# A Study on the Performance of Adaptive Modulation and Cross-layer Design in Cognitive Radio for Fading Channels

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Abstract— Cognitive radio is considered as one of the main and for provisioning dvnamic spectrum/channel allocation in wireless communications. On the other hand several physical layer mechanisms such as adaptive modulation, multiple-input multiple output systems, advanced channel coding and/or combinations of them enhance the capacity of wireless networks. However little effort has been put till now in studying the performance gains of physical layer mechanisms with the presence of cognition capabilities. The incorporation of cognitive mechanisms demands more detailed studies for assessing the impact on the spectral efficiency. To this direction, cross-layer combination of such a physical layer with upper layers should be also considered as a case study in a cognitive wireless environment. In this work we present a study on the spectral efficiency of adaptive modulation and coding which is one of the most promising schemes of applying cognitive radio at the physical layer. Besides, we study a cross-layer combination of adaptive modulation with upper layers in the same cognitive context. We prove that the performance gain of cognitive radio over such a physical layer is not negligible.

Keywords- cognitive radio; flexible channel allocation; adaptive modulation; cross-layer combination; spectral efficiency

### I. INTRODUCTION

The Cognitive Radio (CR) concept brought about the idea to exploit the spectrum holes which result from the underutilization of the electromagnetic spectrum in wireless communications. This fact is corroborated by the Spectrum Policy Task Force of the Federal Communications Commission (FCC) which ascertains that the legacy regulation on spectrum availability begets snags in potential spectrum access by users. More precisely, it was identified that spectrum bands appear to be unoccupied most of the time and some of them are occupied only partially by the primary (or licensed) users [1].

Primary users are assigned with a range of frequency bands but however they do not use them one hundred percent in time or location. As a consequence, a particular communication system which serves its primary users at a specific geographic location may present spectrum holes which could be exploited by using a cognitive radio approach which can identify them. In this way, secondary users that were not being served by the system can access

and exploit the spectrum holes thus improving spectrum utilization.

The spectrum utilization policy of a cognitive radio approach requires the specification of the techniques by which primary users and secondary users will be served. Among the cognitive tasks of such a system the most prominent ones should be the reliable sensing of the spectrum range, the detection of spectrum holes, the estimation of mutual interference between primary and secondary users and the control of their transmission power as well. To this end, we assume a spectrum pooling system with cognition capabilities such as the one presented in [2], which is capable to reliably sense the spectrum range.

A cognitive radio system affects the performance of the physical layer mechanisms e.g.. channel coding or multiple access schemes deployed. In addition the inclusion of cross-layer optimization schemes should also be considered. To this direction, we have chosen to study the performance of adaptive modulation over such a radio environment as well as the performance of it when cross-layer design constraints are imposed in order to sustain a particular quality of service at the data link layer. In order to make an objective performance analysis and evaluation, one should study the cognitive radio system behaviour over a particular fading propagation environment. For this reason, before focusing on the specifics of the performance analysis we calculate the capacity of a cognitive radio system over a flat fading channel with Rayleigh coefficients.

The rest of this paper is organized as follows. In section 2 we provide the utilized cognitive radio system model in a fading environment. Section 3 presents adaptive modulation over cognitive radio focusing on variable rate variable power (VRVP) schemes and taking into account the fading phenomena. In section 4 cross layer design aspects in cognitive radio are discussed. Then in section 5 we provide the performance analysis of the adaptive modulation scheme over cognitive radio under cross-layer design constraints. Finally in section 6 we present the conclusions of our work.

# II. COGNITIVE RADIO SYSTEM MODEL OVER RAYLEIGH FADING CHANNEL

In spectrum pooling systems, the spectrum or channel allocation gives priority to primary users. Subsequently, the



secondary users are assigned the detected spectrum holes. Specifically, the spectrum is divided into N sub-bands and each user l is trying to transmit by controlling its power. The system's transmission is considered as wide-band. In this case, the N sub-bands available to the cognitive radio channel extend to infinite ( $N \rightarrow \infty$ ). The channel is assumed with fading components that is varying slowly in time. In this way the receiver is able to sense and track the channel fluctuations. These fluctuations are called channel gains  $h_l$  and are assumed over a block fading length in order to be able to keep their values constant during a processing block.

