Problem: $(P \to Q) \vdash (P \to (A \to Q))$

1	$(P \to Q)$	Premise
2	$\quad P$	Assumption
3	$\quad\quad A$	Assumption
4	$\quad\quad Q$	1,2 $\to$E
5	$\quad (A \to Q)$	3-4 $\to$I
6	$(P \to (A \to Q))$	2-5 $\to$I

Problem: $(A \vee B) \vdash \sim(\sim A \& \sim B)$

1	$(A \vee B)$	Premise
2	$\quad (\sim A \& \sim B)$	Assumption
3	$\quad\quad A$	Assumption
4	$\quad\quad \sim A$	2 &E
5	$\quad\quad \lightning$	3,4 $\sim$E
6	$\quad\quad B$	Assumption
7	$\quad\quad \sim B$	2 &E
8	$\quad\quad \lightning$	6,7 $\sim$E
9	$\quad \lightning$	1,3-5,6-8 $\vee$E
10	$\sim(\sim A \& \sim B)$	2-9 $\sim$I

Problem: $(A \vee (\exists x)Fx) \vdash (\exists x)(A \vee Fx)$

1	$(A \vee (\exists x)Fx)$	Premise
2	$\quad A$	Assumption
3	$\quad (A \vee Fa)$	2 $\vee$I
4	$\quad (\exists x)(A \vee Fx)$	3 $\exists$I
5	$\quad (\exists x)Fx$	Assumption
6	$\quad\quad Fa$	Assumption
7	$\quad\quad (A \vee Fa)$	6 $\vee$I
8	$\quad\quad (\exists x)(A \vee Fx)$	7 $\exists$I
9	$\quad (\exists x)(A \vee Fx)$	5,6-8 $\exists$E
10	$(\exists x)(A \vee Fx)$	1,2-4,5-9 $\vee$E

Problem: $\vdash (\forall x)(\forall y)((Fx\&{\sim}Fy) \rightarrow {\sim}x = y)$

1	a	Flag
2	$\quad b$	Flag
3	$\quad\quad (Fa\&{\sim}Fb)$	Assumption
4	$\quad\quad\quad a = b$	Assumption
5	$\quad\quad\quad Fa$	3 &E
6	$\quad\quad\quad {\sim}Fb$	3 &E
7	$\quad\quad\quad Fb$	4,5 =E
8	$\quad\quad\quad \lightning$	6,7 $\sim$E
9	$\quad\quad {\sim}a = b$	4-8 $\sim$I
10	$\quad ((Fa\&{\sim}Fb) \rightarrow {\sim}a = b)$	3-9 $\rightarrow$I
11	$(\forall y)((Fa\&{\sim}Fy) \rightarrow {\sim}a = y)$	2-10 $\forall$I
12	$(\forall x)(\forall y)((Fx\&{\sim}Fy) \rightarrow {\sim}x = y)$	1-11 $\forall$I