生物統計 HW4 解答

#1:

In humans, geneticists have identified two sex chromosomes, *R* and *Y*. Every individual has an *R* chromosome, and the presence of a *Y* chromosome distinguishes the individual as male. Thus the two sexes are characterized as *RR* (female) and *RY* (male). Color blindness is caused by a recessive allele on the *R* chromosome, which we denote by *r*. The *Y* chromosome has no bearing on color blindness. Thus relative to color blindness, there are three genotypes for females and two for males:

Female	Male
RR (normal)	*RY* (normal)
Rr (carrier)	*rY* (color-blind)
rr (color-blind)	

A child inherits one sex chromosome randomly from each parent.

(a)

A carrier of color blindness parents a child with a normal male. List the possible genotypes for the child.

Sol):

$RY \times Rr = RR, Rr, RY, Rr$

(b)

What is the probability that a given child born to this couple will be a color-blind male?

Sol):

$\frac{1}{4}$

(c)

If the couple has three children, what is the probability that exactly two are color-blind males?

Sol):

$C_2^3 \times (\frac{1}{4})^2 \times \frac{3}{4} = \frac{9}{64}$

(d)

If the couple has five children, what is the expected number of color-blind males? What is the probability that at most two will be color-blind males? What is the probability that three or more will be color-blind males?

Sol):

$X \sim \mathrm{Bin}(5, \frac{1}{4})$

(1) $E(\mathrm{X})=\frac{5}{4}=1.25$

(2) $P(\mathrm{X}\leq 2)=\sum_{i=0}^{2} C_i^5 (\frac{1}{4})^i (\frac{3}{4})^{5-i}=\frac{243+405+270}{4^5}=\frac{918}{1024}$

(3) $P(\mathrm{X}\geq 3)=1-P(\mathrm{X}\leq 2)=\frac{106}{1024}$

#2:

(a)

In a certain culture, the average number of *Rickettsia typhi* cells (cells which cause typhus) is 5 per 20 square micrometers (1/10,000 of a centimeter). How many such cells would you expect to find in a culture of size 16 square micrometers? What is the probability that none will be found in a 16-square-micrometer culture? What is the probability that at least nine such cells will be found in a culture of this size?

Sol):

$X \sim \mathrm{Bin}(\mathrm{n}=16, \mathrm{p}=\frac{1}{4})$

(1) $\frac{5}{20}\times 16=4$

(2) $P(\mathrm{X}=0)=C_0^{16}\times(\frac{1}{4})^0\times(\frac{3}{4})^{16}\approx 0.01$

(3) $P(\mathrm{X}\geq 9)=\sum_{i=9}^{16} C_i^{16} (\frac{1}{4})^i (\frac{3}{4})^{16-i}\approx 0.007469$

Poisson : $\lambda=0.25, t=16$

(1) $\mu=\lambda t=4$

(2) $P(\mathrm{X}=0)=\frac{e^{-\lambda t}(\lambda\,\mathrm{t})^x}{x!}=\frac{e^{-4}(4)^0}{0!}$ =0.018

(3) $P(\mathrm{X}\geq 9)=\sum_{x=9}^{16}\frac{e^{-4}(4)^x}{x!}=0.02136$

(b)

The probability that a randomly selected baby will be albino is 1/20,000. Of the next 40,000 babies born, what is the probability that none will be albino? What is the probability that at least one will be albino? 8.

用 Binomial 和 Poisson 分析，哪個容易求答案？

Sol):

$$X \sim \text{Bin}(\text{n}=40000, \text{p}=\frac{1}{20000})$$

(1) $P(\text{X}=0)=C_0^{40000}(\frac{1}{20000})(\frac{19999}{20000})^{40000}=0.135$

(2) $P(\text{X}\geq 1)=1-\text{P}(\text{X}=0)=0.865$

$$X \sim Po(40000\times\frac{1}{20000}=2)$$

(1) $P(\text{X}=0)=e^{-2}=0.135$

(2) $P(\text{X}\geq 1)=1-\text{P}(\text{X}=0)=0.865$

使用 Poisson 較易求得答案。

#3:

Let X denote the number of new AIDS cases diagnosed per day at a large metropolitan hospital. Assume that the cumulative distribution for X is

x	0	1	2	3	4	5	6
$F(x)$	.1	.2	.3	.6	.8	.9	1.00

(a)

Find the probability that on a randomly selected day,

i. At most three new cases will be diagnosed.
ii. At least one new case will be diagnosed.
iii. No new cases will be diagnosed.
iv. Between two and four new cases inclusive will be diagnosed.

Sol):

(i)$\text{P}(\text{X}\leq 3)=F(3)=0.6$

(ii)$\text{P}(\text{X}>0)=1-F(0)=0.9$

(iii)$\text{P}(\text{X}=0)=\text{P}(\text{X}\leq 0)=F(0)=0.1$

(iv)$\text{P}(2\leq \text{X}\leq 4)=F(4)-F(1)=0.6$

(b)

Find the density for X. (probability)

Sol):

$f(0) = 0.1$

$f(1) = F(1) - F(0) = 0.1$

$f(2) = F(2) - F(1) = 0.1$

$f(3) = F(3) - F(2) = 0.3$

$f(4) = F(4) - F(3) = 0.2$

$f(5) = F(5) - F(4) = 0.1$

$f(6) = F(6) - F(5) = 0.1$

(c)

Find the average number of cases diagnosed per day.

Sol):

$$\mathrm{E[X]} = \sum_{x=0}^{6} x * f(x) = 3.1$$

(d)

Find σ^2.

Sol):

$\mathrm{Var[X]} = \mathrm{E[(X - E[X])^2]} = \mathrm{E[X^2]} - \mathrm{E[X]^2}$

$$\mathrm{E[X^2]} = \sum_{x=0}^{6} x^2 * f(x) = 12.5$$

So $\mathrm{Var[X]} = 12.5 - 3.1^2 = 2.89$

(e)

Find the standard deviation of X. What physical measurement unit is associated with σ?

Sol):

$\sigma = \sqrt{\sigma^2} = 1.7(\mathrm{cases/day})$

(f)

Find $E(e^X)$

Sol):

$$E[e^X] = \sum_{x=0}^{6} e^x * f(x) \approx 73.24$$

#4:

How many cars? Twenty percent of American households own three or more motor vehicles. You choose 12 households at random.

(a)

What is the probability that none of the chosen households owns three or more vehicles? What is the probability that at least one household owns three or more vehicles?

Sol):

$$P(X = 0) = \binom{12}{0} 0.2^0 * 0.8^{12} = 0.8^{12} \approx 6.87\%$$

$$P(X \geq 1) = 1 - P(X = 0) = 1 - 0.8^{12} \approx 93.13\%$$

(b)

What are the mean and standard deviation of the number of households in your sample that own three or more vehicles?

Sol):

$$\mu = np = 12 * 0.2 = 2.4$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{12 * 0.2 * 0.8} \approx 1.39$$

(c)

What is the probability that your sample count is greater than the mean?

Sol):

$$P(X > 2.4) = P(X \geq 3)$$

$$= 1 - P(X \leq 2)$$

$$= 1 - \binom{12}{0} 0.2^0 * 0.8^{12} - \binom{12}{1} 0.2^1 * 0.8^{11} - \binom{12}{2} 0.2^2 * 0.8^{10}$$

$$\approx 44.17\%$$