

# Joint Opportunistic Power Scheduling and End-to-End Rate Control for Wireless *Ad Hoc* Networks

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**Abstract**—It is known that opportunistic scheduling that accounts for channel variations due to mobility and fading can give substantial improvement over nonopportunistic schemes. However, most work on this subject has focused on single-hop cellular types of architectures. The situation is quite different in *ad hoc* networks due to the inherent multihop nature of transmissions. In this paper, we present a joint opportunistic power scheduling and end-to-end rate control scheme for wireless *ad hoc* networks. We model the time-varying wireless channel as a stochastic process and formulate a stochastic optimization problem, which aims at maximizing system efficiency by controlling the power allocation of each link and the data rate of each user in the system. The joint power scheduling and rate control algorithm is obtained by using stochastic duality and implemented via stochastic subgradient techniques. We illustrate the efficacy of our approach via numerical examples.

**Index Terms**—*Ad hoc* networks, cross-layer optimization, flow control, power control, scheduling, wireless networks.

## I. INTRODUCTION

**I**N CONTRAST to traditional wireless cellular networks in which communication is achieved between the (fixed) access point and (mobile) nodes by using single-hop transmissions, in wireless *ad hoc* networks, communication is typically achieved between (mobile) nodes by using multihop transmissions. Hence, *ad hoc* networks have a substantially different structure from cellular systems and network control schemes for one system are not typically applicable for the other. In fact, in many ways, the structure of *ad hoc* networks is similar to that of wireline networks such as the Internet. For example, both use multihop transmissions for communication and both

require decentralized implementations. However, there also exist fundamental differences. For one, in wireline networks, nodes are static, whereas in *ad hoc* networks, nodes could be mobile, resulting in time-varying channel conditions. In wireline networks, links have a fixed and independent capacity, whereas in wireless networks, the capacity of each link may be time varying and dependent on other links. Hence, network control schemes for wireline networks also cannot be directly applied to wireless *ad hoc* networks. Therefore, there is a real need and also significant interest in studying various problems related to wireless *ad hoc* networks, such as routing, power control, and rate control [1]–[9].

An important lesson that we have learned from the study of (cellular) wireless networks is that by appropriately exploiting the time-varying wireless channel, we can more efficiently utilize radio resources, e.g., increase system performance and decrease the consumption of resources. Hence, there has been a great deal of effort on the development of opportunistic scheduling and power control schemes taking into account these variations in the channel conditions [10]–[17]. The basic idea of opportunistic scheduling is to allocate resources to links when they experience good channel conditions while not allocating resources (if the fairness/performance requirements allow it) to links when they experience poor channel conditions, thus efficiently utilizing radio resources. In such works, the authors exploit the variation of the channel condition of each wireless link and develop optimal scheduling algorithms that maximize the system performance while satisfying a fairness or performance constraint of each user. They show that opportunistic scheduling schemes provide higher system performance than nonopportunistic schemes that do not exploit the variation of the wireless channel.

Opportunistic solutions, i.e., solutions that exploit the channel variability, have primarily been developed for cellular networks [10]–[17] while there has been virtually no work on the subject for wireless *ad hoc* (multihop) networks. In [1]–[4], [6]–[9], resource allocation schemes in *ad hoc* networks have been developed by assuming that the channel condition of each link is constant. In [5], a joint power allocation and routing scheme has been developed for time-varying *ad hoc* networks. This scheme stabilizes the network if the input data rates are within the capacity region. However, it may not provide the optimal solution in terms of network performance (e.g., maximizing network throughput and minimizing power consumption), which is very important in wireless networks.

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In this paper, we study a joint *opportunistic power scheduling* and *end-to-end rate control* problem in wireless *ad hoc* networks. By opportunistic power scheduling, we mean scheduling which user(s) to pick in a time slot as well as determine the power levels at which the selected user(s) should transmit. In particular, we formulate an optimization problem that aims at maximizing network performance, which is defined as the weighted sum of the utilities of all users and power consumption of all nodes in the system, with constraints on the data rate of each user and the power consumption at each node. By solving this problem, we develop a joint opportunistic power scheduling and end-to-end rate control algorithm in which each user adjusts its data rate based on feedback from the system, and the system allocates transmission power to each link considering the demand on rate allocation and the channel condition of the link.

To solve the problem, we will use similar methodologies to those that have been used in [17], such as the dual approach and the use of a stochastic subgradient algorithm. However, the target system (i.e., multihop networks) and the objective (i.e., joint opportunistic power scheduling and end-to-end rate control) of this paper are different from those (i.e., single-hop networks and opportunistic power scheduling) of [17]. Hence, the algorithm that has been developed in [17] cannot be applied to solve our problem, and thus, in this paper, we will develop a new algorithm.

Our algorithm can be decomposed into the system algorithm that is performed by the system to determine transmission power level of each link in each time slot and the user algorithm that is performed by each user with its local information to determine its transmission data rate in each time slot. Moreover, if each link is orthogonal to each other, the system algorithm can be further decomposed into the node algorithm that is performed by each node with its local information. Hence, in this case, our algorithm can be easily implemented in a distributed way. However, for a general interference model, we need a central controller to collect the information of each link and perform the system algorithm, which might not be applicable in some *ad hoc* networks. Thus, in some cases, our algorithm may not be suitable for being implemented in practice. However, even for such cases, since our algorithm can provide an upper bound on the achievable performance of the system, it can be used as a benchmark for the performance of other distributed algorithms.

