

The midterm has at most 10 problem sets; the test time is 100 minutes. The expected time for solving one problem set is about 10-20 mins.

Midterm samplers

1. (15%) Explain the following term, you are graded based on the correctness and completeness
 - a. Entropy
 - b. KL divergence
 - c. Isotropic Gaussian distribution
 - d. Gradient decent method
 - e. Affine properties of Gaussian

2. (10%) Show the Beta distribution and Binomial distribution are a conjugate pair.

3. (10%) Show that L2 norm regularized linear regression is a MAP estimation

4. (10%) Given the pmf of a distribution:

$$P(k \text{ events in interval}) = e^{-\lambda} \frac{\lambda^k}{k!}$$

where

- λ is the average number of events per interval
- e is the number 2.71828... (Euler's number) the base of the natural logarithms
- k takes values 0, 1, 2, ...
- $k! = k \times (k - 1) \times (k - 2) \times \dots \times 2 \times 1$ is the factorial of k .

Please derive the MLE of the mean (i.e., λ) of this pmf.

5. (20%) Given two coins (C_0 , C_1) and in each trial, a certain coin is chosen and tossed for three times. There are totally three trial outcomes: {HHH, HHT, TTT}. Note that the chance of choosing C_0 is k , the chance of C_0 showing H is P_0 , and the chance of C_1 showing H is P_1 .

Use EM algorithm to update the estimates of the parameters for just one round. You should show the process step by step with the initial values of parameters: $k=0.5$, $P_0=0.6$, $P_1=0.1$.