Independent Task Scheduling

Real-Time and Embedded Operating Systems

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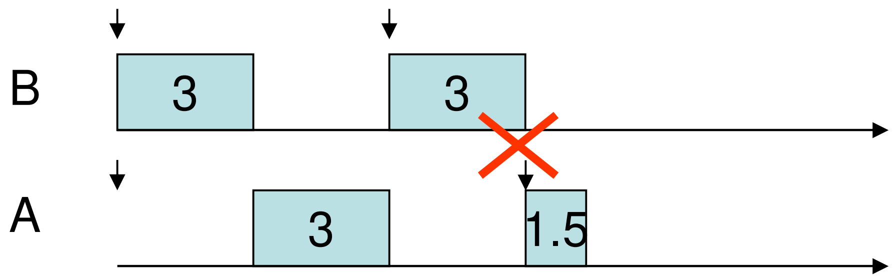
Motivation

- A shared resource should be scheduled
- Take yourself as an example
  - Normally, you have a bunch of things to do, with time pressure
    - Project deadlines, meeting times, class times, and deadlines of bills
  - Some of them regularly recur but some don't
    - Going for lunch at 12:30 everyday
    - Seeing a movie at 8:00pm

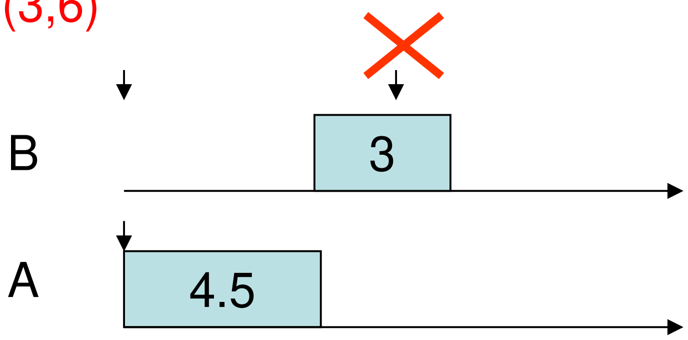
Motivation

- You schedule yourself to meet deadlines
  - Course A: a homework is announced every 9 days, and each costs you 4.5 days
  - Course B: a homework is announced every 6 days, and each costs you 3 days
- You miss deadlines of one course, if you favor either one of the two courses

*A=(4.5,9), B=(3,6)



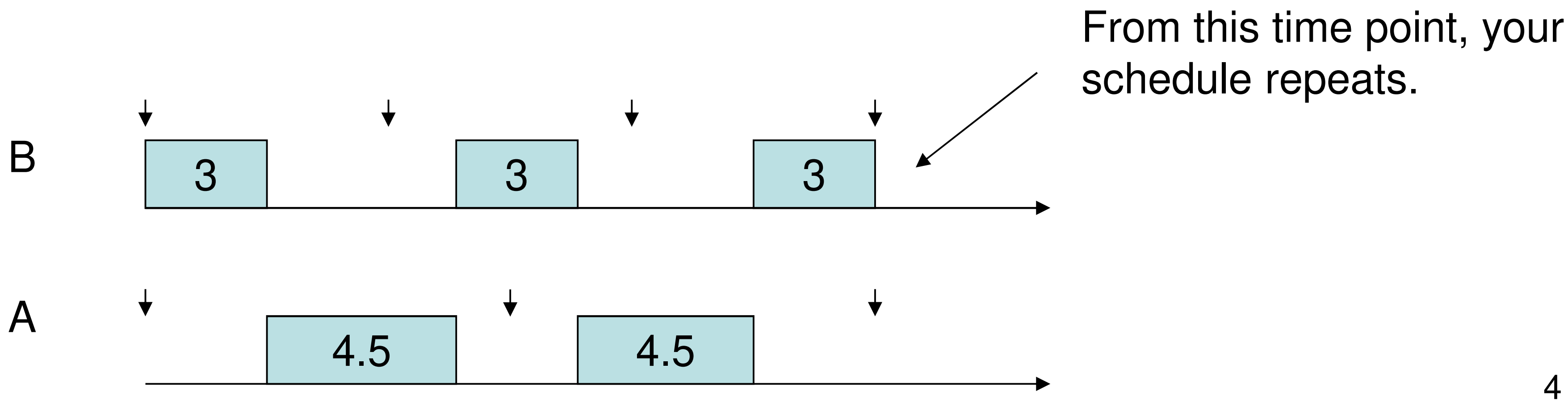
Favor course B



Favor course A

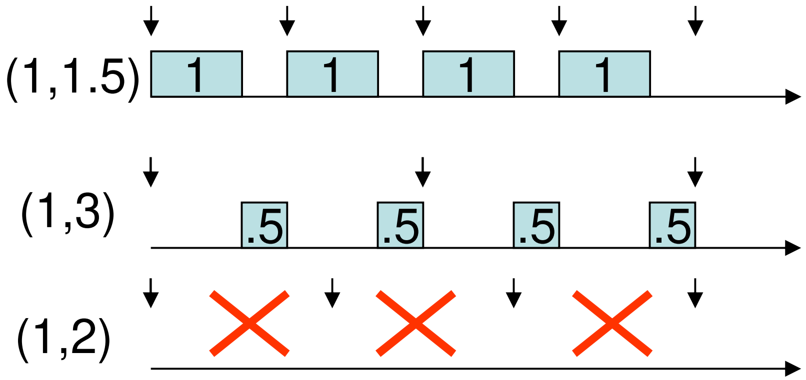
Motivation

- Scheduling to meet deadlines (cont'd.)
  - Course A: (4.5, 9)
  - Course B: (3, 6)
- All deadlines are met if you do the homework whose deadline is the nearest

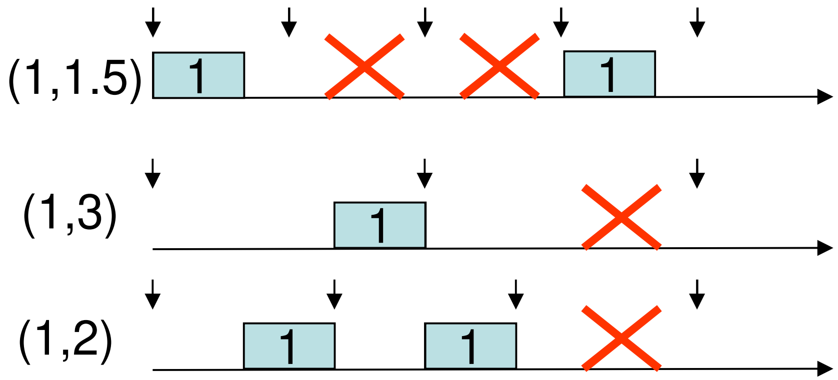


Motivation

- You schedule yourself to survive overloadings
  - (1,2), (1,3), (1,1.5)



Favoring (1,1.5)→(1,3)→(1,2)
2 teachers are happy, 1 will flunk you though…



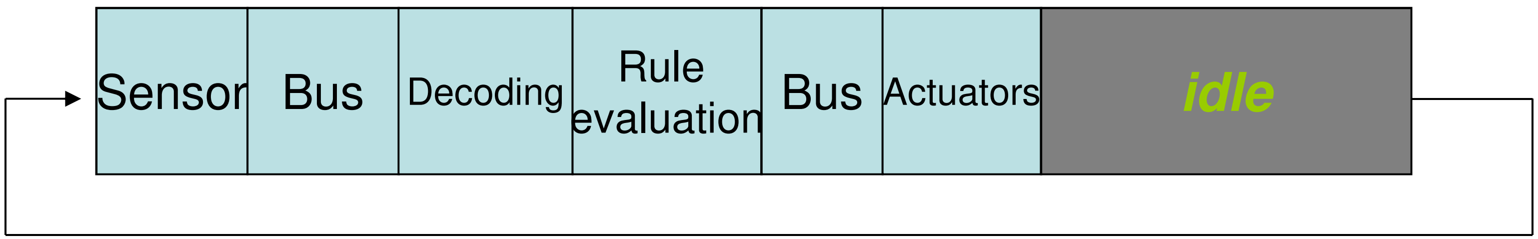
Do whatever has the closest deadline.
You will be in deep trouble…

* Tie breaking is arbitrary

Cyclic-Executive

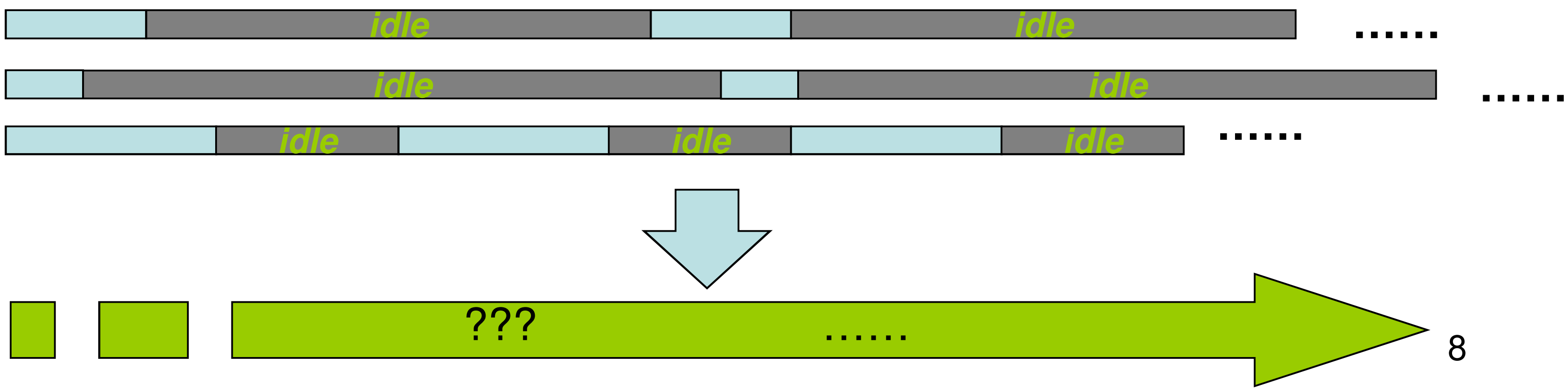
Cyclic Executive

- The system repeatedly executes a static schedule
  - A table-driven approach
- Many existing systems still take this approach
  - Easy to debug and easy to visualize, highly deterministic
  - Hard to program, modify, and upgrade
    - A program should be divided into many pieces (like an FSM)
    - The table needs a revision even if a small modification is made



Cyclic Executive

- The table emulates an infinite loop of routines
  - However, a single independent loop is not enough for complicated tasks
  - Multiple concurrent loops are used
- How large should the table be when there are multiple loops?



