

HW#9

The distribution of diastolic blood pressures for the population of female diabetics between the ages of 30 and 34 has an unknown mean μ_d and standard deviation $\sigma_d = 9.1$ mm Hg. It may be useful to physicians to know whether the mean of this

population is equal to the mean diastolic blood pressure of the general population of females in this age group, 74.4 mm Hg [12].

- What is the null hypothesis of the appropriate test?
- What is the alternative hypothesis?
- A sample of ten diabetic women is selected; their mean diastolic blood pressure is $\bar{x}_d = 84$ mm Hg. Using this information, conduct a two-sided test at the $\alpha = 0.05$ level of significance. What is the p -value of the test?
- What conclusion do you draw from the results of the test?
- Would your conclusion have been different if you had chosen $\alpha = 0.01$ instead of $\alpha = 0.05$?

σ 已知, 用 z test

E. canis infection is a tick-borne disease of dogs that is sometimes contracted by humans. Among infected humans, the distribution of white blood cell counts has an unknown mean μ and a standard deviation σ . In the general population, the mean white blood cell count is $7250/\text{mm}^3$ [13]. It is believed that persons infected with *E. canis* must on average have lower white blood cell counts.

- What are the null and alternative hypotheses for a one-sided test?
- For a sample of 15 infected persons, the mean white blood cell count is $\bar{x} = 4767/\text{mm}^3$ and the standard deviation is $s = 3204/\text{mm}^3$ [14]. Conduct the test at the $\alpha = 0.05$ level.
- What do you conclude?

σ 未知, 用 t test

Hint: 劃線是決定 H_a 單邊或雙邊的 keywords

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Body mass index is calculated by dividing a person's weight by the square of his or her height; it is a measure of the extent to which the individual is overweight. For the population of middle-aged men who later develop diabetes mellitus, the distribution of baseline body mass indices is approximately normal with an unknown mean μ and standard deviation σ . A sample of 58 men selected from this group has mean $\bar{x} = 25.0 \text{ kg/m}^2$ and standard deviation $s = 2.7 \text{ kg/m}^2$ [15].

- Construct a 95% confidence interval for the population mean μ .
- At the 0.05 level of significance, test whether the mean baseline body mass index for the population of middle-aged men who do develop diabetes is equal to 24.0 kg/m^2 , the mean for the population of men who do not. What is the p -value of the test?
- What do you conclude?
- Based on the 95% confidence interval, would you have expected to reject or not to reject the null hypothesis? Why?

好像. 多了 confidence interval 的問題

4. power and sample size

Data from the Framingham Study allow us to compare the distributions of initial serum cholesterol levels for two populations of males: those who go on to develop coronary heart disease and those who do not. The mean serum cholesterol level of the population of men who do not develop heart disease is $\mu = 219$ mg/100 ml and the standard deviation is $\sigma = 41$ mg/100 ml [18]. Suppose, however, that you do not know the true population mean; instead, you hypothesize that μ is equal to 244 mg/100 ml. This is the mean initial serum cholesterol level of men who eventually develop the disease. Since it is believed that the mean serum cholesterol level for the men who do not develop heart disease cannot be higher than the mean level for men who do, a one-sided test conducted at the $\alpha = 0.05$ level of significance is appropriate.

- What is the probability of making a type I error?
- If a sample of size 25 is selected from the population of men who do not go on to develop coronary heart disease, what is the probability of making a type II error?
- What is the power of the test?
- How could you increase the power?
- You wish to test the null hypothesis

$$H_0: \mu \geq 244 \text{ mg/100 ml} \quad (\text{if } \mu_0 = 244 \text{ [0]})$$

against the alternative

$$H_A: \mu < 244 \text{ mg/100 ml}$$

at the $\alpha = 0.05$ level of significance. If the true population mean is as low as 219 mg/100 ml, you want to risk only a 5% chance of failing to reject H_0 . How large a sample would be required?

- How would the sample size change if you were willing to risk a 10% chance of failing to reject a false null hypothesis?

σ known

$$\text{if } \mu_0 = 244$$

set $\beta = \text{Pr}(\text{Accept } H_0 | H_0 \text{ is false})$:

$$\Rightarrow \text{power} = 1 - \beta: \begin{cases} \text{(e)} & \text{power} = 0.95 \\ \text{(f)} & \text{power} = 0.9 \end{cases}$$