Arithmetic versus Geometric Effective Exchange Rates

Author(s): David A. Brodsky

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Bemerkungen

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By

David A. Brodsky

I. Introduction

One of the defining features of the post-Bretton-Woods international monetary system is that countries can no longer effect uniform devaluations or revaluations of their currencies. In particular, a country's rate of exchange in terms of either of the two previous standards of value — gold and the U.S. dollar — provides little information about the movement in its currency vis-à-vis those of the "rest of the world". In order to provide a measure of the "average" movement in a country's exchange rate, the concept of effective exchange rate has therefore come to be widely used in recent years by central banks, international organizations and individual researchers[1].

Effective exchange rate indices are generally defined as weighted averages of individual bilateral exchange rate indices, and there has been considerable discussion in economic literature pertaining to the appropriate weights to be used[2]. Unfortunately, however, this focus on the choice of weights has overshadowed an equally fundamental — and in practice often more significant — issue, namely, the proper mathematical formulation to employ when calculating such indices. Indeed, the most frequently cited reference on the theory of effective exchange rates mentions the subject

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[1] The widespread use of effective exchange rate indices in the 1970s was given strong impetus by the article by Hirsch and Higgins [1970].

[2] See, for example, Artus and Rhomberg [1973], Rhomberg [1976], Morgan Guaranty Trust [1978, 1979], and Honohan [1979]. The most commonly used weights are statistical trade weights based on reported trade flows, although several regularly published indices are based on weights derived from multilateral trade models.

only in a footnote which suggests that the issue is a relatively unimportant one[1].

It will be shown in this paper, however, that the mathematical formulation adopted — specifically, the technique of averaging used — has important quantitative implications for the calculated values of the indices. This finding is of considerable significance, for in recent years effective exchange rates have been widely utilized to analyze exchange rate movements of individual countries. Moreover, a growing number of countries have chosen to *define* their exchange rates through reference to an effective exchange rate index, i.e., by pegging them to a basket of currencies or to the SDR. In this context, it will be seen that the method currently employed for calculating the value of such composite currency units as the SDR and the ECU has an inherent upward bias.

The second section of this paper will examine the different mathematical formulations and will show that, for a given set of weights, there exist three fundamentally different indices of effective exchange rates. The third section will investigate the quantitative importance of differences among the indices, from which it will emerge that the case for using geometric indices is a very strong one. The fourth section will focus on the implications of the findings for those countries which are pegging to baskets of currencies or to the SDR. In the final section a conclusion is drawn.

II. Definitions of Effective Exchange Rates

Effective exchange rates provide a quantitative measure of the change in the "home" country's exchange rate in terms of the currencies of a given set of countries, relative to a specified base period. There is a fundamental ambiguity here, for one can equally well define "the" exchange rate as the price of domestic currency in terms of foreign currency or as the price of foreign currency in terms of domestic currency[2]. While in principle these two definitions would seem to represent essentially the same thing[3], in the next section it will be shown that the results are extremely sensitive to the definition of exchange rate employed.

[1] Rhomberg [1976, p. 95] remarks: "The trade-weighted indices could ... have been calculated as geometric, rather than arithmetic, averages, but this would have introduced an element unfamiliar to users of such indices and would not have notably affected the results."

[2] Exchange rates in the London market have traditionally been quoted "indirectly" (i. e., the price of £ in terms of foreign currencies) while most other markets have used "direct" quotations [Coninx, 1978, p. 10].

