

# Optimal Link Scheduling and Power Control in CDMA Multihop Wireless Networks

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**Abstract**—In this paper, we study the problem of radio resource allocation in wireless multihop networks. The main objective is to develop integrated link scheduling and power control policies to maximize the network throughput. Transmitters in the network are subject to average power constraints per link and peak power constraints per node. Subject to these constraints, we find optimal subsets of concurrent transmitters and associated power levels to maximize network throughput, taking into account multi-access interference. We find that when ambient noise levels are sufficiently high, it is generally desirable to enable simultaneous transmissions, even if they are in close geographic proximity. We compare our results against pure CDMA to illustrate the significant gains of our approach. The results in this paper are applicable to multihop and single hop networks.

**Index Terms**—CDMA, Multihop Networks, Resource Allocation.

## I. INTRODUCTION

In recent years, there has been a great deal of interest and development in supporting data applications in multi-access wireless networks. Wireless devices are however, severely limited in bandwidth and energy and must use their resources judiciously. Resource allocation policies in currently deployed systems like IEEE802.11 though simple and distributed in nature, perform poorly even under moderate loads. Centralized scheduling of node transmissions can greatly improve network capacity, e.g see [6] [3]. Consistent with this, the IEEE802.11 standard includes a *point coordination function* (PCF) which centrally schedules users. The HDR (1EX VDO) standard also schedules transmissions centrally on the downlink channel. The benefits of centralized scheduling for multihop networks are equally rewarding if not more. In this paper, we extend link scheduling from one-hop networks to the multihop paradigm. We model the capacity of a wireless channel as a function of the signal to interference plus noise ratio (SIR) of that channel. For instance, the Shannon capacity of a link  $i$  over a frequency bandwidth  $W$  is  $C(i) = W \log_2(1 + \text{SIR}(i))$  (assuming the noise is Gaussian). The dependence of link rates

on powers naturally raises the possibility of improving capacity through a suitable choice of powers. Before delving into specifics, we define some terms commonly used in this paper.

A directed link  $l \in \{i, j\}$  in a multihop network of  $N$  nodes facilitates data transfer from node  $i$  to node  $j$ . We call any subset of  $L_a$  potential links in a network a *transmission mode*. There are  $2^{L_a}$  transmission modes for an  $L_a$  link network. A list of transmission modes and the relative time-fractions for which they are “used” comprises a link scheduling policy. For each transmission mode, we also need to specify the power used by active transmitters, which constitutes a power control policy. The conventional approach to scheduling links in a multihop networks is to allow nodes far away from each other to transmit simultaneously. Such a link scheduling policy ensures essentially interference-free transmissions, thereby achieving high instantaneous individual link rates. Nodes in the *vicinity*<sup>1</sup> of each other have an edge in the channel contention graph  $G_{ch}$ . Computing a maximum independent set [2] in the channel contention graph requires global topology information, entailing centralized processing. Good approximation algorithms that allow for distributed implementation would naturally be the best recourse. One such example is the *minimum-degree greedy algorithm* [4], a heuristic that chooses nodes with minimum degrees in subgraphs. In our approach, we explicitly model the effects of signal interference, allowing concurrent transmissions in a close geographic proximity. By globally optimizing the transmission modes and the associated power levels, we can achieve much higher throughputs than previous approaches. Specifically, we solve the following resource allocation problem: Find an optimal link scheduling and power control policy to maximize the network throughput, when each link is subject to an average power constraint and each node is subject to a peak power constraint.

The main input to our algorithm is the path-gain matrix of the entire network. This matrix consists of  $N^2$  gain values, one between each pair of nodes. In solving this problem, we only specify the relative time-fractions for which

<sup>1</sup>Typically within a 2-hop distance of each other.

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each transmission mode is active, not the exact time when it is "on". Viewing the time-fractions as weights, transmission modes can be scheduled over slots, using algorithms like deficit round robin.

