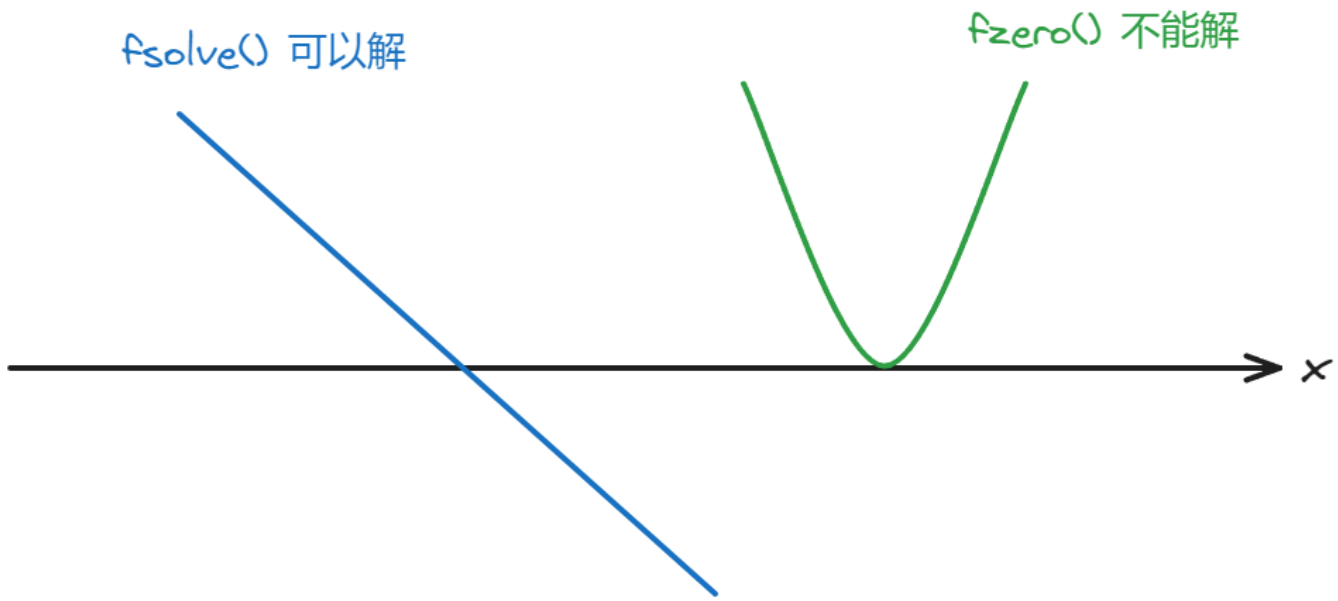
Numerical Analysis Homework 5

網頁版: https://hackmd.io/@Xaio/HkA8utfyA

♪² 找根的方法

- 定義 symbolic variable `sys(x)`, `sys x`
- `solve()` 可以只到 symbolic 的解，`fsolve()` 可以找到 numerical 的解。
- `fzero()` 與 `fsolve()` 相同，但是 `fzero()` 找不到平滑經過的點，但是 `fsolve()` 可以