The rest of this paper is organized as follows: In Section II, we describe the system model considered in this paper and formulate the optimization problem. In Section III, we present the joint opportunistic power scheduling and rate control algorithm, and in Section IV, we present the algorithm for a special case where all links in the system are orthogonal. Numerical results are provided in Section V. Finally, we conclude in Section VI.

## II. SYSTEM MODEL AND PROBLEM

We consider a wireless *ad hoc* network that consists of a set of nodes  $\mathbf{N}$  and a set of links  $\mathbf{L}$ . We denote a link from the transmitter at node  $i$  to the receiver at node  $j$  as  $(i, j)$  and a set of links that emanate from node  $i$  as  $\mathbf{L}_i^{\text{out}}$ . Note that a link is an

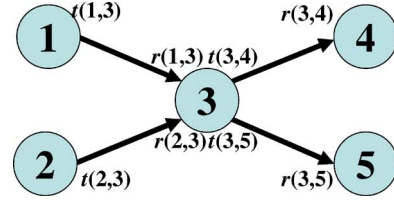


Fig. 1. Virtual transmitter and virtual receiver.

abstract representation of communication between two nodes. There are a set of users  $\mathbf{M}$  in the network, and each user  $m$  has its fixed routing path for communication. We denote a set of links that user  $m$  is using for its communication as  $\mathbf{V}_m$  and a set of users that are using link  $(i, j)$  as  $\mathbf{M}_{i,j}$ . Each user  $m$  has a utility function  $U_m(x_m)$ , where  $x_m$  is its data rate. We assume that  $U_m$  is a continuous and strictly concave function of  $x_m$ . Each node  $i$  has a maximum transmission power level  $P_i^T$ .

In wireless networks, the channel condition of each link is time varying and can be modeled as a stochastic process. We allow the channel condition of each link to vary across time slots and model it as a stationary stochastic process. In a time slot, the system is assumed to be in one of several possible states, in which each state represents one of several possible levels of channel conditions for all links. Each state takes a value from a finite<sup>1</sup> set  $\mathbf{S}$ . We denote the probability that the system is in state  $s$  as  $\pi_s$ .

In *ad hoc* networks, depending on the multiple-access scheme of the system, a node can transmit data to or receive data from multiple nodes simultaneously (e.g., code-division multiple-access scheme), or it can transmit data to or receive data from only one node at a time (e.g., time-division multiple-access scheme). In addition, depending on the duplex scheme of the system, a node can receive data while it transmits data (i.e., full duplex), or it cannot receive data while it transmits data (i.e., half duplex). To model these various systems in a unified framework, we define a virtual transmitter and a virtual receiver corresponding to each link, as shown in Fig. 1. In this figure,  $t(i, j)$  denotes a virtual transmitter of link  $(i, j)$ , and  $r(i, j)$  denotes a virtual receiver of link  $(i, j)$ . Note that although a node can have multiple virtual transmitters or multiple receivers, it may only have one physical transmitter or one physical receiver depending on the system; i.e., virtual transmitters  $t(3, 4)$  and  $t(3, 5)$  and virtual receivers  $r(1, 3)$  and  $r(2, 3)$  may correspond to one physical transmitter and one physical receiver, respectively. We define the path gain from a virtual transmitter  $t(i, j)$  to a virtual receiver  $r(m, n)$  when the system state is  $s$  as  $G_{t(i,j),r(m,n)}^s$ . By appropriately setting  $G_{t(i,j),r(m,n)}^s$ , we can model various systems.

- When a node cannot transmit and receive data simultaneously, we set  $G_{t(i,j),r(m,n)}^s = \infty$ , if  $i = n, \forall s$ .
- When a node can transmit data to or receive data from only one node at a time, we set  $G_{t(i,j),r(m,n)}^s = \infty$ , if  $i = m$  and  $j \neq n$ , or if  $i \neq m$  and  $j = n, \forall s$ .

<sup>1</sup>Note that this assumption is not restrictive, since in a real system, the channel condition of a link is mapped into a level set with a finite number of levels by using quantization.

By using this virtual transmitter and receiver model, we can define the signal-to-interference-plus-noise ratio (SINR) of each link  $(i, j)$ , i.e.,  $\gamma_{i,j}^s$ , when the system is in state  $s$  as

$$\gamma_{i,j}^s(\bar{P}^s) = \frac{G_{t(i,j),r(i,j)}^s P_{i,j}^s}{\theta \sum_{m=1, m \neq i}^M \sum_{n=1, n \neq j}^M G_{t(m,n),r(i,j)}^s P_{m,n}^s + n_{i,j}^s} \quad (1)$$

where

$P_{i,j}^s$	power allocation for link $(i, j)$ when the system is in state $s$ ;
$G_{t(i,j),r(m,n)}^s$	path gain from a virtual transmitter $t(i, j)$ to a virtual receiver $r(m, n)$ when the system is in state $s$ ;
$n_{i,j}^s$	background noise of link $(i, j)$ when the system is in state $s$ ;
$\bar{P}^s = (P_{i,j}^s)_{(i,j) \in \mathbf{L}}$	power allocation vector for all links when the system is in state $s$ ;
$\theta$	orthogonality factor ( $0 \leq \theta \leq 1$ ).