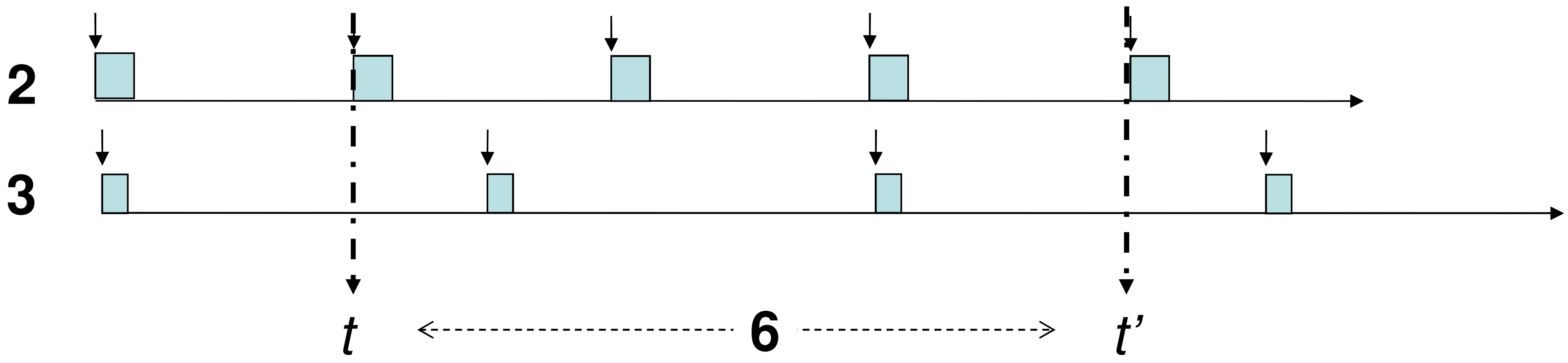
Cyclic Executive

- ***Definition***: The hyper-period (*h*) of a collection of loops is a time window whose length is the least-common-multiplier of the loop lengths

- ***Theorem***: The routines to be executed in a time interval [t,t+x] and those in [t+h,t+h+x] are identical

Cyclic Executive

- ***Proof***:

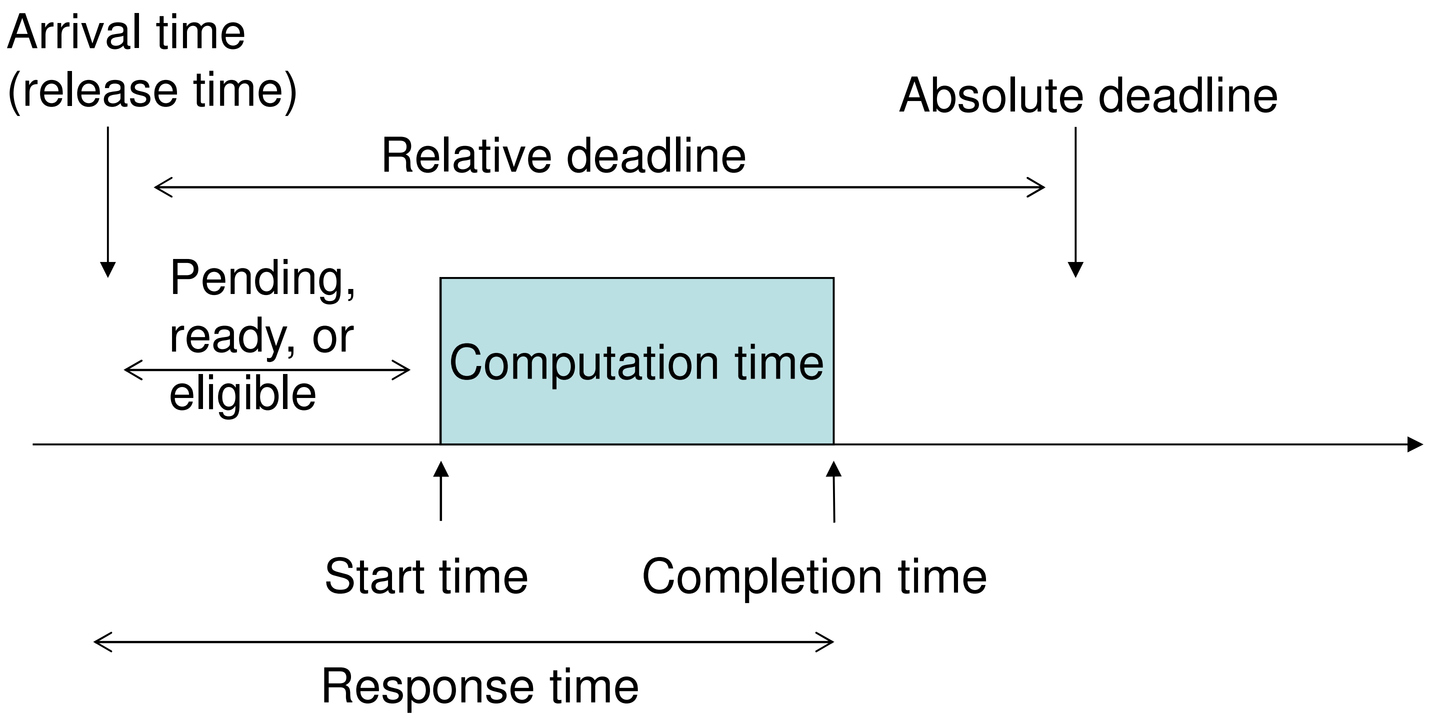


- The arrivals of loops (routines) since *t'* and those since *t* are exactly the same

Rate-Monotonic Scheduling (Fixed-Priority Scheduling)

System Model

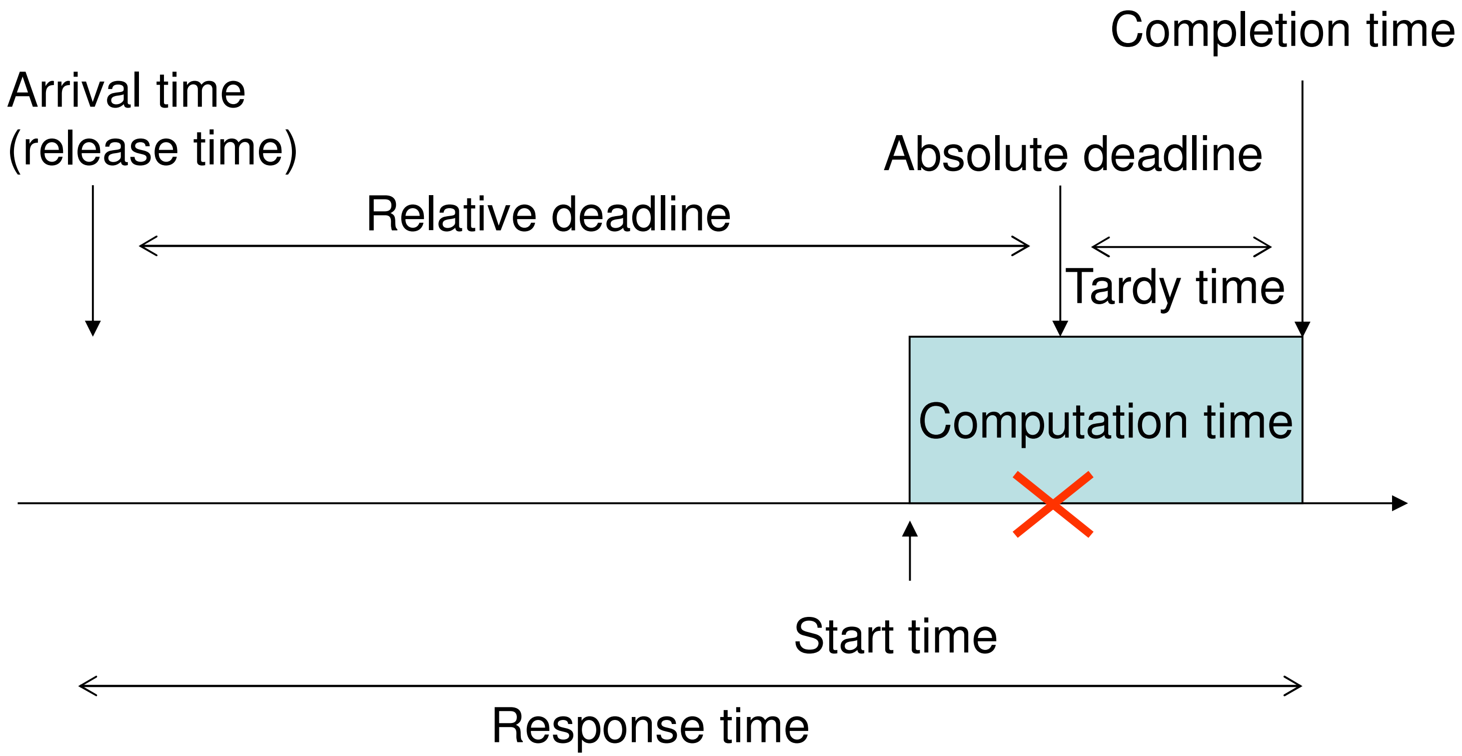
- A job with real-time constraints (non-preemptible)



If a job completes before its deadline, then its deadline is satisfied.

System Model

- A job that misses its deadline



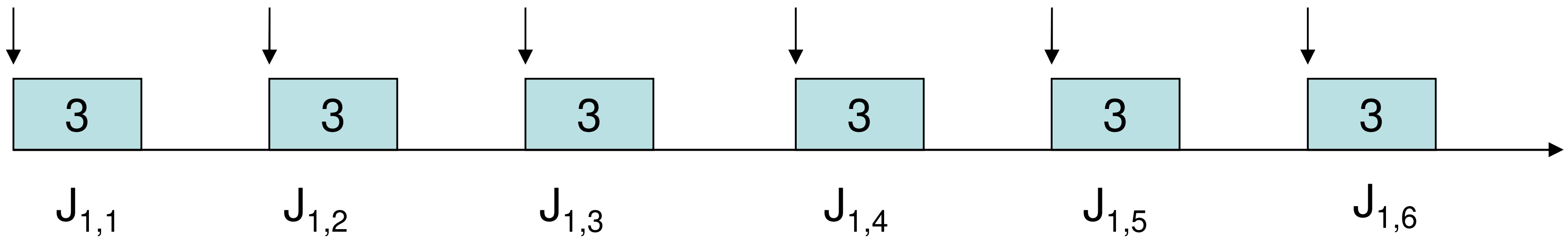
If a job completes after its deadline, then its deadline is violated, or an overflow occurs.

System Model

- A task set is of a fixed number of tasks
  - $\{T_1, T_2, \ldots, T_n\}$
  - Tasks share nothing but the CPU, and they are independent
- A task T_i is a template of jobs, and its recurrences are jobs
  - Every job executes the same piece of code
    - Of course, different input and run-time conditions cause different job behaviors
    - $J_{i,j}$ refers to the j-th job of task T_i
  - The computation time c_i of jobs is bounded and known *a priori*

System Model

- A purely periodic task
  - Jobs of a task T recur every p units of time
  - A job must be completed before the next job arrives
    - Relative deadlines for jobs are, implicitly, the period
  - T is defined as (c,p)



Periodic task T_1=(3,6)

System Model

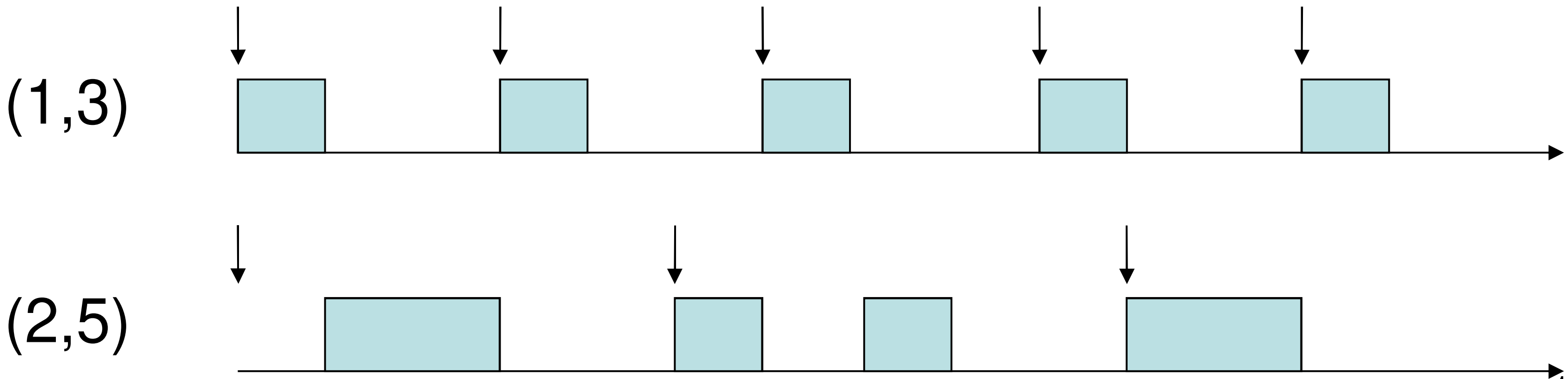
- Priority
  - Reflect the urgency of jobs
  - Jobs of the same task may have the same or different priorities
- Preemption
  - When a high-priority task becomes ready, it preempts the current (lower-priority) task

System Model

- Checklist
  - Periodic tasks
  - Real-time constraints
  - Priority
  - Preemptivity

Rate-Monotonic Scheduling

- A task-level fixed-priority scheduling algorithm
  - All jobs inherit a static priority from its task
- Tasks priorities are proportional to task rates
  - The smaller the period is, the higher the priority is
  - Inversely proportional to task period lengths

