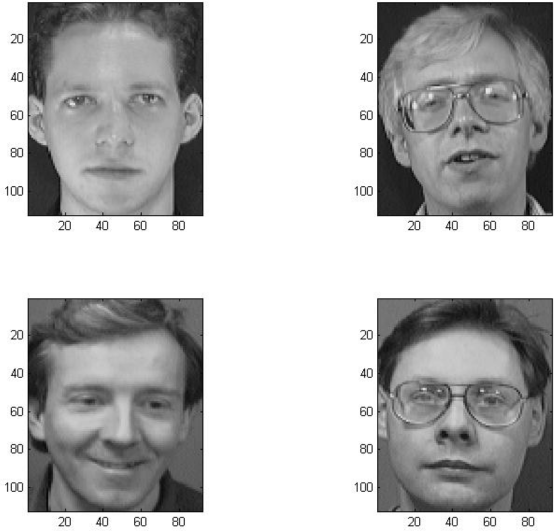
- We are going to build a simple system that does face recognition using PCA.
- Data set: AT&T Lab face image data: 400 images of size 112×92 of 40 different individuals. Each has 10 images.
- Each image is is gray scale, 10,304 pixels, represented by a matrix of size 112×92.
- Each pixel has value ranging from 1 to 255: 255 is white and 1 is black. Anything in between is some shades of grey.

- Our data matrix, in order to work with PCA, has to have dimension $n \times p$.
- Each row corresponds to an observation (image), each column corresponds to some measurement.
- Each image is already a matrix.
- An image will be unfolded to a vector by rows (or columns) of that matrix and line them up to make a long vector.
- We can construct our data matrix from the collections of vectorized images.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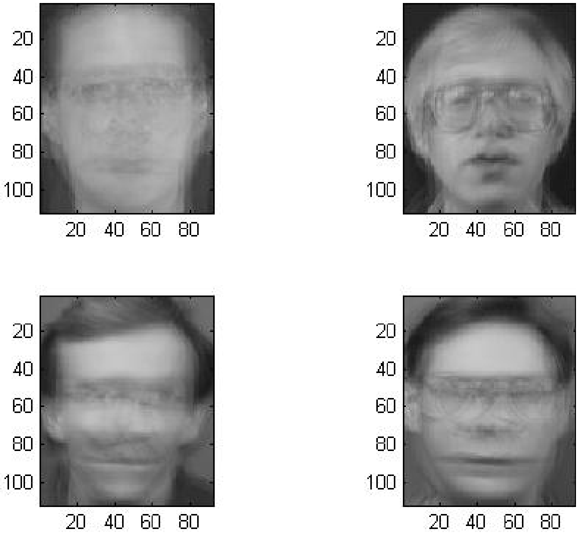
- In Matlab, originally images are usuallly unsigned integer. In order to work with PCA, we have to convert it to double precision.
- We want to figure out what are the principal components of these face images.
- Given a new picture, can we find out which pictures have good match with it?
- Training data has around 320 images, testing data has 80 images.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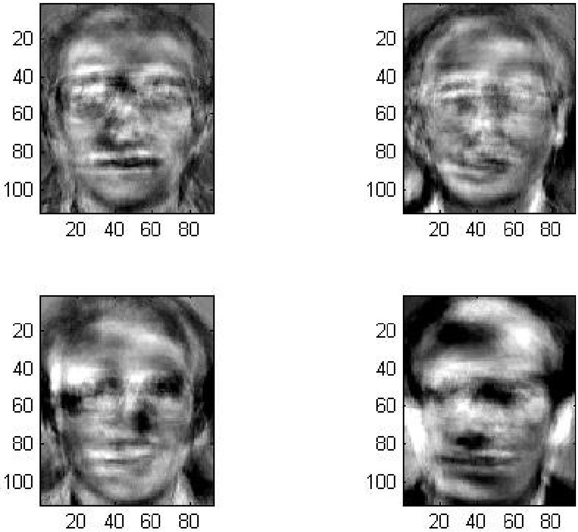
- Lets call the data matrix A and its covariance matrix $C = A^T A$.
- Images of the same person are supposed to have high correlation. Images of different persons are supposed to have low correlation.
- Use PCA to find principal components of matrix C.
- Find eigen values and eigen vectors of C.

- Matrix C usually have high dimensions: 10304×10304 for our data set.
- Calculating the eigen values and eigen vectors of such large matrix is very time consuming.
- One little trick: Eigen values and eigen vectors of C are related to those of $D = AA^T$.
- Let v and λ be eigen vector and eigen values of C, we have:

$$Cv = A^T Av = \lambda v.\text{(multiply both sides with A)}$$
$$AA^T(Av) = A\lambda v = \lambda(Av).$$

- If v and λ be eigen vector and eigen values of $C = A^T A$, then (Av) and λ is eigen vectors and eigen values of $D = AA^T$.
- Vice versa, if v and λ are eigen vectors and eigen values of $D = AA^T$ then $(A^T v)$ and λ are eigen vectors and eigen values of $C = A^T A$.
- Instead calculating eigen values and eigen vectors of large matrix C, we can find those of smaller matrix D.
- $[V, E] = eigs(D, 100, 'lm')$
- Then eigen vectors of C can be calculated by: $A^T V$.

- These eigen vectors are also called eigen faces.
- Each image in the training set can be represented as a linear combination of eigen faces (principal components).
- When a new image turns up, it will be represented as a combination of the basis vectors.
- The system output an image in the directory that is closest in distance with the new image.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- For a new image I, the projection of the image on to the space of the eigen faces (in other words, representing the new image as a linear combination of eigen faces)

$$P_I = I * V$$

- P_I is the new co-ordinate of the new image in the eigen face space.
- The system calculate the distance between the P_I and the images in the directory and find the closest one.