For studying the performance of adaptive modulation, we need to know the impact of the fading channels that can be described by the Rayleigh probability density function (pdf). Since adaptive modulation has been already studied over Nakagami fading channels we can derive the case of Rayleigh fading if we assume m=1 in the Nakagami channel model [3]. In such a case, one should calculate the spectral efficiency when optimal power and rate adaptation is assumed as it happens in [4] over a Rayleigh fading channel. This actually could be done by calculating the spectral efficiency for one user  $C_{1\infty}$  by using as the general probability distribution the Rayleigh instance, the pdf of which is expressed as  $p_{\gamma}(\gamma) = e^{-\gamma/\overline{\gamma}}/\overline{\gamma}$ , where  $\overline{\gamma}$  is the average received SNR. Therefore, we obtain the optimal capacity for one user per unit bandwidth over Rayleigh fading channel in bits/sec/Hz by the following expression:

$$\left\langle C_{l,\infty} \right\rangle_{\text{Rayleigh}} = \log_2(e) \left( \frac{e^{-\gamma_0/\overline{\gamma}}}{\gamma_0/\overline{\gamma}} - \overline{\gamma} \right)$$
 (1)

Moreover, when the Rayleigh channel model is used, the cut-off level is equal to  $\gamma_0$ . Its value is calculated by the average power constraint as defined in Rayleigh channel model using the following formula

$$\int_0^\infty \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) \times p_{\gamma}(\gamma) d\gamma = 1$$
 (2)

Now the spectral efficiency of cognitive radio over Rayleigh channel can be defined. In the case of the cognitive radio system described above, the spectral efficiency is equal to the capacity for each user only for the case of a primary user. In case of secondary users, the spectral efficiency for each user is the instantaneous capacity multiplied by the band factor gain. The band factor gain  $\Delta_{\infty}$  of the cognitive radio system is defined as the band sensed void from user l to user l+1 over the total bandwidth l that is obtained by:

$$\Delta_{\infty} = 1 - \exp(-\frac{\gamma_0}{\overline{\gamma}}) \tag{3}$$

According to [2], considering a system with L users, we should take the sum spectral efficiency that is calculated by:

$$Se_{sum,_{\infty}} = \frac{1 - \Delta_{\infty}^{L}}{1 - \Delta_{\infty}} \cdot C_{1,\infty}$$
 (4)

Figure 1 shows the spectral efficiency of the cognitive radio system under consideration for different number of users over Rayleigh fading channels. In particular, it illustrates the spectral efficiency of optimal power and rate adaptation using the closed-form expression (1) and the sum spectral efficiency of cognitive radio assuming Rayleigh fading channel using the combination of the closed-form expression (4) with  $\gamma_0/\bar{\gamma}$  channel gain. As expected, the cognitive radio assumption yields an increase in capacity that has a considerable value in low to average SNR regions that is ranged between 0.2-0.3 bits/s/Hz. For comparison purposes, we also depict the sum spectral efficiency of cognitive radio when only the primary user is considered that is in fact the capacity of user 1 over Rayleigh fading channel model.

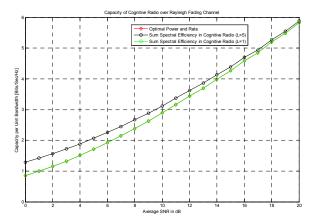


Figure 1. Capacity of Cognitive Radio System over Rayleigh Fading Channel

#### III. ADAPTIVE MODULATION OVER COGNITIVE RADIO

Adaptive modulation is a well-known scheme in which the transceiver is able to adapt its rate and/or power according to channel variations estimated at the receiver side. This scheme has been already studied for wireless communications and specifically over fading channels [3][4]. In particular variable rate and variable power (VRVP) modulations that rely on optimal power control policies were firstly introduced in [4] and the analysis of variable-rate and constant-power (VRCP) instances were presented in [3].