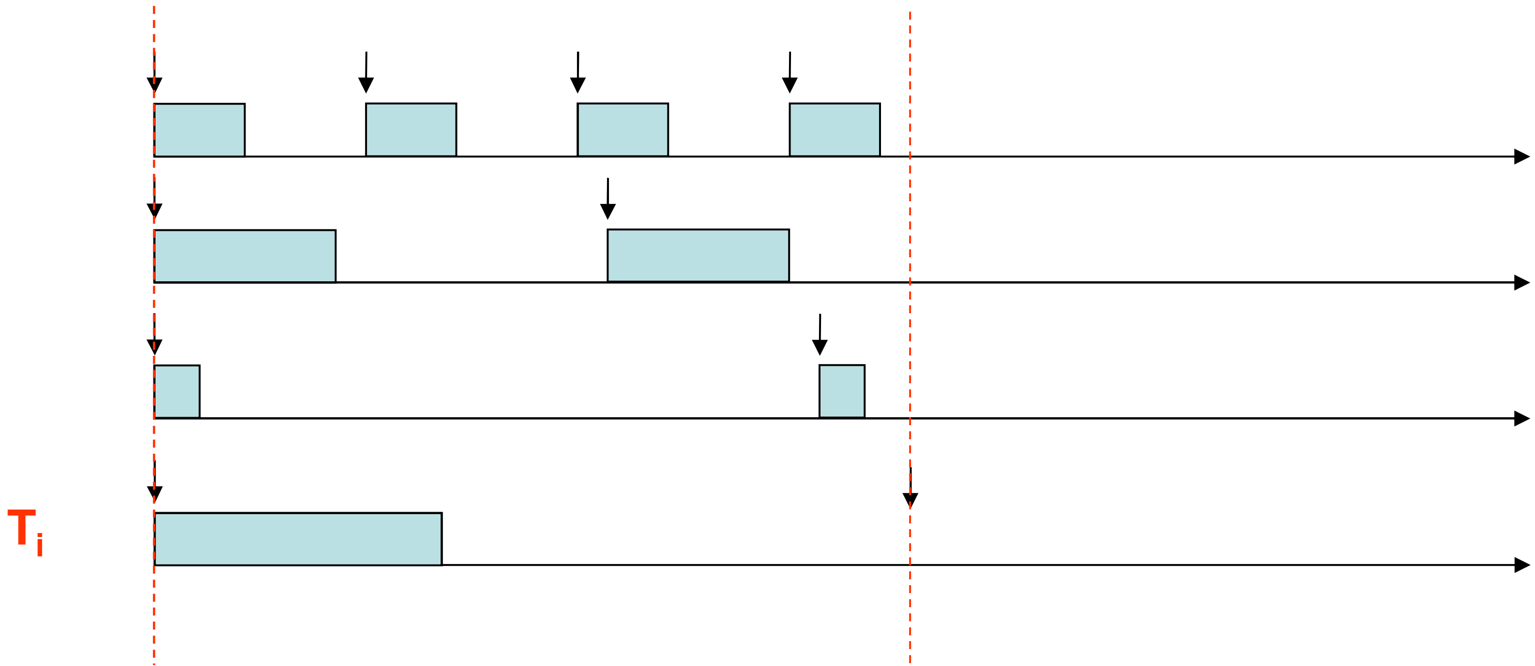


Rate-Monotonic Scheduling

- Does task T_i always meet its deadline?
- Critical instant of task T_i
  - A job $J_{i,c}$ of task T_i released at T_i's critical instant would have the longest response time
  - In this case, to meet the deadline of $J_{i,c}$ would be "the hardest"
  - If $J_{i,c}$ succeeds in satisfying its deadline at the critical instant, then later any job of T_i can succeed

Rate-Monotonic-Scheduling

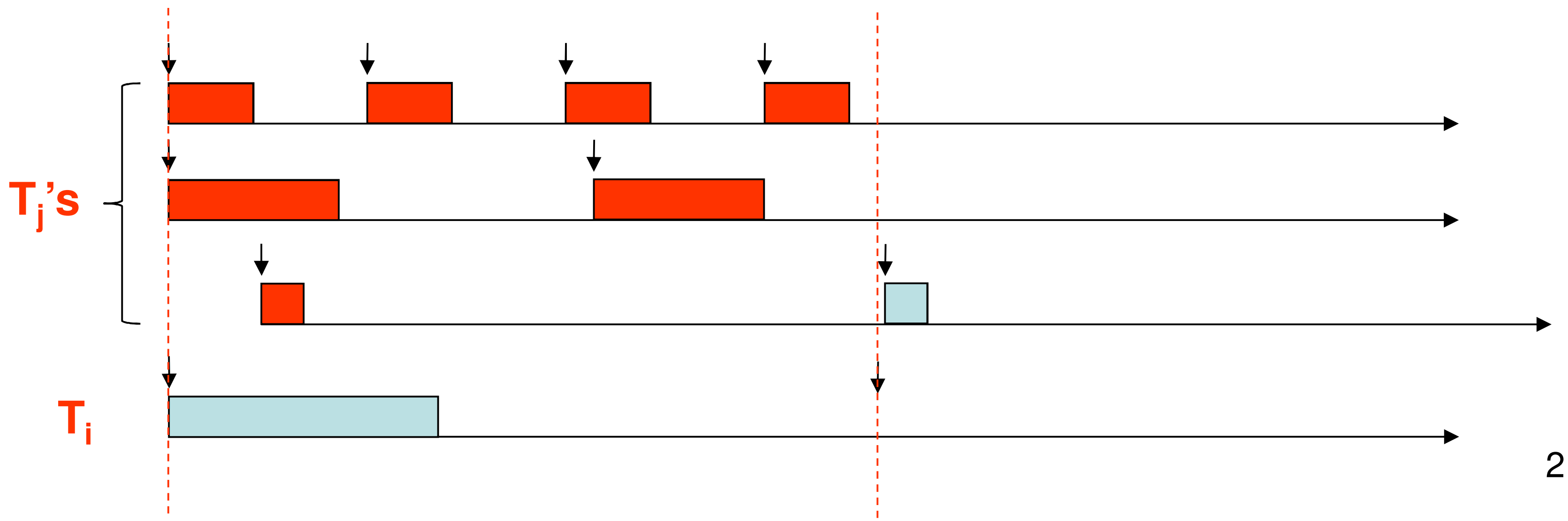
- ***Theorem***: A critical instance of a task T_i occurs when its job $J_{i,c}$ and a job from every higher-priority task are all released at the same time (i.e., in-phase)



Rate-Monotonic-Scheduling

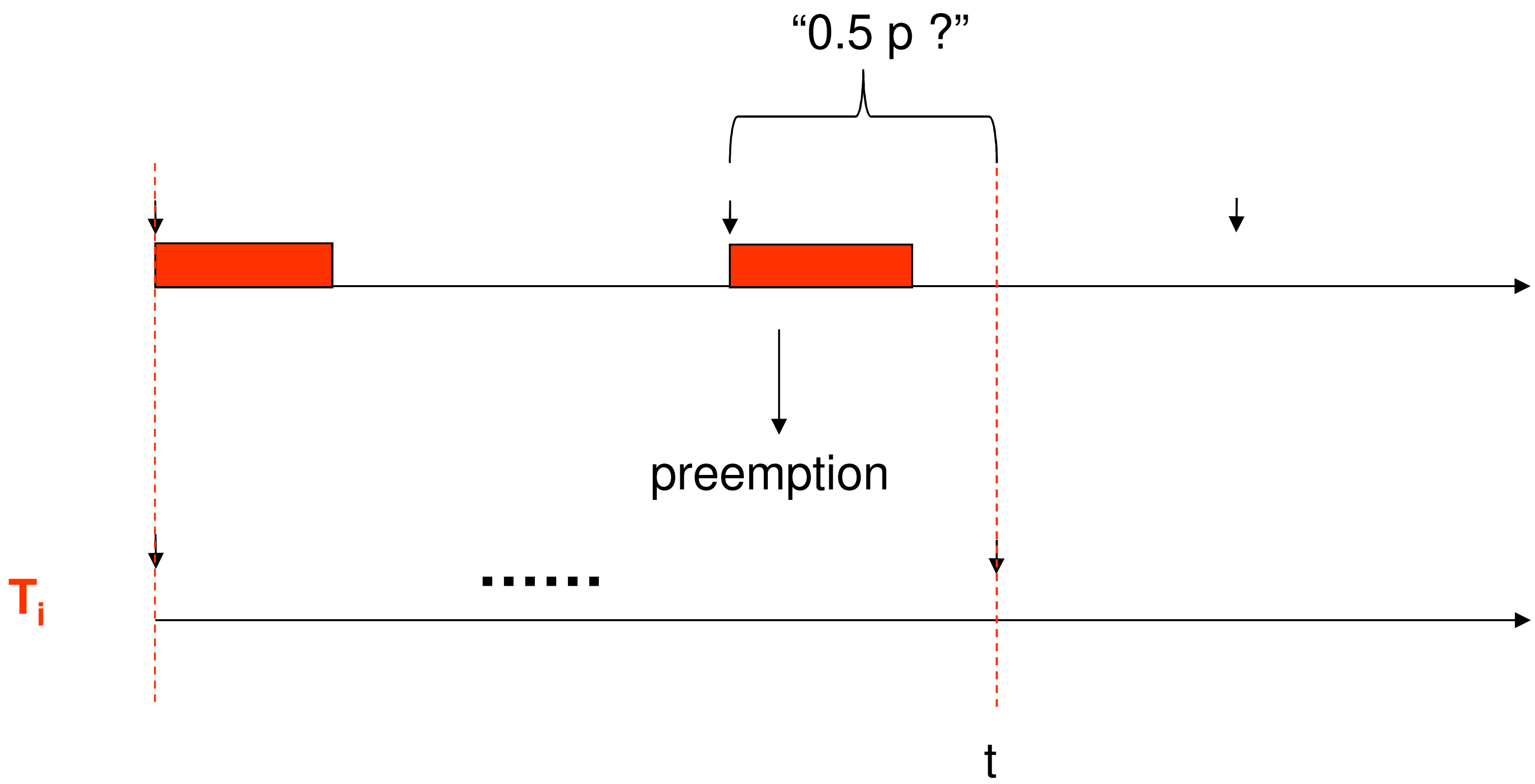
- The "interference" from high-priority tasks in the first period of T_i is never larger than

$$\sum_{j<i} c_j \left\lceil \frac{p_i}{p_j} \right\rceil$$



Rate-Monotonic-Scheduling

- Critical instance: Why ceiling function?

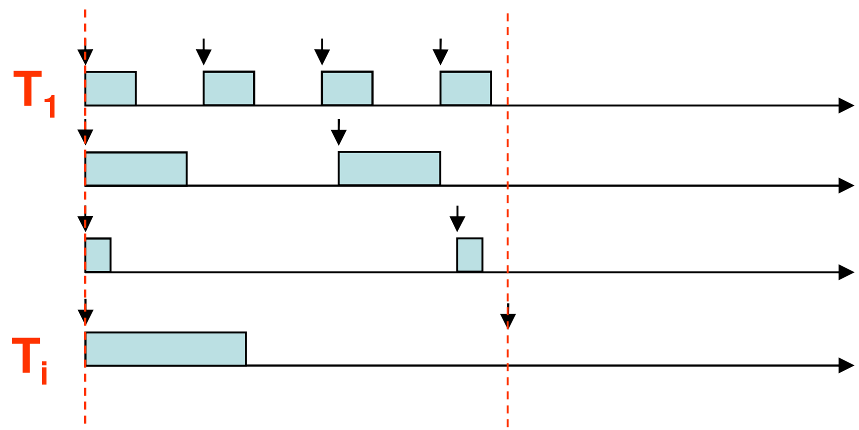


Rate-Monotonic Scheduling

- (Recursive) Response time analysis
  - The response time of a job of T_i at the critical instance can be computed by a recursive function

$$r_0 = \sum_{1 \ldots i} c_i$$

$$r_n = \sum_{1 \ldots i} c_i \left\lceil \frac{r_{n-1}}{p_i} \right\rceil$$



  - Observation: *r* may or may not converge before p_i

Rate-Monotonic Scheduling

- ***Theorem***: A task set={T_1,T_2,…,T_n} is schedulable by RM if and only if the worst-case response times of every task are shorter than their periods

- Observations
  - If every task survives its critical instance (in phase), then with any task phasing all tasks will survive
  - The analysis is an exact schedulability test for RMS
    - Called “Rate-Monotonic Analysis”, RMA for short

Rate-Monotonic Scheduling

- Example: T1=(2,5), T2=(2,7), T3=(3,8)
  - T1:
    - $R_0=2 \le 5$ ok
  - T2:
    - $R_0=2+2=4 \le 7$
    - $R_1=2^* \lceil 4/5 \rceil +2^* \lceil 4/7 \rceil =4 \le 7$ ok
  - T3:
    - $R_0=2+2+3=7 \le 8$
    - $R_1= 2^* \lceil 7/5 \rceil +2^* \lceil 7/7 \rceil + 3^* \lceil 7/8 \rceil =9> 8$ failed

  - Note: every task must succeed in this test!

