Chem/Stat3240: Homework 7a
Matlab

October 17, 2015

1. What is the probability that when three dice are rolled, at least two of the dice have the same value? What is the probability that the value of the third dice roll is strictly between the values of the first two rolls? What is the probability that the third dice roll is >= or <= the first two dice rolls?

   Write a function
   `[ntrialMatch, ntrialVec ]=rollThem(ntrials,diePattern,plotsOn)`

   to simulate games of rolling a die three times (`ntrials` games) and detecting when the die pattern listed above match the three-element integer sequence that is generated in each game, using the input function `diePattern`. Counting those pattern matches is then used to generate an estimate of the probability of those patterns occurring. The input `plotsOn` is a character string to determine whether to produce and save a plot of the empirically computed probabilities of matching the pattern as a function of the number of trials, as shown in the plots below. The outputs `ntrialMatch` and `ntrialVec` are vectors of the computed probabilities of matching, and the associated trial numbers at which those probabilities are computed, the same as used to create the plots below.

   For a given number of games `ntrials`, the function will calculate the empirical probability of the pattern in the diePattern function when the number of trials is a multiple of `nSample`. To calculate what `nSample` should be, you need to determine how large and integer `n` has to be such that `ntrials/n` < 50. That sampling interval will insure you compute and plot approximately 50 emprical probabilities, no mater how large `ntrials` might be. Example plots are shown below.

In a script file called `diePatterns.m` define functions (use anonymous functions in Matlab) `pattern1`, `pattern2`, and `pattern3` for each of the dice combinations described above, with the functions taking a vector input corresponding to the three dice rolls, and returning 1 or 0 depending on whether the input vector satisfies the conditions of the dice combinations specified using compound logical expressions. These will be the functions passed as input arguments to the function `rollThem`.

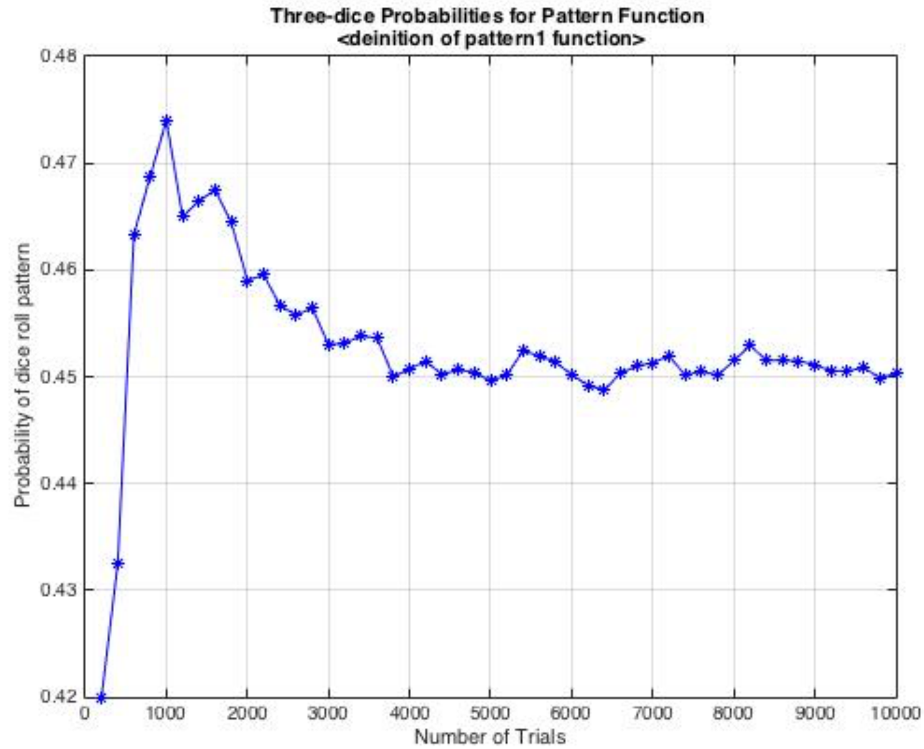
Since there are only a finite number of three-dice combinations, you can analytically calculate the probability of a given combination using nested loops to cycle through all possible combinations and use the functions defined in the diePatterns script to detect how many times those combinations occur. Write a function

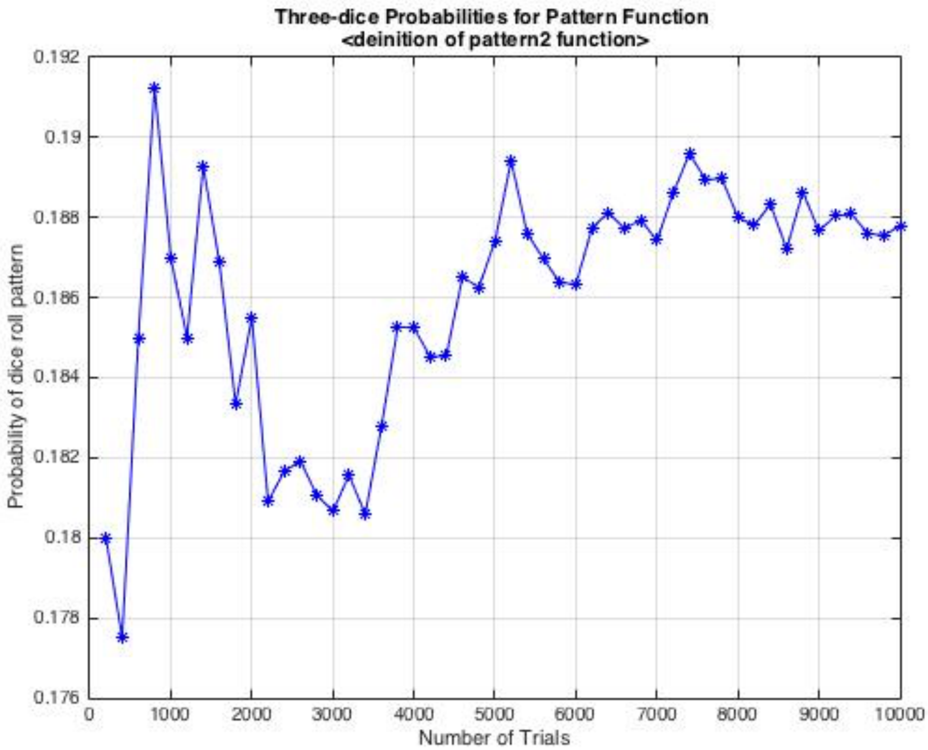
```
[patternMatch, percent ]=countThem(diePattern)
```