[3] It has occasionally been argued that the second definition is more appropriate for the calculation of import-weighted indices since it reflects the price of foreign exchange con-

So let R_{it} denote the price of the home currency at time t in terms of the currency of the i^{th} country, and S_{it} the price of the currency of the i^{th} country at time t in terms of that of the home country. The two exchange rates R_{it} and S_{it} are linked by the equality[1]

(1) $R_{it} = 1/S_{it}$

For a given set of weights w_i (e.g., trade weights), one can then define four possible effective exchange rate indices — two arithmetic and two geometric[2]:

Arithmetic	*Geometric*
$EER1 = 100 \sum_i w_i R_{it}/R_{io}$	$EER3 = 100 \prod_i (R_{it}/R_{io})^{w_i}$
$EER2 = 100/[\sum_i w_i S_{it}/S_{io}]$	$EER4 = 100/[\prod_i (S_{it}/S_{io})^{w_i}]$

Since $1/(R_{it}/R_{io}) = S_{it}/S_{io}$, it follows that EER3 = EER4, so that the following important conclusion emerges: ***a geometric effective exchange rate index is independent of the definition of exchange rate which is employed.*** For a given set of weights there are therefore three possible definitions of effective exchange rate[3]:

EER1 = arithmetic, subsequently denoted as EERA
EER2 = harmonic, subsequently denoted as EERH
EER3 = geometric, subsequently denoted as EERG

Note that both EERA and EERH are "arithmetic" indices, which differ essentially only in the definition of exchange rate employed.

III. Quantitative Differences among the Indices

In practice, the differences among the formulations are far greater than generally recognized: not only are there pronounced differences

fronting domestic importers, and conversely that the first definition is more appropriate for the calculation of export-weighted indices. See, for example, Rhomberg [1976, p. 95] and Honohan [1979, p. 89].

[1] This equality will hold for the exchange rates at a single point in time (e.g., end of a month) but not necessarily for "average" exchange rates over a given period of time, unless the average is defined geometrically. The IMF is not entirely consistent on this point, since average exchange rates for individual currencies are calculated as arithmetic averages, while for the SDR geometric averages are used. For the purposes of this paper, all exchange rates will refer to end-of-month data, so that equality (1) will hold.

[2] Note that for both EER2 and EER4, the reciprocal has been used in order to make these two indices comparable to EER1 and EER3 (i.e., in each case an index larger than 100 signifies an appreciation of the home currency). While EER1 and EER3 could have been inverted instead, this choice in no way affects the analysis presented in the paper.

[3] The terminology follows that employed by Honohan [1979, p. 88f.].

among the levels of the indices at a given point in time, but the two arithmetic indices provide ambiguous measures of the corresponding rates of change. These differences arise in each case from the asymmetric treatment accorded depreciating and appreciating currencies by arithmetic effective exchange rate indices.

1. Levels of the Indices

At each point in time, the three indices must obey the relationship

$$\text{EERH} \leq \text{EERG} \leq \text{EERA} \tag{2}$$

with the strict inequalities holding unless all bilateral exchange rate indices in a particular period are identical[1]. There is a simple explanation for these systematic differences: EERA gives larger weight to currencies which have *depreciated* to a significant extent vis-à-vis the home currency (manifested by a large bilateral exchange rate index R_{it}/R_{io}), while EERH gives larger weight to currencies which have *appreciated* significantly vis-à-vis the home currency (manifested by a large bilateral exchange rate index S_{it}/S_{io}).

This asymmetry arises from the fact that the "distance" between 0 and 1 is much smaller than that between 1 and infinity. Thus, in the case of EERA, an index R_{it}/R_{io} of 10,000 will have a much greater influence on the calculated result than will an index R_{it}/R_{io} of .0001, even though in a very real sense the magnitudes of these two exchange rate changes are identical. Conversely, for EERH the latter type of exchange rate change (which corresponds to an index S_{it}/S_{io} of 10,000) has the greater influence.

In contrast, the geometric index EERG treats depreciating and appreciating currencies in an entirely symmetric manner[2]. We shall henceforth regard EERG as an "unbiased" effective exchange rate index, while the biases of the arithmetic indices EERH and EERA will be defined by the extent to which they differ from EERG.