Let's try
{(1,3),(1,6),(6,12)}   {(3,6),(3.1,9)}

Rate-Monotonic Scheduling

- Proof:
  - If the response time converges at r_n, then the first lowest-priority job completes at r_n
  - If r_n is before p_n , then the first lowest-priority job meets its deadline at its critical instance
  - Since the job survives the critical instace, it always succeed satisfying its deadline under any task phasing

Rate-Monotonic Scheduling

- Test every task for schedulability!!
  - {T1=(3,6),T2=(3.1,9),T3=(1,18)}
  - Response analysis of T3:
    - R0=7.1, R1=10.1, R2=13.2, R3=16.2, R4=16.2<18
    - Does this mean {T1,T2,T3} schedulable?
  - No, T2 fails the test when considering {T1, T2}
    - This task set is not schedulable!!!

Rate-Monotonic Scheduling

- Time complexity
  - $O(n^2 * p_n)$, pseudo-polynomial time
  - Very fast when task periods are harmonically related
  - Would be *extremely slow* when periods of tasks are small and prime to each other

(2,4),(4,7),(1,100)→ T3: 15 interactions, fails

(2,5),(4,7),(1,100)→ T3: 11 interactions, succeeds

(2,4),(9,20),(1,100)→ T3: 3 interactions, succeeds

T1	T2	T3	R0	7	T1	T2	T3	R0	7	T1	T2	T3	R0	12
2	4	1	R1	9	2	4	1	R1	9	2	9	1	R1	16
4	7	100	R2	15	5	7	100	R2	13	4	20	100	R2	18
			R3	21				R3	15				R3	20
			R4	25				R4	19				R4	20
			R5	31				R5	21				R5	20
			R6	37				R6	23				R6	20
			R7	45				R7	27				R7	20
			R8	53				R8	29				R8	20
			R9	61				R9	33				R9	20
			R10	69				R10	35				R10	20
			R11	77				R11	35				R11	20
			R12	85				R12	35				R12	20
			R13	97				R13	35				R13	20
			R14	107				R14	35				R14	20
			R15	120				R15	35				R15	20

Rate-Monotonic Scheduling

- Phenomena
  - Even though RMA is an exact test for fixed-priority scheduling, it is not often used, especially not in dynamic systems, because of its high time complexity
  - RMA is more suitable for static systems
  - Are there any schedulability tests that are efficient enough for on-line implementation?
    - Not slower than polynomial time

Rate-Monotonic Scheduling

- A trivial schedulability test
  - The system accepts a task set T if the following conditions are both true
    - There is only one task
    - $c/p \leq 1$ (CPU utilization LEQ 100%)
  - The algorithm is efficient enough (i.e., O(1))
  - Too pessimistic

Rate-Monotonic Scheduling

- Definition
  - Utilization factor of task T=(c,p) is

$$\frac{c}{p}$$

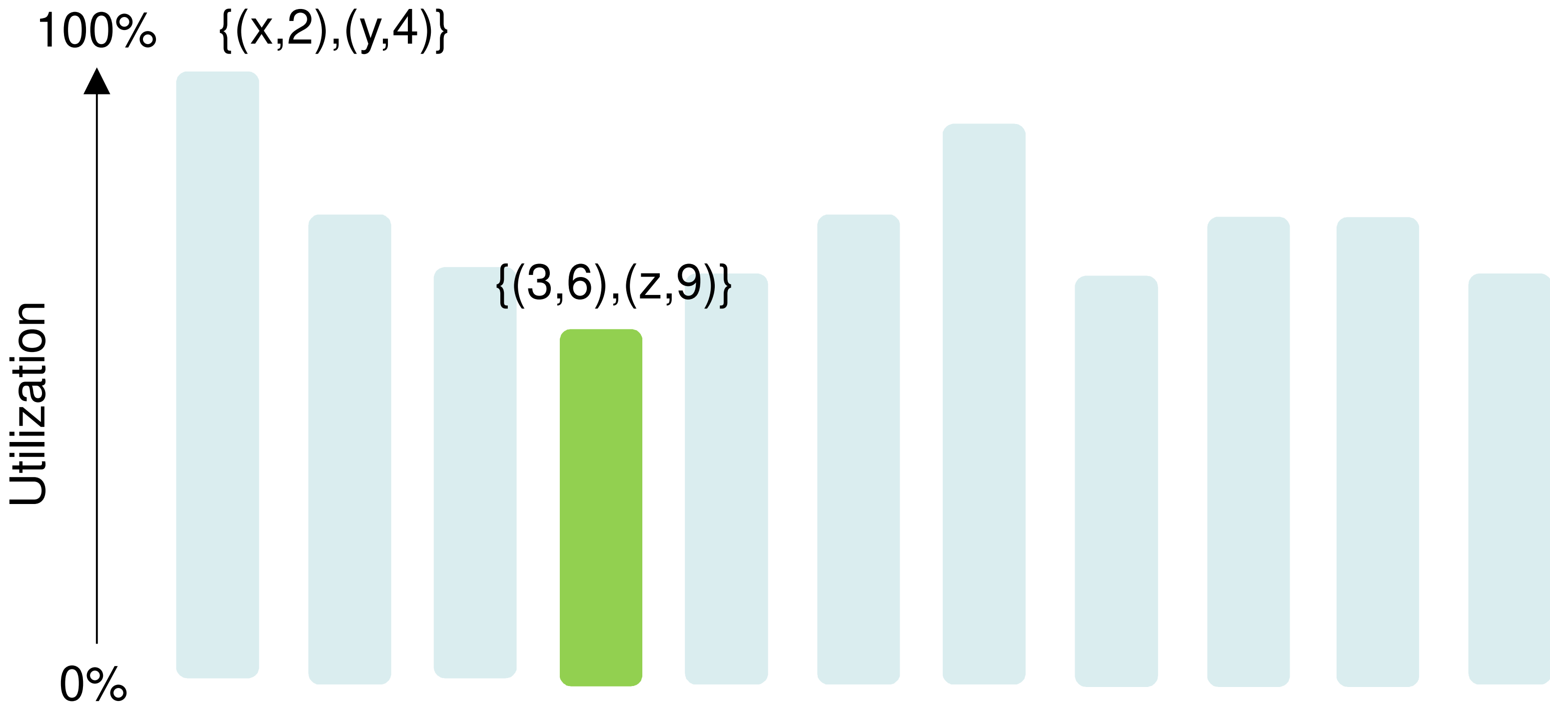
  - CPU utilization of a task set $\{T_1,T_2,\ldots,T_n\}$ is

$$U = \sum_{i=1}^{n} \frac{c_i}{p_i}$$

  - Observation: if the total utilization exceeds 1 then the task set is not schedulable

Rate-Monotonic Scheduling

- To find "the lowest" among "the achievable processor utilizations of different task sets"
  - Achievable utilization is highly related to task periods



Rate-Monotonic Scheduling

- ***Theorem***: [LL73] A task set $\{T_1, T_2, \ldots, T_n\}$ is schedulable by RMS if

$$\sum_{i=1}^{n} \frac{c_i}{p_i} \leq U(n) = n(2^{1/n} - 1)$$

- Observation:
  - If the test succeeds then the task set is schedulable
  - A sufficient condition for schedulability

Rate-Monotonic Scheduling

- Proof: Let us consider two tasks only

T2's 2nd job does not overlap the immediately preceding job of T1

$C_1 \le P_2 - P_1(\lfloor P_2/P_1 \rfloor)$

The largest possible C2 is

$P_2 - C_1(\lceil P_2/P_1 \rceil)$

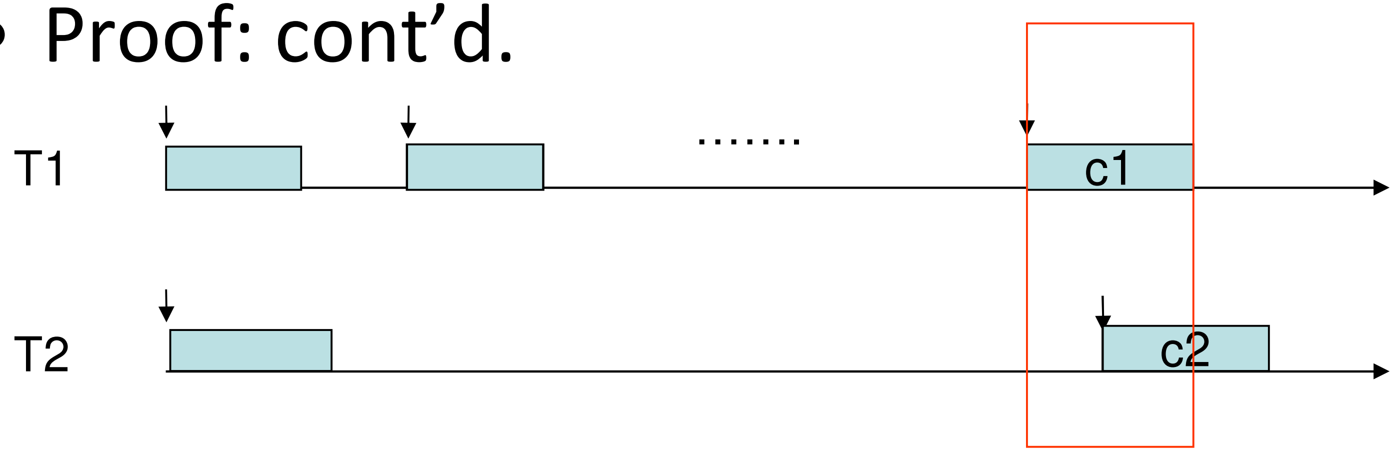
Total utilization factor is

$U = 1 + C_1(1/P_1 - (1/P_2)(\lceil P_2/P_1 \rceil))$

- U monotonically decreases with C_1
    - C1's right-coefficient is negative because $1/P_1 < (1/P_2)(\lceil P_2/P_1 \rceil)$

Rate-Monotonic Scheduling

- Proof: cont'd.



$$C_1 \geq P_2 - P_1(\lfloor P_2/P_1 \rfloor)$$

The largest possible C_2 is

$$-C_1(\lfloor P_2/P_1 \rfloor) + P_1(\lfloor P_2/P_1 \rfloor)$$

Total utilization factor is

$$U = (P_1/P_2)\lfloor P_2/P_1 \rfloor + C_1((1/P_1) - (1/P_2)(\lfloor P_2/P_1 \rfloor))$$

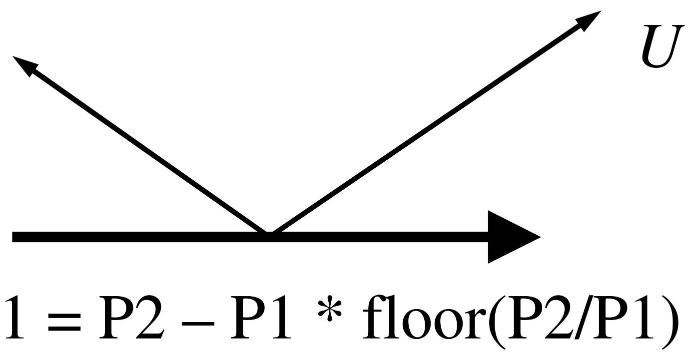
T2's 2nd job overlaps the immediately preceding job of T1

