

Kinetic Mixing and the Supersymmetric Gauge Hierarchy

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Abstract

The most general Lagrangian for a model with two $U(1)$ gauge symmetries contains a renormalizable operator which mixes their gauge kinetic terms. Such kinetic mixing can be generated at arbitrarily high scales but will not be suppressed by large masses. In models whose supersymmetry (SUSY)-breaking hidden sectors contain $U(1)$ gauge factors, we show that such terms will generically arise and communicate SUSY-breaking to the visible sector through mixing with hypercharge. In the context of the usual supergravity- or gauge-mediated communication scenarios with D -terms of order the fundamental scale of SUSY-breaking, this effect can destabilize the gauge hierarchy. Even in models for which kinetic mixing is suppressed or the D -terms are arranged to be small, this effect is a potentially large correction to the soft scalar masses and therefore introduces a new measurable low-energy parameter. We calculate the size of kinetic mixing both in field theory and in string theory, and argue that appreciable kinetic mixing is a generic feature of string models. We conclude that the possibility of kinetic mixing effects cannot be ignored in model-building and in phenomenological studies of the low-energy SUSY spectra.

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1 Introduction

Modern models of supersymmetry (SUSY)-breaking in the minimal supersymmetric Standard Model (MSSM) always involve the division of the full theory into a so-called “hidden” sector and the usual “visible”, or MSSM, sector. This avoids the problem that arises if SUSY is broken in a sector with tree-level couplings to the MSSM, namely the existence of experimentally excluded sum rules on the MSSM sparticle masses [1]. Given the reasonable expectation that SUSY should be broken by non-trivial dynamics in the infrared, the hidden sector must contain a non-abelian gauge group G , and in this case “hidden” implies that there exist no tree-level interactions which couple states charged under G with those charged under the MSSM. In particular, there must not exist fields in the effective Lagrangian (below the scale of SUSY-breaking) which are charged under both gauge groups simultaneously.

Given that SUSY-breaking is generated by hidden-sector dynamics, the most pertinent issue for MSSM phenomenology is the nature of the communication mechanism from the hidden to MSSM sector. The simplest known mechanism is supergravity (SUGRA), which couples the hidden to the visible sector through operators which are Planck-scale suppressed. One assumes that SUSY is broken at some high scale (typically $\Lambda^2 \sim M_Z M_{\text{Pl}}$), so that the SUSY-breaking mass scale which is communicated to the visible sector by SUGRA is $\Lambda^2/M_{\text{Pl}} \sim M_Z$ [2].

Recently another class of models has received attention. These models communicate SUSY-breaking through a cascade of gauge interactions, some of which are those of the MSSM and some those of the hidden sector. Here the SUSY-breaking scale communicated to the visible sector is suppressed by loop factors compared to the scale of SUSY-breaking in the hidden sector, typically taken to be ~ 100 TeV [3, 4, 5].

In either case, it is imperative for the consistency of the model that there not exist operators which couple the larger hidden-sector SUSY-breaking scale to the visible sector without the appropriate loop or M_{Pl} suppressions. Such an operator would pull the weak scale up to the SUSY-breaking scale in the hidden sector, thereby destabilizing the gauge hierarchy.

It has been known for some time that there can exist operators in the effective Lagrangian which perform just such an unwanted task. In the cases usually studied, such operators have the form of tadpoles. Since tadpoles of chiral superfields involve only gauge singlets, it has been noted that the existence of such singlets can have potentially disastrous consequences [6].

In this paper we will consider a new communication mechanism for SUSY-breaking which has the potential to destabilize any model with a $U(1)$ gauge factor in the hidden sector whose D -terms are of order the fundamental SUSY-breaking scale Λ^2 . This communication is provided by a renormalizable operator which is often overlooked, namely the mixing of two separate $U(1)$ gauge kinetic functions. Because this operator is renormalizable, it may be generated by physics at scales far above the SUSY-breaking scale itself, without any suppression by the large mass scale.

Furthermore, we will argue that this operator is generated at one loop in generic field theory models, and is expected to occur quite naturally in realistic string models. It can therefore easily dominate over both SUGRA-mediated and gauge-mediated (GM) soft scalar mass terms. Even in models for which this is not the case, it is still quite possible that this new contribution significantly corrects the usual soft scalar masses generated by SUGRA or GM without destroying the gauge hierarchy.

This paper is organized as follows. In Sect. 2, we discuss the consequences of kinetic mixing for supersymmetric theories. In Sect. 3, we then discuss the sources of kinetic mixing, show how to calculate kinetic mixing effects in string theory, and estimate typical sizes that may be expected within the context of both field theory and string theory. Finally, in Sect. 4, we present our conclusions.