[1] This follows from the strict concavity of the function $f(x) = \log(x)$ (i.e., $f'' < 0$) which implies,

i) $\sum_i w_i \log(R_i) \leq \log(\sum_i w_i R_i)$, i.e., EERG $\leq$ EERA

ii) $\sum_i w_i \log(R_i) = -\sum_i w_i \log(S_i) \geq -\log(\sum_i w_i S_i)$, i.e., EERG $\geq$ EERH.

[2] This property follows from the fact that EERG is a linear function of the component bilateral exchange rate indices:

$$\log(\text{EERG}) = \sum_i w_i \log(R_{it}/R_{io})$$

Thus, "equivalent" depreciations and appreciations cancel one another, since $\log(x) = -\log(1/x)$. In other words, the "logarithmic distance" between 0 and 1 is identical to that between 1 and infinity.

To evaluate the practical importance of these "biases", import-weighted effective exchange rate indices — EERA, EERH and EERG — were calculated for 156 countries, using as weights the shares of the 25 major import partners in 1976[1]. For each country, indices were calculated for May 1976 and June 1979, based in March 1973 — the first month of generalized floating among the major currencies.

Summary information for the 156 countries in the sample is given by the various "world" averages reported in the upper part of Table 1. Regardless of the method of averaging employed — weighted[2] or un-

Table 1 — *Effective Exchange Rate Indices* (March 1973 = 100)

	May 1976			June 1979		
	EERH	EERG	EERA	EERH	EERG	EERA
World Averages						
Effective exchange rates[a]						
(a) unweighted arithmetic	96.3	100.6	209.6	86.3	95.0	451.9
(b) unweighted geometric	90.7	94.1	116.9	76.6	82.6	131.1
(c) import-weighted arithmetic	97.7	99.5	125.7	95.9	100.5	190.4
(d) import-weighted geometric	94.9	96.8	109.7	88.9	93.7	137.9
Selected countries						
Belgium	102.5	104.8	115.8	112.0	119.3	266.7
Brazil	61.4	76.0	782.1	22.9	32.4	1 074.5
Colombia	71.2	82.6	819.2	53.1	67.6	2 019.4
Denmark	103.7	106.2	119.4	99.1	105.6	260.0
Fed. Rep. of Germany	113.1	114.5	116.2	135.9	141.6	149.7
Italy	69.4	71.8	90.7	59.7	65.3	271.1
Spain	89.9	93.5	118.0	82.4	91.2	358.8
Switzerland	137.4	139.2	141.1	164.1	171.7	183.8
Tonga	116.8	325.7	9 158.2	102.1	365.3	24 966.4
United States	103.7	104.5	105.5	100.1	104.1	110.0
Uruguay	33.6	49.0	363.9	13.2	28.4	648.5

[a] Averages calculated on the basis of import-weighted effective exchange rates for 156 countries: EERH refers to harmonic indices, EERG to geometric indices and EERA to arithmetic indices. For (c) and (d) 1976 imports were used as weights.

Source: Calculations based on information available in IMF [b; c].

[1] The trading partners were limited to those which report exchange rate data to the IMF, i.e., excluding all socialist countries but Romania, and the weights were constructed on the basis of information presented in the IMF *Direction of Trade Yearbook 1972—78.*

[2] In calculating the weighted arithmetic and geometric averages, the weight given to each country was proportional to the value of its imports in 1976.

weighted, arithmetic or geometric — for EERA the average of the 156 individual country values is significantly greater than 100: for June 1979, for example, it ranges from 131 (unweighted geometric) to 452 (unweighted arithmetic). In other words, according to EERA, the "average" world currency significantly appreciated over the period from March 1973 to June 1979. This result casts serious doubt on the validity of EERA as a measure of exchange rate change, for virtually by definition the "average" appreciation of world currencies over any period of time is zero. By this standard, the other two indices fare far better, although for each type of average — and in each time period — the value for EERG is considerably closer to 100 than is that for EERH[1].

