

# Midterm Report Problem 10

B1027234 林永濬

B1027236 蕭銘宏

## Problem 10

$$\begin{bmatrix} 13.422 & 0 & 0 & 0 \\ -13.422 & 12.252 & 0 & 0 \\ 0 & -12.252 & 12.377 & 0 \\ 0 & 0 & -12.377 & 11.797 \end{bmatrix} \times \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 750.5 \\ 300 \\ 102 \\ 30 \end{bmatrix}$$

視為  $Ax = b$

$x$ : The resulting chloride concentrations in each of the four lakes.

$b$ : Loadings of chloride to each of the four lakes.

**a) Use the matrix inverse to solve for the concentrations in each of the four lakes.**

### Math

$$Ac = b$$

$$A^{-1}Ac = A^{-1}b$$

$$c = A^{-1}b$$

### MATLAB

使用 `inv(A)` 計算出  $A^{-1}$ ,  $c$

```
A_inv =  
    0.0745    0.0000    0.0000    0.0000  
    0.0816    0.0816    0.0000    0.0000  
    0.0808    0.0808    0.0808    0.0000  
    0.0848    0.0848    0.0848    0.0848  
c =  
    55.916  
    85.741  
    93.116  
   100.237
```

**b) HOW much must the loading to Lake Powell be reduced for the Chloride concentration of Lake Havasu to be 75?**

由題目得知

$$c_4 = 100.237 \Rightarrow 75$$

計算出變化量

$$\Delta c = c_{new} - c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -25.237 \end{bmatrix}$$

$$\Delta b = b_{new} - b = \begin{bmatrix} -x \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

解  $Ax = b$

$$Ax = b$$

$$A\Delta c = \Delta b$$

$$\begin{bmatrix} 13.422 & 0 & 0 & 0 \\ -13.422 & 12.252 & 0 & 0 \\ 0 & -12.252 & 12.377 & 0 \\ 0 & 0 & -12.377 & 11.797 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 0 \\ -25.237 \end{bmatrix} = \begin{bmatrix} -x \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$11.797 \times -25.237 = -x$$

$$x = 297.72$$

## MATLAB

使用 `eqn = A * deltac_c == delta_b` 直接計算出  $x = 297.72$

**c) Using the column-sum norm, compute the condition number and how many suspect digits would be generated by solving this system.**

### Note

#### Column Sum Norm

$$\|A\|_1 = \max_{1 \leq j \leq n} \sum_{i=1}^n |a_{ij}|$$

1. 將每一 row 的值除所有 column 的最大值得到 norm matrix
2. column sum norm = norm matrix 將 column 值取絕對值並加總，並選出最大值

#### Condition Number

$$\text{Cond}[A] = \|A\| \cdot \|A^{-1}\|$$

#### Suspect Digits

$$\text{Digits} = \log_{10}(\text{cond}[A])$$

藉由 MATLAB 計算得出

```
A_norm= 26.844  
A_inv_norm= 0.322  
condition_num = 8.635  
suspect_digits = 0.936
```