## II. SYSTEM MODEL

Consider a system with  $N$  nodes in a CDMA environment. A subset of  $L_a$  transmission links (among the possible  $N(N-1)$  links) constitutes a network topology, and is assumed to be prespecified. We wish to maximize the weighted sum of throughputs on these links, as will be described below in equation (3). For a given link  $l = \{i, j\}$ , the transmitter node  $i$  uses a signal power  $P(l)$ . The path gain from node  $i$  to node  $j$  is given by  $G(i, j)$ . The path gain  $G(i, j)$  models the effects of signal attenuation due to distance, channel fading and shadowing. We assume that the path gains  $G(i, j)$  are constant. Denoting the transmitting and receiving nodes of link  $l$  by  $T(l)$  and  $R(l)$  respectively, the signal to interference and noise ratio (SIR or  $\gamma(l)$ ) for link  $l$  is given by,

$$\gamma(l) = \frac{G(T(l), R(l))P(l)}{\sum_{k: k \neq l} P(k)G(T(k), R(l)) + n_{R(l)}} \quad (1)$$

where  $n_{R(l)}$  is the ambient noise power at node  $R(l)$ . Let the raw data rate of link  $l$  be  $X(l)$ . We assume  $X(l)$  is a linear function of its SIR and the total available bandwidth  $W$ . Assuming the Gaussian approximation [5] to compute the bit-error-rate (BER), the data rate of a link with a tolerable BER of  $10^{-9}$  and using BPSK modulation is given by

$$X(l) = W' \gamma(l) \quad \text{where} \quad W' = \frac{W}{2q \log_2 10}. \quad (2)$$

In fact, we consider a time slotted model, where all slots are of equal duration and transmissions begin and end on slot boundaries. We let  $X_m(l)$  and  $P_m(l)$  be the data rate for link  $l$  and transmission power for  $T(l)$  in slot  $m$ . We then have

$$X_m(l) = W' \left( \frac{G(T(l), R(l))P_m(l)}{\sum_{k: k \neq l} G(T(k), R(l))P_m(k) + n_{R(l)}} \right).$$

The long-term average rate of link  $l$  is then defined as

$$X^{avg}(l) = \liminf_{m \rightarrow \infty} \frac{1}{m} \sum_{k=1}^m X_k(l).$$

The average power consumed by each link  $l$ ,  $P^{avg}(l)$  is the time-average over an infinite horizon

$$P^{avg}(l) = \limsup_{m \rightarrow \infty} \frac{1}{m} \sum_{k=1}^m P_k(l).$$

Let the weights associated with each link be  $\alpha(l)$ . These weights are a reflection of the relative priority of links. We

wish to maximize the average throughput (capacity) of the network,  $f(\vec{X}^{avg})$ , which is simply the weighted sum of long-term average rates:

$$f(\vec{X}^{avg}) = \sum_{l=1}^{L_a} \alpha(l) X^{avg}(l). \quad (3)$$

Each link  $l$  is constrained to expend an average power of at most  $\bar{P}^{avg}(l)$  and each node  $i = T(l)$  is subject to a peak transmit power of at most  $P^{max}(i)$ .

### A. Primal Problem

The *objective* of this paper is to compute transmission schedules for the links in the system and the associated power of their transmitters so as to maximize the system throughput of the wireless network when links are subject to peak and average power constraints. Denoting  $\mathcal{E}(i)$  as the set of links originating from node  $i$ , the primal problem can be succinctly stated as,

$$\max f(\vec{X}^{avg}) \quad \text{subject to the constraints} \quad (4)$$

$$P^{avg}(l) \leq \bar{P}^{avg}(l) \quad \text{and}$$

$$0 \leq \sum_{l \in \mathcal{E}(i)} P_m(l) \leq P^{max}(i) \quad \text{and} \quad (5)$$

$$P_m(l) \geq 0 \quad \text{for all } m \geq 1. \quad (6)$$

Note this optimization involves choosing optimal power levels for each transmitter over each link, every slot. Furthermore, the achieved instantaneous throughput in each slot is not a convex function of the transmission powers in the slot. However, using a duality approach below, we reduce the problem to a convex optimization problem over a single slot.

