

**Solution:**

We can massage the objective function (let's call it  $f_0(u)$ ) in this way:

$$\begin{aligned} f_0(u) &= \frac{1}{n} \sum_{i=1}^n (u^T x_i - u^T \hat{x})^2 \\ &= \frac{1}{n} \sum_{i=1}^n ((x_i - \hat{x})^T u)^2 \\ &= \frac{1}{n} \sum_{i=1}^n (u^T (x_i - \hat{x})) ((x_i - \hat{x})^T u) \\ &= u^T \left( \frac{1}{n} \sum_{i=1}^n (x_i - \hat{x})(x_i - \hat{x})^T \right) u \\ &= u^T \Sigma u \end{aligned}$$