

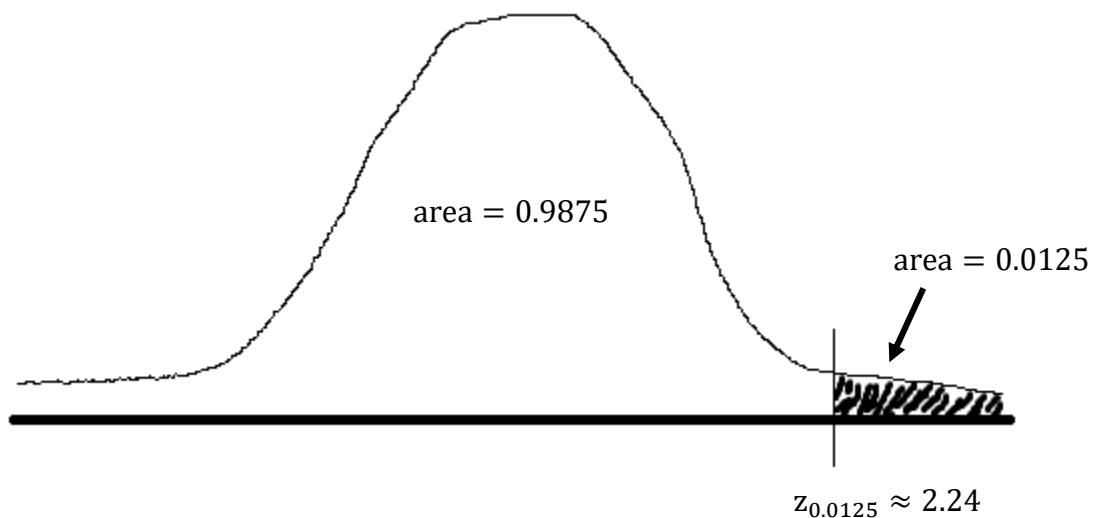
生物統計 HW7 解答

#1.

Find a critical value. The critical value z^* for confidence level 97.5% is not in Table C. Use software or Table B of standard Normal probabilities to find z^* . Include in your answer a copy of Figure 14.4 with $C = 0.975$ that shows how much area is left in each tail when the central area is 0.975.

Sol):

$$\alpha = 0.025 \rightarrow \frac{\alpha}{2} = 0.0125$$



#2.

Pharmaceutical production. A manufacturer of pharmaceutical products analyzes each batch of a product to verify the concentration of the active ingredient. The chemical analysis is not perfectly precise. In fact, repeated measurements follow a Normal distribution with mean μ equal to the true concentration and standard deviation $\sigma = 0.0068$ grams per liter (g/l). Three analyses of one batch give concentrations 0.8403, 0.8363, and 0.8447 g/l. To estimate the true concentration, give a 95% confidence interval for μ .

Sol):

$$X_1 = 0.8403, X_2 = 0.8368, X_3 = 0.8447 \rightarrow \bar{X} = 0.8404$$

$$\sigma = 0.0068, n = 3$$

$$\rightarrow 95\% \text{ C.I. of } \mu = [\bar{X} \pm Z_{0.025} \frac{\sigma}{\sqrt{n}}] = [0.8404 \pm 1.96 \times \frac{0.0068}{\sqrt{3}}] = [0.8327, 0.8481]$$

#3.

IQ test scores. Here are the IQ test scores of 31 seventh-grade girls in a midwest school district:⁴

114	100	104	89	102	91	114	114	103	105	
108	130	120	132	111	128	118	119	86	72	
111	103	74	112	107	103	98	96	112	112	93

(a)

These 31 girls are an SRS of all seventh-grade girls in the school district. Suppose that the standard deviation of IQ scores in this population is known to be $\sigma = 15$. We expect the distribution of IQ scores to be close to Normal. Make a stemplot of the distribution of these 31 scores (split the stems) to verify that there are no major departures from Normality. You have now checked the “simple conditions” to the extent possible.

Sol):

7 | 2 4
7 |
8 |
8 | 6 9
9 | 1 3
9 | 6 8
10 | 0 2 3 3 3 4
10 | 5 7 8
11 | 1 1 2 2 2 4 4 4
11 | 8 9
12 | 0
12 | 8
13 | 0 2

對稱且成鐘型 \Rightarrow 與常態分佈差異不大

(b)

Estimate the mean IQ score for all seventh-grade girls in the school district using a 99% confidence interval.

Sol):

$$\bar{X} = 105.8387, \sigma = 15, n = 31, Z_{0.005} \approx 2.5758$$

$$\rightarrow 99\% \text{ C.I. of } \mu = [\bar{X} \pm Z_{0.005} \frac{\sigma}{\sqrt{n}}] = [105.8387 \pm 2.5758 \times \frac{15}{\sqrt{31}}] = [98.8993, 112.7781]$$

(c)

求 95% 及 90% confidence intervals
看寬度如何變化
Note 寬度 = $2 \times \text{margin of error}$
代表估計的精確度

Sol):

$$90\% \text{ C.I. of } \mu = [\bar{X} \pm Z_{0.05} \frac{\sigma}{\sqrt{n}}] = [105.8387 \pm 1.645 \times \frac{15}{\sqrt{31}}] = [101.4069, 110.2705]$$

$$95\% \text{ C.I. of } \mu = [\bar{X} \pm Z_{0.025} \frac{\sigma}{\sqrt{n}}] = [105.8387 \pm 1.96 \times \frac{15}{\sqrt{31}}] = [100.5583, 111.1191]$$

⇒ 信心水準越高, 信賴區間越寬