- `roots()` 專門找 polynomials 的解

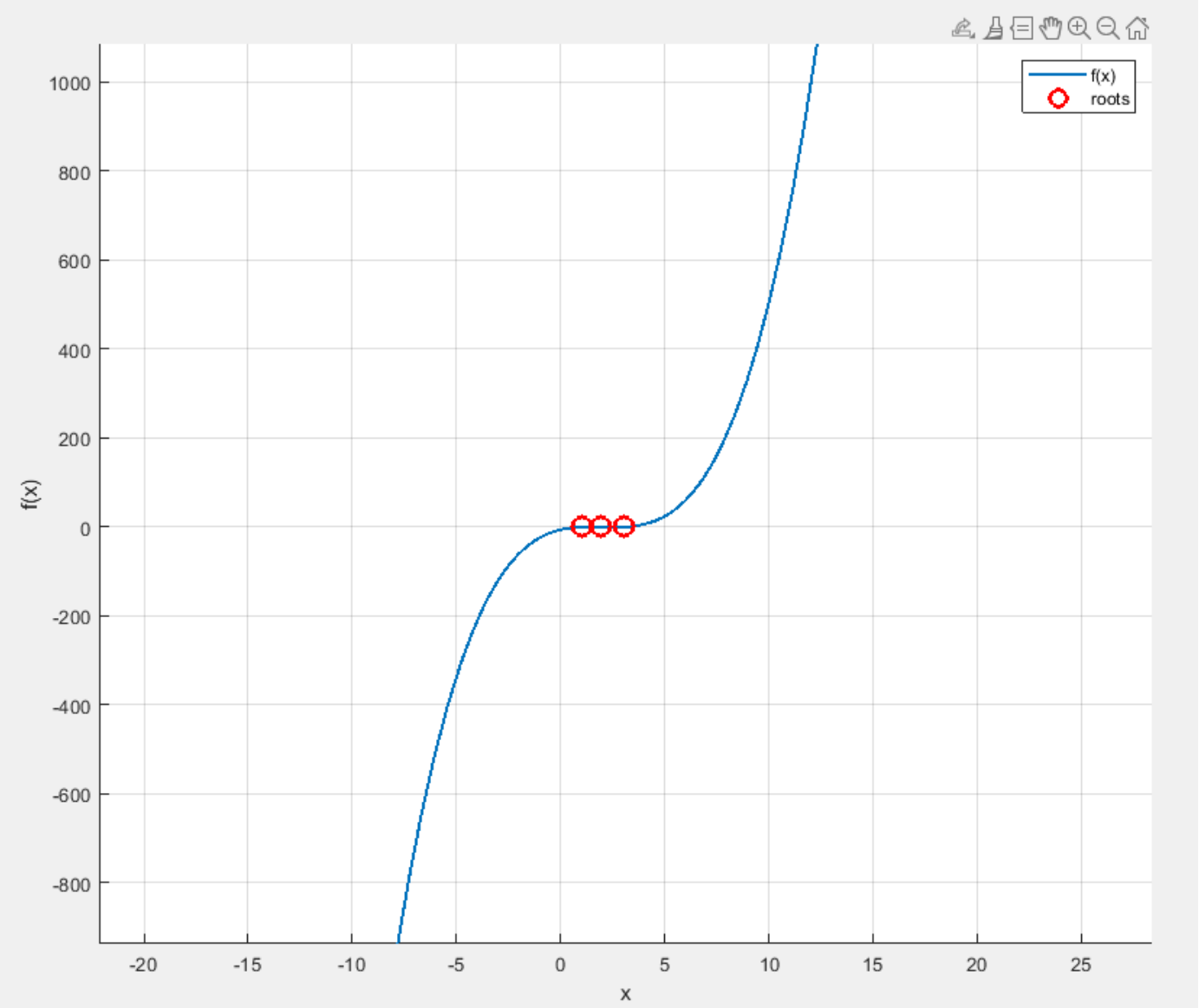
:::spoiler 補充

- 使用 `diff(y)` 直接對 syms 算微分。積分同理，使用 `int()`。
  :::

♪² 1. Determine the highest real root of $f(x)$

$$f(x) = x^3 -6x^2+11x-6.1$$

♪³ 1.a) Graphically



Highest real root: 3.0467

♪³ 1.b) Using the Newton-Raphson method (three iterations, $x_i= 3.5$)

```
Newton-Raphson method
Iteration 1, x = 3.191304
Iteration 2, x = 3.068699
Iteration 3, x = 3.047317
Error: 0.020882%
```

Newton-Raphson method 成功找到解

♪³ 1.c) Using the secant method (three iterations, $x_{i-1}=2.5$ and $x_i=3.5$).

```
Secant method
Iteration 1, x = 4.288889
Iteration 2, x = 5.256667
Iteration 3, x = 6.655075
Error: 118.436904%
```

Secant method 發散

♪³ 1.d) Using the modified secant method (five iterations, $x_i = 3.5$,$\delta= 0.01$).

```
Modified secant method
Iteration 1, x = 3.199597
Iteration 2, x = 3.075324
Iteration 3, x = 3.048818
Error: 0.070165%
```

Modified secant method 成功找到解

♪³ 1.e) Determine all the roots with MATLAB.