2 Supersymmetric Kinetic Mixing

It was realized many years ago [7] that in a theory with two $U(1)$ gauge factors, there can appear in the Lagrangian a term which is consistent with all gauge symmetries and which mixes the two $U(1)$'s. In the basis in which the interaction terms have the canonical form, the pure gauge part of the Lagrangian for an arbitrary $U(1)_a \times U(1)_b$ theory can be written

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F_{(a)}^{\mu\nu} F_{(a)\mu\nu} - \frac{1}{4} F_{(b)}^{\mu\nu} F_{(b)\mu\nu} + \frac{\chi}{2} F_{(a)}^{\mu\nu} F_{(b)\mu\nu} . \quad (2.1)$$

(Throughout this analysis we will work to leading order in χ for simplicity.) In a supersymmetric theory, such a Lagrangian generalizes to*

$$\mathcal{L}_{\text{gauge}} = \frac{1}{32} \int d^2\theta \{W_a W_a + W_b W_b - 2\chi W_a W_b\} \quad (2.2)$$

where W_a and W_b are the chiral gauge field strength superfields for the two gauge symmetries: $W = \overline{D}^2 D V$ for the vector superfield V . In principle, both $U(1)$'s could lie in the hidden sector, or both in the visible, but we will primarily interest ourselves with the case in which $U(1)_a$ is in the visible sector while $U(1)_b$ is in the SUSY-breaking hidden sector. Hypercharge is an example of such a $U(1)_a$.

To bring the pure gauge portion of the Lagrangian to canonical form, one can shift the visible-sector gauge field:

$$V_a^\mu \rightarrow V_a'^\mu = V_a^\mu - \chi V_b^\mu \quad (2.3)$$

which implies that $W_a \rightarrow W_a' = W_a - \chi W_b$. This particular choice of basis is dictated by the assumption that $U(1)_b$ will break once some hidden-sector field with non-zero

*After completing this work we became aware of Ref. [8] in which the existence of the operator $W_a W_b$ was briefly mentioned.

charge receives a vacuum expectation value. In this basis, the gauge Lagrangian is diagonal:

$$\mathcal{L}_{\text{gauge}} = \frac{1}{32} \int d^2\theta \{W'_a W'_a + W_b W_b\} . \quad (2.4)$$

However, the same shift must also be performed in the interaction piece:

$$\begin{aligned} \mathcal{L}_{\text{int}} &= \int d^4\theta \left\{ \varphi_i^\dagger e^{2g_a V_a} \varphi_i + \Phi_i^\dagger e^{2g_b V_b} \Phi_i \right\} \\ &= \int d^4\theta \left\{ \varphi_i^\dagger e^{2g_a V_a + 2g_a \chi V_b} \varphi_i + \Phi_i^\dagger e^{2g_b V_b} \Phi_i \right\} \end{aligned} \quad (2.5)$$

where the final term is in the basis in which the kinetic terms are diagonal. Here we denote by φ_i those chiral superfields charged under only $U(1)_a$ (the visible sector), and by Φ_i those charged under only $U(1)_b$ (the hidden sector).

Eq. (2.5) implies a number of new interactions between the visible and hidden sectors. First, the visible-sector states obtain hidden charges proportional to their visible-sector charges and couplings:

$$\mathcal{D}_b^\mu = \partial^\mu + i(g_b Q_b + g_a \chi Q_a) V_b^\mu . \quad (2.6)$$

Second, the visible-sector fields and their superpartners now couple to the gauginos of the hidden sector:

$$\mathcal{L} = i\sqrt{2} g_a Q_a \chi \varphi_i^\dagger \tilde{\varphi}_i \lambda_b + \text{h.c.} \quad (2.7)$$

Third and finally, upon solving for the D -terms via their equations of motion, one finds:

$$\begin{aligned} D'_a &= -g_a \sum_i Q_{ai} |\varphi_i|^2 \\ D_b &= -g_b \sum_i Q_{bi} |\Phi_i|^2 - \chi g_a \sum_i Q_{ai} |\varphi_i|^2 . \end{aligned} \quad (2.8)$$

The scalar potential is then given by $V = \frac{1}{2} D'_a D'_a + \frac{1}{2} D_b D_b$. Also note that in the presence of a Fayet-Iliopoulos term $\xi \int d^4\theta V$ for $U(1)_b$, we have $D_b \rightarrow D_b + \xi$.

Several points are immediately apparent. First, SUSY-breaking has been communicated to the visible sector via D_b , and there arise new kinetic mixing contributions to the soft masses of visible-sector scalars:

$$(m_i^2)_{\text{KM}} = g_a \chi Q_{ai} \langle D_b \rangle . \quad (2.9)$$

These contributions are in addition to any other induced soft masses, *e.g.*:

$$m_i^2 = (m_i^2)_{\text{GM}} + (m_i^2)_{\text{SUGRA}} + (m_i^2)_{\text{KM}} + g_a Q_{ai} \langle D_a \rangle , \quad (2.10)$$

where the last term is the usual visible-sector D -term. In the MSSM, we have

$$g_a Q_{ai} \langle D_a \rangle = M_Z^2 \cos 2\beta (Q_i \cos^2 \theta_W - Y_i) \quad (2.11)$$

where Q is ordinary electric charge, Y is the hypercharge, and $\tan \beta = \langle H_u \rangle / \langle H_d \rangle$. A reorganization of Eq. (2.10) then leads to

$$\begin{aligned} (m_i^2)_{\text{KM}} + g_a Q_{ai} \langle D_a \rangle &= g_a Q_{ai} (\langle D_a \rangle + \chi \langle D_b \rangle) \\ &= M_Z^2 \cos 2\beta \left[Q_i \cos^2 \theta_W - Y_i \left(1 + \frac{\eta}{\cos 2\beta} \right) \right] \end{aligned} \quad (2.12)$$

where the final equality holds in the MSSM, and where we have defined

$$\eta \equiv \frac{\chi \langle D_b \rangle}{M_Z^2} . \quad (2.13)$$

Thus η is the parameter which signals the importance of kinetic mixing effects.

