GRAPHS

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Contents

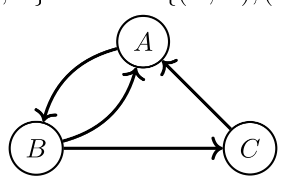
1. Graphs

Definition 1.1. A (directed) *graph* G consists of the following data:

(i) A set V of *vertices*.
(ii) A set $E \subseteq V \times V$ of *edges*.

Remark 1.2. Recall that a binary relation on a set V can be encoded as any subset of $V \times V$. Thus, a graph is a set V equipped with a binary relation E.

Example 1.3. Let $V = \{A, B, C\}$ and $E = \{(A, B), (B, A), (B, C), (C, A)\}$.

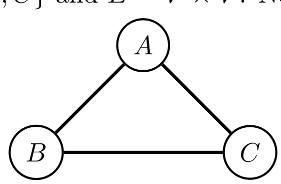


Definition 1.4. A graph (V, E) is *undirected* if the binary relation E is symmetric.

Remark 1.5. Example 1.3 is not an undirected graph since $(B, C) \in E$, but $(C, B) \notin E$. Similarly, $(C, A) \in E$, but $(A, C) \notin E$.

Remark 1.6. In an undirected graph, we drop the arrow tips as they convey no additional information.

Example 1.7. Let $V = \{A, B, C\}$ and $E = V \times V$. Note that E is symmetric.



Definition 1.8. Let G be a graph. A *walk* is a sequence of vertices and edges defined inductively as follows:

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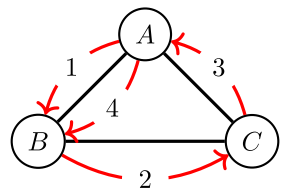
(i) If $(v_1, v_2) \in E$ then the sequence $v_1, (v_1, v_2), v_2$ is a walk.
(ii) If $(v_n, v_m) \in E$ and $v_1, (v_1, v_2), v_2, \ldots, v_n$ is a walk then $v_1, (v_1, v_2), v_2, \ldots, v_n, (v_n, v_m), v_m$ is a walk.

Example 1.9. Let G be the graph defined in Example 1.7. The sequence

$$A, (A, B), B, (B, C), C, (C, A), A, (A, B), B$$

is a walk.



Each red arrow represents a step of the walk.

2. BFS

3. DFS