生物統計 **HW5** 解答

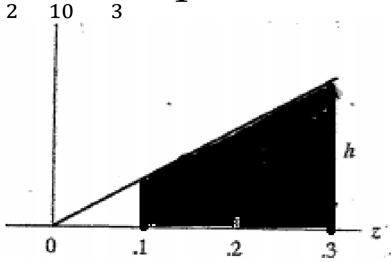
#1.

1. Assume that the density for the random variable Z, the number of cc's of a drug to be prescribed for the control of epileptic seizures, is as shown in Figure 5.4*a*.
   (*a*) Verify that for the area under the graph of f to be 1, h, the height of the triangle, must be $\frac{20}{3}$. *Hint:* The formula for the area of a triangle is $A = \frac{1}{2}$(base)(height).
   (*b*) In Figure 5.4*a* shade the area corresponding to the probability that at least .1 cc should be prescribed.
   (*c*) What probability is represented by the shaded area of Figure 5.4*b*?
   (*d*) What probability is represented by the shaded area of Figure 5.4*c*?
   (*e*) What probability is represented by the shaded area of Figure 5.4*d*?
   (*f*) If you knew the areas in parts *c* and *d*, how would you find the area in part *e*?
   (*g*) Approximate the average dosage required by estimating the balance point by inspection.
   (*h*) It can be shown that the equation of the density for Z is

   $$f(z) = 200z/9 \qquad 0 \le z \le .3$$

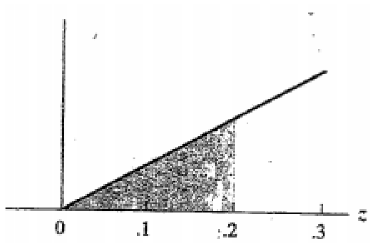
   Use this information to find the value of $f(z)$ when $z = .2$. Now find the area of the shaded region in Figure 5.4*b*, thus finding the probability that at most .2 cc of drug should be prescribed to control seizures.
   (*i*) Use the method of part *h* to find the probability that at most .1 cc of the drug should be prescribed.
   (*j*) Use the information from parts *h* and *i* to find the probability that between .1 cc and .2 cc of the drug should be prescribed.

(a) $\frac{1}{2} * \frac{3}{10} * \frac{20}{3} = 1$

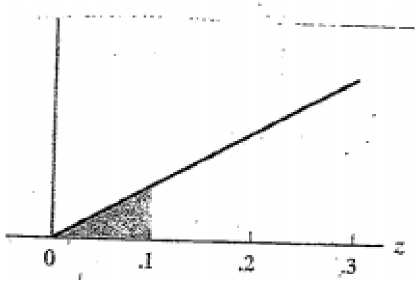


(b)

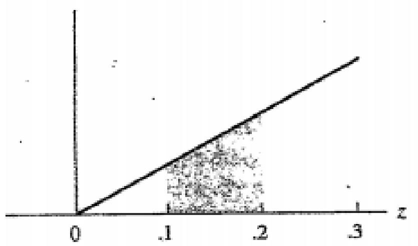
(c) $P(Z \le 0.2)$



(d) $P(Z \le 0.1)$



(e) $P(0.1 \leq Z \leq 0.2)$



(f) $\text{Area}(e) = \text{Area}(c) - \text{Area}(d)$

(g) $\left(\text{Slope of } f(z)\right) = \frac{20}{3} / \frac{3}{10} = \frac{200}{9}$, so $f(z) = \frac{200}{9} z$

$$E[Z] = \int_0^{0.3} z * f(z)\, dz = 0.2 \quad \left(\int_0^{0.3} z * f(z)\, dz = 0.5 \text{ 的解是中位數！}\right)$$

(h) $f(0.2) = \frac{200*0.2}{9} = \frac{40}{9}$, $\text{Area} = \frac{1}{2} * \frac{2}{10} * \frac{40}{9} = \frac{4}{9}$

(i) $\frac{1}{2} * \frac{1}{10} * f(0.1) = \frac{1}{2} * \frac{1}{10} * \frac{20}{9} = \frac{1}{9}$

(j) $\frac{4}{9} - \frac{1}{9} = \frac{1}{3}$

#2.

Uniform distribution. A random variable whose density is flat is said to be *uniformly* distributed. The equation for the curve is $f(x) = c$ where c is a constant. Such densities are easy to handle because the areas involved are always rectangular. The equation for the area of a rectangle is *area* = *base* · *height*. Assume that the random variable X, the time is minutes that it takes a nurse to respond to a patient's call, is uniformly distributed over the interval 0 to 5 minutes. The density is shown in Figure 5.8.

