生物統計 HW2 解答

#1:

A family has four children. Identifying each child by sex only, use a tree to find the 16 possible birth orders of the children. Assume that each child is just as likely to be a boy as a girl. Find the probability of each of these events.

- A: The first child is a boy
- B: Exactly two of the four are boys
- C: The oldest and youngest are boys
- D: Two are girls and three are boys

Sol):

$$\mathrm{P(boy)} = \mathrm{P(girl)} = \frac{1}{2}$$

$$\mathrm{P(A)} = \frac{1}{2}, \mathrm{P(B)} = \frac{\frac{4!}{2!2!}}{2^4} = \frac{3}{8}, \mathrm{P(C)} = \frac{2^2}{2^4} = \frac{1}{4}, \mathrm{P(D)} = 0$$

#2:

In guinea pigs, short hair (L) is dominant to long hair (l) and black fur (B) is dominant to albino fur (b). A female which is black with short hair is mated to a male that is albino with long hair.

(a) What are the possible genotypes for the female? What are the possible genotypes for the male?

(b) [illegible] List [illegible] the possible outcomes for the offspring.

(c) Find the probability of obtaining an albino with short hair

(d) Find the probability of obtaining a black fur with long hair.

Sol):

(a) Female: BBLL、BbLL、BBLl、BbLl ; male: bbll

(b) BbLl、bbLl、Bbll、bbll

(c) Let A≡ obtaining an albino with short hair

If the genotype of mother is BBLL, then $\mathrm{P(A)} = 0$

If the genotype of mother is BbLL, then $\mathrm{P(A)} = \frac{1}{2}$

If the genotype of mother is BBLl, then $\mathrm{P(A)} = 0$

If the genotype of mother is BbLl, then, $\mathrm{P(A)} = \frac{1}{4}$

(d) Let A≡ obtaining a black fur with long hair

If the genotype of mother is BBLL, then $P(A) = 0$

If the genotype of mother is BbLL, then $P(A) = 0$

If the genotype of mother is BBLl, then $P(A) = \frac{1}{2}$

If the genotype of mother is BbLl, then, $P(A) = \frac{1}{4}$

#3:

Two body colors, gray (*E*) and ebony (*e*), are recognized in fruit flies with gray being dominant. Two wing types are also noted, normal (*V*) and short, or vestigial (*v*), with normal being dominant. Homozygous flies (*EEVV* and *eevv*) are mated to form double-heterozygous offspring (*EeVv*). These are allowed to mate.

(5分)

a. List all possible outcomes.

b. 15分

If a fly is selected at random from among a large number of flies resulting from the above experiment, what is the probability that it will have normal wings? What is the probability that it will be ebony and have short wings? What is the probability that it will *not* be ebony with short wings?

Sol):

(a) 題意為 EeVv × EeVv：

EEVV、EeVV、EEVv、EeVv、eeVV、eeVv、Eevv、EEvv、eevv，共九種。

(此題若寫成 EEVV × EeVv 跟 eevv × EeVv 扣 1 分，子代會有 7 種可能)

(b) EeVv × EeVv：

$P(normal\ wings) = \frac{3}{4}$

$P(\text{ebony and short wings}) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

$P(\text{not be ebony with short wings}) = 1 - \frac{1}{16} = \frac{15}{16}$

EEVV × EeVv and eevv × EeVv：

$P(normal\ wings) = \frac{3}{4}$

$P(\text{ebony and short wings}) = \frac{4}{32} = \frac{1}{8}$

$P(\text{not be ebony with short wings}) = 1 - \frac{1}{8} = \frac{7}{8}$

#4:

In a study of alcoholics, it was found that 40% had alcoholic fathers and 6% had alcoholic mothers. Forty-two percent had at least one alcoholic parent. What is the probability that a randomly selected alcoholic will
(*a*) Have both parents alcoholic?
(*b*) Have an alcoholic mother if the father is alcoholic?
(*c*) Have an alcoholic mother but not an alcoholic father?
(*d*) Have an alcoholic mother if the father is not alcoholic?

Sol):

Let A ≡ had alcoholic father ; B ≡ had alcoholic mother

$P(A) = 0.4, P(B) = 0.06, P(A \cup B) = 0.42$

(a) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.04$
(b) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.04}{0.4} = 0.1$
(c) $P(B \cap A^c) = P(B) - P(A \cap B) = 0.06 - 0.04 = 0.02$
(d) $P(B|A^c) = \frac{P(B \cap A^c)}{P(A^c)} = \frac{0.02}{1-0.4} = \frac{1}{30} = 0.0\overline{3}$

#5:

A study indicates that 10% of the population in the United States is 65 years old or older and that 1% of the total population is afflicted with mild heart failure. Furthermore, 10.4% of the population is age 65 or older or suffers from mild heart failure. An individual is selected at random.
(*a*) Find the probability that the individual is 65 or older and suffering from mild heart failure.
(*b*) If an individual is 65 or older, what is the probability that the person will suffer mild heart failure?
(*c*) If an individual is under age 65, what is the probability that he or she will suffer mild heart failure?

Sol):

Let A ≡ 65 years old or older; B ≡ afflicted with heart failure

$P(A) = 0.1, P(B) = 0.01, P(A \cup B) = 0.104$

(a) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.006$
(b) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.006}{0.1} = 0.06$
(c) $P(B|A^c) = \frac{P(B \cap A^c)}{P(A^c)} = \frac{P(B) - P(A \cap B)}{1 - P(A)} = \frac{0.01 - 0.006}{0.9} = 0.00\overline{4}$

#6:

In a study of high school students, each subject was asked to roll a die and then flip a coin. If the coin came up heads, the subject was to answer question A below and if tails, question B.

A: Did the die land on an even number?

B: Have you ever smoked marijuana?

In a group of 50 subjects, 35 answered yes. Use this information to approximate the probability that a student randomly selected from this group has smoked marijuana.

Sol):

Let A ≡ answered question A; B ≡ answered question B

$$\mathrm{P}(answered\ yes) = \mathrm{P}(answered\ yes|A)\mathrm{P}(\mathrm{A}) + \mathrm{P}(answered\ yes|B)\mathrm{P}(\mathrm{B})$$

$$\rightarrow \frac{35}{50} = \frac{3}{6} \times \frac{1}{2} + P(smoked\ marijuana) \times \frac{1}{2}$$

$$\rightarrow P(smoked\ marijuana) = 0.9$$