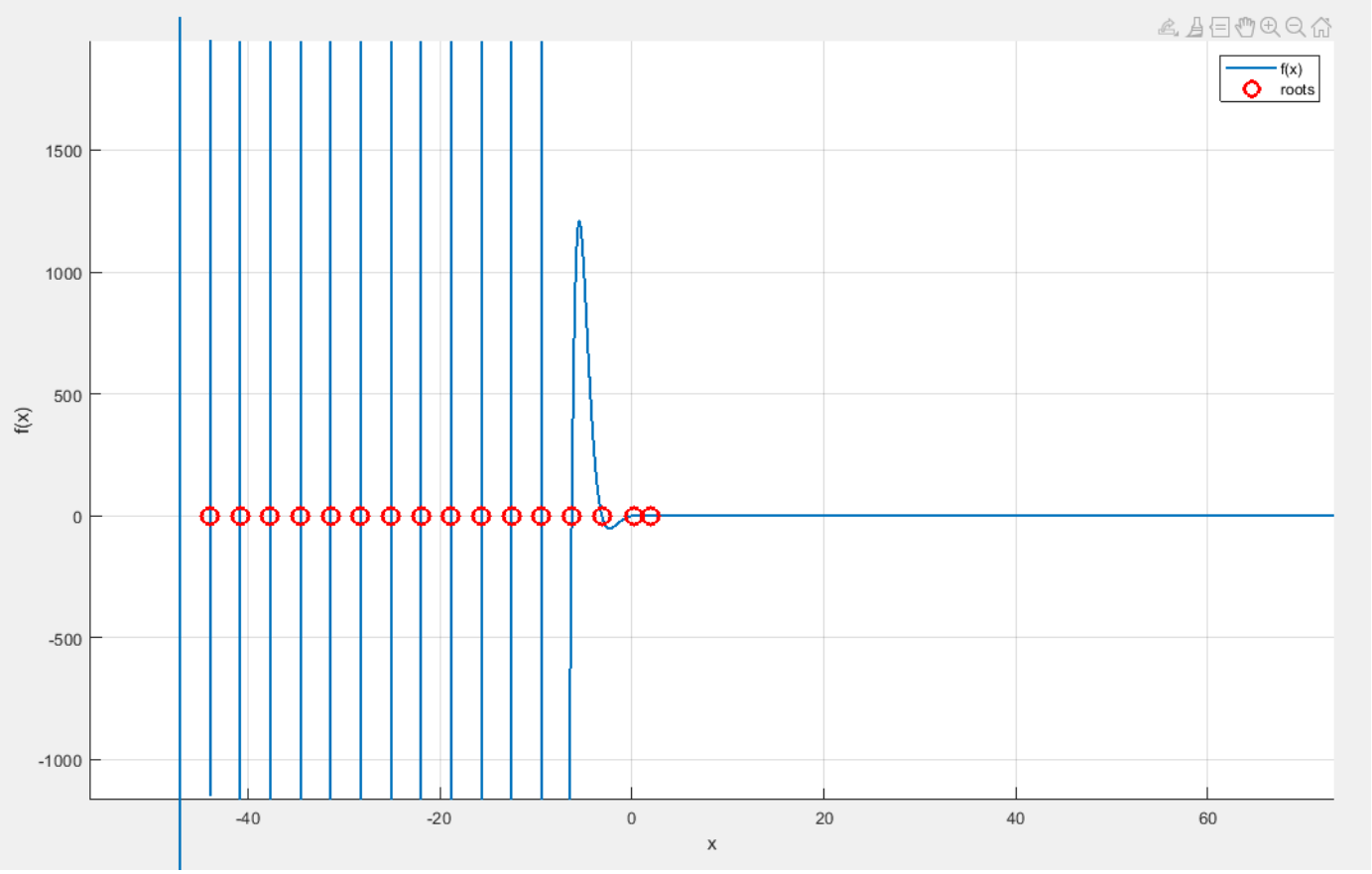
```
    1.0544
    1.8990
    3.0467
```

♪² 2. Determine the lowest real root of $g(x)$

$$g(x) : 7\sin(x)e^{-x}-1$$

$g(x) < 0$ 有無窮多解，在此的 lowest real root 只討論 $x>0$ 的情形

♪³ 1.a) Graphically



♪³ 2.b) Using the Newton-Raphson method (three iterations, $x_i= 0.3$)

```
Newton-Raphson method
Iteration 1, x = 0.144376
Iteration 2, x = 0.169409
Iteration 3, x = 0.170179
Error: 0.000422%
```

Newton-Raphson method 成功找到解

♪³ 2.c) Using the secant method (three iterations, $x_{i-1}=0.5$ and $x_i=0.4$).

```
Secant method
Iteration 1, x = 0.797218
Iteration 2, x = 1.960000
Iteration 3, x = 2.035892
Error: 1096.316989%
```

Secant method 找到非負次小的解 1.893，與此解的誤差為 7.55%

♪³ 2.d) Using the modified secant method (five iterations, $x_i = 3.5$,$\delta= 0.01$).

```
Modified secant method
Iteration 1, x = -5.682098
Iteration 2, x = -7.519326
Iteration 3, x = -6.790712
Error: 4090.311458%
```

Modified secant method 找到 $x<0$ 次大的解 -6.2829，與此解的誤差為 8.083%

♪³ 2.e) Determine all the roots with MATLAB.

有無窮多解，x 在 [-45, 2] 的範圍內有下列這些解

roots	roots	roots	roots
0.170179993752879	-3.14772834675799	-6.28291845844458	-9.42478948913899
-12.5663701161672	-15.7079632894778	-18.8495559206084	-21.9911485751688
-25.1327412287166	-28.2743338823082	-31.4159265358979	-34.5575191894877
-37.6991118430775	-40.8407044966673	-43.9822971502571	1.89305902941322