There are three distinct cases to consider, depending on whether in Eq. (2.12) we have $\eta \gg 1$, $\eta \sim 1$, or $\eta \ll 1$. If $\eta \gg 1$, then kinetic mixing is the dominant messenger of SUSY-breaking, leading to two related disasters. First, the hierarchy is destabilized, with the scalar masses being pulled far above M_Z to the hidden-sector SUSY-breaking scale. Second, since $U(1)_a$ anomaly cancellation requires that both signs of $U(1)_a$ charge exist, at least one state must receive negative mass-squared, thereby breaking the symmetries under which it transforms. For the case of hypercharge, this is obvious, since the states of the MSSM carry both signs of Y . Thus kinetic mixing with Y cannot be the primary means for communicating SUSY-breaking to the visible world.

If $\eta \sim 1$, then kinetic mixing provides a measurable correction to the soft scalar masses. It is noteworthy that these corrections are flavor-independent and do not directly produce additional flavor-changing neutral current (FCNC) effects. Extracting such contributions from the usual SUGRA and/or GM soft masses is likely to be quite involved. For example, there are no simple soft mass sum rules which directly measure η without reference to the original ($\eta = 0$) soft masses. Typical relations are of the form:

$$m_{\tilde{e}_L}^2 + m_{\tilde{\nu}}^2 = 2(m_L^2)_{\eta=0} - \frac{1}{2}M_Z^2(\cos 2\beta + \eta) \quad (2.14)$$

where $(m_L^2)_{\eta=0}$ needs to be given by a theory which in turn is presumably calibrated by other observables.

The third and final case, with $\eta \ll 1$, is trivial. Here the effects of kinetic mixing are small either because χ itself is very small, or because the hidden-sector D_b -term is much less than the intrinsic SUSY-breaking scale itself. An interesting example with $\langle D_b \rangle \sim M_Z$ is provided by models in which SUSY-breaking is communicated to the visible world by a $U(1)_X$ whose anomalies are cancelled via the Green-Schwarz mechanism [9]. In such models, kinetic mixing in the D -terms provides only a small correction to the masses of the MSSM particles unless $\chi \sim \mathcal{O}(1)$. (We will argue in the next section that such large values of χ are indeed possible.) Also note that one must still verify that loops of $U(1)_X$ gauginos coupled to the MSSM particles do not induce large masses proportional to the F -components in the hidden sector.

How small must χ be in the generic case in order to avoid either destabilizing a model, or at least providing large corrections to it (*i.e.*, to avoid $\eta \gtrsim 1$)? As an example, let us consider two standard cases in which we again identify $U(1)_a$ with the $U(1)$ of hypercharge: that of SUGRA-mediated SUSY-breaking in which we assume $\langle D_b \rangle \sim \Lambda^2 \sim M_Z M_{\text{Pl}}$, and that of gauge-mediated (GM) SUSY-breaking in which we assume $\langle D_b \rangle \sim (\alpha_Y/4\pi)^{-2} M_Z^2 \sim (100 \text{ TeV})^2$. In these two cases, stability of the weak scale puts upper bounds on χ :

$$|\chi| \lesssim \begin{cases} (M_Z/M_{\text{Pl}})/g_Y & \sim 10^{-16} & (\text{SUGRA}) \\ (\alpha_Y/4\pi)^2/g_Y & \sim 10^{-6} & (\text{GM}) \end{cases} \quad (2.15)$$

Since χ is dimensionless and no new symmetries necessarily arise as $\chi \rightarrow 0$, such small values of χ are unnatural from the point of view of 't Hooft and require either some conspiracy or some new symmetry of the theory. This second possibility will be further explored in the next section.

3 Kinetic Mixing in Field Theory and String Theory

Having argued that non-zero χ can lead to large corrections to the soft scalar masses, we now consider the typical magnitude of χ which one would expect to be generated both in field theory and in string theory.

3.1 Expected magnitude of kinetic mixing in field theory

Let us first consider the generation of χ in an effective field theory context. Kinetic mixing can be generated at one loop when there exist states which are simultaneously charged under both $U(1)$ gauge factors. Consider a chiral superfield of mass m with charges Q_a and Q_b under the two $U(1)$ gauge factors. Such a chiral superfield contributes to the two-point polarization diagram of Fig. 1:

$$\Pi_{ab}^{\mu\nu}(\mu) = \frac{g_a g_b}{16\pi^2} Q_a Q_b \log\left(\frac{m^2}{\mu^2}\right) [k^\mu k^\nu - k^2 g^{\mu\nu}] \quad (3.1)$$

In the effective Lagrangian, this generates the operator $\frac{1}{2} F_{\mu\nu}^{(a)} F^{(b)\mu\nu}$ with coefficient [7]

$$\chi(\mu) = -\frac{g_a g_b}{16\pi^2} Q_a Q_b \log\left(\frac{m^2}{\mu^2}\right) \quad (3.2)$$

