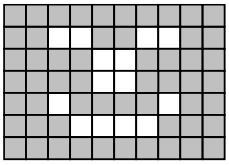
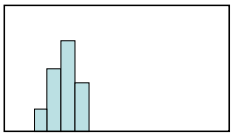
1. [4%] Consider the region of white pixels in the image here. How many connected components are there according to 4-connectivity and 8-connectivity, respectively?



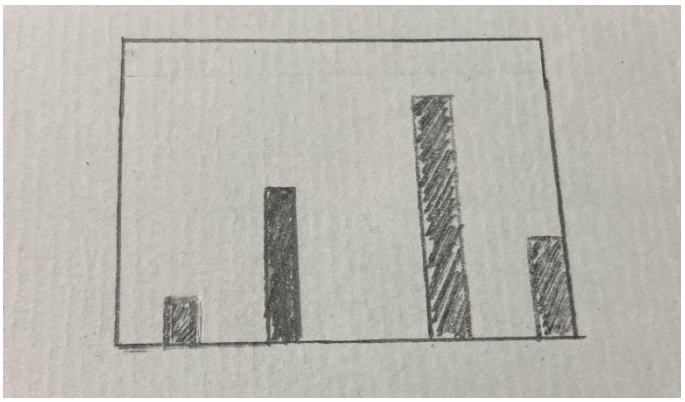
4-connectivity: 6
8-connectivity: 2

2. [15%] For one 16-gray-level image, its intensity histogram is shown to the right:



(a) Sketch the likely resulting histogram after histogram equalization. Explain the reasoning behind your sketch.
(b) If you want to enhance this image with power-law intensity transform, what kind of gamma value (>1, =1, or <1) should you use? Explain briefly.
(c) Noise, if present in this image, will be enhanced after histogram equalization. Explain briefly.

(a)



Reasoning behind sketching:
(1)The number of peaks is four.
(2)The height of each bar is the same as the original one.
(3)The interval between peaks should be proportional to the height of bars.

(b)

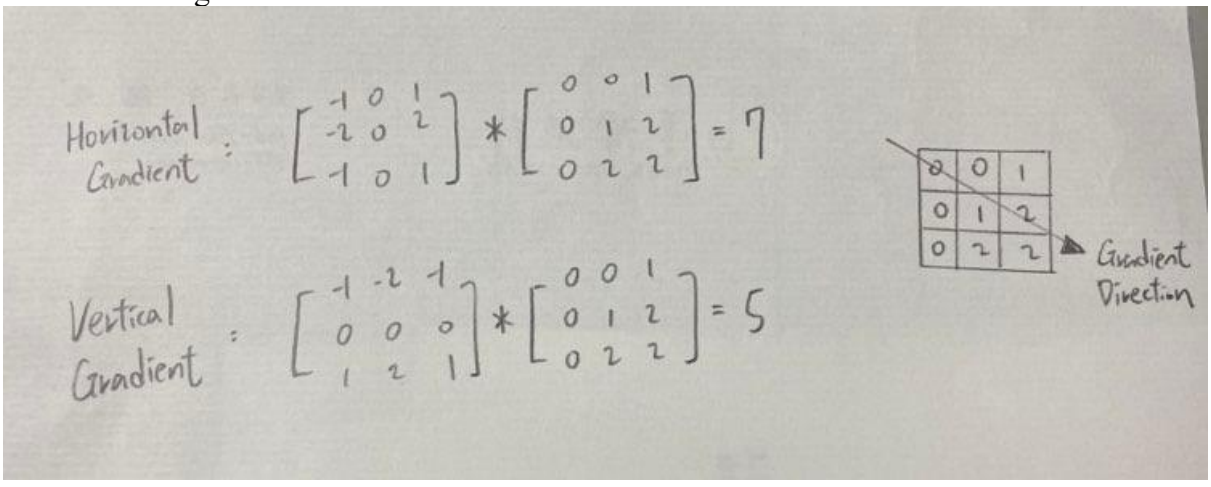
gamma < 1
Because the image is dark, in order to generate a brighter image, we should use gamma <1 to map a narrow range of dark input values into a wider range of output values.

(c)

The contrast of noise will be enhanced as well, because the differences between noise and its original value will be enlarged.

3. [10%] Use Sobel filters to compute the gradient direction of the center pixel of the 3x3 region shown to the right.

0	0	1
0	1	2
0	2	2



4. [10%] Answer the following questions regarding CMY and CMYK color models:
(a) What colors do the letters in CMY and CMYK stand for?
(b) How are the CMY colors related to RGB?
(c) CMY is the standard color space used in color printing. Why using CMY instead of RGB?
(d) Explain the reason why CMYK, instead of CMY, is used in modern color printers?