As in [2] and [3], we assume that the data rate that can be achieved is a linear function of the SINR. The data rate  $r_{i,j}^s$  for link  $(i, j)$  when the system is in state  $s$  is thus defined as

$$r_{i,j}^s(\bar{P}^s) = W \gamma_{i,j}^s(\bar{P}^s) \quad (2)$$

where  $W$  is the bandwidth of the system.

We use a weighted sum of the utilities of all users and the average power consumption of all nodes as a measure of system performance. The system performance is thus defined as

$$F(\bar{x}, \bar{P}) = \sum_{m \in \mathbf{M}} a_m U_m(x_m) - \sum_{s \in \mathbf{S}} \pi_s \sum_{i,j:(i,j) \in \mathbf{L}} b_{i,j} P_{i,j}^s \quad (3)$$

where  $a_m$  and  $b_{i,j}$  are nonnegative constants,  $\bar{x} = (x_m)_{m \in \mathbf{M}}$ , and  $\bar{P} = (P_{i,j}^s)_{i,j:(i,j) \in \mathbf{L}, s \in \mathbf{S}}$ . Then, the optimization problem that we study in this paper is formulated as

$$\begin{aligned} (\text{P}) \quad & \max_{\bar{x}, \bar{P}} F(\bar{x}, \bar{P}) \\ \text{s.t.} \quad & x_m^{\min} \leq x_m \leq x_m^{\max}, \quad m \in \mathbf{M} \\ & \sum_{s \in \mathbf{S}} \pi_s r_{i,j}^s(\bar{P}^s) \geq \sum_{k \in \mathbf{M}_{i,j}} x_k, \quad (i, j) \in \mathbf{L} \\ & \sum_{s \in \mathbf{S}} \pi_s \sum_{j:(i,j) \in \mathbf{L}_i^{\text{out}}} P_{i,j}^s \leq P_i^a, \quad i \in \mathbf{N} \\ & \bar{P}_i^s \in \mathbf{P}_i^s, \quad i \in \mathbf{N}, s \in \mathbf{S} \end{aligned}$$

where  $x_m^{\min}$  and  $x_m^{\max}$  are minimum and maximum data rates for user  $m$ ,  $P_i^a$  is the maximum average power consumption of node  $i$ ,  $\bar{P}_i^s = (P_{i,j}^s)_{j:(i,j) \in \mathbf{L}_i^{\text{out}}}$ ,  $\mathbf{P}_i^s = \{(P_{i,j}^s)_{j:(i,j) \in \mathbf{L}_i^{\text{out}}} \mid \sum_{j:(i,j) \in \mathbf{L}_i^{\text{out}}} P_{i,j}^s \leq P_i^T, 0 \leq P_{i,j}^s \leq P_i^T, \forall j: (i, j) \in \mathbf{L}_i^{\text{out}}\}$ , and  $P_i^T$  is the maximum transmission power limit of node  $i$ . Hence, by solving this problem, we can obtain the joint power scheduling and rate allocation that maximizes the system performance under constraints on the minimum and maximum data rates of each user (i.e., the first constraint), the capacity

of each link (i.e., the second constraint), the average power consumption, and the maximum transmission power limit of each node (i.e., the third and fourth constraints).

In Section III, we will develop the algorithm that solves problem (P). An important question that is not the focus of this paper is how does one ensure feasibility, i.e., how do we ensure that there exists a power scheduling and rate control policy that satisfies the constraints. This can be achieved by adopting an appropriate call admission control strategy. However, the development of such a strategy is outside the scope of this paper. Instead, in this paper, we will assume that the system has an admission control policy in place that ensures feasibility and focus on the power scheduling and rate control problem.

### III. ALGORITHM

When solving problem (P), if we already knew the underlying probability distribution for the system states (i.e.,  $\pi_s$ ,  $\forall s \in \mathbf{S}$ ), the problem would be equivalent to a deterministic optimization problem. However, in practice, we do not have such *a priori* knowledge. Thus, we need to develop an algorithm that will work even without such *a priori* knowledge of the underlying probability distribution for the system states. To this end, we consider the dual of problem (P) and develop the algorithm that solves the dual problem even without such a prior knowledge. However, we must note that problem (P) may not be a convex optimization problem. This implies that there might be a duality gap between problem (P) and its dual, and thus, we cannot guarantee to obtain the optimal power scheduling and rate allocation that solves problem (P) by using the dual approach. We will come back to this issue later in this section; instead, here, we first focus on solving the dual problem of problem (P).