### B. Duality Approach

We exploit a *convex duality* approach to solve the primal problem. Defining a set of dual variables  $\vec{\beta} = \{\beta(1), \beta(2), \dots, \beta(L_a)\}$  for each link, the dual objective function is

$$g(\vec{\beta}) = \max \{ f(\vec{X}^{avg}) + \sum_{l=1}^{L_a} \beta(l) (\bar{P}^{avg}(l) - P^{avg}(l)) \} \\ \text{subject to constraints (5) and (6).}$$

Note the absence of the average power constraints in the definition of  $g(\vec{\beta})$ . For any non-negative vector  $\vec{\beta}$ , it is clear that the maximum value of the objective function in the optimization problem (4) is upper bounded by  $g(\vec{\beta})$ . This observation leads to the dual optimization problem,

$$\min g(\vec{\beta}) \quad \text{subject to} \quad \{\vec{\beta} \geq \mathbf{0}\} \quad (7)$$

Computation of  $g(\vec{\beta})$  involves optimizing over the set of link powers  $\{0 \leq \vec{P}_k \leq \vec{P}^{\max}\}$  in every single slot  $k$ . Using the linearity of  $f(\cdot)$  and the fact that the definition of  $g(\vec{\beta})$  does not directly incorporate average power constraints, it is clear that  $g(\vec{\beta})$  can be evaluated by optimizing over a single slot. i.e.  $g(\vec{\beta}) = h(\vec{\beta})$  where we define  $h(\vec{\beta})$  below. Defining

$$Q(\vec{P}_1, \vec{\beta}) = \sum_{l=1}^{L_a} [\alpha(l)X_1(l) + \beta(l)(\bar{P}^{avg}(l) - P_1(l))] \quad (8)$$

the dual function over a single slot  $h(\vec{\beta})$  is simply

$$h(\vec{\beta}) = \max Q(\vec{P}_1, \vec{\beta}) \quad \text{subject to} \\ \sum_{l \in \mathcal{E}(i)} P_1(l) \leq P^{max}(i) \text{ and } P_1(l) \geq 0.$$

Therefore we consider the simplified dual problem

$$\min h(\vec{\beta}) \quad \text{subject to} \quad \{\vec{\beta} \geq 0\} \quad (9)$$

### III. SOLVING THE DUAL PROBLEM

In order to solve the dual problem (9), we must first compute the dual objective function. Consider the polyhedral set  $S_P = \{\vec{P} : \sum_{l \in \mathcal{E}(i)} P(l) \leq P^{max}(i) \text{ for all } i \in [1, N] \text{ and } 0 \leq P(l) \text{ for all } l \in \mathcal{E}\}$ . Let  $M$  be the number of extreme points of  $S_P$ , and let  $\vec{P}_m^{ext}$ ,  $m = 1, 2, \dots, M$  denote the extreme points of  $S_P$ . By definition, each point in  $S_P$  can be represented as a convex combination of  $\vec{P}_m^{ext}$ ,  $m = 1, 2, \dots, M$ . An upper bound on  $M$  is  $2^{L_a}$ . If  $M_i$  is the number of links emanating from node  $i$ , then  $M = \prod_{i=1}^N (1 + M_i)$ .

Using the fact that  $Q(\vec{P}_1, \vec{\beta})$  for a fixed  $\vec{\beta}$ , is convex in each variable  $P_1(l)$ , the following lemma can be proven

*Lemma 1:*

$$h(\vec{\beta}) = \max\{Q(\vec{P}_m^{ext}, \vec{\beta}) : m = 1, 2, \dots, M\} \quad (10)$$

It is interesting to note that  $Q(\vec{P}_1, \vec{\beta})$  is not a convex function of  $\vec{P}_1$ , only convex in each coordinate of  $\vec{P}_1$ .

