1. $\mu = 40.125$ mm

$\sigma = 0.002$ mm

$\frac{X_1+X_2+X_3+X_4}{n} = \bar{X}$

$\mu_{\bar{x}} = \mu = 40.125$ (mm) #

$\sigma_{\bar{x}}\ \frac{\sigma}{\sqrt{n}} = \frac{0.002}{\sqrt{4}} = \frac{0.002}{2} = 0.001$ #

2.

(a)

$\bar{X} = \frac{X_1+X_2+X_3}{3}$

$\mu_{\bar{x}} = \mu = 123$

$\sigma_{\bar{x}} = \frac{0.08}{\sqrt{3}} = 0.046 \quad \therefore N(123, \sigma_{\bar{x}} = 0.046)$ #

(b)

$P(X \geq 124) = P\left(\frac{X-\mu}{\sigma/\sqrt{n}} \geq \frac{124-123}{0.046}\right) = P\left(Z \geq \frac{1}{0.046}\right) = P(Z \geq 21.73) \approx 0$ #

3.

(a) $\mu_{\bar{x}} = \mu = 0.9$

$\sigma_{\bar{x}} = \frac{0.15}{\sqrt{125}} = 0.0134 \quad \therefore N(0.9, \sigma_{\bar{x}} = 0.0134)$ #

(b)

$P(\bar{X} \geq L) = 0.01 \Rightarrow P\left(\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \geq \frac{L-\mu}{\frac{\sigma}{\sqrt{n}}}\right) = 0.01$

$\Rightarrow P\left(Z \geq \frac{L-\mu}{\frac{\sigma}{\sqrt{n}}}\right) = 0.01$

$\Rightarrow 1 - P\left(Z \leq \frac{L-\mu}{\frac{\sigma}{\sqrt{n}}}\right) = 0.99$

$\therefore \frac{L-\mu}{\frac{\sigma}{\sqrt{n}}} = 2.33 \Rightarrow L = 2.33 \times \frac{\sigma}{\sqrt{n}} + \mu = 2.33 \times 0.0134 + 0.9$

$= 0.93122$ #

4. $\mu = 1.5 \quad \sigma = 0.75$.

(a)

Yes.

(b) $\because \mu_{\bar{x}} = \mu = 1.5$ people/car. $\quad \therefore N(1.5, \sigma_{\bar{x}} = 0.028)$ ※

$\sigma_{\bar{x}} = \frac{0.75}{\sqrt{700}} = 0.028$

(c)

$P(700\mu_{\bar{x}} \geq 1075)$.

$= P(\mu_{\bar{x}} \geq 1.536)$

$= P\left(\frac{\mu_{\bar{x}} - \mu}{\frac{\sigma}{\sqrt{n}}} \geq \frac{1.536 - 1.5}{\frac{0.75}{\sqrt{700}}}\right) = P\left(Z \geq \frac{0.036}{0.0283}\right) = P(Z \geq 1.27)$.

$= 1 - P(Z < 1.27) = 1 - 0.8980 = 0.102$ ※

5.

(a)

$\binom{15}{3} (0.2)^3 (0.8)^{12} = \frac{15 \times 14 \times 13}{3 \times 2} \times 0.2^3 \times 0.8^{12} = 0.25$ ※

(b)

$\binom{15}{0} 0.2^0 \, 0.8^{15} +$

$\binom{15}{1} 0.2^1 \, 0.8^{14} +$

$\binom{15}{2} 0.2^2 \, 0.8^{13} +$

$\binom{15}{3} 0.2^3 \, 0.8^{12} = 0.0352 + 0.132 + 0.231 + 0.25 = 0.6482$ ※

6.

1. $p(x>15) \le \frac{10}{15} = \frac{2}{3} = 0.67$ #

2. $\sigma = 2$

$p(5<\bar{x}<15) = p(-5<\bar{x}-10<5) = p(|\bar{x}-10|<5) = 1-p(|\bar{x}-10| \geq 5)$

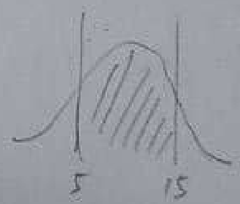
$\because p(|x-10| \geq 5) \le \frac{2^2}{5^2} = \frac{4}{25}$

$\therefore 1-p(|x-10| \geq 5) \geq \frac{21}{25} = 0.84$ #

3.

$p(5<\bar{x}<15) = p(\bar{x}<15) - p(\bar{x}<5)$

$= p\left(\frac{\bar{x}-\mu}{\sigma} < \frac{15-10}{2}\right) - p\left(\frac{\bar{x}-\mu}{\sigma} < \frac{5-10}{2}\right)$



$= p(Z<2.5) - p(Z<-2.5)$

$= 0.9938 - 0.0062 = 0.9876$ #

紫比雪夫定理說 $p(5<\bar{x}<15) \geq 0.84$

$\Rightarrow$ 實際機率 $0.9876 \geq 0.84$

$(0.9876 - 0.84 = 0.1476)$ #