In general, at scales μ far from m , it is necessary to resum the large logarithms using the renormalization group equations (RGE's):

$$\begin{aligned} \frac{dg_a}{dt} &= \frac{1}{16\pi^2} g_a^3 b_{aa} \\ \frac{dg_b}{dt} &= \frac{1}{16\pi^2} g_b \left(g_b^2 b_{bb} - 2g_a g_b \chi b_{ab} \right) \\ \frac{d\chi}{dt} &= \frac{1}{16\pi^2} \left(g_a^2 \chi b_{aa} + g_b^2 \chi b_{bb} - 2g_a g_b b_{ab} \right) \end{aligned} \quad (3.3)$$

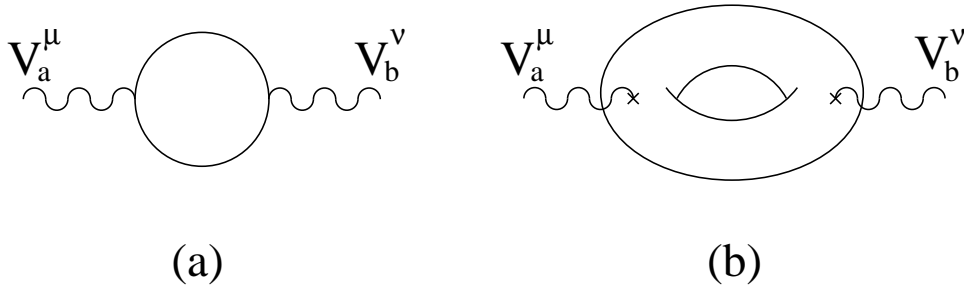


Figure 1: (a) The one-loop diagram which contributes to kinetic mixing in field theory, and (b) its generalization to string theory.

where $b_{ab} \equiv Q_a Q_b$ for the single superfield. For two or more superfields, we take $b_{ab} \equiv \sum_i Q_a^{(i)} Q_b^{(i)}$. Note that these RGE's hold only in the limit $\chi \ll 1$; expressions valid for all χ can be found in Refs. [10, 11]. As shown in Ref. [10], large values of χ (*i.e.*, $0.1 \lesssim \chi \lesssim 1$) can quite easily be generated by such renormalization group running in realistic models, thereby changing the low-energy phenomenology substantially.

In hidden-sector models, $Q_a Q_b$ is by definition zero for the states below the SUSY-breaking scale. Thus, at one loop, non-zero values of χ are not generated in the effective field theory below this scale. Nonetheless it is entirely possible that non-zero χ can be generated as a threshold effect in the full theory at scales far above the SUSY-breaking scale, a region about which one has in principle very little knowledge. In the low-energy theory, such a value would not be suppressed by powers of the high mass scale.

In order to estimate the typical size of such an effect, let us consider a toy model with two chiral superfields of charges (Q_a, Q_b) and $(Q_a, -Q_b)$ and masses m and m' respectively. Their joint contribution to χ has the form:

$$\chi = -\frac{g_a g_b}{16\pi^2} Q_a Q_b \log \left(\frac{m^2}{m'^2} \right). \quad (3.4)$$

At scales far below m and m' , this constitutes a threshold correction. For $m - m' \sim m$, charges of $\mathcal{O}(1)$, and gauge couplings in the range $1/60 \simeq \alpha_Y(M_Z) \lesssim \{\alpha_a, \alpha_b\} \lesssim \alpha_Y(M_{\text{GUT}}) \simeq 1/25$, this leads to a contribution to χ of typical magnitude

$$10^{-2} \lesssim \chi \lesssim 10^{-3}. \quad (3.5)$$

This is only mildly dependent on the mass scale of the states through the running of the couplings. Given the bounds in Eq. (2.15), this clearly constitutes a large effect.

One may wonder if it is possible to prevent the appearance of kinetic mixing by requiring some property of the spectrum. Indeed there are two possibilities.

Clearly, if one or both of the $U(1)$ gauge factors sits within an unbroken non-abelian gauge symmetry, then kinetic mixing is not possible. However, one could suppose that the spectrum of matter states fills out unsplit non-abelian multiplets

despite having broken off a $U(1)$ gauge factor from the full non-abelian gauge symmetry. While such a spectrum would indeed provide for an exact cancellation of kinetic mixing because of the tracelessness of the $U(1)$ generators on the non-abelian multiplets, the presumed mass degeneracy of the states within such multiplets is not stable against radiative corrections and therefore non-zero kinetic mixing is generated after all. We will see in the next section that this is exactly the situation in certain string models.

Another possibility is to forbid non-zero χ by imposing a discrete symmetry. Such a symmetry would be a cousin of charge conjugation which acts non-trivially on only the hidden $U(1)$. In particular, consider embedding $U(1)_b$ at large scales into a non-abelian group G in which there exists a G -transformation Γ that inverts $U(1)_b$. Let us suppose that when G breaks, it leaves, in addition to $U(1)_b$, Γ as an unbroken discrete gauge symmetry. In the low-energy theory, Γ acts on $U(1)_b$ (but not on the theory as a whole) as a charge conjugation. In this case the states charged under $U(1)_b$ appear in degenerate conjugate pairs, and no χ can be induced. Equivalently, the kinetic mixing operator itself is forbidden by the symmetry. It is worth noting that the types of symmetries which can forbid the existence of a Fayet-Iliopoulos term are exactly of this form. It is also very interesting that such a symmetry necessarily implies the existence of stable “Alice strings” with their delocalized charged excitations [12]. Examples of such models include the $SU(6) \times U(1)$ SUSY-breaking model in Ref. [5] and the visible-sector left-right model of Ref. [13].