#### A. Minimizing the Dual Objective function

The dual objective function  $g(\vec{\beta})$  is the *pointwise maximum* of  $2^{L_a}$  affine functions. Therefore,  $h(\vec{\beta})$  is a piecewise linear (convex) continuous function of  $\vec{\beta}$ .

$$h(\vec{\beta}) = \max_{1 \leq m \leq M} \{\mathbf{a}_m \vec{\beta} + \mathbf{b}_m\} \quad (11)$$

where  $\mathbf{a}_m = \bar{P}^{avg} - \vec{P}_m^{ext}$  and  $\mathbf{b}_m = Q(\vec{P}_m^{ext}, \vec{0})$ . We have developed an iterative descent algorithm to solve the dual problem (9). Exploiting the piecewise linearity of  $h(\vec{\beta})$ , our algorithm converges in a finite number of iterations. Alternately, by introducing a slack variable, (9) can be reduced to a linear program and be efficiently solved using interior

point methods. Solving the dual problem yields  $\vec{\beta}^*$  and  $\mathbf{P}^{*,i}$ , such that

$$\min\{h(\vec{\beta}) : \vec{\beta} \geq 0\} = h(\vec{\beta}^*) = Q(\vec{P}^{*,i}, \vec{\beta}^*) \quad \text{for } i \in [1, \hat{K}]$$

The vectors  $\vec{P}^{*,i}$  are extreme points of  $S_P$  and are called the optimal transmission modes. They denote a collection of subsets of links and their associated power levels of the transmitters.

### IV. OPTIMAL LINK SCHEDULING POLICY

The solution to the dual problem yields us a set of optimal transmission modes. The complementary slackness condition ensures that links with  $\beta^*(l) > 0$  meet their average power constraints with equality. At least one user will meet his average power constraint with equality in the optimal solution. Suppose  $L_{eq}$  is defined as  $L_{eq} = \{l : \beta^*(l) > 0\}$ , and let  $T = |L_{eq}|$ . Using the principle of strong duality [1], there exists an optimal schedule  $\lambda^* = \{\lambda^*(1), \lambda^*(2), \lambda^*(\hat{K})\}$  such that the equations below are satisfied.

$$\begin{aligned} \sum_{i=1}^{\hat{K}} \lambda^*(i) P^{*,i}(l) &= P^{avg}(l) \quad \text{for all links } l \in L_{eq} \\ \sum_{i=1}^{\hat{K}} \lambda^*(i) P^{*,i}(l) &< P^{avg}(l) \quad \text{for all links } l \notin L_{eq} \\ \sum_{i=1}^{\hat{K}} \lambda^*(i) &= 1, \quad \text{where } \lambda^*(i) \geq 0 \end{aligned}$$

Note that  $\hat{K}$  is not necessarily equal to  $T + 1$ , so the above system of equations may be under or over determined. If  $\hat{K} \leq T + 1$ , we compute  $\lambda^*$  by inverting a  $\hat{K} \times \hat{K}$  matrix. If  $\hat{K} > T + 1$ , we invert a series of  $R_{max} \times R_{max}$  matrices, corresponding to different subsets of optimal transmission modes, where the cardinality of the subsets are equal to  $R_{max} \leq T + 1$ . Note that the matrices involved have elements which are either 0 or 1 and are sparse, thereby simplifying the computation. We terminate upon finding a solution, which is guaranteed by the principle of strong duality.

The optimal policy consists of time sharing the optimal transmission modes, where the fraction of time that transmission mode  $\vec{P}^{*,i}$  is used is equal to  $\lambda^*(i)$  for each  $i \in [1, \hat{K}]$ . In every slot, if link  $l$  is included in the enabled transmission mode, then the node  $T(l)$  transmits at its peak power  $P^{max}(T(l))$  to a single receiver  $R(l)$ .

#### A. Reducing Complexity

The algorithm developed to solve the dual problem considers at most  $\prod_{i=1}^N (1 + M_i)$  transmission modes, where  $M_i$  is the number of links emanating from node  $i$ . However, in the optimal schedule we have no more than  $L_a + 1$

transmission modes. The complexity of minimizing the dual objective function increases quickly with  $L_a$ . Thus, reducing the set of possible transmission modes considered can greatly reduce complexity. A natural consequence of our algorithm for a network of nodes with omni-directional antennas is that, no node transmits or receives data at the same time in the optimal schedule due to high self interference. This observation can reduce the number of candidate transmission modes significantly. In general, in many cases it is possible using simple heuristics to eliminate many "obviously inefficient" transmission modes from consideration, since they are unlikely to be used in an optimal policy. The resulting policies we find can be shown to be optimal among all policies which are constrained to use only the set of chosen candidate transmission modes.