While space does not permit reporting of the results for all 156 individual countries, the lower part of Table 1 draws attention to some of the more interesting findings. For Brazil, Colombia and Uruguay, for example, both EERH and EERG reveal a significant *depreciation* of the domestic currency in the period from March 1973 to June 1979, while EERA indicates a tremendous *appreciation* (represented by indices of 1,075, 2,019 and 649, respectively) over the same period. Other cases are equally noteworthy, including that of Tonga, whose EERA in June 1979 was 24,966, as compared to 102 and 365, respectively, for EERH and EERG. It is important to note, however, that this problem is by no means restricted to developing countries, as there are a number of developed countries — e.g., Belgium, Denmark, Italy, Spain and, to a lesser extent, the Federal Republic of Germany, Switzerland and the United States — whose calculated effective exchange rate indices differ markedly according to the mathematical formulation employed.

These results illustrate that when there are significant differences among the indices, it is EERA which is most likely to be "out of line". In other words, over the period under review the upward bias imparted to EERA by depreciating currencies was significantly larger than the downward bias imparted to EERH by appreciating currencies. This result can be attributed in large part to the fundamental asymmetry in recent years between the world's "strong" and "weak" currencies. Thus, measuring exchange rates in terms of SDRs (an "average" world currency) one finds that between March 1973 and June 1979 nineteen currencies depreciated to a greater extent than the Swiss franc, the world's strongest currency over this period[2], appreciated. In particular, the Chilean peso

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[1] If for all countries exports equalled imports, the import-weighted geometric average of EERG would in principle be 100.

[2] I.e., the SDR index of the Swiss franc (Swiss francs per SDR) was 54.92, which corresponds in (geometric) magnitude to a depreciation represented by an index of 182.08.

depreciated by more than 2,000 times the amount by which the Swiss franc appreciated, while for Argentina the corresponding factor was 340.

2. Rates of Change of the Indices

In analyzing indices of effective exchange rates, it is often of considerable interest to examine not only the level of the index at a particular point in time — which measures the change since the base period — but also the *rate* at which the currency is effectively depreciating or appreciating. For a time series of geometric effective exchange rates, the relative change between any two points can be measured by simply comparing the corresponding levels of EERG. In other words, for a given set of weights, the percentage change is independent of the base period and hence is uniquely determined[1].

In contrast, neither of the two arithmetic indices — EERA and EERH — provides an unambiguous measure of the rate of change, since for a given set of weights the result depends on the choice of an initial (or base) period[2]. This can be illustrated as follows: The relative change in EERA between periods 1 and 2, when it is based in period 0, is

$$\begin{aligned} EERA_2/EERA_1 &= [\sum_i w_i R_{i2}/R_{io}] / [\sum_i w_i R_{i1}/R_{io}] \\ &= \sum_i w_i^* R_{i2}/R_{i1} \end{aligned}$$

where the "effective" weights w_i^* are defined by

$$(3) \qquad w_i^* = [w_i R_{i1}/R_{io}] / [\sum_i w_i R_{i1}/R_{io}] = [w_i R_{i1}/R_{io}]/EERA_1$$

Similarly, the relative change in EERH between periods 1 and 2 is equal to

$$EERH_2/EERH_1 = 1 \Bigg/ \frac{\sum_i w_i S_{i2}/S_{io}}{\sum_i w_i S_{i1}/S_{io}} = 1/[\sum_i w_i^{**} S_{i2}/S_{i1}]$$

where

$$(4) \qquad w_i^{**} = [w_i S_{i1}/S_{io}] / [\sum_i w_i S_{i1}/S_{io}] = [w_i S_{i1}/S_{io}] / [1/EERH_1]$$

Equations (3) and (4) show that for both EERA and EERH the relative change between periods 1 and 2 is calculated by using not the original weights w_i but a set of weights reflecting exchange rate movements

[1] This property follows at once from the fact that $[R_{i2}/R_{io}]/[R_{i1}/R_{io}] = R_{i2}/R_{i1}$.