Apart from this one exception, we will now argue that the “ultimate” high-scale theory, namely string theory, naturally leads to a generation of $\chi \neq 0$.

3.2 Kinetic mixing in string theory: General formalism

Since realistic string models often have gauge groups containing many $U(1)$ gauge factors as well as infinite towers of massive (Planck-scale) string states, such theories serve as an ideal laboratory for estimating the magnitude of kinetic mixing effects. First, we shall show how kinetic mixing effects can be calculated in string theory. Then, we shall discuss the typical sizes that string theory predicts for such effects.

We begin, however, with few preliminary remarks concerning generic properties of string spectra. Unlike the case in field theory, the tree-level string spectrum consists of states whose masses are quantized in units of the Planck mass. Only the states of lowest energy (the massless states) are observable. Furthermore, in string theory, the freedom to alter or redefine the charges of states is extremely limited, for one must satisfy a number of worldsheet consistency constraints stemming from worldsheet conformal invariance, modular invariance, and worldsheet supersymmetry. One therefore seeks to construct a particular string theory (or “string model”) which satisfies all of these constraints and which is also “realistic” — *i.e.*, whose massless states are those of the MSSM or its extensions coupled to $N = 1$ supergravity. Indeed, once a particular string model is constructed that satisfies all of these

constraints, its entire spectrum of massless and massive string states is completely fixed, and their quantum numbers cannot be altered. Thus string theories provide a very rigid structure in which certain phenomenological properties such as kinetic mixing may be meaningfully tested.

With this in mind, our goal is to repeat the one-loop field-theoretic calculation discussed above, only now in a string-theoretic context. Certain parts of this calculation are straightforward. As illustrated in Fig. 1, the string generalization of the field-theoretic vacuum polarization diagram is a torus amplitude with two vertex operators inserted on the worldsheet (corresponding to the gauge bosons of the $U(1)_a$ and $U(1)_b$ gauge factors). All of the string states (both massless and massive) propagate in the torus and thereby contribute to this one-loop diagram. However, in order to evaluate this diagram in a fully consistent way in string theory, we must perform this calculation in a non-trivial background which satisfies the string equations of motion and which, in particular, takes into account not only the contributions from the gauge interactions but also their gravitational back-reaction in the presence of a suitable regulator [14, 15]. Indeed, *a priori*, both gauge and gravitational terms contribute in generating a non-zero value of χ in the string-derived low-energy effective Lagrangian \mathcal{L} in Eq. (2.1).

Such string calculations have previously been performed in the case that the two inserted gauge bosons come from the *same* $U(1)_a$ gauge factor; in such cases the result describes the coefficient of the corresponding $F_{\mu\nu}^{(a)} F^{(a)\mu\nu}$ factor in \mathcal{L} , and is relevant for the study of the so-called “heavy string threshold contributions” to the corresponding gauge coupling g_a [14, 15, 16]. Recall that in such cases, the effect of this one-loop torus amplitude is to correct the gauge coupling $g_a(\mu)$ at a scale μ according to

$$\frac{1}{g_a^2(\mu)} = k_a \frac{1}{g_X^2} + \frac{b_a}{16\pi^2} \ln \frac{M_X^2}{\mu^2} + \frac{1}{16\pi^2} \Delta_a . \quad (3.6)$$

Here M_X and g_X are the unification scale and coupling (here to be identified with the string scale and coupling), while k_a is the $U(1)_a$ normalization factor. This normalization factor is analogous to the string Kač-Moody level which appears for non-abelian gauge groups [17], and is defined analogously as the coefficient of the double-pole term in the worldsheet operator-product expansion (OPE) of the $U(1)_a$ current J_a with itself:

$$J_a(z) J_a(w) = \frac{k_a/2}{(z-w)^2} + \text{regular} . \quad (3.7)$$

For example, in the case of hypercharge $U(1)_Y$, the MSSM predicts $k_Y = 5/3$. Finally, the threshold correction Δ_a in Eq. (3.6) is defined as

$$\Delta_a \equiv k_a \mathcal{Y} + \int_{\mathcal{F}} \frac{d^2\tau}{\text{Im } \tau} \left[B_a(\tau) - b_a \right] . \quad (3.8)$$

Here \mathcal{Y} is a so-called “universal term” which receives contributions from both gravitational back-reaction and universal gauge oscillators [18, 15]; the integral over the complex parameter τ in the modular region \mathcal{F} represents a summation over conformally inequivalent tori; b_a is the one-loop beta-function coefficient (calculated by considering only the massless string spectrum); and $B_a(\tau)$ represents a supertrace over *all* string states with arbitrary left- and right-moving spacetime masses M_L and M_R :

$$B_a(\tau) \equiv \text{Str} \left\{ \left(\frac{1}{12} - \overline{Q_H}^2 \right) Q_a^2 \overline{q}^{\alpha' M_R^2} q^{\alpha' M_L^2} \right\} , \quad q \equiv e^{2\pi i \tau} . \quad (3.9)$$

Here Q_a is the gauge charge operator for the gauge group factor $U(1)_a$, α' is the Regge slope, and $\overline{Q_H}$ is the spacetime helicity operator. In this context, recall that in field theory, the one-loop β -function coefficient can be written as $b_a = \text{Str} [(1/12 - h^2) Q_a^2]$ where h is the helicity operator (with $h = 0$ for scalars, $\pm 1/2$ for fermions, and ± 1 for vectors). Likewise, in Eq. (3.9), $\overline{Q_H}$ is the analogous helicity operator for states of arbitrarily high spin. Since we are working in the context of string theories with spacetime supersymmetry, we can omit the factor of $1/12$ in Eq. (3.9) since its contribution is proportional to $\text{Str} \mathbf{1} = 0$.

