Homework 5

1. Suppose that the p.d.f. of a random variable X is as follows:

$$f(x) = \begin{cases} cx^2 & \text{for } 1 \le x \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the value of the constant c and sketch the p.d.f.

   (b) Find the value of $\Pr(X > \frac{3}{2})$

2. Suppose that the p.d.f. of a random variable X is as follows:

$$f(x) = \begin{cases} \frac{1}{8}x & \textit{for } 0 \le x \le 4 \\ 0 & \textit{otherwise.} \end{cases}$$

   (a) Find the value of t such that $\Pr(X \le t) = 1/4$

   (b) Find the value of t such that $\Pr(X \ge t) = 1/2$

3. Show that there does not exist any number c such that the following function $f(x)$ would be a p.d.f.:

$$f(x) = \begin{cases} \frac{c}{x} & \textit{for } 0 < x < 1 \\ 0 & \textit{otherwise.} \end{cases}$$

4. Bigger mammals tend to carry their young longer before birth. The length of horse pregnancies from conception to birth varies according to a roughly Normal distribution with mean 336 days and standard deviation 3 days. Use the 68-95-99.7 rule to answer the following questions.

   (a) Almost all (99.7%) horse pregnancies fall within what range of lengths?

   (b) What percent of horse pregnancies are longer than 339 days?

5. Use **Z table** to find the proportion of observations from standard Normal distribution that falls in each of the following regions. In each case, sketch a standard Normal curve and shade of the area representing the region.

   (a) $z < 1.85$ (c) $z > 1.85$

   (b) $z > -0.66$ (d) $-0.66 < z < 1.85$

6. Find the z value that satisfies each of the following conditions (report the value of z that comes closest to satisfying the condition). In each case, sketch a standard Normal curve with your value of z marked on the axis.

   (a) 20% of the observations fall below z.

   (b) 30% of the observations fall below z.

   (c) 30% of the observations fall above z.