We define a Lagrangian function associated with problem (P) as

$$\begin{aligned} L(\bar{x}, \bar{P}, \bar{\mu}, \bar{\lambda}) &= \sum_{m \in \mathbf{M}} a_m U_m(x_m) - \sum_{s \in \mathbf{S}} \pi_s \sum_{i,j:(i,j) \in \mathbf{L}} b_{i,j} P_{i,j}^s \\ &+ \sum_{i,j:(i,j) \in \mathbf{L}} \mu_{i,j} \left( \sum_{s \in \mathbf{S}} \pi_s r_{i,j}^s(\bar{P}^s) - \sum_{k \in \mathbf{M}_{i,j}} x_k \right) \\ &+ \sum_{i \in \mathbf{N}} \lambda_i \left( P_i^a - \sum_{s \in \mathbf{S}} \pi_s \sum_{j:(i,j) \in \mathbf{L}_i^{\text{out}}} P_{i,j}^s \right) \\ &= \sum_{m \in \mathbf{M}} a_m U_m(x_m) - \sum_{s \in \mathbf{S}} \pi_s \sum_{i \in \mathbf{N}} \sum_{j:(i,j) \in \mathbf{L}_i^{\text{out}}} b_{i,j} P_{i,j}^s \\ &+ \sum_{s \in \mathbf{S}} \pi_s \sum_{i \in \mathbf{N}} \sum_{j:(i,j) \in \mathbf{L}_i^{\text{out}}} \mu_{i,j} r_{i,j}^s(\bar{P}^s) \\ &- \sum_{m \in \mathbf{M}} \sum_{i,j:(i,j) \in \mathbf{V}_m} \mu_{i,j} x_m \\ &+ \sum_{i \in \mathbf{N}} \lambda_i \left( P_i^a - \sum_{s \in \mathbf{S}} \pi_s \sum_{j:(i,j) \in \mathbf{L}_i^{\text{out}}} P_{i,j}^s \right) \end{aligned}$$

$$= \sum_{s \in \mathbf{S}} \pi_s \left\{ \sum_{m \in \mathbf{M}} \left( a_m U_m(x_m) - \sum_{i,j:(i,j) \in \mathbf{V}_m} \mu_{i,j} x_m \right) + \sum_{i \in \mathbf{N}} \sum_{j:(i,j) \in \mathbf{L}_i^{\text{out}}} (-b_{i,j} P_{i,j}^s + \mu_{i,j} r_{i,j}^s (\bar{P}^s) - \lambda_i P_{i,j}^s) + \sum_{i \in \mathbf{N}} \lambda_i P_i^a \right\}$$

where  $\bar{\mu} = (\mu_{i,j})_{i,j:(i,j) \in \mathbf{L}}$ ,  $\bar{\lambda} = (\lambda_i)_{i \in \mathbf{N}}$ ,  $\bar{P} = (\bar{P}_i^s)_{i \in \mathbf{N}, s \in \mathbf{S}}$ ,  $\mathbf{L}_i^{\text{out}}$  is a set of links that emanate from node  $i$ ,  $\mathbf{M}_{i,j}$  is a set of users that are using link  $(i, j)$ , and  $\mathbf{V}_m$  is a set of links that user  $m$  is using. Then, the dual problem is defined as

$$(D) \quad \min_{\bar{\mu} \geq 0, \bar{\lambda} \geq 0} Q(\bar{\mu}, \bar{\lambda})$$

where

$$Q(\bar{\mu}, \bar{\lambda}) = \max_{\bar{x} \in \mathbf{X}, \bar{P} \in \mathbf{P}} L(\bar{x}, \bar{P}, \bar{\mu}, \bar{\lambda}) \quad (4)$$

$\mathbf{X} = \{(x_m)_{m \in \mathbf{M}} | x_m^{\min} \leq x_m \leq x_m^{\max}, \forall m \in \mathbf{M}\}$ , and  $\mathbf{P} = \{(\bar{P}_i^s)_{i \in \mathbf{N}, s \in \mathbf{S}} | \bar{P}_i^s \in \mathbf{P}_i^s, \forall i \in \mathbf{N}, s \in \mathbf{S}\}$ . We first consider the problem in (4) for given  $\bar{\mu}$  and  $\bar{\lambda}$ . Since  $L(\bar{x}, \bar{P}, \bar{\mu}, \bar{\lambda})$  is separable in  $s$ ,  $\bar{x}(\bar{\mu}) = (x_m(\bar{\mu}))_{m \in \mathbf{M}}$  and  $\bar{P}(\bar{\mu}, \bar{\lambda}) = (\bar{P}_i^s(\bar{\mu}, \bar{\lambda}))_{i \in \mathbf{N}, s \in \mathbf{S}}$  (where  $\bar{P}_i^s(\bar{\mu}, \bar{\lambda}) = (\bar{P}_i^s(\bar{\mu}, \bar{\lambda}))_{i \in \mathbf{N}}$ ) solve the problem in (4) if and only if

$$x_m(\bar{\mu}) = \arg \max_{x_m^{\min} \leq x_m \leq x_m^{\max}} \left\{ a_m U_m(x_m) - \sum_{i,j:(i,j) \in \mathbf{V}_m} \mu_{i,j} x_m \right\} \quad m \in \mathbf{M} \quad (5)$$

and (6), shown at the bottom of the page, is obtained by using (1) and (2). Note that for a given system state  $s$ , the above problems are deterministic, and we can solve them without knowledge of the underlying probability distribution.

