Problem: $(P \rightarrow Q) \vdash (P \rightarrow (A \rightarrow Q))$

| 1 | $(P \rightarrow Q)$ | Premise |
|---|---|---|
| 2 | $P$ | Assumption |
| 3 | $A$ | Assumption |
| 4 | $Q$ | 1,2 →E |
| 5 | $(A \rightarrow Q)$ | 3-4 →I |
| 6 | $(P \rightarrow (A \rightarrow Q))$ | 2-5 →I |

Problem: $(A \vee B) \vdash \sim(\sim A \& \sim B)$

| 1 | $(A \vee B)$ | Premise |
|---|---|---|
| 2 | $(\sim A \& \sim B)$ | Assumption |
| 3 | $A$ | Assumption |
| 4 | $\sim A$ | 2 &E |
| 5 | $\bot$ | 3,4 ⊥I |
| 6 | $B$ | Assumption |
| 7 | $\sim B$ | 2 &E |
| 8 | $\bot$ | 6,7 ⊥I |
| 9 | $\bot$ | 1,3-5,6-8 ∨E |
| 10 | $\sim(\sim A \& \sim B)$ | 2-9 ∼I |

Problem: $(A \lor (\exists x)Fx) \vdash (\exists x)(A \lor Fx)$

| 1 | $(A \lor (\exists x)Fx)$ | Premise |
|---|---|---|
| 2 | $A$ | Assumption |
| 3 | $(A \lor Fa)$ | 2 $\lor$I |
| 4 | $(\exists x)(A \lor Fx)$ | 3 $\exists$I |
| 5 | $(\exists x)Fx$ | Assumption |
| 6 | $Fa$ | Assumption |
| 7 | $(A \lor Fa)$ | 6 $\lor$I |
| 8 | $(\exists x)(A \lor Fx)$ | 7 $\exists$I |
| 9 | $(\exists x)(A \lor Fx)$ | 5,6-8 $\exists$E |
| 10 | $(\exists x)(A \lor Fx)$ | 1,2-4,5-9 $\lor$E |

Problem: $\vdash (\forall x)(\forall y)((Fx \& \sim Fy) \to \sim x = y)$

| 1 | $a$ | Flag |
|---|---|---|
| 2 | $b$ | Flag |
| 3 | $(Fa \& \sim Fb)$ | Assumption |
| 4 | $a = b$ | Assumption |
| 5 | $Fa$ | 3 &E |
| 6 | $\sim Fb$ | 3 &E |
| 7 | $Fb$ | 4,5 =E |
| 8 | $\perp$ | 6,7 $\perp$I |
| 9 | $\sim a = b$ | 4-8 $\sim$I |
| 10 | $((Fa \& \sim Fb) \to \sim a = b)$ | 3-9 $\to$I |
| 11 | $(\forall y)((Fa \& \sim Fy) \to \sim a = y)$ | 2-10 $\forall$I |
| 12 | $(\forall x)(\forall y)((Fx \& \sim Fy) \to \sim x = y)$ | 1-11 $\forall$I |