- U monotonically increases with C_1

Rate-Monotonic Scheduling

- Proof: Cont'd.
  - It can be found that the minimal U occurs at

$$C_1 = P_2 - P_1 \left(\left\lfloor P_2 / P_1 \right\rfloor \right)$$



  - By some differentiation, the minimal achievable utilization is

$$U(2) = 2(2^{1/2} - 1)$$

Rate-Monotonic Scheduling

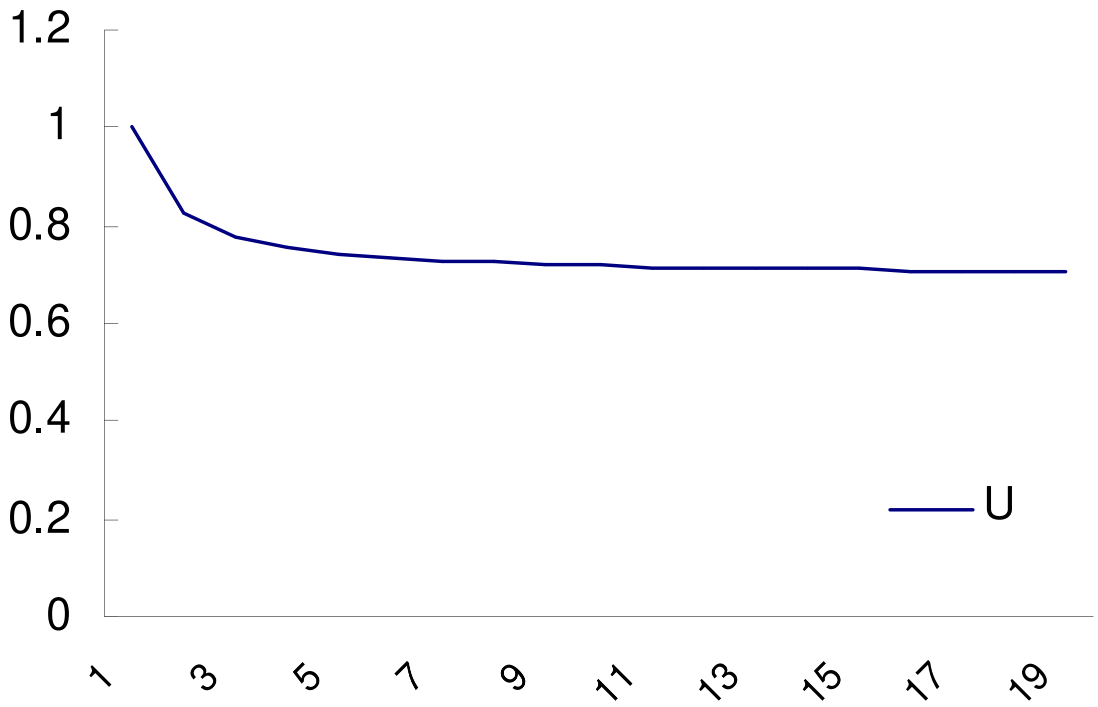
- The generalized result for *n* tasks is

$$U(n) = n(2^{1/n} - 1)$$

- If a task set of n tasks whose total utilization is not larger than U(n), then this task set is guaranteed to be schedulable by RM
  - The time complexity of the test is O(n), which is efficient enough for on-line implementation

Rate-Monotonic Scheduling

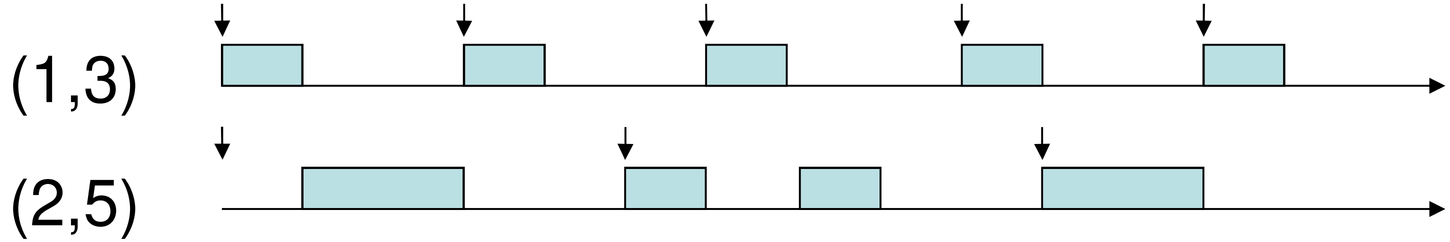
- When x → infinitely large, U(x)→0.68



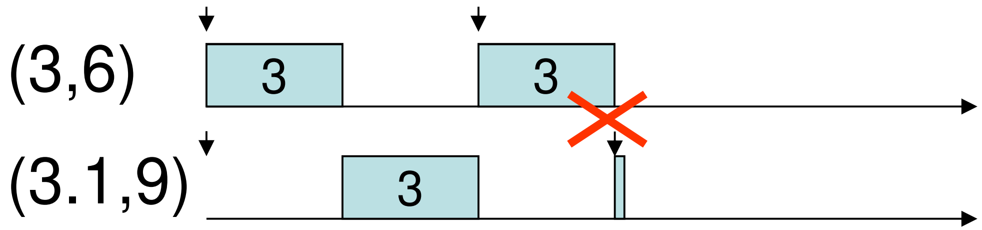
1	1
2	0.828427
3	0.779763
4	0.756828
5	0.743492
6	0.734772
7	0.728627
8	0.724062
9	0.720538
10	0.717735
11	0.715452
12	0.713557
13	0.711959
14	0.710593
15	0.709412

Rate-Monotonic Scheduling

- Example 1: (1,3), (2,5)
  - Utilization =0.73 ≦ U(2)=0.828

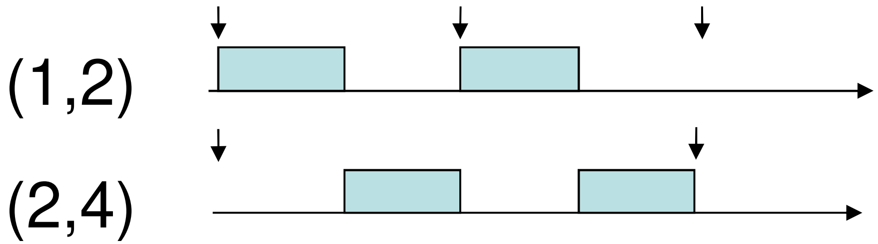


- Example 2: (3,6), (3.1,9)
  - Utilization =0.84 > U(2)=0.828



Rate-Monotonic Scheduling

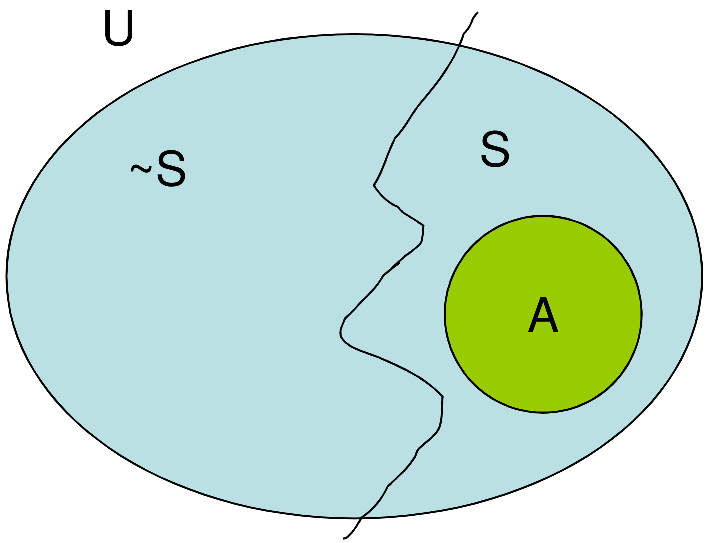
- Example 3: (1,2), (2,4)
  - Utilization =100%>U(2)=0.828



- Example 2 and 3 shows that, we know nothing about those task sets of total utilization > the utilization bound!

Rate-Monotonic Scheduling

- Sufficient but not necessary
  - Utilization test provides a fast way to check if a task set is schedulable
  - A task set that fails the utilization test could be schedulable!



U: universe of task sets
~S: task sets unschedulable by RM
- Example 2

S: task sets schedulable by RM
- Example 1 and Example 3

A: Those can be found by utilization test
- Example 1

Example 3 is in S-A

Rate-Monotonic Scheduling

- Summary
  - Explicit prioritization over tasks
  - To decide task sets' schedulability is costly
  - Sufficient tests were developed for fast admission control

Earliest-Deadline First (Dynamic-Priority Scheduling)

Earliest-Deadline-First Scheduling

- Definition
  - Feasible
    - A set of tasks is feasible if there exists some way to schedule the tasks without any deadline violations
  - Schedulable
    - Given a scheduling algorithm A
    - A set of tasks is schedulable by A, if algorithm A successfully schedule the tasks without any deadline violations
- Observations
  - A feasible task set may not be schedulable (by RM)
  - If a task set is schedulable by some algorithm A, then it is feasible

Earliest-Deadline-First Scheduling

- If for an algorithm, schedulable ←→ feasible
  - then it is a universal scheduling algorithm

- What are the universal scheduling algorithms for periodic and preemptive uniprocessor systems?
  - EDF
  - LLF/LSF
  - …

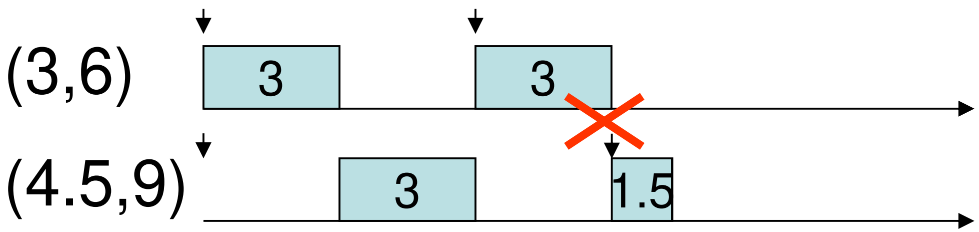
Earliest-Deadline-First Scheduling

- EDF always picks a ready job whose deadline is the earliest for execution
  - The earlier the deadline of a job is, the more urgent the job is
  - “Priorities” among tasks change from time to time
  - Better to avoid using the term “priority” for EDF since there is no explicit definition; use urgency or importance instead