We now solve the dual problem (D). However, to minimize  $Q(\bar{\mu}, \bar{\lambda})$ , we need explicit knowledge of the underlying probability distribution, which is infeasible to obtain in practice. To overcome this difficulty, we use a stochastic subgradient method [18], [19] that is defined by the following iterative processes:

$$\mu_{i,j}^{(n+1)} = [\mu_{i,j}^{(n)} - \alpha^{(n)} v_{\mu_{i,j}}^{(n)}]^+, \quad \forall i, j : (i, j) \in \mathbf{L} \quad (7)$$

and

$$\lambda_i^{(n+1)} = [\lambda_i^{(n)} - \alpha^{(n)} v_{\lambda_i}^{(n)}]^+, \quad \forall i \in \mathbf{N} \quad (8)$$

where  $[a]^+ = \max\{0, a\}$ , and  $v_{\mu_{i,j}}^{(n)}$  and  $v_{\lambda_i}^{(n)}$  are some random variables. Let the sequences of solutions  $\bar{\mu}^{(0)}, \bar{\mu}^{(1)}, \dots, \bar{\mu}^{(n)}$  and  $\bar{\lambda}^{(0)}, \bar{\lambda}^{(1)}, \dots, \bar{\lambda}^{(n)}$  be generated by (7) and (8), respectively. Let  $v_{\mu_{i,j}}^{(n)}$  and  $v_{\lambda_i}^{(n)}$  be chosen such that

$$E \left\{ v_{\mu_{i,j}}^{(n)} | \bar{\mu}^{(0)}, \dots, \bar{\mu}^{(n)}, \bar{\lambda}^{(0)}, \dots, \bar{\lambda}^{(n)} \right\} = \partial_{\mu_{i,j}} Q(\bar{\mu}^{(n)}, \bar{\lambda}^{(n)})$$

and

$$E \left\{ v_{\lambda_i}^{(n)} | \bar{\mu}^{(0)}, \dots, \bar{\mu}^{(n)}, \bar{\lambda}^{(0)}, \dots, \bar{\lambda}^{(n)} \right\} = \partial_{\lambda_i} Q(\bar{\mu}^{(n)}, \bar{\lambda}^{(n)})$$

respectively, where  $E\{v\}$  is the expected value of a random variable  $v$ , and  $\partial_{\mu_{i,j}} Q(\bar{\mu}^{(n)}, \bar{\lambda}^{(n)})$  and  $\partial_{\lambda_i} Q(\bar{\mu}^{(n)}, \bar{\lambda}^{(n)})$  are subgradients of  $Q(\bar{\mu}, \bar{\lambda})$  at  $\bar{\mu} = \bar{\mu}^{(n)}$  and  $\bar{\lambda} = \bar{\lambda}^{(n)}$  with respect to  $\mu_{i,j}$  and  $\lambda_i$ , respectively. Then,  $v_{\mu_{i,j}}^{(n)}$  and  $v_{\lambda_i}^{(n)}$  are called stochastic subgradients of  $Q(\bar{\mu}, \bar{\lambda})$  at  $\bar{\mu} = \bar{\mu}^{(n)}$  and  $\bar{\lambda} = \bar{\lambda}^{(n)}$  with respect to  $\mu_{i,j}$  and  $\lambda_i$ , respectively. By using these iterative procedures with a sequence of step sizes that satisfy

$$\alpha^{(n)} \geq 0, \sum_{n=0}^{\infty} \alpha^{(n)} = \infty, \quad \text{and} \quad \sum_{n=0}^{\infty} (\alpha^{(n)})^2 < \infty$$

the dual variables  $\bar{\mu}^{(n)}$  and  $\bar{\lambda}^{(n)}$  can be shown to converge to the optimal solutions that solve the dual problem (D) with probability one [18], [19]. To apply this method to solve problem (D), we need to know the stochastic subgradients of  $Q(\bar{\mu}, \bar{\lambda})$ , i.e.,  $v_{\mu_{i,j}}^{(n)}$  and  $v_{\lambda_i}^{(n)}$ , and they are obtained as in [16] and [17] by

$$v_{\mu_{i,j}}^{(n)} = r_{i,j}^{s(n)} \left( \bar{P}^{s(n)}(\bar{\mu}^{(n)}, \bar{\lambda}^{(n)}) \right) - \sum_{k \in \mathbf{M}_{i,j}} x_k^{(n)}(\bar{\mu}^{(n)}) \quad (9)$$

and

$$v_{\lambda_i}^{(n)} = P_i^a - \sum_{j:(i,j) \in \mathbf{L}_i^{\text{out}}} P_{i,j}^{s(n)}(\bar{\mu}^{(n)}, \bar{\lambda}^{(n)}) \quad (10)$$

where  $s^{(n)}$  is a system state at iteration  $n$ , and  $x_k^{(n)}(\bar{\mu}^{(n)})$  and  $\bar{P}^{s(n)}(\bar{\mu}^{(n)}, \bar{\lambda}^{(n)}) = (\bar{P}_i^{s(n)}(\bar{\mu}^{(n)}, \bar{\lambda}^{(n)}))_{i \in \mathbf{N}}$  are solutions to the problems in (5) and (6), respectively, with  $\bar{\mu} = \bar{\mu}^{(n)}$  and  $\bar{\lambda} = \bar{\lambda}^{(n)}$ . Note that this stochastic subgradient method does