[2] In calculating effective exchange rate indices, the most common practice has been to use a single set of weights for the entire period under consideration, with the first observation serving as the base period and the weights representing trade values in one of the intermediate periods.

from period 0 to period 1. For EERA, currencies which have depreciated (vis-à-vis the home currency) by more than the "average" — or have appreciated by less than the average — will be given more weight in calculating the relative change in subsequent time periods, while currencies depreciating by less than the average (or appreciating by more than the average) will be given less weight. Conversely, in the case of EERH it is currencies which appreciate by more than the average which are given increased weight, while those appreciating by less than the average are given decreased weight.

In order to evaluate the practical importance of the non-uniqueness of relative changes, Table 2 presents information, for the same 156 countries, on the percentage change (expressed as an annual rate) from the end of May 1979 to the end of June 1979. For each index, the corresponding rate of change was calculated for three choices of base period: March 1973, May 1976 and May 1979.

Table 2 — *Annual Rate of Change of World Effective Exchange Rate Indices, May 1979 to June 1979*

	EERH based in			EERG based in			EERA based in		
	March 1973	May 1976	May 1979	March 1973	May 1976	May 1979	March 1973	May 1976	May 1979
Average effective exchange rates[a]									
(a) unweighted arithmetic	-7.5	-7.7	-5.7	-5.5	-5.5	-5.5	13.9	-1.4	-5.3
(b) unweighted geometric	-11.4	-11.6	-9.7	-9.4	-9.4	-9.4	5.7	-5.7	-9.3
(c) import-weighted arithmetic	-1.5	-1.5	0.5	0.7	0.7	0.7	19.8	4.5	0.8
(d) import-weighted geometric	-4.3	-4.3	-2.4	-2.2	-2.2	-2.2	13.6	1.5	-2.0

[a] Averages calculated on the basis of import-weighted effective exchange rates for 156 countries: EERH refers to harmonic indices, EERG to geometric indices and EERA to arithmetic indices. For (c) and (d) 1976 imports were used as weights.

Source: See Table 1.

The figures illustrate that, as noted above, for EERG the choice of base period has no effect on the calculated rate of change. For both EERA and EERH, however, the choice of base period influences — often significantly — the result, with the discrepancies in the rates of change of

EERA being considerably greater than those for EERH. In the case of the weighted-arithmetic averages, for example, basing EERA in March 1973 indicates a world average rate of change of 19.8 per cent, compared to 0.8 per cent when the indices were based in May 1979; for EERH, the corresponding figures were — 1.5 per cent and 0.5 per cent.

For individual countries, the dependence on the base period is most pronounced for those in which there were significant differences in the levels of EERH, EERG and EERA. In Ecuador, for example, when EERA is based in March 1973 it indicates an annual rate of appreciation of 111 per cent between May and June 1979, while a base of May 1976 suggests only a 5.1 per cent appreciation and a base of May 1979 reveals a *depreciation* of 7.5 per cent[1]. For EERH the results are less dramatic, although there are several cases in which the choice of base period significantly affects the calculated rate of change. In Paraguay, for example, a base of either March 1973 or May 1976 suggests a rate of depreciation of about 20 per cent, as compared to less than 2 per cent when the index is based in May 1979.

It is also interesting to note that the differences among the rates of change calculated using May 1979 as a base are uniformly small. Indeed, for the 156 countries in the sample, all four average rates of change of EERG differ from those of both EERH and EERA by less than 0.3 per cent. This result suggests that the inherent inconsistencies which characterize EERH and EERA could be reduced by frequently re-basing the indices, so that the weights are not allowed to substantially diverge from their initial levels. However, the existence of such a means of "rectifying" EERA and EERH can be taken as an argument for using a geometric effective exchange rate index in the first instance, since in the limit such frequent re-basing would produce indices virtually indistinguishable from EERG[2].

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[1] The extreme result for Ecuador's EERA is due to the fact that the implied weights of Argentina and Chile as of May 1979 (86.3 and 10.9 per cent, respectively) bear little resemblance to their original values (2.3 and 1.7 per cent, respectively).