As described above the achievable performance of cognitive radio depends on the band factor gain that implies the cognition capabilities of each user to sense holes in spectrum range employing the corresponding channel allocation technique. As specified in [2], this capability is

related to the frequency variation that is expressed by the channel fading distribution. Since a power control policy is employed the frequency variation depends on the cut-off fade level.

Thus it is rational to deploy the VRVP case for studying the performance of adaptive modulation over cognitive radio, since we adopt the aforementioned cognitive radio system which is based on optimal power control policy for channel allocation. In particular it seems that by relying on the spectral efficiency of VRVP case of adaptive modulation we should achieve significant performance gains [4]. The VRVP scheme adopts MQAM constellations denoted as a set  $\{M_j : j = 0, 1, ..., m\}$  that can be chosen according to the fade level  $\gamma$  during the symbol period. In this set, the M<sub>0</sub> choice means no data transmission. Each constellation is associated with a fading region that is emerged by the division of the range  $\gamma$  into m+1 regions called fading regions. Thus, when the fading level is set in the  $j^{th}$  region then the constellation  $M_i$  is chosen and in consequence the current data rate of the adaptable system is  $\log_2 M_i$ . However, since the transmit power  $S(\gamma)$  should also be adapted in order to retain the average power constraint  $\overline{S}$  the received SNR should be equal to  $\gamma \cdot S(\gamma)/S$ . Thus, the scheme should be able to decide by which rate will transmit in the next period and which will be the transmit power either. Both rate and power are controlled under the required BER value (target BER) needed to be offered at the physical layer by the adaptive modulation technique.

Afterwards, the target of such a system, that involves power control policy, is to maximize the spectral efficiency subject to the power constraint that is given by the following expression:

$$\frac{S(\gamma)}{\overline{S}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma \cdot K}, & \gamma \ge \gamma_0 / K \\ 0, & \gamma < \gamma_0 / K \end{cases}$$
(5)

It is obvious that in this case the cut-off fade level is equal to  $\gamma_0/K$  where K is given by  $K = -1.5/\ln(5BER)$  and BER is the target BER. Therefore, if we denote  $\gamma_0/K$  as  $\gamma_\kappa$  the spectral efficiency is maximized when the data rates fall in the fading region with optimum power allocation expressed as  $\gamma/\gamma_\kappa$ . In this case, this dependency between the band factor gain and the optimum power control policy in order to maximize the spectral efficiency of VRVP adaptive modulation, is expressed as:

$$\left\langle \Delta_{\infty} \right\rangle_{VRVP} = 1 - \exp(-\gamma_0 / K)$$
 (6)

Taking into account the sum spectral efficiency of cognitive radio as calculated by (4), we derive the sum spectral efficiency of adaptive modulation over cognitive radio:

$$\left\langle Se_{sum_{\infty}}\right\rangle_{VRVP} = \frac{1 - \left\langle \Delta_{\infty}\right\rangle_{VRVP}^{L}}{1 - \left\langle \Delta_{\infty}\right\rangle_{VBVP}} \cdot \left(\frac{R}{B}\right) \tag{7}$$

where  $S_e$  is the spectral efficiency of adaptive modulation at the physical layer. This formula is one of the prerequisites for studying the cross-layer scheme that involves adaptive modulation at the physical layer.

## IV. CROSS-LAYER DESIGN ASPECTS IN COGNITIVE RADIO

Cross-layer design has attracted a lot of attention lately since its adoption can enhance the performance of wireless networks significantly [5]. Furthermore it is suggested that cross-layer design combined with adaptive modulation at the physical layer and automatic-repeat request (ARQ) at the data link layer increases spectral efficiency under certain performance constraints [6]. This cross-layer approach aims to redesign the adaptive modulation scheme at the physical layer having into consideration a specific threshold on the number of erroneous packets. The affordable number of erroneous packets is imposed by the quality of service needed to be retained at the upper layers. Subsequently, the system limits the number of packet retransmissions in order to keep the predefined quality of service criteria. Figure 2 presents the cross-layer design model that is assumed in this paper where packet retransmissions are controlled by a hybrid ARQ protocol.