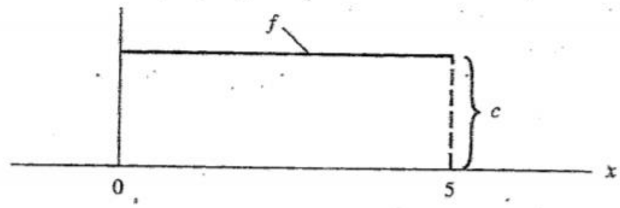
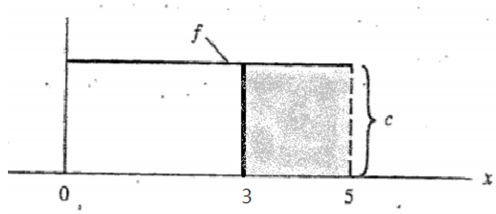


FIGURE 5.8
The random variable X, the time required for a nurse to respond to a patient's call, is uniformly distributed over a 5-minute interval.

(*a*) Verify that in this case $c = \frac{1}{5}$.
(*b*) Shade the area corresponding to the probability that the response time exceeds 3 minutes.
(*c*) Find the probability pictured in part *b*.
(*d*) What is the average response time?

(a) $\int_0^5 c\,dx = 1 \Rightarrow 5c = 1 \Longrightarrow c = \frac{1}{5}$

(b) 

(c) $\int_3^5 c\,dx = 2c = 2 * \frac{1}{5} = \frac{2}{5}$

(d) 題意為求平均 $\int_0^5 cxdx = \frac{1}{2} * \mathrm{c} * 5^2 = \frac{1}{2} * \frac{1}{5} * 5^2 = \frac{5}{2}$

#3.

> IQ test scores. The Wechsler Adult Intelligence Scale (WAIS) is the most common "IQ test." The scale of scores is set separately for each age group and is approximately Normal with mean 100 and standard deviation 15. According to the 68–95–99.7 rule, about what percent of people have WAIS scores
>
> (a) above 100? (b) above 145? (c) below 85?

(a) $P(X > 100) = P\left(Z > \frac{100-100}{15}\right) = P(Z > 0) = 50\%$

(b) $P(X > 145) = P\left(Z > \frac{145-100}{15}\right) = P(Z > 3) \approx 0.15\%$

(c) $P(X < 85) = P\left(Z < \frac{85-100}{15}\right) = P(Z < -1) \approx 16\%$

#4:

(a)

> Low IQ test scores. Scores on the Wechsler Adult Intelligence Scale (WAIS) are approximately Normal with mean 100 and standard deviation 15. People with WAIS scores below 70 are considered mentally retarded when, for example, applying for Social Security disability benefits. According to the 68–95–99.7 rule, about what percent of adults are retarded by this criterion?

Sol):

$$\Pr(X < 70) = \Pr\left(\frac{X-110}{15} < \frac{70-110}{15}\right) = \Pr(Z < -2) \approx 0.0228$$

(b)

> 如果資優定義為 IQ超過平均二倍標準差.
> 請問此智商是多少?(要查表)

Sol):

$$110 + 2 \times 15 = 130$$

#5:

Pregnancy length in horses. Bigger mammals tend to carry their young longer before birth. The length of horse pregnancies from conception to birth varies according to a roughly Normal distribution with mean 336 days and standard deviation 3 days. Use the 68–95–99.7 rule to answer the following questions:

(a)

Almost all (99.7%) horse pregnancies fall within what range of lengths?

Sol):

$$336-3\times 3=327, 336+3\times 3=345$$
$$\rightarrow (327,345)$$

(b)

What percent of horse pregnancies are longer than 339 days?

Sol):

$$\Pr(X>339)=\Pr(\frac{X-336}{3}>\frac{339-336}{3})=\Pr(Z>1)\approx 0.16$$

#6:

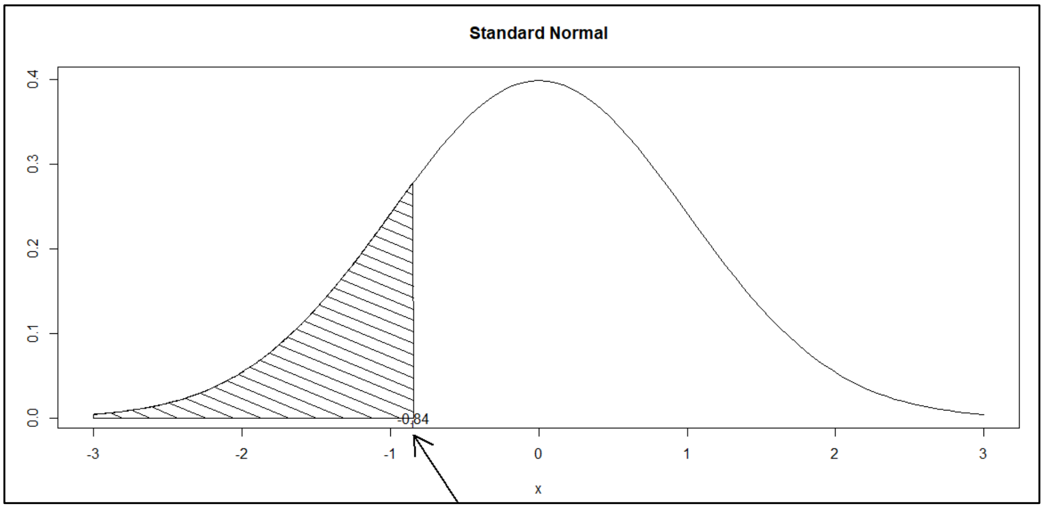
Using Table B. Find the z value that satisfies each of the following conditions (report the value of z that comes closest to satisfying the condition). In each case, sketch a standard Normal curve with your value of z marked on the axis.

