生物統計 Homework 2

1. **Getting into college.** Ramon has applied to both Princeton and Stanford. He thinks the probability that Princeton will admit him is 0.4, the probability that Stanford will admit him is 0.5, and the probability that both will admit him is 0.2.
   (a) Make a Venn diagram with the probabilities given marked.
   (b) What is the probability that neither university admits Ramon?
   (c) What is the probability that he gets into Stanford but not Princeton?

2. Consider the experiment of Example 2.2.5. Suppose that the parents are each genotype *SsFf*.
   (*a*) Draw the tree to represent the possible outcomes for the offspring.
   (*b*) Use the tree to find the probability that
      i. The child will be albino with attached earlobes
      ii. The child will have normal skin and free earlobes
      iii. The child will be albino
      iv. The child will have free earlobes

3. In the pea plant, tallness (T) is dominant to shortness (t). Yellow seeds (Y) are dominant to green (y), and the round shape (W) is dominant to wrinkled (w). Suppose that two plants are cross-matched. One has genotype *TTYYWw* and the other *TtYyWw*.
   (*a*) Describe each of the parent plants relative to the three characteristics mentioned.
   (*b*) Draw a tree to represent the possible ways in which the cross-match can occur. (*Hint:* Extend the idea of Example 2.2.5 to a six-stage process.)
   (*c*) Describe the plant associated with each path through the tree.
   (*d*) Use the tree to find the probability that the cross-match will result in a tall pea plant.
   (*e*) What is the probability that the cross-match will result in a tall, yellow, wrinkled pea plant?
   (*f*) What is the probability that the cross-match will produce a green pea plant?

4. **Preparing for the GMAT.** A company that offers courses to prepare would-be MBA students for the Graduate Management Admission Test (GMAT) finds that 40% of its customers are currently undergraduate students and 60% are college graduates. After completing the course, 50% of the undergraduates and 70% of the graduates achieve scores of at least 600 on the GMAT. Draw a tree diagram that organizes this information. What percent of all customers score at least 600 on the GMAT?

5. **Screening job applicants.** A company retains a psychologist to assess whether job applicants are suited for assembly-line work. The psychologist classifies applicants as A (well suited), B (marginal), or C (not suited). The company is concerned about event D: an employee leaves the company within a year of being hired. Data on all people hired in the past five years give these probabilities:

$$P(A) = 0.4 \qquad P(B) = 0.3 \qquad P(C) = 0.3$$
$$P(A \text{ and } D) = 0.1 \qquad P(B \text{ and } D) = 0.1 \qquad P(C \text{ and } D) = 0.2$$

Sketch a Venn diagram of the events A, B, C, and D and mark on your diagram the probabilities of all combinations of psychological assessment and leaving (or not) within a year. What is $P(D)$, the probability that an employee leaves within a year?

6. *Randomized response method of getting honest answers to sensitive questions.* This is a method used to guarantee an individual that answers to sensitive questions will be anonymous, thus encouraging a truthful response. It operates as follows. Two questions A and B are posed, one of which is sensitive and the other not. The probability of receiving a yes to the nonsensitive question must be known. For example, one could ask

A: Does your Social Security number end in an odd digit? (Nonsensitive)

B: Have you ever intentionally filed a fraudulent insurance claim? (Sensitive)

We know that $P[\text{answer yes} \mid \text{answered } A] = \frac{1}{2}$. We wish to approximate $P[\text{answer yes} \mid \text{answered } B]$. The subject is asked to flip a coin and answer A if the coin comes up heads and answer B if it is tails. In this way, the interviewer does not know which question the subject is answering. Thus a yes answer is not incriminating. There is no way for the interviewer to know whether the subject is saying, "Yes, my Social Security number ends in a odd digit," or "Yes, I have intentionally filed a fraudulent claim." The percentage of subjects in the group answering yes is used to approximate $P[\text{answer yes}]$.

(*a*) Use the fact that the event "answer yes" is the event "answer yes and answered A" or "answer yes and answered B" to show that $P[\text{answer yes} \mid \text{answered } B]$ equals

$$\frac{P[\text{answer yes}] - P[\text{answer yes} \mid \text{answered } A]P[\text{answered } A]}{P[\text{answered } B]}$$

(*b*) If this technique is tried on 100 subjects and 60 answered yes, find the approximate probability that a person randomly selected from the group has intentionally filed a fraudulent claim.