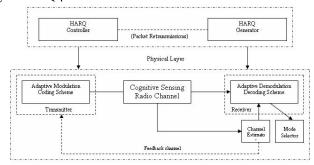


Figure 2. Cognitive radio and cross-layer design model with adaptive modulation

In this scheme the performance at the physical layer is equal to the spectral efficiency of adaptive modulation, without taking into account the cross-layer design constraints, divided by the average number of retransmissions at the data link layer. This number is related to the imposed constraints. Thus, the average spectral efficiency in this case is obtained by

$$\overline{S}_{e,cld} = \frac{\overline{S}_{e,physical}}{\overline{T}(p,T_t^{\max})}$$
 (8)

where  $\overline{S}_{e,physical}$  denotes the spectral efficiency at the physical layer which is actually the spectral efficiency of the AMC-only scheme and  $\overline{T}$  refers to the average number of

transmissions that are required per packet. For more details on the denominator's definition, the reader can refer to [6].

Taking into account the analysis made in section 3 we assume that the  $\overline{S}_{e,physical}$  of the system coincides with the VRVP one. Having defined this spectral efficiency at the physical layer, we need to calculate the average number of retransmissions at the data link layer under the imposed constraints. To this end, we consider 5 regions for the VRVP scheme with BPSK, QPSK, 16-QAM and 64-QAM as the candidate modulations of the adaptive system with the first region to correspond to no transmission at all. However, this calculation is based on the average packet error rate (PER) at the data link layer which in sequel depends on the switching thresholds of the AMC scheme. Using the thresholds derived from the corresponding crosslayer design, the performance results do not conclude in rational values. This is related to the fact that the switching thresholds are lower than the optimum VRVP reveals.

To overcome this issue one should fit equation (8) properly and particularly the denominator in order to obtain the performance of adaptive modulation under the cross-layer constraints. Specifically, the new switching thresholds (i.e. when power control policy is considered) are obtained from  $\gamma_j = \gamma_K \cdot M_j$  where  $M_j : j = 0,1,...,m$  according to the number of m+1 regions. The  $\gamma_K$  optimization values have been already obtained by equation (5) when the  $S_{e,physical}$  is calculated [4].

Tables I and II list the values of the switching thresholds derived considering the optimum control policy and those derived without considering it. We observe that the incorporation of power control lowers significantly the switching thresholds for each modulation scheme deployed.

The following constraints should be imposed according to the cross-layer design presented in [6]:

- The maximum allowable number of transmissions per information packet is  $T_t^{\text{max}}$
- The probability of unsuccessful reception after  $T_t^{\max}$  transmissions is no greater than packet loss  $p_{loss}$ .

In this context, a packet loss means that a packet is dropped after a maximum number of transmissions which is actually  $T_i^{\max} + 1$  since the first transmission in HARQ corresponds to error detection only. In addition to that, when the PER should be retained in a target value (e.g. instead of BER as in AMC-only), the switching thresholds  $\gamma_j$  are obtained as follows

$$\gamma_0 = 0$$

$$\gamma_j = \frac{1}{g_j} \ln(\frac{a_j}{P_{target}}) \tag{9}$$

$$\gamma_{i+1} = +\infty$$

where  $a_j$  and  $g_j$  are obtained by the curve fitting of PER values as explained in [6]. The values listed in both

Tables I and II as well as the performance results quoted in the following section are obtained taking  $p_{loss} = 0.01$ .

