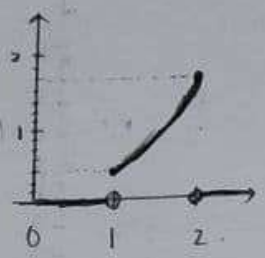
1. $f(x) = \begin{cases} cx^2 & \text{for } 1 \le x \le 2 \\ 0 & \text{other.} \end{cases}$

(a) $\int_1^2 cx^2\,dx = 1 \Rightarrow \frac{c}{3}x^3\Big|_1^2 = 1 \Rightarrow \frac{8}{3}c - \frac{1}{3}c = 1 \Rightarrow c = \frac{3}{7}$ #



$f(1) = \frac{3}{7} \times 1^2 = \frac{3}{7}$

$f(2) = \frac{3}{7} \times 2^2 = \frac{12}{7} = 1\frac{5}{7}$

(b) $P(X > \frac{3}{2}) = 1 - P(X \le \frac{3}{2}) = 1 - \int_0^{\frac{3}{2}} \frac{3}{7}x^2\,dx = 1 - \frac{1}{7}x^3\Big|_0^{\frac{3}{2}}$

$= 1 - \frac{1}{7} \cdot (\frac{3}{2})^3 = 0.518$ #

2.

$f(x) = \begin{cases} \frac{1}{8}x, & 0 \le x \le 4 \\ 0, & \text{other.} \end{cases}$

(a) $\int_0^t \frac{1}{8}x\,dx = \frac{1}{4} \Rightarrow \frac{1}{16}x^2\Big|_0^t = \frac{1}{4} \Rightarrow \frac{t^2}{16} = \frac{1}{4} \Rightarrow t = 2$ #

(b) $\int_t^4 \frac{1}{8}x\,dx = \frac{1}{2} \Rightarrow \frac{1}{16}x^2\Big|_t^4 = \frac{1}{2} \Rightarrow 1 - \frac{1}{16}t^2 = \frac{1}{2} \Rightarrow t = 2\sqrt{2}$ #

3. 假設 C 存在，使得：

$\int_0^1 \frac{c}{x}\,dx = 1 \Rightarrow c\log x\Big|_0^1 = 1 \Rightarrow c(\log 1 - \log 0) = 1 \Rightarrow c \cdot \infty = 1$

$\because c = \frac{1}{\infty}$ $\therefore$ C不存在 $\because$ 假設錯誤 $\therefore$ 不存在 C #

4.

$\mu = 336$.

$\sigma = 3$

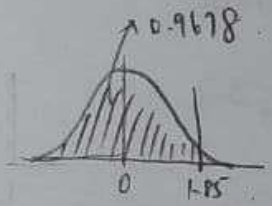
(a) $(336 - 3\times3,\ 336 + 3\times3) = (327, 345)$ #

(b) $339 - 336 = 3$ 1個標準差

$\frac{1 - 0.68}{2} = 0.16$ $\therefore$ 16% #

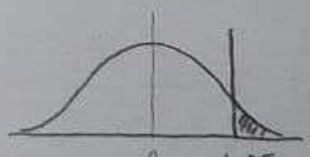
5.

(a) $P(Z < 1.85) = 0.9678$ #



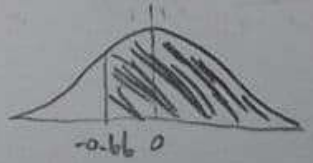
(b) $P(Z > 1.85) = 1 - 0.9678$

$= 0.0322$ #



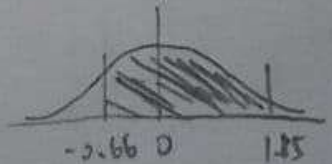
(c) $P(Z > -0.66) = 1 - 0.2546$

$= 0.7454$ #



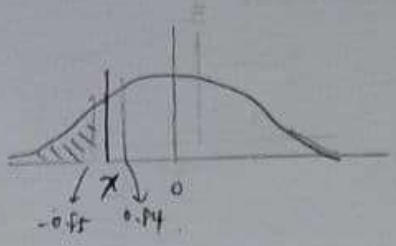
(d) $P(-0.66 < Z < 1.85) =$

$1 - 0.2546 - 0.0322 = 0.7132$ #



6.

(a)

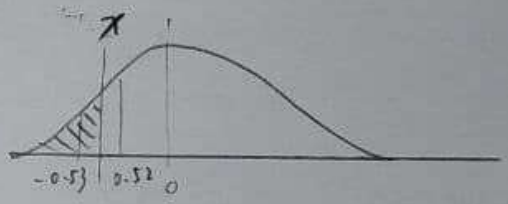


$-0.85 \quad X \quad -0.84$

$0.1977 \quad 0.2 \quad 0.2005$

$$\frac{0.2-0.1977}{0.2005-0.1977} = \frac{X-(-0.85)}{-0.84-(-0.85)} \Rightarrow \frac{0.0023}{0.0028} = \frac{X+0.85}{0.01} \Rightarrow X = -0.8418 \#$$

(b)

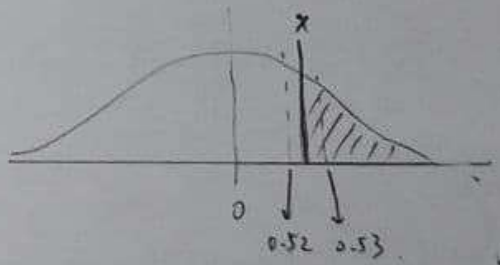


$-0.53 \quad X \quad -0.52$

$0.2981 \quad 0.3 \quad 0.3015$

$$\frac{0.3-0.2981}{0.3015-0.2981} = \frac{X-(-0.53)}{-0.52-(-0.53)} \Rightarrow \frac{0.0019}{0.0034} = \frac{X+0.53}{0.01} \Rightarrow X = -0.5244 \#$$

(C)



$0.52 \quad X \quad 0.53$

$0.6985 \quad 0.7 \quad 0.7019$

$$\frac{0.7-0.6985}{0.7019-0.6985} = \frac{Y-0.52}{0.53-0.52}$$

$$= \frac{0.0015}{0.0034} = \frac{X-0.52}{0.01} \Rightarrow X = 0.5244 \#$$