(a) CMY: Cyan, Magenta, Yellow
CMYK: Cyan, Magenta, Yellow, Black

(b) $\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$

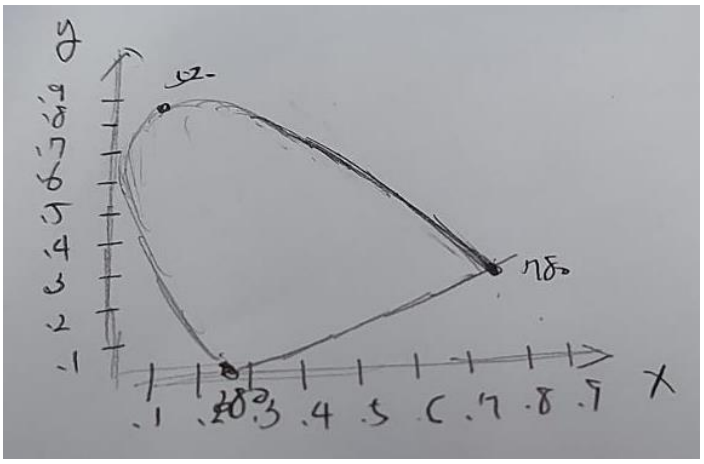
(c) The reason why we see the color of ink is that the ink absorbs certain colors from the light. Therefore, we use the complement of RGB, which is CMY, to represent the color space in printing.

(d) CMY by itself can't generate pure black color, so black (K) is added in order to generate pure black.

5. [10%] Regarding color gamuts:
(a) A color gamut is displayed in a 2-D space with x and y coordinates. What do the two axes stand for, and how are they related to the light sensing cells in the retina?
(b) Approximately sketch the "region of all visible colors" in this x-y space.
(c) What is the reason that the color gamuts of typical color displays are triangles in this space? What do the three corners of the triangle stand for?

(a) x and y are normalized X and Y in the CIE XYZ color space.
They represent the ratio of response from the "red" and "green" retina cones, respectively.

(b)



(c) Typical displays use the combination of three light sources, RGB, to generate colors.
The three corners are the pure R (red), G (green), and B (blue) light color.

6. [10%] Regarding image sharpening:
(a) Give a typical 3x3 Laplacian filter.
(b) Explain how the filter can be used for image sharpening.
(c) Explain what the visual effect of "Mach band" is, and how it is related to image sharpening.

(a)

0	-1	0
-1	4	-1
0	-1	0

(b)

0	0	0
0	1	0
0	0	0

+

0	-1	0
-1	4	-1
0	-1	0

Laplacian Filter

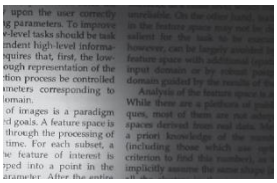
=

0	-1	0
-1	5	-1
0	-1	0

Sharpening Filter

(c) Mach band effect means the edges of darker objects next to lighter objects appear lighter by the human visual system. The reason for this effect is that the human visual system automatically enhances the contrast of edges.
Image sharpening can be achieved by increasing the intensity changes of edges.

7. [15%] Consider the methods for automatic threshold selection.
(a) List the steps of the Basic Global Method as described in the textbook/lecture.
(b) Explain how you can extend the procedure in (a) to cases with multiple thresholds.
(c) Uneven illumination often causes problem in threshold selection. An example is shown to the right. Describe a method to improve the thresholding of this image so that the text in the image can be extracted.

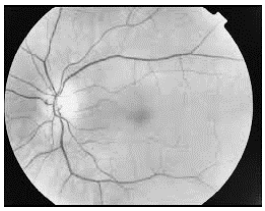


(a)
Initial,
Threshold T, the average gray level of the image
Iteration,
1. compute m1 and m2, the average gray levels for those pixels with gray levels above and below T, respectively.
2. replace T with $(m_1 + m_2)/2$
3. when the change of T between iterations is below some small predefined tolerance, terminate the iteration

(b)
If we have n thresholds, $T_1, T_2, \dots, T_n$, T_i were i/n $* maxPixelValue$,
then we can compute $m_0, m_1, \dots, m_n$,
m_i were average gray level between T_{i-1} and T_i
Iteration computes $T_1 = (m_0 + m_1)/2$, $T_2 = (m_1 + m_2)/2$, $\dots, T_n = (m_1 + m_2)/2$

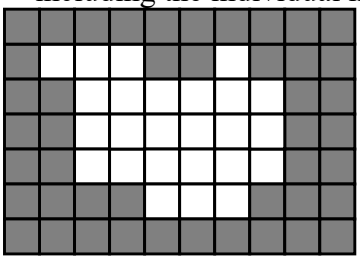
(c)
We can use adaptive thresholding, divide the image into many small regions, then do local threshold.