## V. SIMULATION RESULTS AND DISCUSSION

In this section we compare the performance of our algorithm with CDMA for a network topology of 8 single hop links as shown in figure 1. The peak power of transmitters are set to  $P^{max}(i) = 1$  Watt each. Links are assigned equal weights i.e.  $\alpha(i) = \frac{1}{8}$ . The signal attenuation between nodes is modeled by the inverse square law. i.e.  $G(i, j) = \frac{1}{d(i, j)^2}$ . We compare the throughput achieved by the optimal policy against a pure CDMA policy as a function of ambient noise and the average-to-peak power ratio. All users in the CDMA policy transmit at their average power  $\bar{P}^{avg}$  every slot. Figure 2 depicts the ratio of the throughput of our policy to that of the CDMA policy as a function of ambient noise for different values of average-to-peak power ratio, namely  $\bar{P}^{avg} = \frac{P^{max}}{6}$ ,  $\frac{P^{max}}{4}$  and  $\frac{P^{max}}{2}$ . The ambient noise is assumed to be the same at all the receivers. The x-axis in figure 2 is the logarithm of the ambient noise value normalized by the peak received power in the network.

In low noise regimes, the throughput of the optimal policy is higher than CDMA by at least a factor of 85 for all average-to-peak power ratios considered. The optimal policy in low noise regimes schedules good links one at a time in TDMA sequence. In this regime, CDMA performs poorly due to high mutual interference between transmissions. As the ambient noise level increases, the optimal policy schedules links far away from each other simultaneously. Each transmission mode is active for non-trivial fractions of time. In fact in high noise regimes, the optimal policy schedules a large number of simultaneous transmissions; the throughput achieved by the CDMA policy therefore approaches to within a factor of 2 of the optimal policy. For sufficiently high noise beyond that indicated in the graph, CDMA achieves the optimal throughput.

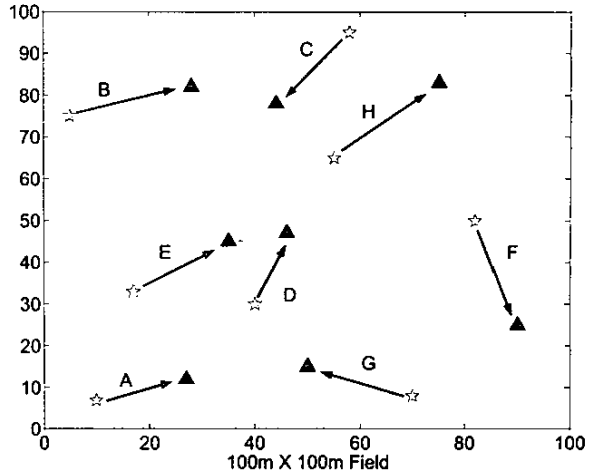


Fig. 1. Network Topology: 16 Nodes, 8 Links.

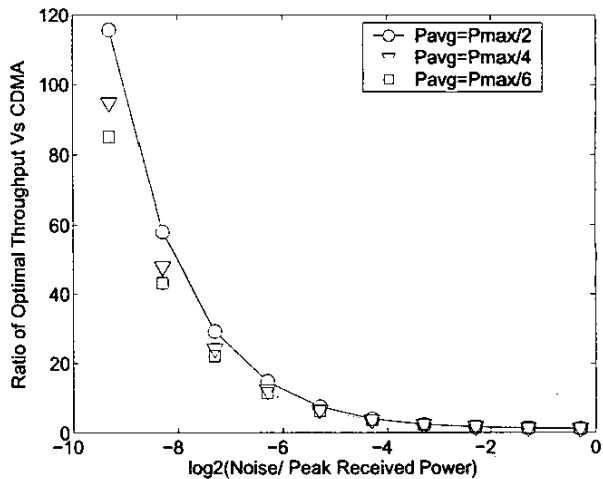


Fig. 2. Ratio of Optimal Throughput Vs CDMA.