$$\begin{aligned} \bar{P}^s(\bar{\mu}, \bar{\lambda}) &= \arg \max_{\bar{P}^s \in \mathbf{P}^s} \left\{ \sum_{i \in \mathbf{N}} \sum_{j:(i,j) \in \mathbf{L}_i^{\text{out}}} (-b_{i,j} P_{i,j}^s + \mu_{i,j} r_{i,j}^s(\bar{P}^s) - \lambda_i P_{i,j}^s) \right\} \\ &= \arg \max_{\bar{P}^s \in \mathbf{P}^s} \left\{ \sum_{i \in \mathbf{N}} \sum_{j:(i,j) \in \mathbf{L}_i^{\text{out}}} W \mu_{i,j} \frac{G_{t(i,j), r(i,j)}^s P_{i,j}^s}{\sum_{m=1, m \neq i}^M \sum_{n=1, n \neq j}^M G_{t(m,n), r(i,j)}^s P_{m,n}^s + n_{i,j}^s} - (b_{i,j} + \lambda_i) P_{i,j}^s \right\}, \quad s \in \mathbf{S} \end{aligned} \quad (6)$$

not require *a priori* knowledge of the underlying probability distribution for system states. It only requires the information of the system state at the current iteration, which can be obtained by measuring the channel condition of each link at the current iteration.

Thus far, we have developed an algorithm that solves the dual problem (D) even without *a priori* knowledge of system states. However, as we have mentioned before, due to the duality gap, we cannot guarantee that the dual approach will result in an optimal solution for our original optimization problem (P). Fortunately, we are able to show in the following proposition that as the randomness in the system increases, the duality gap decreases. This implies that as the randomness in the system increases (i.e., as we have a finer granularity of the quantization for the channel condition of each link), the dual solution approaches that of the original problem (P).

**Proposition 1:** Let  $\sup(P)$  be the optimal value of problem (P) and  $\inf(D)$  be the optimal value of the corresponding dual problem. If  $\sup_s \pi_s (\sum_{m \in \mathbf{M}} U_m(x_m) - \sum_{i,j:(i,j) \in \mathbf{L}} b_{i,j} P_{i,j}^s) \rightarrow 0$  and  $\sup_s \pi_s (r_{i,j}^s(P^s) - \sum_{k \in \mathbf{M}_{i,j}} x_k) \rightarrow 0$ ,  $\forall (i,j) \in \mathbf{L}$  as  $S \rightarrow \infty$ , then  $\sup(P) - \inf(D) \rightarrow 0$  as  $S \rightarrow \infty$ .

*Proof:* This can be proven in the same way as in [17, Proof of Prop. 1]. ■

From this result, we deduce that when the randomness of the system is large, the same power scheduling algorithm that has been developed in this paper can be used to obtain a good approximation to the optimal solution, although the primal problem cannot be formulated as a convex optimization problem. In Section V, we provide numerical results to illustrate the effectiveness of our solution.

To implement the proposed algorithm, we can decompose it as a system problem that is solved by the system and user problems each of which is solved by each user. First, based on parameters in time slot  $n$ , i.e.,  $s = s^{(n)}$ ,  $\bar{\mu} = \bar{\mu}^{(n)}$ , and  $\bar{\lambda} = \bar{\lambda}^{(n)}$ , the system solves the problem in (6) and obtains the power allocation for each link in the current time slot. Note that the problem in (6) may not be a convex optimization problem since the objective function might not be a concave function. However, since the objective function is a convex function in each variable  $P_{i,j}^s$ , the optimal solution of the problem can be obtained at an extreme point of  $\bar{\mathbf{P}}^s$  [3]. Hence, if a node transmits data, it transmits data to only one link at a time at its maximum power level  $P_i^T$ . By using this property, the optimal solution to the problem in (6) can be found easily, although it is not a convex optimization problem. The data rate for each link is obtained by using (2) based on the power allocation. After allocating power and data rate, based on the power consumption of each node and the aggregate data rate of the users that are using each link in the current time slot, the system updates parameters  $\bar{\lambda}$  and  $\bar{\mu}$  for the next time slot by using (7)–(10). Next, the data rate of each user is determined by the user itself based on its local information. In time slot  $n$ , each user  $m$  determines its data rate  $x_m^{(n)}$  by solving the problem in (5). To solve the problem, the user requires the sum of  $\mu_{i,j}^{(n)}$ 's,  $i, j : (i,j) \in \mathbf{V}_m$ , i.e., the sum of  $\mu_{i,j}^{(n)}$ 's corresponding to its routing path. This can be obtained by feedback from the nodes that are on its routing path.

#### IV. SPECIAL CASE: SYSTEM WITH ORTHOGONAL LINKS

If all links are orthogonal, we can set  $\theta = 0$  in (1), and the SINR of link  $(i,j)$  in (1) can be rewritten as

$$\gamma_{i,j}^s(P_{i,j}^s) = \frac{G_{t(i,j),r(i,j)}^s P_{i,j}^s}{n_{i,j}^s}. \quad (11)$$

Hence, there is no interference between links. In this case, we can show that the system problem can be further decomposed as several node problems where each node solves its own problem that requires only its local information. Furthermore, problem (P) becomes a convex optimization problem. Hence, there is no duality gap between problem (P) and problem (D), and we can always obtain the optimal power scheduling and rate allocation by using the proposed algorithm.

