Show that the maximizer for this problem is equal to v_1, where v_1 is the eigenvector corresponding to the largest eigenvalue λ_1. Also show that optimal value of this problem is equal to λ_1.

Solution:

We start by invoking the spectral decomposition of $\Sigma = V\Lambda V^T$, which is a symmetric positive semi-definite matrix.