Confidence Interval 講義与作業

Confidence interval的意義

CONFIDENCE INTERVAL FOR THE MEAN OF A NORMAL POPULATION

Draw an SRS of size n from a Normal population having unknown mean μ and known standard deviation σ. A level C **confidence interval for μ** is

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$$

The critical value z^* is illustrated in Figure 14.4 and found in Table C.

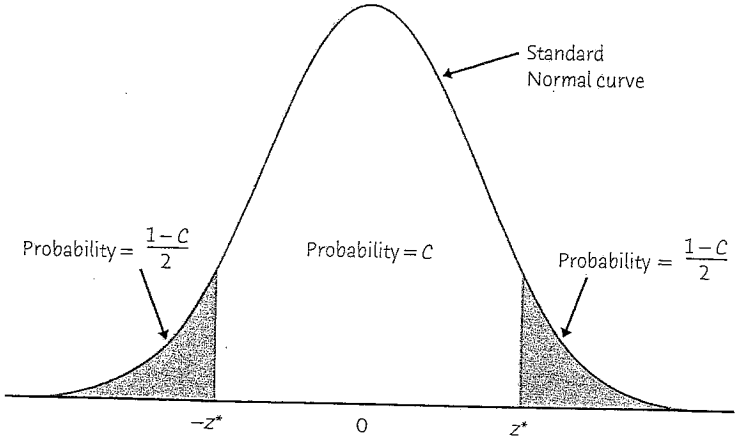
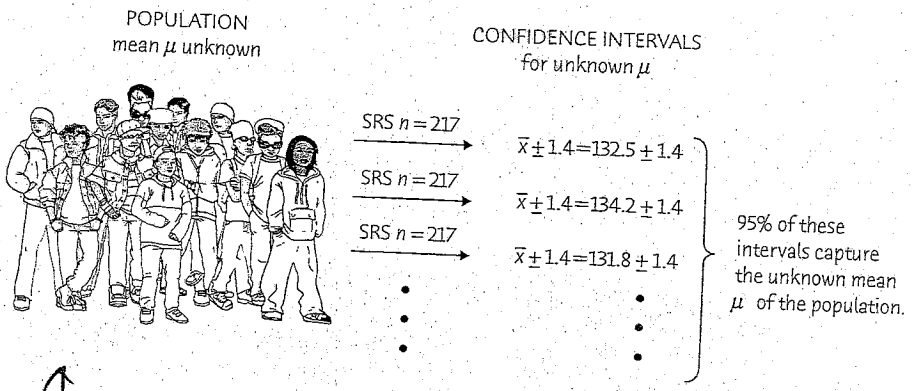
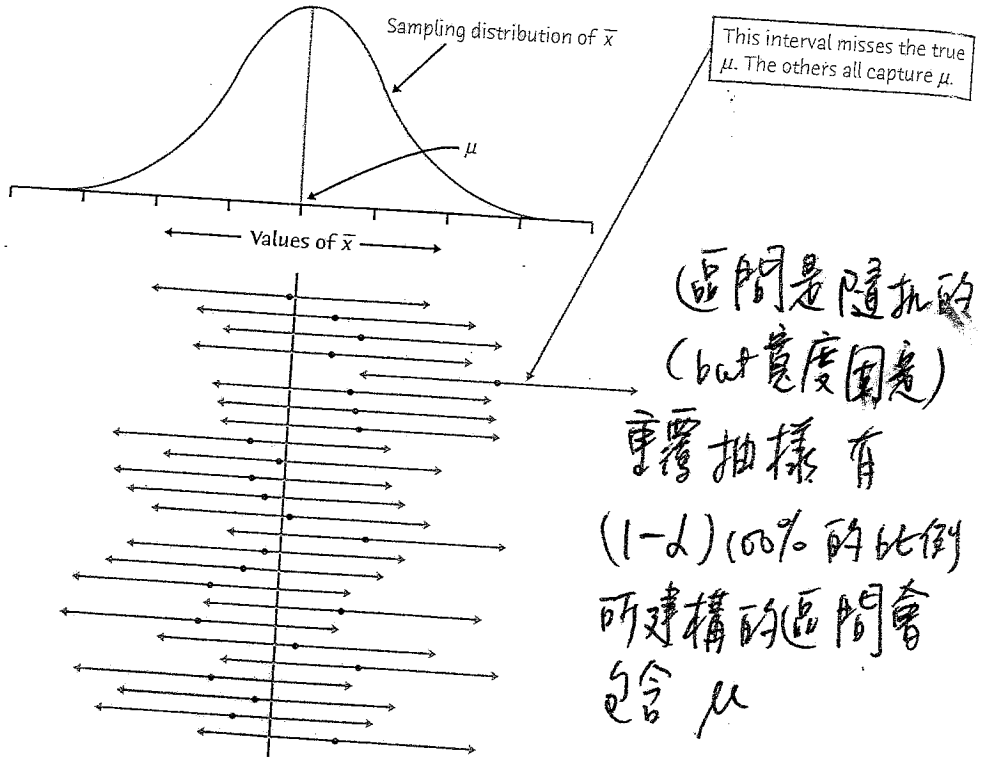