8. [10%] A little about mathematical morphology:

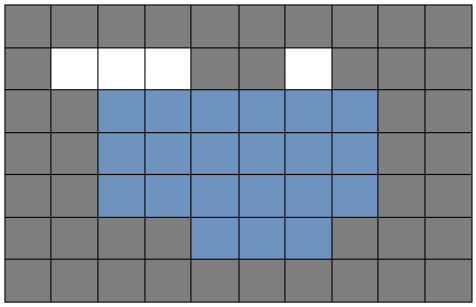


(a) For the binary image below (white pixels as "1" and gray pixels as "0"), indicate the "1" pixels after morphology opening with a 3x3 structuring element (all 9 pixels in the 3x3 included). You can directly mark those pixels in the "image".

(b) Top-hat and bottom-hat transforms are typical used to extract thin structures in images; an example is shown to the right. Describe the steps involved in this case, including the individual morphological operators used.



(a)



(b) In this case, we use bottom-hat to keep dark objects, $B_{hat}(f) = (f \cdot b) - f$

$\cdot$ is closing in gray-scale morphology.

b is

1	1	1
1	1	1
1	1	1

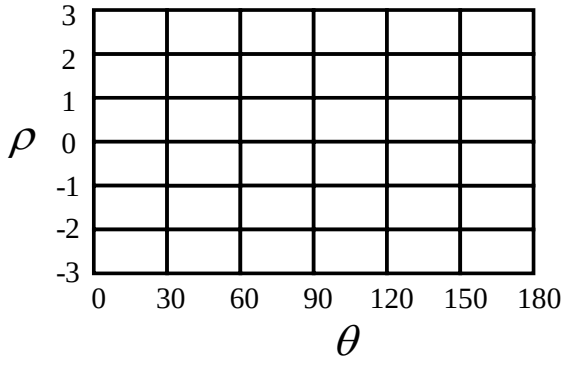
and the subtraction is actually subtraction of intensities, not set difference.

9. [16%] An image contains only two foreground pixels at (-1,0) and (1,1). Use Hough transform to find the equation of the most significant line. The line equation in polar coordinates is $x\cos\theta + y\sin\theta = \rho$.

(a) Why do we use the polar-coordinate representation of lines, not the simpler $y=ax+b$?

(b) Fill the given accumulation bins below. The resolutions of the accumulation bins are 30° for θ and one pixel for ρ. Use cos(45°)=sin(45°)=0.7, cos(75°)=sin(15°)=0.25, and cos(15°)=sin(75°)=0.95.

(c) Determine the equation of the most significant line.



(a) In $y = ax + b$, the range of a is (-inf, inf), which is unbounded

(b)

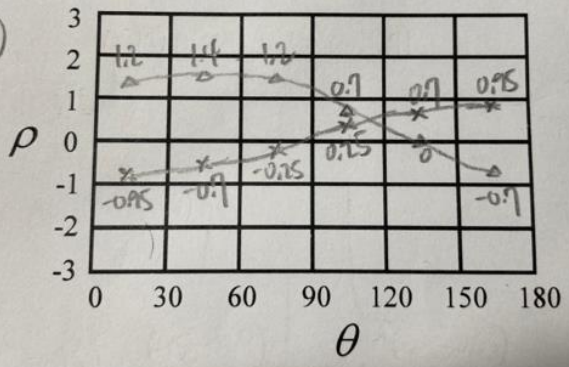
$x\cos\theta + y\sin\theta = \rho$

$(-1,0)$: ×

$(1,1)$: △

(b)
$$\begin{cases} -\cos\theta = \rho \\ \cos\theta + \sin\theta = \rho \end{cases}$$

(θ, ρ)



$15°$
$$\begin{cases} -0.95 = \rho \\ 0.95 + 0.25 = \rho \end{cases}$$

$45°$
$$\begin{cases} -0.7 = \rho \\ 0.7 + 0.7 = \rho \end{cases}$$

$105°$
$$\begin{cases} 0.25 = \rho \\ -0.25 + 0.95 = \rho \end{cases}$$

$75°$
$$\begin{cases} -0.25 = \rho \\ 0.25 + 0.95 = \rho \end{cases}$$

$135°$
$$\begin{cases} 0.7 = \rho \\ -0.7 + 0.7 = \rho \end{cases}$$

$165°$
$$\begin{cases} 0.95 = \rho \\ -0.95 + 0.25 = \rho \end{cases}$$

(c)

(c) The accumulation cell with highest total: $\theta = 105°$, $\rho = 0.5$

$x\cos(105°) + y\sin(105°) = 0.5$

$\Rightarrow -0.25x + 0.95y = 0.5$