圖形識別 HW2

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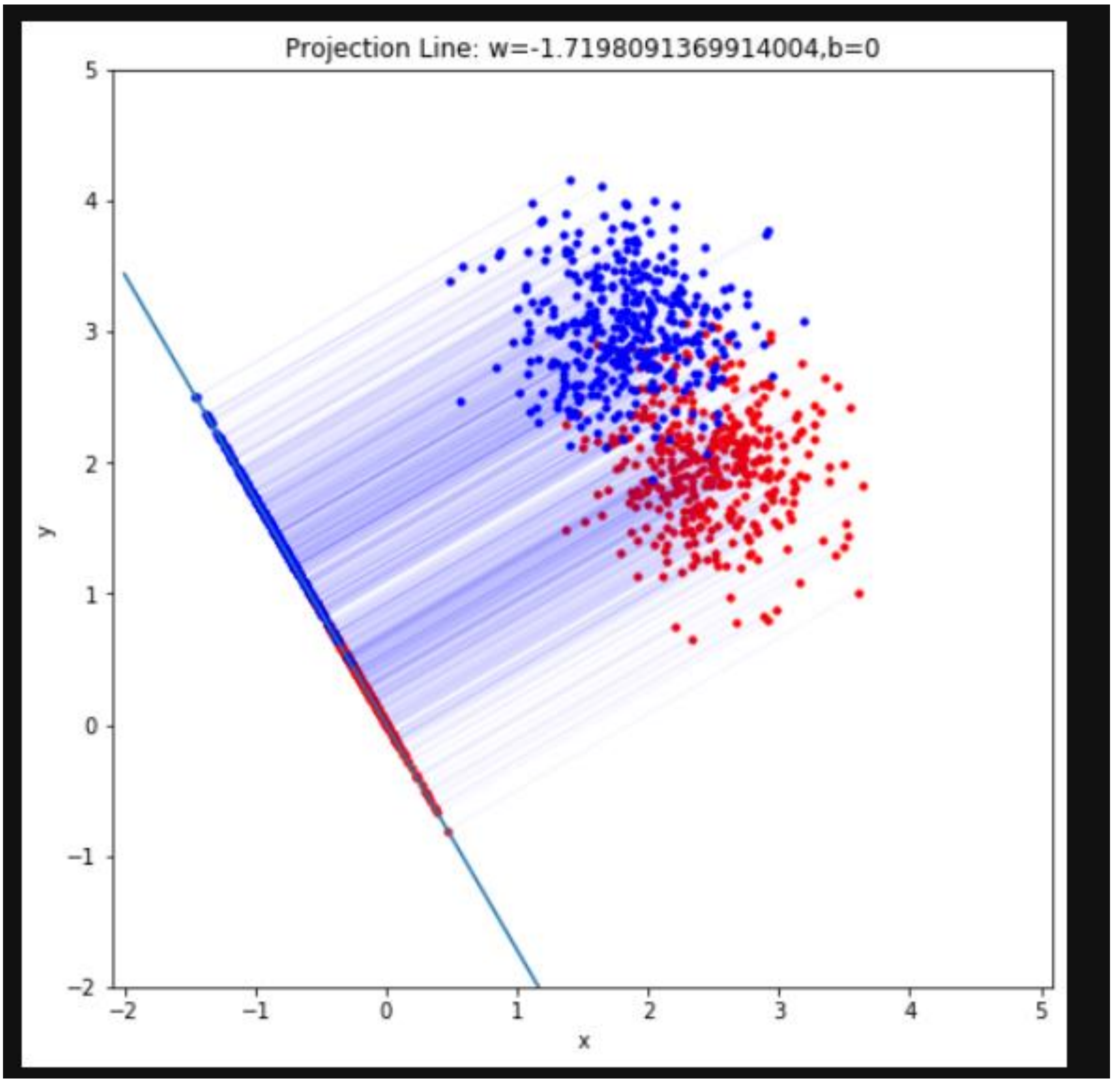
Part.1 Coding

1. mean vector of class 1: [2.47107265 1.97913899] mean vector of class 2: [1.82380675 3.03051876]
2. Within-class scatter matrix SW: [[140.40036447 -5.30881553] [-5.30881553 138.14297637]]
3. Between-class scatter matrix SB: [[0.41895314 -0.68052227] [-0.68052227 1.10539942]]
4. Fisher's linear discriminant: [[-0.00432865] [0.00744446]]
5. Accuracy of test-set 0.912
6. 