(a)

20% of the observations fall below z.

Sol):

$\Pr(Z < z) = 0.2 \rightarrow z \approx -0.84$

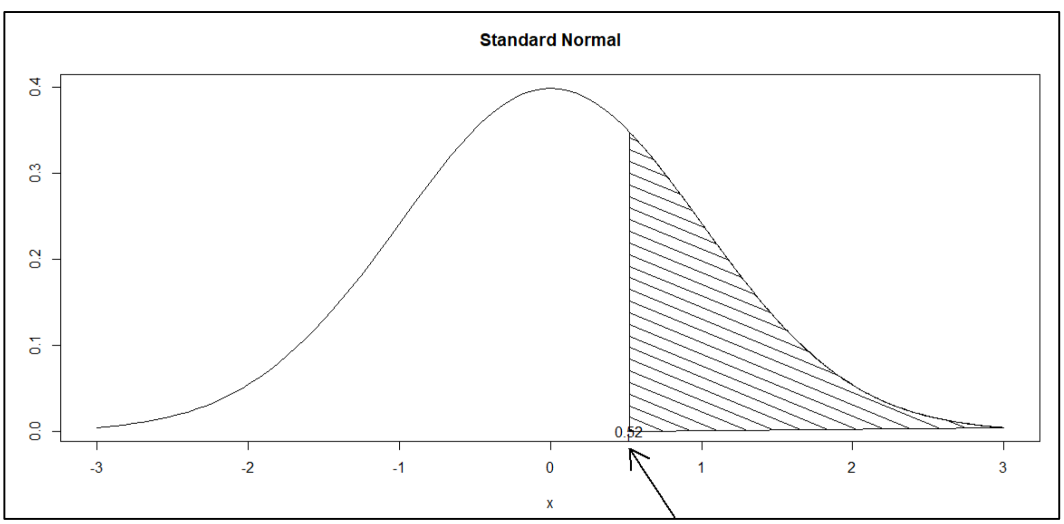


(b)

30% of the observations fall above z.

Sol):

$\Pr(Z > z) = 0.3 \rightarrow z \approx 0.52$

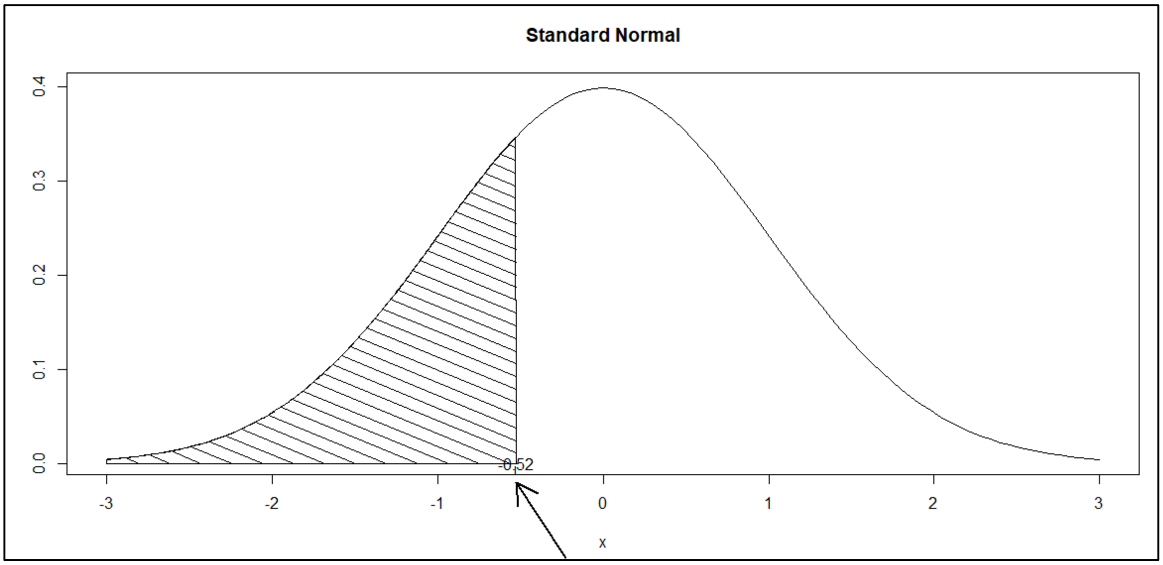


(c)

30% of the observations fall below z.

Sol):

$\Pr(Z < z) = 0.3 \to z \approx -0.52$



(d)

50% of the observations fall between z.

Sol):

$\Pr(-z < Z < z) = 0.5$

$\to \Pr(Z < z) - \Pr(Z < -z) = \Pr(Z < z) - [1 - \Pr(Z < z)] = 2\Pr(Z < z) - 1 = 0.5$

$\to \Pr(Z < z) = 0.75 \to z \approx 0.67$