For the experiment with  $\bar{P}^{avg} = \frac{P^{max}}{4}$ , the optimal schedule in very low noise regimes (abscissa:  $-9.3$  to  $-6.3$ ) is a simple TDMA sequence of links  $A$ ,  $C$ ,  $D$  and  $E$  for equal fractions of time. The throughput maximizing policy is unfair in such regimes. In noise regimes (abscissa:  $-5.3$  to  $-4.3$ ), the transmission modes  $A$ ,  $\{B, G\}$ ,  $C$  and  $D$  are active for equal time-fractions. In moderate noise regimes (abscissa:  $-4.3$  to  $-3.3$ ), the optimal policy schedules transmission modes  $\{A, C\}$ ,  $\{B, G\}$ ,  $D$ , and  $E$  for equal time-fractions. In high noise regimes (abscissa:  $-3.3$  to  $-0.3$ ), the optimal transmission modes are:  $\{A, H\}$ ,  $\{B, D\}$ ,  $\{C, G\}$ , and  $\{E, F\}$ ; they are active for equal time-fractions. As we relax the average power constraint on links, (i.e. increase  $\bar{P}^{avg}$ ) we find that there are four simultaneous

transmissions per transmission mode. For the experiment with  $\bar{P}^{avg} = \frac{P^{max}}{2}$ , transmission modes  $\{A, B, G, H\}$  and  $\{C, D, E, F\}$  are active for equal durations in high noise regimes (abscissa:  $-0.3$ ).

In low noise regimes, we observe a high improvement in throughput with increasing average-to-peak power ratios. This is so because the optimal policy schedules good links exclusively for dominant fractions of time (almost equal to their average-to-peak power ratios). At higher ambient noise levels, there is relatively little improvement in throughput with increasing average power. The optimal policy as noted earlier, schedules a large number of simultaneous transmissions in such regimes. The relative fairness of service among links improves with increasing ambient noise. The total average power consumption of the optimal policy increases with increasing ambient noise. However, the average power expended by each link in the optimal policy never exceeds the maximum average power  $\bar{P}^{avg}$ . For low noise levels, the optimal policy consumes significantly less power than CDMA while achieving substantially higher throughput (than CDMA). Since the optimal schedule for low noise regimes is a simple TDMA of good links, some of the average power constraints are not active in such regimes and results in low total average power consumption. At the other extreme of the noise horizon, the optimal policy consumes nearly as much power as CDMA, but only achieves slightly higher rates. The above results and observations emphasize the need for adapting the throughput maximizing strategy with changing ambient noise. In practice, the channel conditions would change with time. We believe that our algorithms developed in this paper can be easily extended to maximizing network throughput over block fading wireless channels.

## VI. CONCLUSION

We have developed an integrated link scheduling and power allocation policy for a multihop network that maximizes network throughput subject to peak and average power constraints per node, assuming constant channels. Our scheduling and power control algorithms can support significantly higher data rates than current day approaches to radio resource allocation. Our optimal power control algorithm is such that each node either transmits to a single receiver at its peak power or does not transmit at all. It is therefore simple to implement in a real network. The throughput maximizing policy time-shares a small number of transmission modes ( $\leq L_a + 1$ ) for non-trivial durations. The optimal transmission modes and their relative durations of activity depend on the level of ambient noise and average-to-peak power ratios. The optimal policy in low noise regimes schedules links with good channels in a TDMA sequence for durations roughly equal to their average-to-peak power ratios. In low to moderate noise regimes, the optimal policy consumes considerably less total average power than

CDMA. In moderate to high noise regimes, the optimal strategy schedules multiple simultaneous transmissions even though they may be in close geographic proximity. In sufficiently high noise regimes and high average power limits, we find that pure CDMA is the optimal transmission policy. The ambient noise which models extraneous interference and thermal noise is hard to control over shared unlicensed spectrums. The adaptation of the transmission policy with ambient noise is therefore essential to radio resource allocation over shared frequency bands.

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