

Problem: $(P \rightarrow Q) \vdash (P \rightarrow (A \rightarrow Q))$

1		$(P \rightarrow Q)$	Premise
2			
2			P Assumption
3			
3			
3			A Assumption
4			
4			
4			Q 1,2 \rightarrow E
5			$(A \rightarrow Q)$ 3-4 \rightarrow I
6		$(P \rightarrow (A \rightarrow Q))$	2-5 \rightarrow I

Problem: $(A \vee B) \vdash \sim(\sim A \& \sim B)$

1		$(A \vee B)$	Premise
2			
2			$(\sim A \& \sim B)$ Assumption
3			
3			
3			A Assumption
4			
4			$\sim A$ 2 &E
5			\wedge 3,4 \wedge I
6			
6			B Assumption
7			
7			$\sim B$ 2 &E
8			\wedge 6,7 \wedge I
9			\vee 1,3-5,6-8 \vee E
10		$\sim(\sim A \& \sim B)$	2-9 \sim I

Problem: $(A \vee (\exists x)Fx) \vdash (\exists x)(A \vee Fx)$

1	$(A \vee (\exists x)Fx)$	Premise
2	A	Assumption
3	$(A \vee Fa)$	2 $\vee I$
4	$(\exists x)(A \vee Fx)$	3 $\exists I$
5	$(\exists x)Fx$	Assumption
6	Fa	Assumption
7	$(A \vee Fa)$	6 $\vee I$
8	$(\exists x)(A \vee Fx)$	7 $\exists I$
9	$(\exists x)(A \vee Fx)$	5,6-8 $\exists E$
10	$(\exists x)(A \vee Fx)$	1,2-4,5-9 $\vee E$

Problem: $\vdash (\forall x)(\forall y)((Fx \& \sim Fy) \rightarrow \sim x = y)$

1	a	Flag
2	b	Flag
3	$(Fa \& \sim Fb)$	Assumption
4	$a = b$	Assumption
5	Fa	3 $\&E$
6	$\sim Fb$	3 $\&E$
7	Fb	4,5 $=E$
8	\wedge	6,7 $\sim E$
9	$\sim a = b$	4-8 $\wedge I$
10	$((Fa \& \sim Fb) \rightarrow \sim a = b)$	3-9 $\rightarrow I$
11	$(\forall y)((Fa \& \sim Fy) \rightarrow \sim a = y)$	2-10 $\forall I$
12	$(\forall x)(\forall y)((Fx \& \sim Fy) \rightarrow \sim x = y)$	1-11 $\forall I$