FIGURE 14.4 The critical value z^* is the number that catches central probability C under a standard Normal curve between $-z^*$ and z^*.



重覆抽樣. 每次抽 n=217人, 每次都得一個 interval



區間是隨机的
(but 信度固定)
重覆抽樣 有
(1−α)100% 的比例
所建構的區間會
包含 μ

上課用的符号

$C = 1-\alpha$ (通常用調用 $0.95=C$)

z^* = 尾端機率 = $\frac{\alpha}{2}$ 的切点值 = $z_{\alpha/2}$

$Pr(Z > z_{\alpha/2}) = \frac{\alpha}{2}$

誤差範圍 = margin of error = $z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

EXAMPLE 14.3 Healing of skin wounds

STATE: Biologists studying the healing of skin wounds measured the rate at which new cells closed a razor cut made in the skin of an anesthetized newt. Here are data from 18 newts, measured in micrometers (millionths of a meter) per hour:[3]

29	27	34	40	22	28	14	35	26
35	12	30	23	18	11	22	23	33

This is one of several sets of measurements made under different conditions. We want to estimate the mean healing rate for comparison with rates under other conditions.

FORMULATE: We will estimate the mean rate μ for all newts of this species by giving a 95% confidence interval.

SOLVE: We should start by checking the conditions for inference. For this first example, we will find the interval, then discuss how statistical practice deals with conditions that are never perfectly satisfied.

The mean of the sample is $\bar{x} = 25.67$. As part of the "simple conditions," suppose that from past experience with this species of newts we know that the standard deviation of healing rates is 8 micrometers per hour. For 95% confidence, the critical value is $z^* = 1.960$. A 95% confidence interval for μ is therefore

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} = 25.67 \pm 1.960 \frac{8}{\sqrt{18}}$$
$$= 25.67 \pm 3.70$$
$$= 21.97 \text{ to } 29.37$$

CONCLUDE: We are 95% confident that the mean healing rate for all newts of this species is between 21.97 and 29.37 micrometers per hour.

這裡"暫時"令 $\sigma = 8$ 為已知
下次上課會教 σ 未知時的方法

Home work

1.

Find a critical value. The critical value z^* for confidence level 97.5% is not in Table C. Use software or Table B of standard Normal probabilities to find z^*. Include in your answer a copy of Figure 14.4 with $C = 0.975$ that shows how much area is left in each tail when the central area is 0.975.

查 normal 表. 求 $z_{\frac{0.025}{2}}$

2

Pharmaceutical production. A manufacturer of pharmaceutical products analyzes each batch of a product to verify the concentration of the active ingredient. The chemical analysis is not perfectly precise. In fact, repeated measurements follow a Normal distribution with mean μ equal to the true concentration and standard deviation $\sigma = 0.0068$ grams per liter (g/l). Three analyses of one batch give concentrations 0.8403, 0.8363, and 0.8447 g/l. To estimate the true concentration, give a 95% confidence interval for μ.

3

IQ test scores. Here are the IQ test scores of 31 seventh-grade girls in a midwest school district:[4]

114	100	104	89	102	91	114	114	103	105	
108	130	120	132	111	128	118	119	86	72	
111	103	74	112	107	103	98	96	112	112	93

(a) These 31 girls are an SRS of all seventh-grade girls in the school district. Suppose that the standard deviation of IQ scores in this population is known to be $\sigma = 15$. We expect the distribution of IQ scores to be close to Normal. Make a stemplot of the distribution of these 31 scores (split the stems) to verify that there are no major departures from Normality. You have now checked the "simple conditions" to the extent possible.

(b) Estimate the mean IQ score for all seventh-grade girls in the school district using a 99% confidence interval.

(c) 求 95% 和 90% confidence interval
看寬度如何變化
Note 寬度 = 2 × margin of error
代表估計的精確度