First, the problem in (6) can be rewritten as

$$\begin{aligned} \bar{P}^s(\bar{\mu}, \bar{\lambda}) &= \arg \max_{\bar{\mathbf{P}}^s \in \mathbf{P}^s} \left\{ \sum_{i \in \mathbf{N}} \sum_{j:(i,j) \in \mathbf{L}_i^{\text{out}}} -b_{i,j} P_{i,j}^s + \mu_{i,j} r_{i,j}^s(P_{i,j}^s) - \lambda_i P_{i,j}^s \right\} \\ &= \arg \max_{\bar{\mathbf{P}}^s \in \mathbf{P}^s} \left\{ \sum_{i \in \mathbf{N}} \sum_{j:(i,j) \in \mathbf{L}_i^{\text{out}}} \left( W \mu_{i,j} \frac{G_{t(i,j),r(i,j)}^s}{n_{i,j}^s} - b_{i,j} - \lambda_i \right) P_{i,j}^s \right\}, \quad s \in \mathbf{S}. \end{aligned} \quad (12)$$

The above problem is separable in  $i$ . Hence,  $\bar{P}^s(\bar{\mu}, \bar{\lambda}) = (\bar{P}_i^s(\bar{\mu}, \bar{\lambda}))_{i \in \mathbf{N}}$  maximizes it if and only if

$$\begin{aligned} \bar{P}_i^s(\bar{\mu}, \bar{\lambda}) &= \arg \max_{\bar{\mathbf{P}}^s \in \mathbf{P}^s} \left\{ \sum_{j:(i,j) \in \mathbf{L}_i^{\text{out}}} \left( W \mu_{i,j} \frac{G_{t(i,j),r(i,j)}^s}{n_{i,j}^s} - b_{i,j} - \lambda_i \right) P_{i,j}^s \right\} \\ &\quad i \in \mathbf{N}, \quad s \in \mathbf{S}. \end{aligned} \quad (13)$$

Each node  $i$  solves its problem [i.e., the problem in (13)] and obtain power and rate allocation for links that are emanating from it. The solution of the problem in (13) can be obtained as

$$P_{i,j}^s = \begin{cases} P_i^T, & \text{if } j = \arg \max_{k \in \mathbf{L}_i^{\text{out}}} \left\{ W \mu_{i,k} \frac{G_{t(i,k),r(i,k)}^s}{n_{i,k}^s} - b_{i,k} - \lambda_i > 0 \right\} \\ W \mu_{i,k} \frac{G_{t(i,k),r(i,k)}^s}{n_{i,k}^s} - b_{i,k} - \lambda_i > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Moreover, (9) can be rewritten as

$$v_{\mu_{i,j}}^{(n)} = r_{i,j}^{s^{(n)}} \left( P_{i,j}^{s^{(n)}} \left( \bar{\mu}^{(n)}, \bar{\lambda}^{(n)} \right) \right) - \sum_{k \in \mathbf{M}_{i,j}} x_k^{(n)} \left( \bar{\mu}^{(n)} \right). \quad (14)$$

To solve problems in (10) and (14), we only need local information corresponding to node  $i$ . Hence, node  $i$  can update





TABLE III  
COMPARISON OF THE AVERAGE POWER CONSUMPTIONS  
FOR THE MODE 1 SYSTEM

Node	1	2	3	4	5	6	Total
Opp.	0.14	0.49	0.03	0.03	0.01	0.01	0.72
Non-opp.	0.51	0.82	0.20	0.20	0.10	0.10	1.94

TABLE IV  
COMPARISON OF THE ACHIEVED DATA RATES  
FOR THE MODE 1 SYSTEM

User	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$
Opp.	1.20	3.58	1.21	3.68	3.13
Non-opp.	1	1	1	1	1

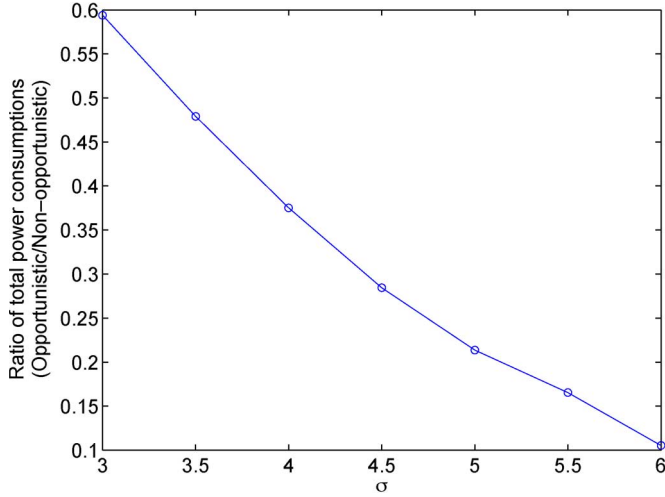


Fig. 3. Ratio of the total power consumption of the opportunistic scheme to that of the nonopportunistic scheme.

where  $|M_{i,j}|$  is the number of the users that are using link  $(i, j)$ . Hence, in this scheme, each user is allocated the same data rate as in our previous simulation with  $a = 0$ . We provide the average power consumption for each node and the total average power consumption of the network in each scheme in Table III. As shown in Table III, our opportunistic scheme requires less power from each node than its nonopportunistic counterpart to achieve the same performance.

