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Standard Map Project

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July 5, 2019

1 General Statement

Dr. Mezic discussed the concept of the ergodic partition earlier today. In this project we will apply the method to the Chirikov standard map. Further information on these methods can be found in the following references [MW99; LM10; LM15] .

1.1 1-Function Approximation

(a) Produce a 1-function approximation of the ergodic partition of the standard map:

$$I' = I + \delta sin(2\pi\varphi),$$
$$\varphi' = \varphi + I + \delta sin(2\pi\varphi),$$

for $\delta = 0.17$ by computing the time averages f_i^* of the the functions $f_1 = sin(2\pi\varphi)$ and $f_2 = sin(3\pi\varphi)$ using a grid of initial conditions of sizes $10 \times 10, 100 \times 100, 400 \times 400$ with 100 and 1000 iterations of the map. Plot the results over the state space.

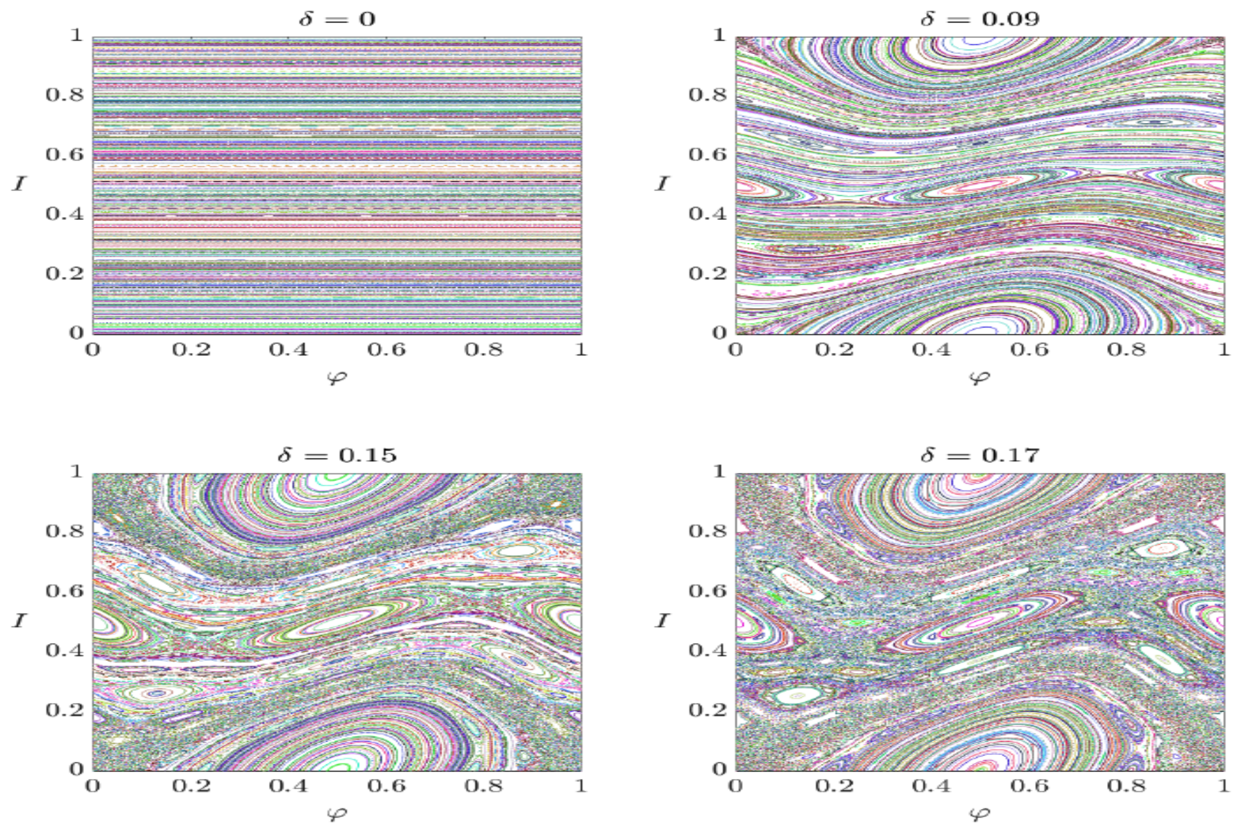

Figure 1: State-space portrait for the standard map for different values of δ. Each trajectory corresponds to a different color.

(b) Which of the two functions was a better choice of observable? Describe why.

(c) Describe the best results obtained in terms of the state-space structure.

(d) How does the spatial scale of state-space structures change as you refine the discretization from 100 to 16000 initial conditions?

(e) How does the picture change when you use 1000 iterates instead of 100?

1.2 2-Function Approximation

(f) Produce a 2-function approximation to the ergodic partition of the standard map by considering time-averages of the functions f_2 and $f_3 = cos(12\pi\varphi)cos(2\pi I)$.

1.3 Harmonic Averages

(g) For the frequency $\omega = \frac{1}{2}$, compute the harmonic averages of the function $f_4 = cos(2\pi(I+\varphi))$

1.4 Finite Section

(h) Now use the finite section method to approximate eigenfunctions and eigenvalues of the Koopman operator by considering the Krylov sequence of f_4 under the dynamics. Plot the eigenvectors as functions of initial conditions. Discuss the eigenvalues and eigenvectors in comparison with previous calculations that used averages.

References

[MW99] Igor Mezic and Stephen Wiggins. "A method for visualization of invariant sets of dynamical systems based on the ergodic partition". In: *Chaos (Woodbury, N.Y.)* 9 (Apr. 1999), pp. 213–218. DOI: 10.1063/1.166399.

[LM10] Zoran Levnajic and Igor Mezic. "Ergodic Theory and Visualization I: Mesochronic Plots for Visualization of Ergodic Partition and Invariant Sets". In: *Chaos (Woodbury, N.Y.)* 20 (Sept. 2010), p. 033114. DOI: 10.1063/1.3458896.

[LM15] Zoran Levnajic and Igor Mezic. "Ergodic theory and visualization. II. Fourier mesochronic plots visualize (quasi)periodic sets". In: *Chaos (Woodbury, N.Y.)* 25.5 (May 2015), p. 053105. ISSN: 1054-1500. DOI: 10.1063/1.4919767. URL: https://doi.org/10.1063/1.4919767.