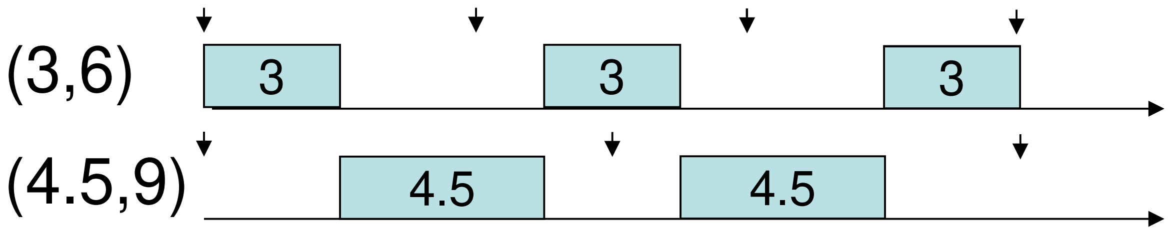
Earliest-Deadline-First Scheduling

- Example

Not schedulable by RM



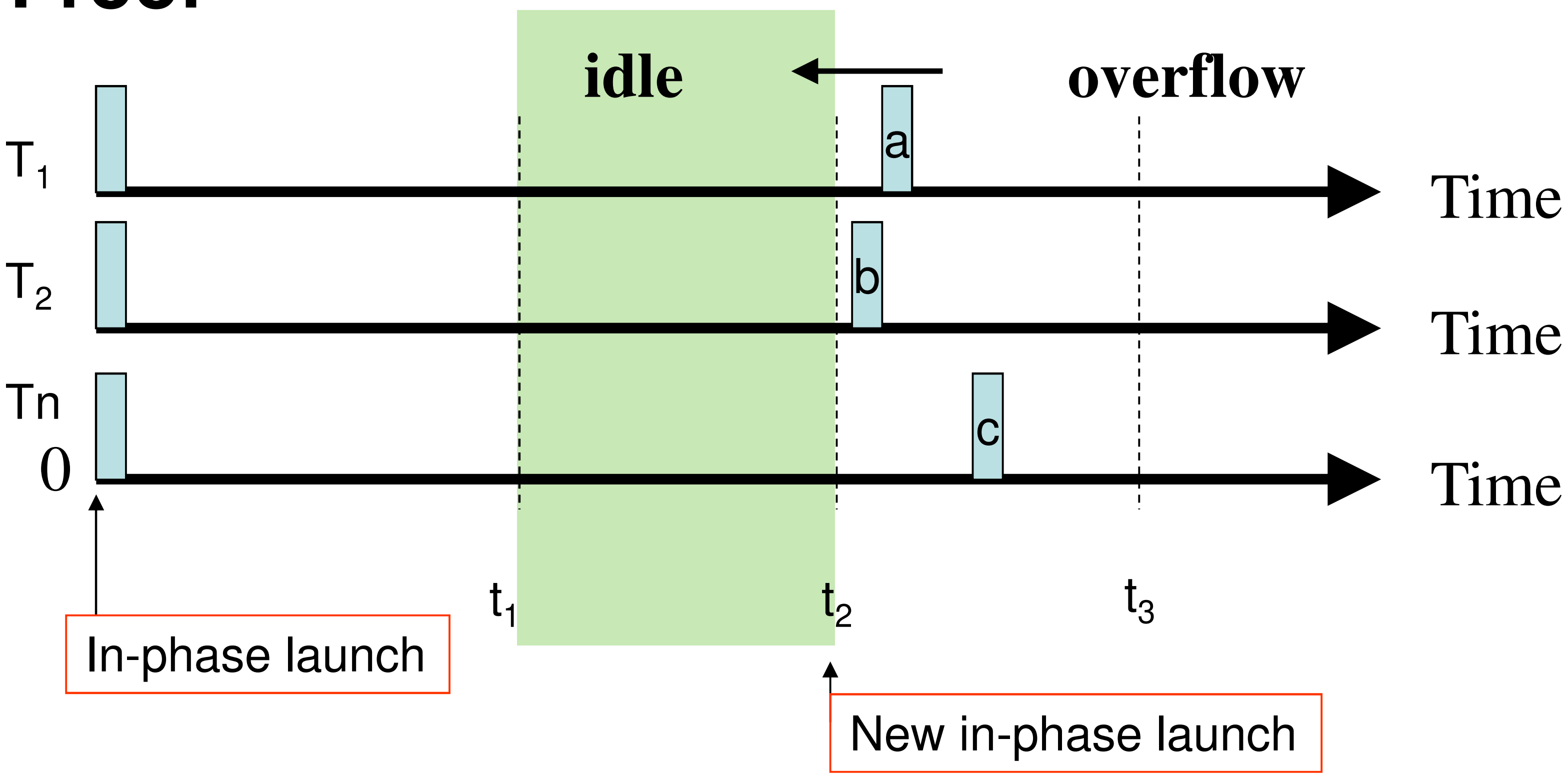
Schedulable by EDF



Earliest-Deadline-First Scheduling

- Observation: The critical instance of a task with EDF is the same as that with RM
-
- ***Lemma***: With EDF, there is no idle time before an overflow
  - This is a very strong statement that implies the optimality of EDF in terms of schedulability

Proof



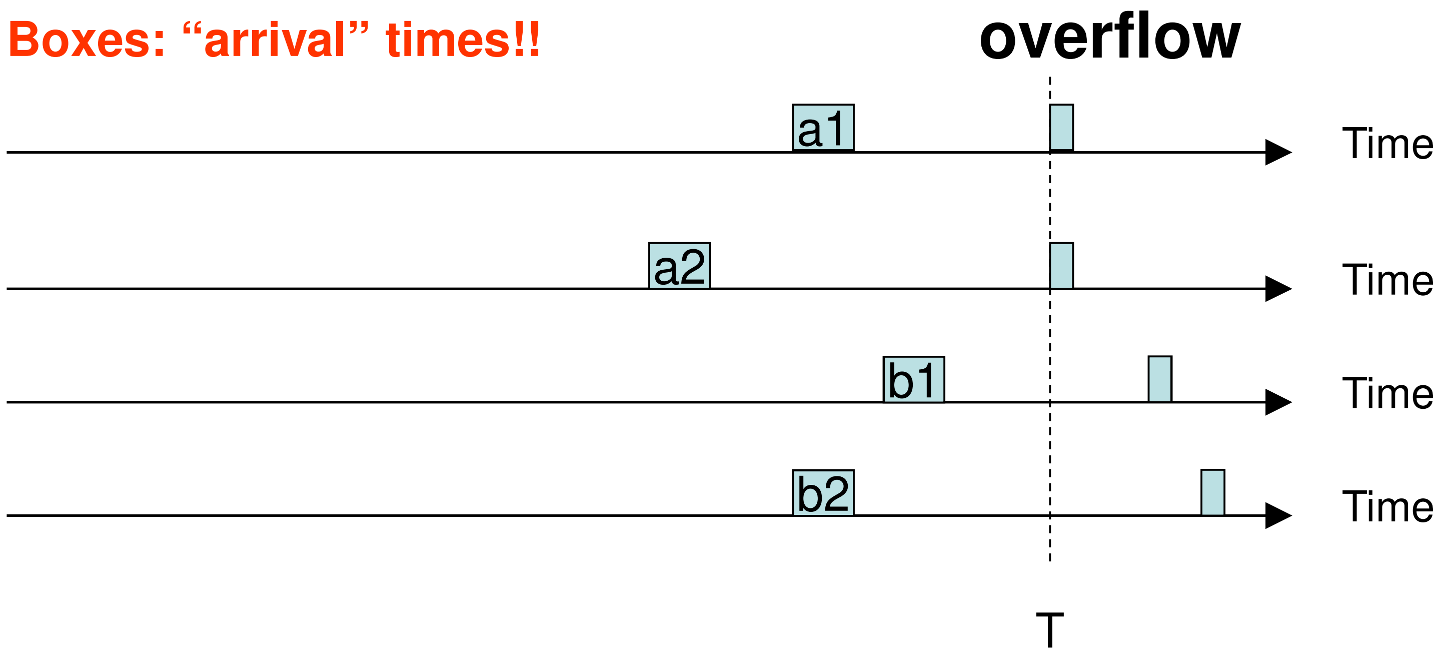
- Consider in-phase launching of all tasks. Suppose that there is an overflow at time t_3, and the processor idles between t_1 and t_2
- If we move "a" forward to be aligned to t_2, the overflow would occur earlier than it was (i.e., at or before t_3)
  - That is because EDF's discipline: moving forward means promoting the urgency of T_1's jobs
- By repeating the above action, jobs a,b, and c can be aligned at t_2 , forming another in-phase launch
  - → that contradicts the assumption! From t_2 on, there is no idle until the overflow

Earliest-Deadline-First Scheduling

- ***Theorem***: A set of tasks is schedulable by EDF if and only if its total CPU utilization is no more than 1

- Observation: → is easy, ← requires some reasoning similar to the proof of the last theorem

Boxes: “arrival” times!!



←: suppose that U<=1 but the system is not schedulable by EDF

- Suppose that there is an overflow at time T
  - Jobs a’s have deadlines **at** time T
  - Job b’s have deadlines **after** time T

- Case A: non of job b’s is executed before T
  - The total computational demand between [0,T] is

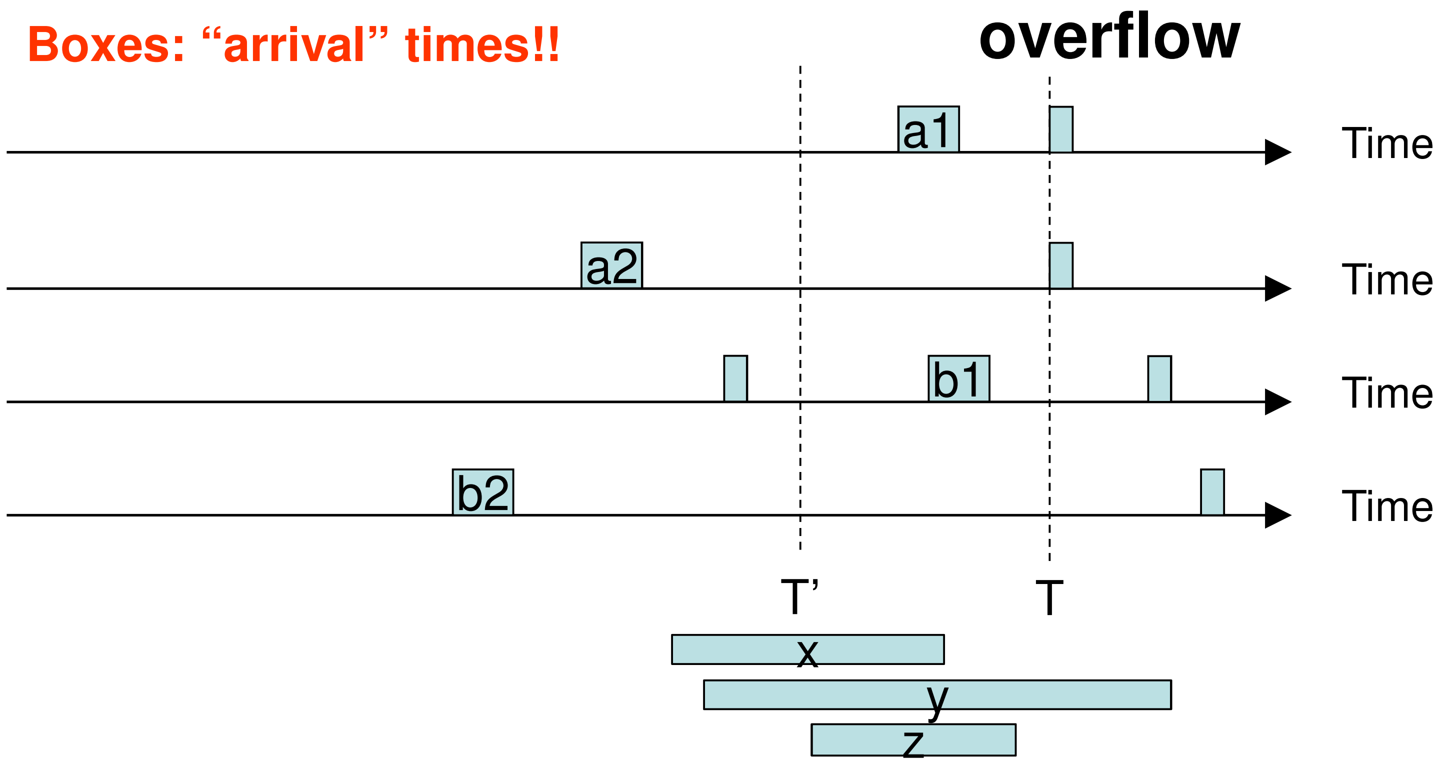
$$C_1(\lfloor T/P_1 \rfloor) + C_2(\lfloor T/P_2 \rfloor) + \ldots + C_n(\lfloor T/P_n \rfloor)$$

  - Since there is no idle before an overflow

$$C_1(\lfloor T/P_1 \rfloor) + C_2(\lfloor T/P_2 \rfloor) + \ldots + C_n(\lfloor T/P_n \rfloor) > T$$

  - That implies U>1
  - →←

Boxes: “arrival” times!!