Given these results, it is relatively straightforward to generalize the calculation to the case when the two vertex-operator insertions correspond to *different* $U(1)$ gauge factors. Taking $\chi = 0$ at the unification scale at tree level, one then finds that the string spectrum naturally generates a non-vanishing value for χ at one loop given by

$$\left\{ \frac{\chi}{g_a g_b} \right\} (\mu) = \frac{b_{ab}}{16\pi^2} \ln \frac{M_X^2}{\mu^2} + \frac{1}{16\pi^2} \Delta_{ab} . \quad (3.10)$$

Note that according to the RGE's in Eq. (3.3), it is the combination $\chi/g_a g_b$ which runs analogously to the usual gauge couplings $1/g_a^2$. Thus we see that Eq. (3.10) is indeed the string analogue of the field-theoretic expression given in Eq. (3.2). The first term in Eq. (3.10) represents the contribution from the massless string states, with b_{ab} serving as their “mixed” beta-function coefficient

$$b_{ab} = - \text{Str}_{\text{massless}} \overline{Q_H}^2 Q_a Q_b , \quad (3.11)$$

while the second term in Eq. (3.10) is the kinetic mixing threshold correction due to the infinite tower of massive string states. This threshold correction is defined as

$$\Delta_{ab} = k_{ab} \mathcal{Y} + \int_{\mathcal{F}} \frac{d^2 \tau}{\text{Im } \tau} \left[B_{ab}(\tau) - b_{ab} \right] . \quad (3.12)$$

Here \mathcal{Y} is the same universal term that appears in the above gauge coupling calculation, and k_{ab} is defined analogously to Eq. (3.7) via the OPE between the $U(1)$ currents J_a and J_b :

$$J_a(z) J_b(w) = \frac{k_{ab}/2}{(z-w)^2} + \text{regular} . \quad (3.13)$$

Note that $k_{aa} = k_a$. Likewise, $B_{ab}(\tau)$ is the kinetic mixing supertrace

$$B_{ab}(\tau) \equiv - \text{Str } \overline{Q_H}^2 Q_a Q_b \overline{q}^{\alpha' M_R^2} q^{\alpha' M_L^2} . \quad (3.14)$$

Thus, since Δ_{ab} contributes as a threshold correction due to purely massive string states, it is this object which is our primary focus.

As indicated in Eq. (3.12), Δ_{ab} generally contains two separate contributions. However, while the second term in Eq. (3.12) is highly model-dependent, the first term $k_{ab}\mathcal{Y}$ is universal and thus can be evaluated in a general setting. Indeed, even though \mathcal{Y} generally receives contributions from both gravitational back-reaction and gauge oscillators, calculating such contributions is unnecessary in the present case because taking $\chi = 0$ at tree level is tantamount to requiring that $U(1)_a$ and $U(1)_b$ be orthogonal gauge factors in our string model. This in turn implies that OPE's between their currents should vanish, or that $k_{ab} = 0$. This can also be seen by writing our two $U(1)$ gauge factors as linear combinations of the 22 elementary $U(1)$ worldsheet currents J_i ($i = 1, \dots, 22$) of the heterotic string,

$$U(1)_a = \sum_{i=1}^{22} a_i^{(a)} J_i , \quad U(1)_b = \sum_{i=1}^{22} a_i^{(b)} J_i , \quad (3.15)$$

for it is then straightforward to show that the one-loop universal contributions are all proportional to

$$k_{ab} \equiv 2 \sum_i a_i^{(a)} a_i^{(b)} . \quad (3.16)$$

Indeed, since it is known [17] that $k_a = 2 \sum_i [a_i^{(a)}]^2$, we see that once again we have the relation $k_{aa} = k_a$. However, orthogonality of the $U(1)_a$ and $U(1)_b$ gauge factors explicitly means that $\vec{a}^{(a)} \cdot \vec{a}^{(b)} = 0$. This again implies that $k_{ab} = 0$.

Thus, we conclude that only the second term in Eq. (3.12) can contribute to one-loop kinetic mixing effects in string theory.

3.3 Kinetic mixing in string theory: Expected magnitude

Given these results, we now wish to determine the expected size of the kinetic mixing parameter χ in string theory. In other words, given two orthogonal $U(1)$ gauge factors in a given string model, how large a mixing contribution Δ_{ab} will typically be generated by summing over the contributions of all of the massive string states?

