

Problem:  $(P \rightarrow Q) \vdash (P \rightarrow (A \rightarrow Q))$

1		$(P \rightarrow Q)$	Premise
2			
2			$P$ Assumption
3			
3			
3			$A$ Assumption
4			
4			
4			$Q$ 1,2 $\rightarrow$ E
5			$(A \rightarrow Q)$ 3-4 $\rightarrow$ I
6		$(P \rightarrow (A \rightarrow Q))$	2-5 $\rightarrow$ I

Problem:  $(A \vee B) \vdash \sim(\sim A \& \sim B)$

1		$(A \vee B)$	Premise
2			
2			$(\sim A \& \sim B)$ Assumption
3			
3			
3			$A$ Assumption
4			
4			$\sim A$ 2 &E
5			$\perp$ 3,4 $\sim$ E
6			
6			$B$ Assumption
7			
7			$\sim B$ 2 &E
8			$\perp$ 6,7 $\sim$ E
9			$\perp$ 1,3-5,6-8 $\vee$ E
10		$\sim(\sim A \& \sim B)$	2-9 $\sim$ I

Problem:  $(A \vee (\exists x)Fx) \vdash (\exists x)(A \vee Fx)$

1	$(A \vee (\exists x)Fx)$	Premise
2	$A$	Assumption
3	$(A \vee Fa)$	2 $\vee$ I
4	$(\exists x)(A \vee Fx)$	3 $\exists$ I
5	$(\exists x)Fx$	Assumption
6	$Fa$	Assumption
7	$(A \vee Fa)$	6 $\vee$ I
8	$(\exists x)(A \vee Fx)$	7 $\exists$ I
9	$(\exists x)(A \vee Fx)$	5,6-8 $\exists$ E
10	$(\exists x)(A \vee Fx)$	1,2-4,5-9 $\vee$ E

Problem:  $\vdash (\forall x)(\forall y)((Fx \& \sim Fy) \rightarrow \sim x = y)$

1	$a$	Flag
2	$b$	Flag
3	$(Fa \& \sim Fb)$	Assumption
4	$a = b$	Assumption
5	$Fa$	3 $\&$ E
6	$\sim Fb$	3 $\&$ E
7	$Fb$	4,5 $=$ E
8	$\perp$	6,7 $\sim$ E
9	$\sim a = b$	4-8 $\sim$ I
10	$((Fa \& \sim Fb) \rightarrow \sim a = b)$	3-9 $\rightarrow$ I
11	$(\forall y)((Fa \& \sim Fy) \rightarrow \sim a = y)$	2-10 $\forall$ I
12	$(\forall x)(\forall y)((Fx \& \sim Fy) \rightarrow \sim x = y)$	1-11 $\forall$ I