TABLE I. SWITCHING THRESHOLDS BASED ON OPTIMUM POWER CONTROL POLICY

Average	No	BPSK	QPSK	16-	64-
SNR	Transmissi			QAM	QAM
(dB)	on				
7	0	3.0767	6.087	12.1076	18.1282
10	0	3.8863	6.8966	12.9172	18.9378
13	0	4.4948	7.5051	13.5257	19.5463
16	0	4.9346	7.9449	13.9655	19.9861
20	0	5.2745	8.2848	14.3054	20.326
25	0	3.8571	6.8674	12.888	18.9086

TABLE II. SWITCHING THRESHOLDS WITHOUT CONSIDERING OPTIMUM POWER CONTROL POLICY

	No Transmiss ion	BPSK	QPSK	16- QAM	64- QAM
Transmis sion1	0	8.2206	11.2566	18.0596	24.1094
Transmis sion2	0	7.6765	10.7201	17.4979	23.5206
Transmis sion3	0	7.3765	10.4247	17.1872	23.1932

#### V. NUMERICAL RESULTS

In figure 3 we provide the performance of the cross-layer combination of VRVP adaptive modulation with HARQ. The red line shows the VRVP scheme designed under the cross-layer constraints. This is actually the case with no retransmissions. The dashed lines refer to the cases with 1, 2 or 3 retransmissions respectively i.e. when an upper limit on the number of retransmissions has been imposed where we can easily observe the performance gain. We have to note here that HARQ protocols which adopt a bounded number of retransmissions are also known as truncated ARQ protocols [7]. Finally with blue line and diamonds we depict the performance of the VRVP with 5 regions without considering the cross-layer design constraints and the new switching thresholds.

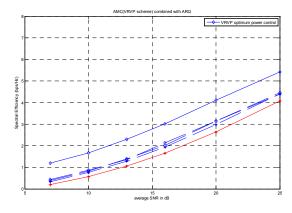


Figure 3. Performance of cross-layer combination of VRVP and HARQ over cognitive radio

It is evident that the power control policy imposed from the channel allocation in cognitive radio gives lower switching thresholds in adaptive modulation at the physical layer. The sum spectral efficiency, when cognitive radio system is considered, is provided next.

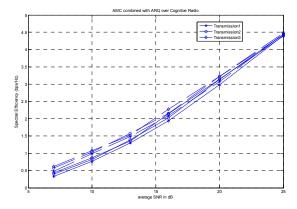


Figure 4. Impact of cross-layer design over cognitive radio

Figure 4 shows the performance of cross-layer combination between VRVP and HARQ over cognitive radio. The dashed lines depict the results over cognitive radio while the solid lines illustrate the results of figure 3 i.e. without cognition. We notice that with this set up the spectral efficiency is improved in low average SNR regions of fading channels while it could be considered negligible in the high average SNR regions. In low average SNR regions the performance gain is close to 0.2 bit/s/Hz.

In the sequel we complement our analysis and results by comparing them with those derived from the cross-layer design that was presented in [6] where the authors do not consider cognitive radio channel allocation. Figure 5 presents our results with dashed lines and the results that were obtained based on the assumptions of [6] in solid lines where we have considered all the possible values for the number of retransmissions.

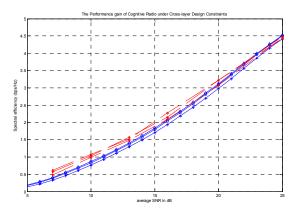


Figure 5. Comparison of cross-layer combined system over cognitive radio vs. the conventional one

The performance increase when we deploy cognitive radio under cross-layer design constraints is obvious and again we achieve improved spectral efficiency, mainly in low average SNR regions of fading channels. This is an important conclusion of our study since in [6] the authors do not use any power control policy and thus it would be possible for our results to be worse than those presented in there.

### VI. CONCLUSIONS

This work considers cross-layer design over cognitive radio with spectrum sensing and pooling capabilities. Initially, we obtain the spectral efficiency of cognitive radio in a fading channel environment. After that we assume the adaptive modulation at the physical layer and we derive the spectral efficiency of such a scheme over radio with cognition capabilities. Afterwards, we study the performance of the above scheme under constraints imposed by a specific cross-layer design model. The numerical results and the performance analysis that we carried out corroborate the advantages of the exploitation of cognitive radio schemes together with a cross-layer design that combines adaptive modulation and hybrid automatic-repeat request at the physical and data link layer respectively. In particular, in fading channels, it yields a considerable increase of the system's spectral efficiency in low-average SNR regions.

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