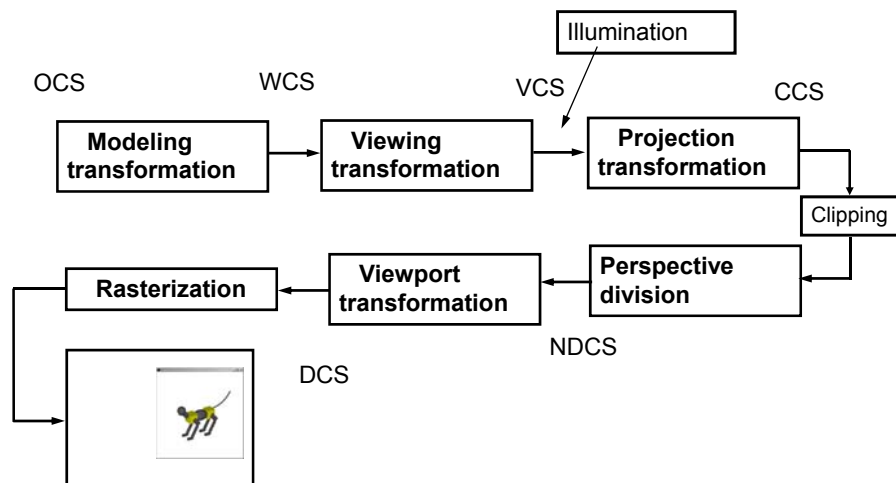
*end*

*end*

*end*



## Completion of the Z-Buffer Graphics Pipeline





## What Have We Ignored?

### Some local phenomena

- shadows — every point is illuminated by every light source
- attenuation — intensity falls off with square of distance to light
- transparent objects — light can be transmitted through surface

### Global illumination

- reflections of objects in other objects
- indirect diffuse light — ambient term is just a hack

### Realistic surface detail

- can make an orange sphere
- but it doesn't have the texture of the real fruit

### Realistic light sources

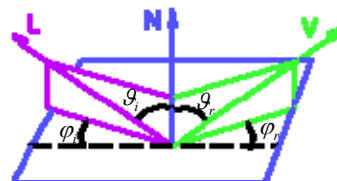
## Advanced Concepts

### *Physics-based illumination models*

### *Bidirectional reflectance distribution function: BRDF*

$$\rho(\vartheta_i, \varphi_i, \vartheta_r, \varphi_r, \lambda)$$

$\lambda$ : light wavelength



# Global Illumination

*Computing light interface between all surfaces*

Courtesy of Henrik Wann Jensen

*Radiosity*

*Ray tracing*



## Radiosity

*Physics-based*

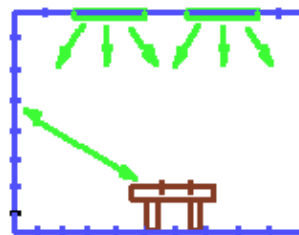
- heat transfer
- illumination engineering

*Suited for diffuse reflection*

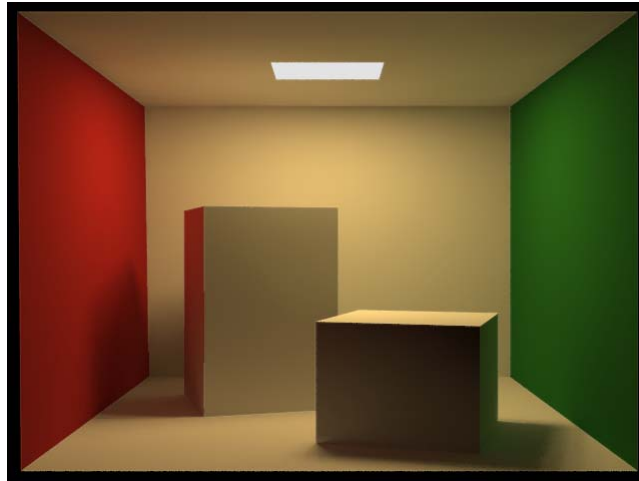
- Infinite inter-reflections

*Area light sources*

- Soft shadows



## Example



## Radiosity Algorithm

*Break scene into small patches (polygons)*

*Assume uniform reflection and emission per patch*

*Energy balance for all patches:*

Light leaving surface = Emitted light + Reflected light

## Notation

- **Flux:** energy per unit time ( $W$ )
- **Radiosity**  $B$ : exiting flux density ( $W/m^2$ ) for surfaces
- $E$ : exiting flux density for light sources
- **Reflectivity**  $R$ : fraction of incoming light that is reflected (unitless)
- **Form factor**  $F_{i,j}$ : fraction of energy leaving polygon  $A_i$  and arriving at polygon  $A_j$ 
  - determined by the geometry of polygons  $i$  and  $j$

## Energy Balance

$$\overbrace{B_i A_i}^{\text{Light leaving surface}} = \overbrace{E_i A_i}^{\text{Emitted light}} + \overbrace{R_i \sum_j B_j F_{j,i} A_j}^{\text{Reflected light}}$$

Therefore

$$B_i = E_i + R_i \sum_j B_j F_{j,i} \frac{A_j}{A_i}$$

Now  $F_{j,i} A_j = F_{i,j} A_i$  (form-factor reciprocity)

Therefore

$$B_i = E_i + R_i \sum_j B_j F_{i,j}$$

or

$$E_i = B_i - R_i \sum_j B_j F_{i,j}$$

## Linear System

*Assume constant radiosity polygons (n of them)*

*Compute form factors  $F_{ij}$  for  $1 \leq i, j \leq n$*

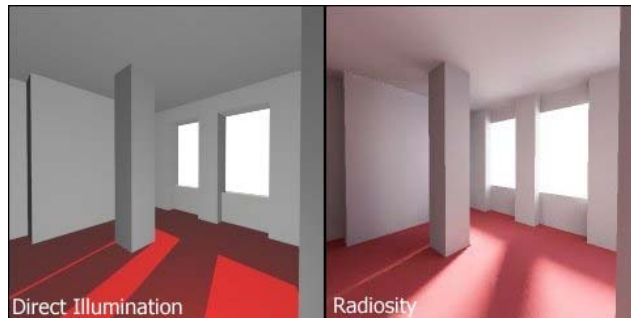
*Assemble a system of n linear equations:*

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix} = \begin{bmatrix} 1 - R_1 F_{1,1} & -R_1 F_{1,2} & \dots & -R_1 F_{1,n} \\ -R_2 F_{2,1} & 1 - R_2 F_{2,2} & \dots & -R_2 F_{2,n} \\ \vdots & \vdots & \vdots & \vdots \\ -R_n F_{n,1} & \dots & R_n F_{n,(n-1)} & 1 - R_n F_{n,n} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix}$$

*n x n matrix*

*Solve the system for the exiting fluxes  $B_i$*

## Comparison Between Direct Illumination and Radiosity



## Radiosity Summary

*Object space algorithm*

*Suited for diffuse (inter-)reflections*

*Area light sources with nice, soft shadows*