In order to estimate the size of such effects, one can calculate Δ_{ab} for different orthogonal $U(1)$ gauge factors in a variety of four-dimensional string models. In general, for string models with moduli of order one, it turns out that one typically obtains quite sizable results, with Δ_{ab} lying in the range

$$10^{-1} \lesssim |\Delta_{ab}| \lesssim 10^{+1} ; \quad (3.17)$$

moreover, in generic models with larger moduli, $|\Delta_{ab}|$ increases dramatically. We stress that the results in Eq. (3.17) indeed record the contributions from only the

massive string states, for any undesired kinetic mixings due to massless string states are explicitly removed from the integrand of Eq. (3.12) through the subtraction of the mixed beta-function coefficient b_{ab} . Thus, *a priori*, Eq. (3.17) sets the scale for the expected contributions to kinetic mixing from the massive states in string theory. This in turn implies that at the weak scale, we have

$$3g_b \times 10^{-4} \lesssim \chi(M_Z) \lesssim 3g_b \times 10^{-2}. \quad (3.18)$$

Here we have interpreted $U(1)_a$ as hypercharge, and have used Eq. (3.10) to translate from Δ_{ab} to χ . As can be seen from comparison with Eq. (2.15), this is a very large effect which has the potential to destabilize the supersymmetric gauge hierarchy.

However, one possible objection to this result is the fact that it is not sufficient to demand mere orthogonality of the different $U(1)$ gauge factors. Indeed, we are interested in the case when these $U(1)$ gauge factors are also presumably *hidden* from each other. Of course, if two $U(1)$ gauge factors are truly hidden from each other in this way, then we have $Q_a Q_b = 0$ for each state in the string spectrum. This in turn implies that all of the above terms vanish, and that no kinetic mixing is generated at one loop. However, it turns out that imposing this condition is far too severe in the context of string theory — we should really only require that the *massless* (*i.e.*, observable) string states satisfy $Q_a Q_b = 0$. Indeed, given a particular choice of charge assignments for the massless states, the string self-consistency constraints do not generally permit all massive string states to satisfy $Q_a Q_b = 0$. In such cases we would then have $b_{ab} = 0$, but χ would still receive contributions from Δ_{ab} as in Eqs. (3.10) and (3.12).

Unfortunately, no realistic string models have been constructed for which $Q_a Q_b = 0$ for all states in the massless spectrum. Indeed, in string theory, it has become traditional to refer to a gauge group as “hidden” merely if it does not couple to those states in the massless spectrum which correspond to the Standard Model particles — such hidden gauge symmetries can nevertheless couple to the extra states beyond the Standard Model which also appear in the massless spectrum and which have Standard Model quantum numbers. We will therefore refer to such groups as being only “semi-hidden”. Thus, we see that in general, semi-hidden $U(1)$ gauge group factors can still give rise to mixed coefficients $b_{ab} \neq 0$. As indicated in the first term of Eq. (3.10), this may serve as an additional string-theoretic source for kinetic mixing. To be conservative, however, in what follows we shall focus only on the remaining terms, namely the corrections Δ_{ab} due to the *massive* string states.

In order to calculate Δ_{ab} for the case when $U(1)_a$ and $U(1)_b$ are semi-hidden relative to each other, we have evaluated Eq. (3.12) within the context of various special semi-realistic string models which have been constructed in the literature [19, 20, 21]. All of these string models have $N = 1$ supersymmetry, three chiral generations, and phenomenologically interesting gauge groups such as $SU(3) \times SU(2) \times U(1)_Y$ and $SO(6) \times SO(4)$. Moreover, they also give rise to semi-hidden gauge symmetries including extra $U(1)$ gauge factors. We thus seek to calculate the mixing that takes

place in these models between the hypercharge $U(1)_Y$ and one of these extra semi-hidden $U(1)$ gauge factors. Indeed, such a calculation is completely analogous to the gauge-coupling threshold correction calculations that were performed for these string models in Ref. [22].

The result we obtain, however, is quite surprising: in each of those realistic string models, it turns out that Δ_{ab} in Eq. (3.12) vanishes exactly! This occurs *in spite* of the fact that these models contain states (even massless states) which simultaneously carry both hypercharge and semi-hidden $U(1)$ charge. Indeed, even though such states exist in these models, their contributions nevertheless cancel level-by-level across the entire massless and massive string spectra.

It is easy to see why such cancellations arise in these particular models. In the $SO(6) \times SO(4)$ string model [19], for example, this cancellation arises due to the field-theoretic GUT mechanism discussed previously: the hypercharge $U(1)$ generator is embedded within the larger non-abelian group structure $SO(6) \times SO(4)$, and therefore the trace over all multiplets cancels exactly (or equivalently, the kinetic mixing term in the effective Lagrangian would not be invariant under the full non-abelian $SO(6) \times SO(4)$ gauge symmetry, and thus cannot exist). A similar phenomenon arises even in those string models [20, 21] whose low-energy gauge groups are already broken down to $SU(3) \times SU(2) \times U(1)_Y$ at the Planck scale.

However, this cancellation is ultimately unstable against a variety of effects beyond tree level because the exact cancellation of Δ_{ab} relies on the exact tree-level degeneracy of the states at each mass level. Effects that can destroy this exact degeneracy include GUT symmetry breaking, Dine-Seiberg-Witten shifts of the string ground state (which are generally necessary in order to break anomalous $U(1)$ gauge symmetries and restore spacetime supersymmetry [23]), and low-energy supersymmetry breaking. In fact, even if these effects do not destroy the degeneracy, renormalization group flow down to low energies inevitably will. For example, in the string model of Ref. [20], the contribution to kinetic mixing from extra massless electroweak doublet states beyond the MSSM is cancelled by those from extra Standard Model singlets and color triplets. While it is noteworthy that such cancellations arise in the first place, they are clearly unstable against radiative corrections. Thus, since such cancellations are not protected by any symmetries, they must be viewed as accidental artifacts of the tree-level spectrum.

