Given some non-linear function g, calculate $\nabla_w y$. Don't forget to use δ's.

Solution: The output y is given by the function:

$$y = g(s_y) = g(w_{hy}h + w_{xy}x) = g(w_{hy}g(s_h) + w_{xy}x) = g(w_{hy}g(w_{xh}x) + w_{xy}x)$$

To calculate ∇y we need all the partial derivatives $\frac{\partial y}{\partial w_h}, \frac{\partial y}{\partial w_{hy}}, \frac{\partial y}{\partial w_{xy}}$. We'll start with the ones closest to the output.

$$\frac{\partial y}{\partial w_{hy}} = \frac{\partial y}{\partial s_y} \cdot \frac{\partial s_y}{\partial w_{hy}} = g'(s_y)h = \delta_y h$$

$$\frac{\partial y}{\partial w_{xy}} = \frac{\partial y}{\partial s_y} \cdot \frac{\partial s_y}{\partial w_{xy}} = g'(s_y)x = \delta_y x$$

$$\frac{\partial y}{\partial w_{xh}} = \frac{\partial y}{\partial s_y} \cdot \frac{\partial s_y}{\partial w_{xh}} = g'(s_y)\frac{\partial}{\partial w_{xh}}(w_{hy}h + w_{xy}x)$$

$$= g'(s_y)(\frac{\partial}{\partial s_h}(w_{hy}g(s_h))\frac{\partial s_h}{\partial w_{xh}} + \frac{\partial}{\partial w_{xh}}w_{xy}x)$$

$$= w_{hy}g'(s_y)g'(s_h)\frac{\partial s_h}{\partial w_{xh}}$$

$$= w_{hy}g'(s_y)g'(s_h)\frac{\partial(w_{xh}x)}{\partial w_{xh}}$$

$$= w_{hy}g'(s_y)g'(s_h)x = w_{hy}\delta_y g'(s_h)x$$