Part2.Questions

1.

$$\frac{\partial L}{\partial w} = 0$$

$$\Rightarrow \frac{\partial w^T}{\partial w}(m_2 - m_1) + \frac{\partial w^T w}{\partial w} \cdot \lambda = 0$$

$$\Rightarrow \begin{bmatrix} \frac{\partial w_1}{\partial w_1} & \frac{\partial w_2}{\partial w_1} & \cdots & \frac{\partial w_n}{\partial w_1} \\ \vdots & & & \\ \frac{\partial w_1}{\partial w_n} & \frac{\partial w_2}{\partial w_n} & \cdots & \frac{\partial w_n}{\partial w_n} \end{bmatrix} (m_2 - m_1) + \begin{bmatrix} \frac{\partial w_1^2}{\partial w_1} & \frac{\partial w_2^2}{\partial w_1} & \cdots & \frac{\partial w_n^2}{\partial w_1} \\ \vdots & & & \\ \frac{\partial w_1^2}{\partial w_n} & \frac{\partial w_2^2}{\partial w_n} & \cdots & \frac{\partial w_n^2}{\partial w_n} \end{bmatrix} \cdot \lambda = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots 0 & 1 \end{bmatrix} (m_2 - m_1) + \begin{bmatrix} 2w_1 & 0 & \cdots & 0 \\ 0 & 2w_2 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & & \cdots 0 & 2w_n \end{bmatrix} \cdot \lambda = 0$$

$$\Rightarrow I \cdot (m_2 - m_1) + 2\lambda (w^T I) = 0$$

$$\Rightarrow (m_2 - m_1) + 2\lambda (I^T w) = 0$$

$$\Rightarrow (m_2 - m_1) + 2\lambda w = 0$$

$$\Rightarrow w = \frac{-1}{2\lambda}(m_2 - m_1)$$

$$\Rightarrow w \propto (m_2 - m_1)$$

2.

$$a = \ln \frac{P(x|C_1)P(C_1)}{P(x|C_2)P(C_2)} \quad \cdots (8)$$

$$= \ln \frac{e^{-\frac{1}{2}(x-\mu_1)^T \Sigma^{-1}(x-\mu_1)} \cdot \frac{1}{2\pi^D |\Sigma|^{\frac{1}{2}}} \cdot P(C_1)}{e^{-\frac{1}{2}(x-\mu_2)^T \Sigma^{-1}(x-\mu_2)} \cdot \frac{1}{2\pi^D |\Sigma|^{\frac{1}{2}}} \cdot P(C_2)} \quad \cdots (7)$$

$$= -\frac{1}{2}(x-\mu_1)^T \Sigma^{-1}(x-\mu_1) + \frac{1}{2}(x-\mu_2)^T \Sigma^{-1}(x-\mu_2) + \ln \frac{P(C_1)}{P(C_2)} \quad \cdots (6)$$

$$= \frac{1}{2}\left[x^T \Sigma^{-1}(\mu_1 - \mu_2) + \mu_1^T \Sigma^{-1}(x-\mu_1) - \mu_2^T \Sigma^{-1}(x-\mu_2) \right] + \ln \frac{P(C_1)}{P(C_2)} \quad \cdots (5)$$

$$= \frac{1}{2}\left[x^T \Sigma^{-1}(\mu_1 - \mu_2) + (\mu_1^T - \mu_2^T)\Sigma^{-1} x \right] - \frac{1}{2}\mu_1^T \Sigma^{-1}\mu_1 + \frac{1}{2}\mu_2^T \Sigma^{-1}\mu_2 + \ln \frac{P(C_1)}{P(C_2)} \quad \cdots (4)$$

$$= x^T \Sigma^{-1}(\mu_1 - \mu_2) + w_0 \quad \cdots (3)$$

$$= x^T w + w_0 \quad \cdots (2)$$

$$= w^T x + w_0 \quad \cdots (1)$$

結論 $p(C_1|x) = \sigma(a) = \sigma(w^T x + w_0)$.

$$w = \Sigma^{-1}(\mu_1 - \mu_2) \quad (由\ (3))$$

$$w_0 = -\frac{1}{2}\mu_1^T \Sigma^{-1}\mu_1 + \frac{1}{2}\mu_2^T \Sigma^{-1}\mu_2 + \ln \frac{P(C_1)}{P(C_2)} \quad (由\ (4)$$