We now simulate our opportunistic scheme with  $x_m^{\min} = 1$ ,  $x_m^{\max} = 10$ , and  $a_m = 1$  for each user  $m$ , and  $b_{i,j} = 0$  for each link  $(i, j)$ . We further set the maximum average power consumption  $P_i^a$  for each node  $i$  as the average power consumption of each node in the nonopportunistic scheme in Table III, i.e.,  $P_0^a = 0.511$ ,  $P_1^a = 0.818$ , and so on. Since we set  $a_m = 1$  and  $b_{i,j} = 0$ , our algorithm maximizes the sum of the utilities of all users without considering power consumption. However, the power consumption in each node is controlled by the constraint on its maximum average power consumption, which is the same value as the average power consumption of each node in the nonopportunistic scheme. Hence, this simulation illustrates the performance gain of our scheme over the nonopportunistic scheme when both schemes consume the same amount of power. Table IV shows the achieved data rate of each user in each scheme, and our scheme provides a higher data rate to each user than the nonopportunistic scheme. In Figs. 3 and 4,

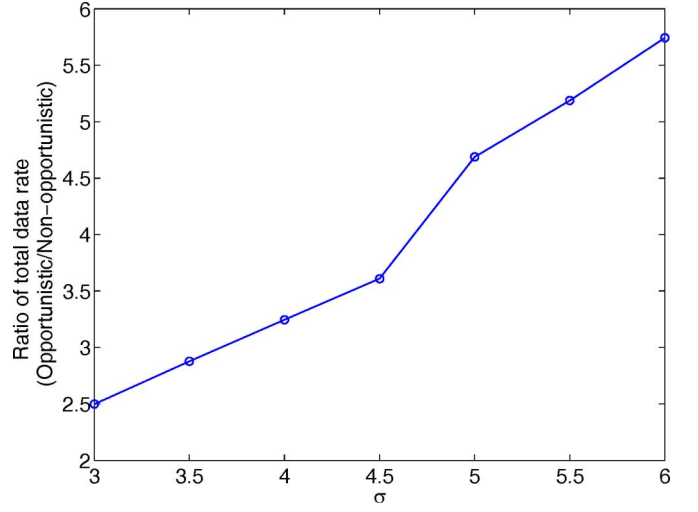


Fig. 4. Ratio of the total data rate of the opportunistic scheme to that of the nonopportunistic scheme.

TABLE V  
COMPARISON OF THE TOTAL SYSTEM UTILITIES OF THE  
MODE 2 AND MODE 3 SYSTEMS

$\theta$	0	0.2	0.4	0.6	0.8	1
Mode 2	5.11	3.26	2.82	2.62	2.55	2.48
Mode 3	4.39	3.1	2.76	2.66	2.5	2.49

we simulate the same scenarios for various values of  $\sigma$ , which is the standard deviation of  $K_{t(i,j),r(m,n)}^s$  in (15), and we compare the ratio of the total system power consumption (i.e., the sum of the power consumption of each node) and the total system data rate (i.e., the sum of the data rate of each user) of our opportunistic scheme to those of the nonopportunistic scheme, respectively. They show that as the randomness of the channel condition of each link increases, the performance gain of our opportunistic scheme over the nonopportunistic scheme also increases. Hence, as the variation of the channel condition of each link gets larger, it is more important to appropriately exploit it for system performance.

In Table V, we simulate mode 2 and mode 3 systems and compare their performances. We set  $a_m = 1$ ,  $\forall m$ , and  $b_{i,j} = 0$ ,  $\forall i, j : (i, j) \in \mathbf{L}$ , and provide the total system utilities for each system by varying the orthogonality factor  $\theta$ . As expected, the mode 2 system provides higher total system utilities than the mode 3 system. Although we allow multiple transmissions in the mode 2 system, due to the property of the optimal transmission as we have studied in Section III, a node in the mode 2 system always transmits data to only one node at a time. Hence, mode 2 and mode 3 systems have the same transmission strategy, and the only difference between them is that a node in the mode 2 system can receive data from multiple nodes at a time, whereas a node in the mode 3 system can receive data from only one node at a time. However, although a node is allowed to receive data from multiple nodes at a time in the mode 2 system, when the orthogonality factor is large, the optimal strategy is to receive data from only one node at a time, since as the orthogonality factor increases, interference between two transmissions also increases. Hence, as the orthogonality factor increases, the performance difference between these two

systems decreases, and when the orthogonality factor is large, they provide almost the same performance.

## VI. CONCLUSION AND FUTURE WORK

In wireless networks, the channel condition of the communication link is time varying, and exploiting this property is important in improving the system performance. In this paper, we have modeled the channel condition as a stochastic process and formulated a stochastic optimization problem that aims at maximizing the performance of the system while satisfying the constraints on the data rate of each user and power consumption of each node. Our problem formulation can be applied to various modes of wireless *ad hoc* networks by simply modifying parameter settings. By using a dual approach and the stochastic subgradient algorithm, we have developed a joint opportunistic power scheduling and end-to-end rate control algorithm for wireless *ad hoc* networks. Numerical results show that by using our scheme, i.e., by opportunistically exploiting the time-varying channel condition of each link, we can improve system performance while reducing power consumption at each node in the system. However, as shown in this paper, to obtain the optimal solution that exploits instantaneous time-varying channel condition of each link in *ad hoc* networks, in general, we need to have a centralized algorithm or possibly a distributed algorithm with a global information, which might be infeasible in most of situations in *ad hoc* networks. Hence, it will be a good future research topic to develop a distributed algorithm, in which each node requires only local information and performs its own algorithm locally. This distributed algorithm may provide a suboptimal solution. In this case, the algorithm developed in this paper can be used to measure its efficiency.

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