Given this situation, we would like to calculate the amount of kinetic mixing that will be induced in these models by all of these effects. *A priori*, we would need to calculate the exact spectrum of massless and massive string states after all of the above effects have been included. We would then use this complete spectrum as an input into Eq. (3.12), thereby iteratively calculating the true value of χ . Unfortunately, such a calculation is beyond present capabilities.

However, one can estimate the mass splittings generated in the string spectrum by each of the above effects. To be conservative, we will estimate the contributions from the splittings of only the *massive* states; it is clear that splittings of the massless

states would yield a very large effect. Our procedure will be as follows. For simplicity, we assume that the contributions of the states in the tree-level spectrum cancel pairwise in Δ_{ab} . We can then use the results in Eq. (3.4), together with an estimate of the splitting Δm between m and m' , to determine the resulting contribution to χ from each pair. Each of the effects which splits the pairs has a characteristic scale associated with it. Mass splittings due to GUT symmetry breaking are typically $\Delta m \sim 10^{16}$ GeV. In the case of vacuum shifting, while it is true that at least one scalar field obtains a vacuum expectation value of approximately 10^{17} GeV, the effects on the remaining states are typically suppressed due to discrete symmetries which forbid low-dimension terms in the superpotential. Indeed, a typical mass scale for such splittings has been argued [24] to be as low as 10^{11} GeV $\lesssim \Delta m \lesssim 10^{14}$ GeV. Next, in the case of splittings induced by SUSY-breaking, the appropriate scale is the fundamental scale of SUSY-breaking in the hidden sector, since by definition the states are charged under the $U(1)_b$ which couples directly to the SUSY-breaking sector. In the supergravity case we have $\Delta m \sim 10^{11}$ GeV, while in the gauge-mediated case we have $\Delta m \sim 10^4$ GeV. Finally, in all three of the above cases, RGE flows generally produce additional splittings which will be of the same order as their respective splitting scales Δm .

The above Δm splittings then produce the following estimates for χ :

$$\frac{|\chi|}{g_b C} \sim \begin{cases} 10^{-4} & \text{(GUT)} \\ 10^{-6} - 10^{-9} & \text{(vacuum shift)} \\ 10^{-9} & \text{(SUGRA)} \\ 10^{-16} & \text{(GM)} . \end{cases} \quad (3.19)$$

In the above estimates, g_b is the hidden-sector $U(1)_b$ coupling, while the quantity C parametrizes the effect of the summation over all newly split pairs of states throughout the string spectrum. As such, we estimate the combined effect of such a summation to be approximately

$$10^1 \lesssim C \lesssim 10^2 . \quad (3.20)$$

We emphasize again that in this estimate, we are being conservative by ignoring the effects of the splittings of massless string states which can also be sizable. We also point out that the effects in Eq. (3.19) are not exclusive of each other; a given string model will typically be subject to a simultaneous combination of these effects.

Thus, our conclusions regarding the size of kinetic mixing in string theory are as follows. In general, the expected scale for kinetic mixing effects is given by Eq. (3.18). However, in certain semi-realistic string models, there is an accidental cancellation of kinetic mixing effects in the tree-level spectrum. In such cases, non-zero kinetic mixing will then be generated by the effects considered above, and the resulting magnitudes for kinetic mixing from each effect are given in Eq. (3.19). We see from these results that there exists considerable variation in the possible amounts of kinetic mixing. Nevertheless, as can be seen by comparing these results with those of Eq. (2.15), they are all in the interesting range for low-energy phenomenology.

4 Conclusions

We have demonstrated the existence of an important new mechanism for the communication of supersymmetry breaking from a hidden sector to the visible sector. This mechanism applies when there exists in the hidden sector a $U(1)$ gauge symmetry that is not isolated from SUSY-breaking, and it relies on a D -term interaction which is induced by the supersymmetric generalization of kinetic mixing.

The new contributions to the soft squared masses for scalars are proportional to hypercharge Y . In cases where the D -term of the hidden $U(1)$ is larger than the weak scale (in particular, of order the scale of fundamental SUSY-breaking), this leads to phenomenologically disastrous consequences unless the mixing parameter χ is very small. In particular, one must have $|\chi| \lesssim 10^{-6}$ in the case of gauge-mediated models with D -terms $\sim (100 \text{ TeV})^2$, and $|\chi| \lesssim 10^{-16}$ in the case of supergravity-mediated models with D -terms $\sim (10^{10} \text{ GeV})^2$.

We have argued that since χ is a renormalizable interaction, its value is sensitive to physics at all mass scales. In particular, substantial values of χ can be generated at arbitrarily high mass scales in both field theory and string theory contexts. We have shown how to calculate the amount of kinetic mixing generated in string theory, and found that estimates of the magnitude of χ in both field theory and string theory often lead to values in excess of the above limits.

Thus we conclude that the kinetic mixing parameter χ should be considered as a very natural additional measurable parameter describing the soft SUSY-breaking spectrum.

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