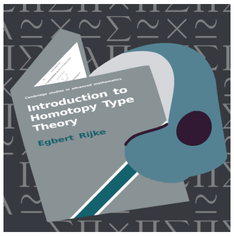

Worksheet 1

HoTTEST Summer School 2022

The HoTTEST TAs , and

4th July 2022

1 (⋆)

State the introduction and elimination rules for

1. ×-types
2. →-types
3. ∏-types

$$\frac{\Gamma \vdash a : \tau_1 \quad \Gamma \vdash b : \tau_2}{\Gamma \vdash (a,b) : \tau_1 \times \tau_2} \qquad \frac{\Gamma \vdash e : \tau_1 \times \tau_2}{\Gamma \vdash \pi_1 e : \tau_1} \qquad \frac{\Gamma \vdash e : \tau_1 \times \tau_2}{\Gamma \vdash \pi_2 e : \tau_2}$$

$$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x : \tau_1 .\ e : \tau_1 \to \tau_2} \qquad \frac{\Gamma \vdash e_1 : \tau_1 \to \tau_2 \quad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1 e_2 : \tau_2}$$

$x : \tau_1$ $= B(a)$

2 (⋆)

Fill in this proof tree:

$$\dfrac{\dfrac{\dfrac{}{a : A, b : B \vdash a : A} \qquad \dfrac{}{\underline{\qquad\qquad}}}{\underline{\qquad\qquad} \vdash \underline{\qquad\qquad} : A \times B}}{\dfrac{a : A \vdash \lambda(b : B).(a,b) : \underline{\qquad\qquad}}{\cdot \vdash \lambda(a : A).\lambda(b : B).(a,b) : A \to B \to A \times B}}$$

3 (⋆⋆)

Write a proof tree ending with a term of type $A \times B \to B \times A$ in the empty context.

$$\frac{x : A \times B \vdash (\pi_2 x, \pi_1 x)}{\vdash \lambda x : A \times B . (\pi_2 x, \pi_1 x) : A \times B \to B \times A}$$

4 (⋆⋆)

For problems 2 and 3, what is the *logical* content of the proof tree? That is, under the "types are theorem" interpretation of Curry-Howard, what theorems have we proven?

Next, what is the *computational* content of the proof tree? That is, under the "programs are proofs" interpretation of Curry-Howard, what programs have we written?

5 (⋆ ⋆ ⋆)

Define the **swap** function $\sigma_{A,B}$ of type

$$\sigma_{A,B} : \left(\prod_{x:A}\prod_{y:B} C(x,y)\right) \to \left(\prod_{y:B}\prod_{x:A} C(x,y)\right)$$

and show that $\sigma_{B,A} \circ \sigma_{A,B}$ is (definitionally) equal to the identity.

$$\equiv \lambda p : \prod_{x:A}\prod_{y:B} C(x,y).\ p$$