- Scalar: a number, real, integer, or complex, $\mathbb{R}$
- Vector: a single row or column of numbers (usually of the same type), denoted by lower case letters. E.g: $a = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \end{pmatrix}$ Vector in $\mathbb{R}^n$ is an ordered set of n real numbers. In the above example, $a \in \mathbb{R}^5$.
- Matrix: array of numbers with dimensions are the number of rows and columns. E.g: $X = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5. \end{pmatrix} \in \mathbb{R}^{2\times 3}$ Denoted by capital letter in this course. A square matrix has number of rows equal number of columns. Denote entry in the ith row and jth column: X_{ij}. Symmetric matrix: $X_{ij} = X_{ji}$ Diagonal matrix: $X_{ii} \neq 0$. Upper triangular matrix: $X_{ij} \neq 0, j \geq i$

All above in Matlab

- Scalar: can be initilized simply as:

```
a=1
```

- Vector: initalize vector

```
a=zeros(5,1)
b=ones(1,5)
c=1:5
```

- Matrix:

```
X=zeros(2,3)
X'
X[1,2]
```

- Indexing

```
X(1,:)
X(:,2)
X > 1
```

Vector and matrix operations

- Transpose of a vector or matrix: $a = \begin{pmatrix} 1 & 2 \end{pmatrix}$ $a^T = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$
- To do substraction and addition, vector or matrix has to have same dimesion.

```
a=1:6;b=2:7;a - b
A=[1 2;3 4]; B=[5 10;4 7]; A + B
```

- Matrix matrix product:$A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{n \times p}$: C= AB.
  $C_{ij} = \sum_{k=1}^{n} A_{ik} B_{kj}$, $C \in \mathbb{R}^{m \times p}$
  Condition: number of columns of A = number of rows of B.
  Notice: $AB \neq BA$,

Vector matrix product

- Matrix matrix product example: $A \in \mathbb{R}^{3\times 3}, B \in \mathbb{R}^{3\times 2}$
  $$\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \\ C_{31} & C_{32} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} \times \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ B_{31} & B_{32} \end{pmatrix}$$
  $C_{11} = A_{11}B_{11} + A_{12}B_{21} + A_{13}B_{31}$
  $C_{12} = A_{11}B_{12} + A_{12}B_{22} + A_{13}B_{32}$
- Not commutative: $AB \neq BA$
- Associative: (AB)C=A(BC).
- Multiplication and transpose: $(AB)^T = B^T A^T$
- Identity matrix: Square diagonal matrix with all diagonal elements = 1. AI=IA=A

Matrix vector operations

- Matrix vector product is a special case of matrix matrix product.
- Dot or inner product of two vectors with same dimension: $x, y \in \mathbb{R}^{p\times 1}$:$\langle x, y\rangle = \sum_{i=1}^{p} a_i b_i = x^T y$. Sum of elementwise product of the two vector.

```
a=1:3;b=3:5; dot(a,b)
```

Vector norm

- Norm of a vector $\|x\|$ is a measure of "length" of the vector.

$$\|x\|_p = (\sum_{i=1}^{n} |x_i|^p)^{1/p} \tag{1}$$

- Common norm: $|.|_1$ or ℓ_1 , $|.|_2$ or ℓ_2, $|x|_1 = \sum_{i=1}^{n} |x_i|, |x|_1 = \sqrt{\sum_{i=1}^{n} x_i^2}$
  `x=[1;-2;1]; norm(x,1) norm(x)`
- Infinity norm $|x|_\infty = max_i x_i$
- Zero norm: $|x|_0 =$ number of nonzero elements of x

Orthogonal

- Two vectors x and y are orthogonal if $\langle x, y \rangle = 0$, $\|x\|_2 \neq 0, \|y\|_2 \neq 0$
- Two vectors x and y are orthonormal if $\langle x, y \rangle = 0$, $\|x\|_2 = 1, \|y\|_2 = 1$
- Orthogonal matrices: if every pair of columns are orthogonal.
- Orthonormal matrices: if every pair of columns are orthonormal.
- If A is an orthonormal matrices, $A^T A = AA^T = \mathbb{I}$

Inverse matrix

- Inverse of a square matrix $A \in \mathbb{R}^{n\times n}$: A^{-1} or inv A: $AA^{-1} = \mathbb{I} = A^{-1}A$.
- Non singular matrix has inverse.
- Determinant of a square matrix: $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$ $det(A) = A_{11}A_{22} - A_{21}A_{12}$ For $A \in \mathbb{R}^{n\times n}$: for any fix i: $1 \leq i \leq n$ $det(A) = \sum_{j=1}^{n}(-1)^{i+j}A_{ij}det(A^{ij})$, where A^{ij} is the submatrix obtained by removing ith row and jth column.
- det(AB)=det(A)det(B), $det(A+B) \neq det(A) + det(B)$

Linear independence and matrix rank

- Vectors $x_1, x_2, \cdots, x_n$ are linear independence if $c_1x_1 + c_2x_2 + \cdots + c_nx_n = 0$ implies $c_1 = c_2 = \cdots = c_n = 0$ or Xc=0 implies c=0.
- $X \in \mathbb{R}^{m\times n}$ rank(X) is the maximal number of linearly independen columns, or maximal number of linearly independent rows.
  $rank(X) \leq min(m, n)$.
- if rank(X) =n: X has full column rank.
- $X \in \mathbb{R}^{n\times n}$, rank(X)=n if and only if X is nonsingular (has inverse). $X \in \mathbb{R}^{n\times n}$, rank(X)=n if and only if $det(X) \neq 0$. If $rank(X) < n$, X is singular.