Homework 3

判斷 "Independence"

1. **Prosperity and education.** Call a household prosperous if its income exceeds $100,000. Call the household educated if the householder completed college. Select an American household at random, and let A be the event that the selected household is prosperous and B the event that it is educated. According to the Current Population Survey, $P(A) = 0.138$, $P(B) = 0.261$, and the probability that a household is both prosperous and educated is $P(A \text{ and } B) = 0.082$.

   (a) Find the conditional probability that a household is educated, given that it is prosperous.

   (b) Find the conditional probability that a household is prosperous, given that it is educated.

   (c) Are events A and B independent? How do you know?

2. Mathematically, these two definitions for the relative odds can be shown to be equivalent.

   Consider the following data, taken from another study of the risk factors for breast cancer. This one is a case-control study that examines the effects of the use of oral contraceptives [13]. In a *case-control study*, investigators start by identifying groups of individuals with the disease (the cases) and without the disease (the controls). They then go back in time to determine whether the exposure in question was present or absent for each individual. Among the 989 women in the study who had breast cancer, 273 had previously used oral contraceptives and 716 had not.

(a) In this case, is it possible to approximate RR? Explain.

(b) Some idea of the impact of exposure can be obtained by $\frac{P(E|D)}{P(E|D')}$ where E is the event of exposure, D is the event of disease.

Can each of the conditional probabilities involved in this ratio be estimated? If so, evaluate and interpret the ratio.

3. Patients immobilized for a substantial amount of time can develop deep vein thrombosis (DVT), a blood clot in a leg or pelvis vein. DVT can have serious adverse health effects and can be difficult to diagnose. On its website, drug manufacturer Pfizer reports the outcome of a study looking at the effectiveness of the drug Fragmin (dalteparin) compared with that of a placebo in preventing DVT in immobilized patients.

   In a double-blind, multinational study, severely immobilized patients were randomly assigned to receive daily subcutaneous injections of either Fragmin or a placebo for 12 to 14 days and were followed for 90 days. The results, in number of patients experiencing a complication from DVT (including death), are summarized in the table below.

	Treatment outcome		
	Complication	No complication	Sample size
Fragmin	42	1476	1518
Placebo	73	1400	1473

(a) 求因使用 Fragmin 而 Outcome 爲 Complication 之 Relative Risk and Odds Ratio.

(b) 兩者是否有相關性？(Outcome 是否與用藥有關)

4. The following table shows the density for the random variable X, the number of wing beats per second of a species of large moth while in flight.

x	6	7	8	9	10
$f(x)$	.05	.1	.6	.15	?

(*a*) Find $f(10)$.
(*b*) Find $P[X \le 8]$. Interpret this probability in the context of this problem.
(*c*) Find $P[X < 8]$.
(*d*) Find $P[X \ge 7]$.
(*e*) Find $P[X > 7]$.

5.

A desk lamp produced by The Luminar Company was found to be defective (*D*). There are three factories (*A*, *B*, *C*) where such desk lamps are manufactured. A manager is responsible for investigating the source of found defects. This is what the manager knows about the company's desk lamp production and the possible source of defects:

Factory	% of total production	Probability of defective lamps
A	0.40 = P(A)	0.025 = P(D | A)
B	0.30 = P(B)	0.015 = P(D | B)
C	0.30 = P(C)	0.012 = P(D | C)

The manager would like to answer the following question: If a randomly selected lamp is defective, what is the probability that the lamp was manufactured in factory C?