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數據所

1. $n=16$, $S=0.24$, $\bar{X}=13.82$, $\mu=14$, $\alpha=0.05$

$H_0: \mu=14 \quad H_a: \mu<14$

Reject H_0 if $\bar{X}_{obs} < \mu_0 - t_{15,0.05}\frac{S}{\sqrt{n}} \Leftrightarrow \bar{X}_{obs} < 14 - 1.753\frac{0.24}{\sqrt{16}}$

$\Leftrightarrow \bar{X}_{obs} < 13.89482$

When $\bar{X}_{obs} = 13.82 \Rightarrow$ reject H_0 ※

2. $\mu=45$, $n=9$ $\bar{X}=49.2$, $S=3.5$, $\alpha=0.05$

$H_0: \mu=45$ v.s. $H_a: \mu\neq 45$

Reject H_0 if $\bar{X}_{obs} < \mu_0 - t_{8,0.025}\frac{S}{\sqrt{n}}$ or $\bar{X}_{obs} > \mu_0 + t_{8,0.025}\frac{S}{\sqrt{n}}$

$\Leftrightarrow \bar{X}_{obs} < 45 - 2.306\frac{3.5}{\sqrt{9}}$ or $\bar{X}_{obs} > 45 + 2.306\frac{3.5}{\sqrt{9}}$

$\Leftrightarrow \bar{X}_{obs} < 42.31$ or $\bar{X}_{obs} > 47.69$

When $\bar{X}_{obs} = 49.2 \Rightarrow$ reject H_0 ※

3.
(a) $\mu=1$

$\bar{X} = (0.95+1.02+1.01+0.98)/4 = 0.99$

$S = \left[\left((0.99-0.95)^2+(1.02-0.99)^2+(1.01-0.99)^2+(0.99-0.98)^2\right)/3\right]^{\frac{1}{2}} = 0.0316$

$\left[\mu - t_{n-1,\frac{\alpha}{2}}\frac{S}{\sqrt{n}} , \mu + t_{n-1,\frac{\alpha}{2}}\frac{S}{\sqrt{n}}\right] = \left[1 - t_{3,0.025}\frac{0.0316}{\sqrt{4}} , 1 + t_{3,0.025}\frac{0.0316}{\sqrt{4}}\right]$

$= [1-0.05031 , 1+0.05031] = [0.93969, 1.04031]$

※

(b)

$H_0: \mu = 1 \quad H_a: \mu \neq 1$

Reject H_0 if $\mu_0 \notin [\bar{X}_{obs} - t_{n-1,\frac{\alpha}{2}} \frac{S}{\sqrt{n}}, \bar{X}_{obs} + t_{n-1,\frac{\alpha}{2}} \frac{S}{\sqrt{n}}]$

When $\bar{X}_{obs} = 0.99$, $1 \in [0.99 - 0.05031, 0.99 + 0.05031]$

$, 1 \in [0.93969, 1.04031]$

$\Rightarrow$ fail reject H_0 #

4.

(a.) $\bar{X} = (2.8 + 3.2 + 3.75 + 3.1 + 2.95 + 3.4)/6 = 3.2$

$S = \sqrt{\frac{\sum_{i=1}^{n}(X_i - X)^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{6}(X_i - 1)^2}{5}} = 0.3391$

$[\bar{X} - t_{n-1,\frac{\alpha}{2}} \frac{S}{\sqrt{n}}, \bar{X} + t_{n-1,\frac{\alpha}{2}} \frac{S}{\sqrt{n}}] = [3.2 - t_{5,0.025} \frac{0.3391}{\sqrt{6}}, 3.2 + t_{5,0.025} \frac{0.3391}{\sqrt{6}}]$

$= [3.2 - 2.571 \times \frac{0.3391}{\sqrt{6}}, 3.2 + 2.571 \times \frac{0.3391}{\sqrt{6}}] = [2.844, 3.556.]$ #

(b).

$\alpha = 0.01$

$[\bar{X} - t_{5,0.005} \frac{S}{\sqrt{n}}, \bar{X} + t_{5,0.005} \frac{S}{\sqrt{n}}] = [3.2 - 4.032 \frac{0.3391}{\sqrt{6}}, 3.2 + 4.032 \frac{0.3391}{\sqrt{6}}] = [2.6418, 3.7581]$

$\because 3.7582 - 2.6418 = 1.1163$

$3.5560 - 2.8440 = 0.7120 \quad \therefore$ increase #

(c) $n = 24$.

$[3.2 - t_{23,0.025} \frac{0.31}{\sqrt{24}}, 3.2 + t_{23,0.025} \frac{0.31}{\sqrt{24}}] = [3.2 - 2.069 \frac{0.3391}{\sqrt{24}}, 3.2 + 2.069 \frac{0.3391}{\sqrt{24}}]$

$= [3.0568, 3.3432]$

$\because 3.3432 - 3.0568 = 0.2864$

$3.5560 - 2.8440 = 0.7120 \quad \therefore$ decrease #

5.

a. t- distribution., because when σ^2 is know $\frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$

when σ^2 is unknow $\frac{\bar{X}-\mu_0}{s/\sqrt{n}} \sim T_{n-1}$ #

b.

$$\text{p-value} = 2p\left(T_{19} > \left|\frac{\bar{X}_{obs}-\mu_0}{s/\sqrt{n}}\right|\right)$$

$$= 2p\left(T_{19} > \left|\frac{17.5-15}{5.9/\sqrt{20}}\right|\right)$$

$$= 2p(T_{19} > 1.8949) \in (0.05, 0.1)$$ #

C.

Since p-value $> \alpha = 0.05$, we fail to reject H_0 #

d

$H_0: \mu = 15$, $H_a: \mu < 15$.

p-value $= p\left(T_{19} > \frac{17.5-15}{5.9/\sqrt{20}}\right) = P(T_{19} > 1.8949) \in (0.025, 0.05)$

Since the p-value $\in (0.025, 0.05) < \alpha = 0.05$, we reject H_0 if $H_a: \mu < 15$

So We fail reject H_0 if $H_a: \mu \geq 15$ #

e

$H_0: \mu = 15$, $H_a: \mu > 15$.

P-value $= p(T_{19} > 1.895) \in (0.025, 0.05) < \alpha = 0.05$, We reject H_0 if $H_a: \mu > 15$

So We fail reject H_0 if $H_a: \mu \leq 15$ #