Case B: some of job b’s are executed before T

- Because an overflow occurs at T, the violated jobs must be a’s
  - Right before T, there must be some job a’s being executed
  - Let in [T’,T] there is no job b’s being executed
- Before T’
  - Job x: already completed, Job y: not affecting a’s, Job z: will interfere a’s
- Back to [T’,T], the total computation demand is no less than

  $C_1(\lfloor T - T'/P_1 \rfloor) + C_2(\lfloor T - T'/P_2 \rfloor) + \ldots + C_n(\lfloor T - T'/P_n \rfloor)$

  - Since there is no idle before the deadline violation, so

$$C_1(\lfloor T - T'/P_1 \rfloor) + C_2(\lfloor T - T'/P_2 \rfloor) + \ldots + C_n(\lfloor T - T'/P_n \rfloor) > T - T'$$

  - →←

Earliest-Deadline-First Scheduling

- Summary
  - A universal scheduling algorithm for real-time periodic tasks
  - Urgency of tasks is dynamic
    - But static for jobs
  - Job-level fixed-priority scheduling

Independent Task Scheduling

- Summary
  - Tasks share nothing but the CPU
    - Periodic and preemptive
  - Priority-driven scheduling vs. deadline-driven scheduling
    - Robustness vs. utilization
  - Admission control policies
    - On-line tests vs. exact tests

Comparison

	RM	EDF
Optimality	Optimal for fixed-priority scheduling	Universal
Schedulability test	Exact test is slow (PP), conservative tests O(n)	O(n) for exact test
Algorithm time complexity	O(1) job insertion is possible	Both job insertion and dispatch take O(log n) time
Overload survivability	High and predictable	Low and unmanageable**
Responsiveness	High priority tasks always have shorter response time	Non-intuitive to reach conclusions
Ease of implementation	Pretty simple	Relatively complicated
Run-time overheads (like preemption)	Low	High Is it true?

Advanced Topics

Advanced Topics

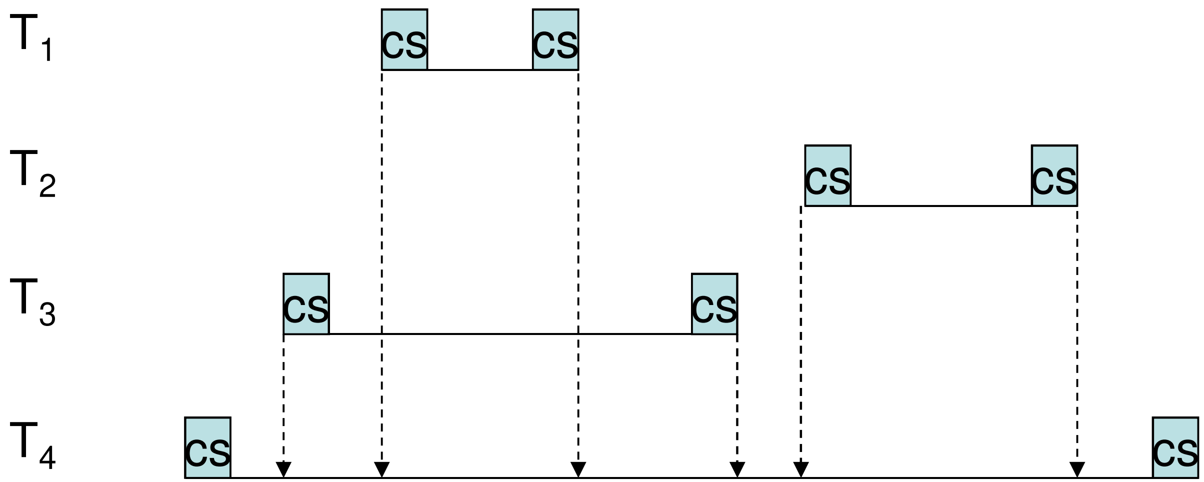
- RMS vs. EDF, revisited
  - Cxtsw cost
  - Optimality
- Earliest-Deadline-First, revisited
  - What's its limitations?
- Rate-Monotonic Scheduling, revisited
  - Harmonically-related tasks
  - More on critical instant
  - Arbitrary task phasing
  - Arbitrary priority assignments
- Cycle-based scheduling
- Periods vs. Deadlines

Context-Switch Overhead

- Context-switch overheads under RM
  - A job preempts and is preempted by other jobs
  - Preemption introduces context-switch overheads
  - How to take the cxtsw overheads into account?
  - The cxtsw overheads should be associated with the preempting job, not the preempted job
    - Let the time cost of a cxtsw operation be *x*
    - The computation time c should be added to 2x

Context-Switch Overhead

- Context-switch overheads under RM
  - The execution time of a task should be added to 2x
  - ***Proof***. A task always preempts and resumes to the same task



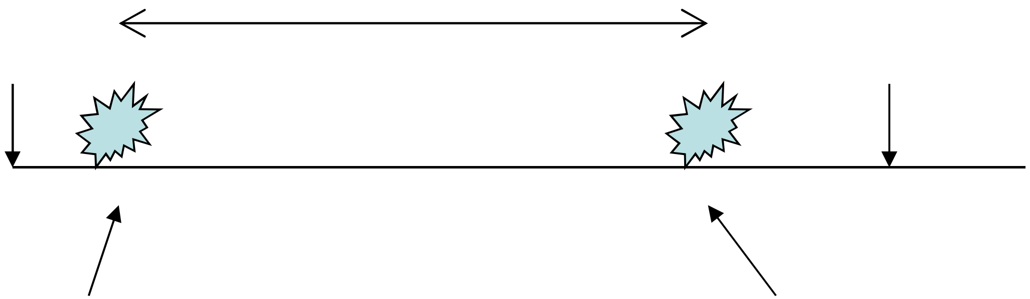
Like an LIFO

Context-Switch Overhead

- Preemption overhead under EDF
  - ***Paradox***: because EDF is dynamic-priority scheduling, any two arbitrary tasks can preempt each other and tasks may have higher cxtsw overheads
  - ***Fact***: A job can only be preempted by the jobs with shorter periods
  - Context switch overheads of a job under EDF are the same as that under RM (i.e., 2x)

Context-Switch Overhead

- Preemption overheads
  - EDF



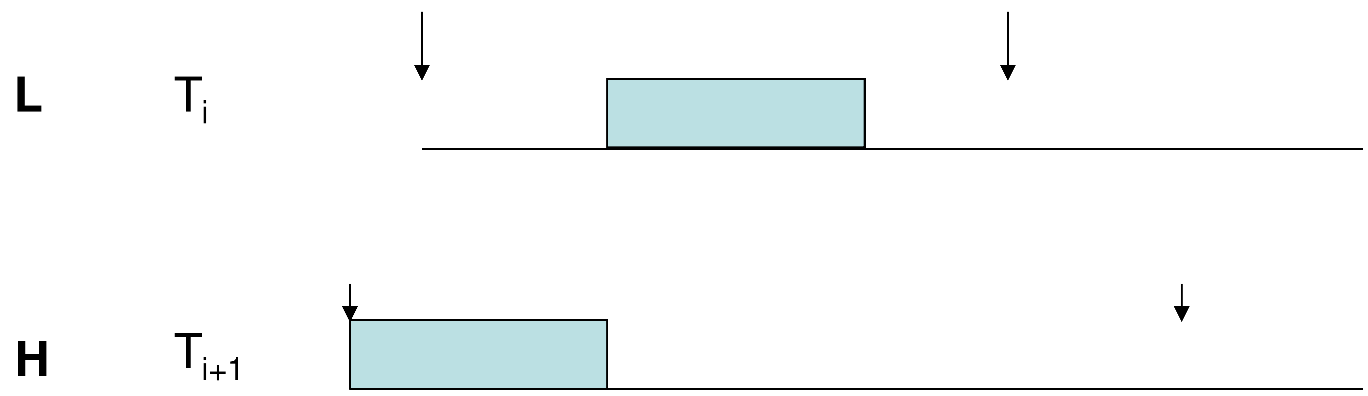
If the job is "preempted", then the preempting job arrives later than the preempted job

If the job is previously "preempted", then the preempting job has an earlier deadline than the preempted job

  - Then what makes EDF differemt from RMS?
    - A job may be "delayed" by a job having a longer period (see the example of (3,6),(4.5,9) for EDF)

Optimality of RM

- ***Theorem***: If a task set is schedulable by fixed-priority scheduling with an arbitrary priority assignment, then the task set is schedulable by RM
- ***Proof***. To swap priorities until it becomes RM

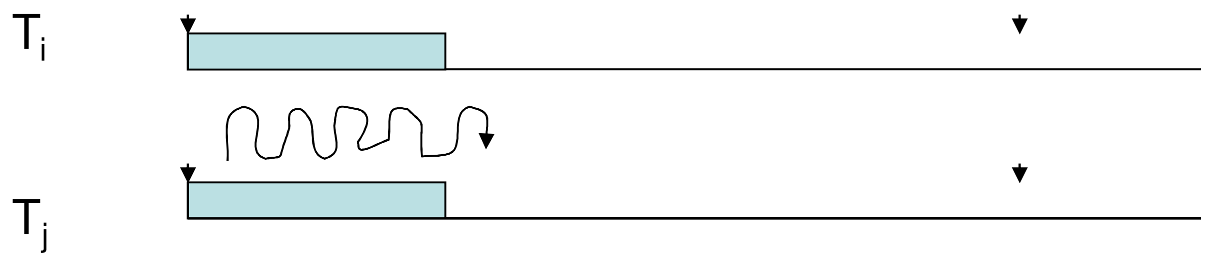


Optimality of RM

- Arbitrary task priority assignment
  - Under an arbitrary task priority assignment, the utilization test is no longer applicable!
    - Because it is weaker than RM… U test is for RM only
    - However, RMA can still be adopted
    - Let’s exercise RMA for L=(1,4) and H=(3,8)

Optimality of EDF

- EDF is universal to periodic, preemptive tasks
  - Least-Slack-Time (LST) or Least-Laxity First (LLF) is also universal to periodic, preemptive tasks
    - At any time instant, run the job having the least slack time
    - Let's try {(8,16),(8,18)}
    - Problem of LST: highly frequent context switches



Optimality of EDF

- The good(s) of EDF
  - [Liu and Layland] EDF is universal to periodic, preemptive tasks with arbitrary arrival times
  - [Jackson’s Rule] EDF is optimal to non-periodic, non-preemptive jobs whose ready times are all 0
  - [Horn’s Rule] EDF is optimal to preemptive and non-periodic jobs with arbitrary arrival times

Optimality of EDF

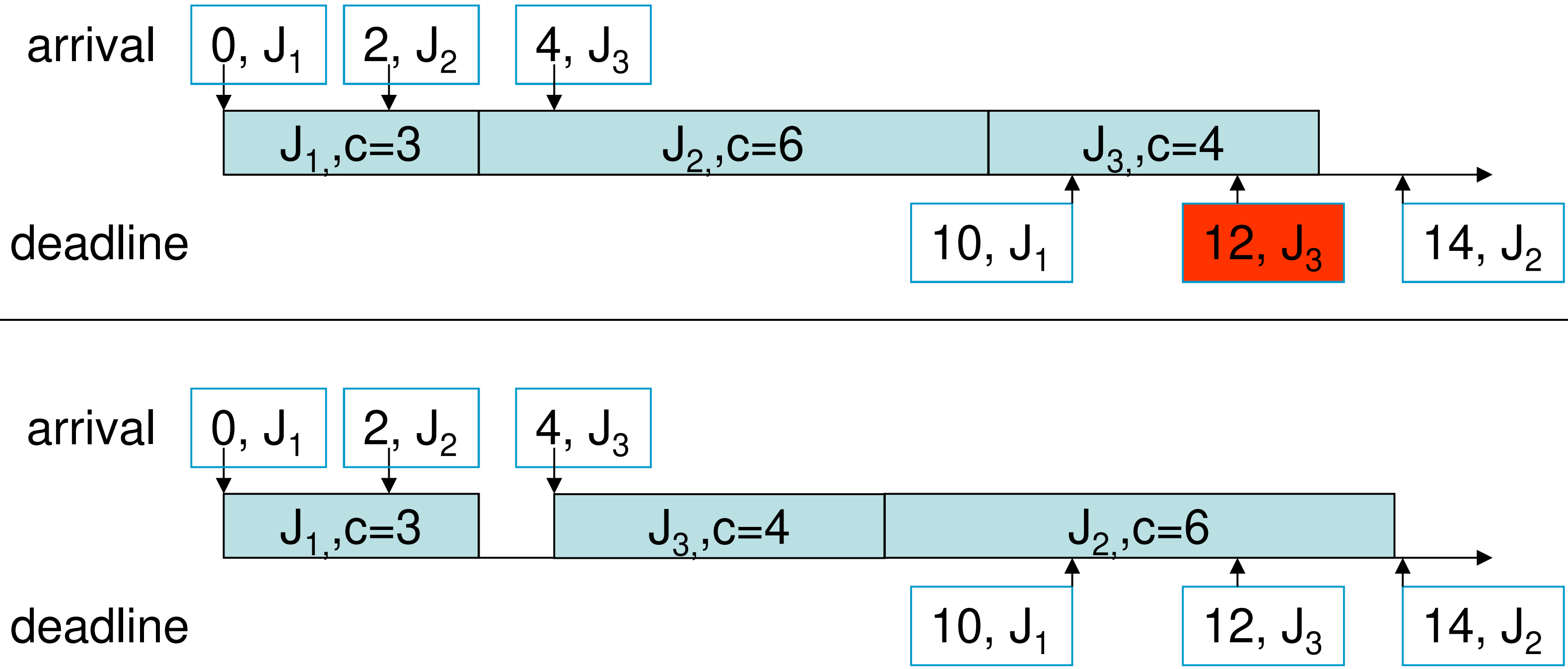
- The bad(s) of EDF
  - [Jeffay] EDF is not optimal to periodic, non-preemptive tasks with arbitrary arrival times (NP-complete)
  - [Gary and Johnson] EDF is not optimal to non-periodic, non-preemptive jobs with arbitrary arrival times (NP-complete)
  - [Mok] EDF is not optimal for multiprocessor scheduling (NP-complete, without task migration)