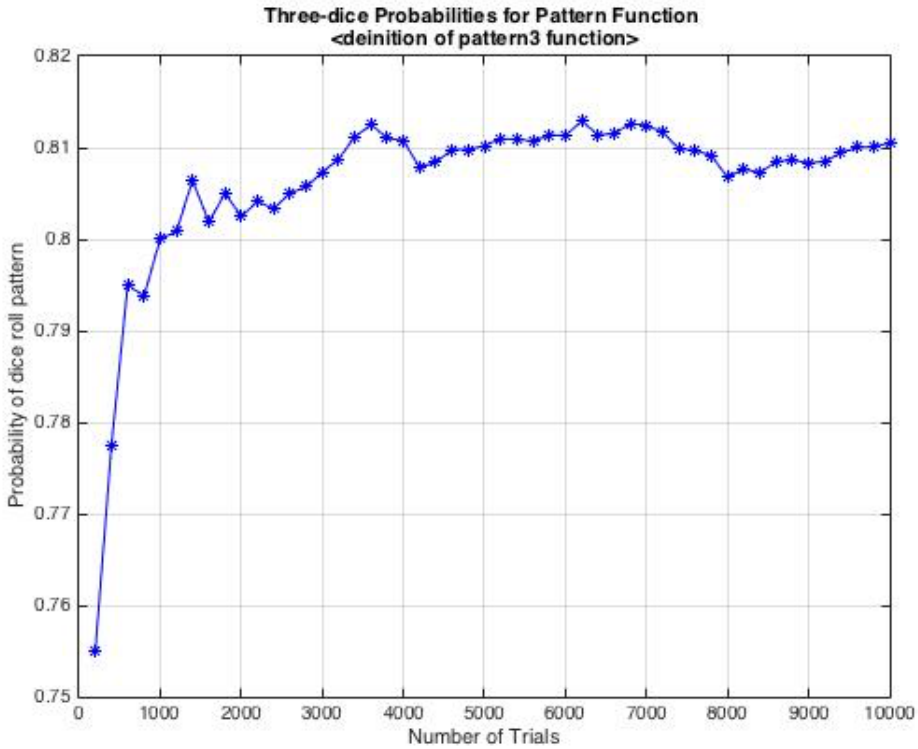
that takes an input `diePattern` function and returns the number of matches to that pattern , `patternMatch`, and the ratio of `patternMatch` to the total number of combinations. Verify that these are the numbers that the `rollThem` plots appear to converge to, as seen below.

Once your `countThem` function has passed the Cody tests, you can use it with the `myCountThemTest.m` and write in the definitions for the functions `pattern1`, `pattern2`, and `pattern3` to verify your function definitions in the script `diePatterns.m` work. Use those functions as inputs to the function `rollThem` to reproduce the pdf files below.

Submit your function files `rollThem` and `countThem` to Cody, as well as to the collab site, along with the `diePatterns` script and the pdfs of the four plots shown above to Collab site.


Three-dice Probabilities for Pattern Function
<deinition of pattern1 function>
Probability of dice roll pattern
Number of Trials
0.48
0.47
0.46
0.45
0.44
0.43
0.42
0 1000 2000 3000 4000 5000 6000 7000 8000 9000 10000


Three-dice Probabilities for Pattern Function
<deinition of pattern2 function>
Probability of dice roll pattern
Number of Trials
0.192
0.19
0.188
0.186
0.184
0.182
0.18
0.178
0.176
0 1000 2000 3000 4000 5000 6000 7000 8000 9000 10000



Analytic Answer based on Probability Theory

The probability that three dice have different values are $6/6 * 5/6 * 4/6 = 5/9$, so the probability that at least two have the same value is $1 - 5/9 = 4/9 \approx 0.4444$.

Since three different dice generates different values with probability 5/9, and which one is in between of the other two is in equally likelihood, so the probability that the third one is in between of the first two is $(5/9)/3 = 5/27 \approx 0.1852$.

Addendum:

1. The output of your diePattern function should be a numeric value of 1 or zero. To implement these functions in Matlab as anonymous functions (in just a single line), you have to convert the output of a logical expression used to detect the specific die pattern (logical 1 or 0) to numeric values of 1 or 0. The `double` command would be useful

here. In Mathematica, you can define a single line function with the `If` command and explicitly set the function output.