Optimality of EDF

- [Jeffay] An interesting observation on non-preemptible EDF
  - Consider non-preemptive, periodic tasks (3,5) and (4,10) both become ready at time 0
  - Consider the same two tasks with release times 1 and 0

Optimality of EDF

- [Gary and Johnson] EDF is not optimal to non-periodic, non-preemptive jobs with arbitrary arrival times (NP-complete)

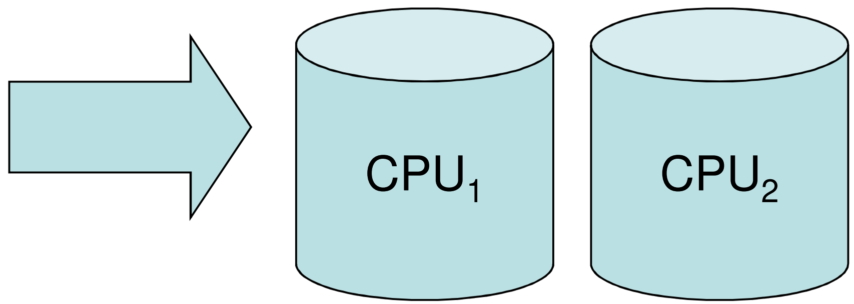


Optimality of EDF

- Limitation of EDF
  - [Mok] EDF is not optimal for multiprocessor scheduling (NP-complete, without task migration)

{(5,10), (5,10), (8,12)}

EDF with load balancing

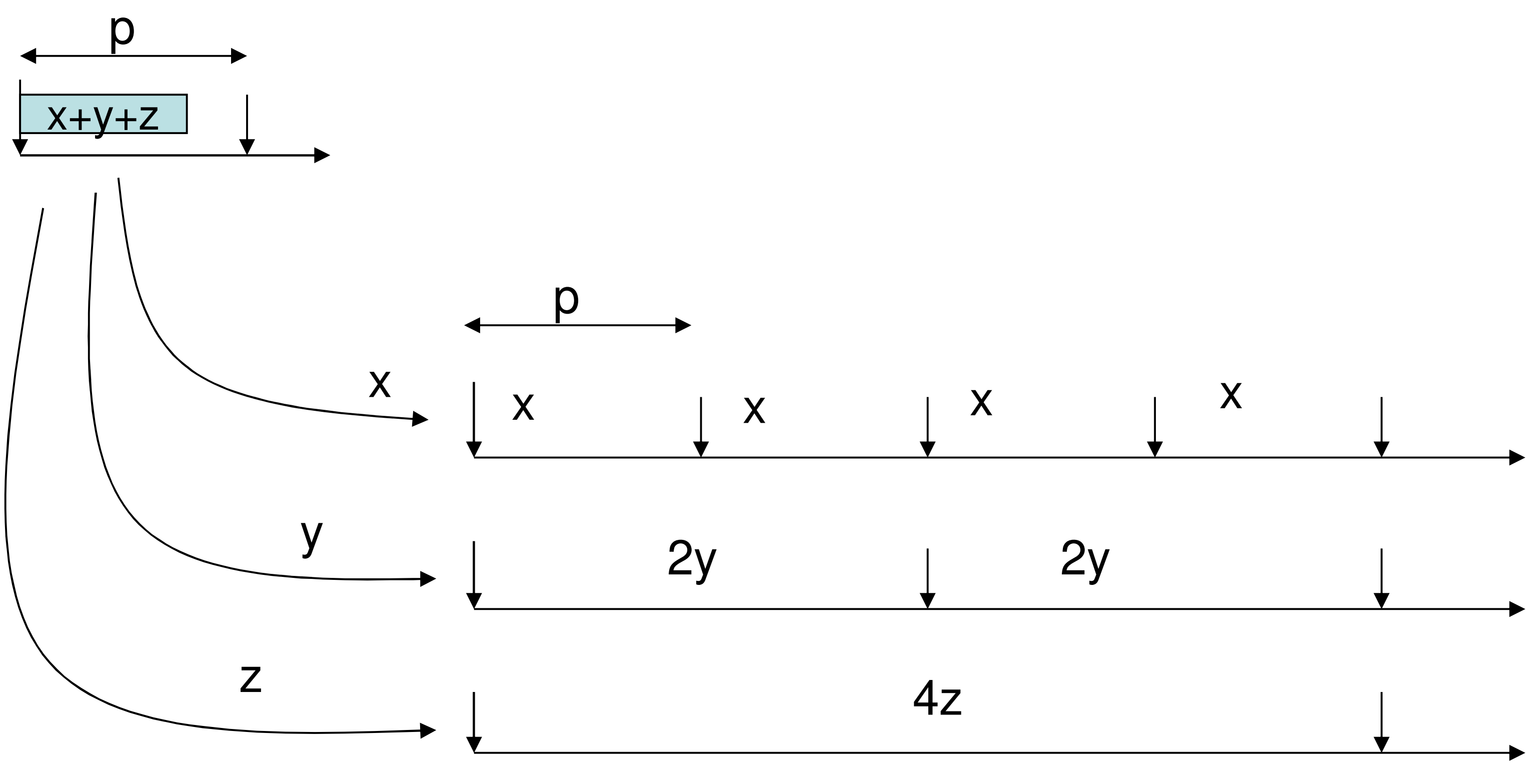


Harmonically-related tasks

- Harmonic chain *H* is a set of tasks in which a task period can be divided by shorter task periods
- Given a set of tasks $S = \{T_1, T_2, ..., T_n\}$ and harmonic chain $H = \{T'_1, T'_2, ..., T'_m\}$. If $\{(\sum_{T'_i \in H} c'_i (p'_1 / p'_i), p_1)\} \cup S$ is schedulable then $H \cup S$ is schedulable.
  - E.g., {(1,4),(1,8),(1,7),(1,16)}
    - {(1+0.5+0.25,4),(1,7)}
  - Useful in RM: a harmonic chain is represented by a task and thus small *n* in *U(n)* can be used

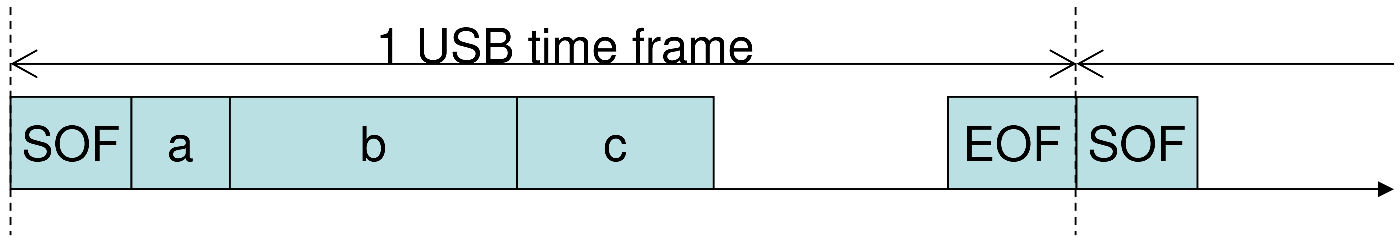
Harmonically-related tasks

– ***Proof***.



Cycle-Based Scheduling

- Cycle-based scheduling (A.K.A. Frame-based scheduling)
  - Many I/O buses divides time into frames
    - Requests are periodically services for every frame
  - A representative example: USB 1.1
    - USB use 1ms time frame to service isochronous requests
    - A transfer rate *r* KB/s is translated as to transfer $\lceil (r*1024)/1000 \rceil$ bytes every 1 ms frame
    - Very simple admission control: request sizes should not exceed the capacity of one time frame



SOF+a+b+c+EOF <= 1500 bytes (1ms for 1500 bytes)

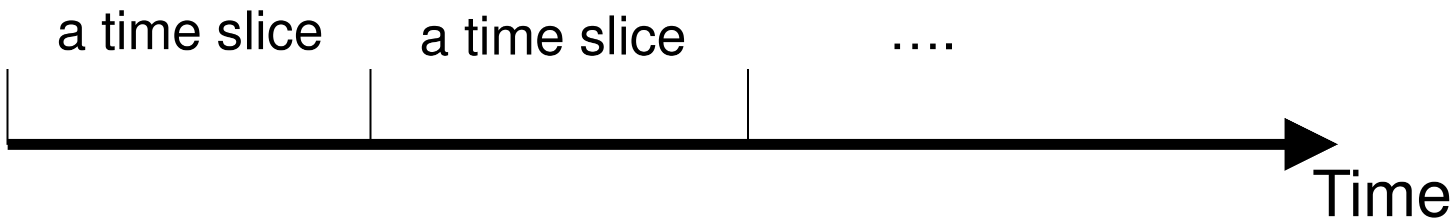
Cycle-Based Scheduling

- Cycle-based scheduling
  - Different from purely periodic tasks, tasks in cycle-based scheduling have the same period, i.e., the frame size.
  - If we care about "bandwidth" only , then cycle-based scheduling is very useful!

Cycle-Based Scheduling

Theorem: Given a set of m tasks, it is schedulable by some priority-driven scheduler if $U \leq 1$.

Proof.



For every time slice, τ_i receives a share of is c_i/p_i .

Within p_i , τ_i receives c_i!!

Could you disprove this paradox?

End of Chapter 1

Supplemental stuff

Task Phasing: RM

- Preemptive and periodic task with RM
  - If tasks are not in phase, critical instant might not happen
  - Let’s see (4.5, 9) and (3, 6)
    - Arrival times: 0, 1.5
  - Scheduling tasks in phase is harder than not in phase
    - → ok, ← not ok

Deadline vs Period: RM

- Pre-period deadlines with preemptible “fixed-priority scheduling”
  – Deadline-monotonic scheduling (DM) is optimal
  – A sufficient test

$$\sum \frac{c_i}{d_i} \leq U(n)$$

- Post-period deadlines with preemptible RM
  – Ouch…

- RMA is still an exact test for the two cases!!

Deadline vs Period: RM

- RM with arbitrary priority assignment
  - (iff) response time analysis
  - (N/A) utilization test

- Pre-period deadlines (use DM)
  - Response time analysis (iff)
  - Deadline utilization test (sufficient)
- Post-period deadlines (use DM)
  - Response time analysis (iff)
  - Utilization test (N/A)

Deadline vs Period: EDF

$$\sum \frac{c_i}{d_i} \leq 1$$

Density function

- Pre-period deadlines with preemptible EDF
  - Sufficient
- Post-period deadlines with preemptible EDF
  - Sufficient

Deadline vs Period: EDF

- “Iff” test for pre-period deadlines with preemptible EDF

$$\forall L > 0, \sum_{i=1}^{n} \left\lfloor \frac{L+T_i-Di}{T_i} \right\rfloor C_i \leq L$$

  – Pseudo polynomial time

- EDF schedulability test becomes much harder if deadline <> period !

Deadline vs Period: EDF

- EDF
  - Density function (sufficient)
    - Pre-period deadlines
    - Post-period deadlines
  - Demand-bounded function (iff)
    - Pre-period deadlines