[2] Using the fact that for small x, $x \sim \log(1 + x)$, the period-to-period relative change in EERG can be expressed as

$$
\begin{aligned}
(EERG_t - EERG_{t-1})/EERG_{t-1} &= \log(EERG_t/EERG_{t-1}) \\
&= \sum_i w_i \log(R_{it}/R_{it-1}) = -\sum_i w_i \log(S_{it}/S_{it-1}) \\
&= \sum_i w_i (R_{it} - R_{it-1})/R_{it-1} = 1/[\sum_i w_i (S_{it} - S_{it-1})/S_{it-1}]
\end{aligned}
$$

and the last two expressions are precisely the arithmetic and harmonic average rates of change of the individual bilateral exchange rate indices.

IV. Pegging to a Basket of Currencies

1. The General Case

In many cases, countries which have continued to peg their currency to that of a single country have been confronted with unanticipated (and often undesired) changes in their (trade-weighted) effective exchange rates. In recent years a growing number of countries have therefore chosen instead to peg their currencies to a basket of currencies — where the basket is generally defined to consist of the currencies of the major trading partners — or to the SDR, which itself is defined as a basket of currencies[1]. In each case, the objective is to bring a greater degree of stability to the traded goods sector[2].

The general definition of pegging to a basket is that given by Crockett and Nsouli [1977], who define it to be "stabilising the efffective exchange rate of a currency, where the effective exchange rate is a suitable average of market rates vis-à-vis the currencies of the trading partners". From the preceding discussion, however, it is clear that for a given set of weights there are (at least) three separate ways of pegging to a basket of currencies.

Thus, if a country wishes to peg to a basket of currencies by stabilizing the arithmetic notion of an effective exchange rate (i.e., EERA), it continually revises its exchange rate so that EERA, defined with respect to a given base period, does not change[3]. In practical terms, at any moment a country need only determine its exchange rate vis-à-vis some numéraire currency, since its exchange rates with respect to all other currencies are then determined uniquely by triangulation. Taking the numéraire to be the dollar, a country wishing to maintain a constant EERA can determine its appropriate dollar exchange rate as follows: defining E_{it} = price of one dollar in terms of the i^{th} currency at time t and D_t = price of the home currency at time t in terms of dollars,

[1] Of the 142 countries whose exchange rate regimes were classified by the IMF as of 31 December 1981, 21 were pegging to a basket of currencies and 15 were pegging to the SDR [IMF, c, Feb. 1982, p. 13]. In addition, several countries were managing their exchange rates through reference to an effective exchange rate index.

[2] For a discussion of the advantages of pegging to a basket, see Crockett and Nsouli [1977, pp. 131—137]. Explanations given by central banks for their decisions to peg to a basket can be found in: Bank of Finland [1978], Central Bank of Sweden [1979], Reserve Bank of New Zealand [1979] and Central Bank of Malta [1979].

[3] For practical reasons, most countries which peg to a basket of currencies maintain their appropriate effective exchange rates within narrow margins, rather than maintaining a completely fixed index. This is analogous to the way in which, under the Bretton Woods system, the market rate of a country's currency was allowed to move within narrow margins of its declared par value.

then $R_{it} = E_{it}D_t$, where R_{it} is as defined previously. A constant EERA implies that the following equation holds continuously:

$$1 = \sum_i w_i R_{it}/R_{io}, \quad \text{which in turn is equivalent to}$$

$$1 = \sum_i (w_i E_{it}/E_{io}) D_t/D_o$$

so that the dollar exchange rate at time t is determined by

$$(5) \qquad D_t = D_o/[\sum_i w_i E_{it}/E_{io}]$$

Equation (5) has a clear intuitive interpretation. The denominator is precisely the effective exchange rate, relative to the base period, which would obtain if the home country's rate were pegged to the dollar. When the dominator increases, it is necessary for the home country to reduce its dollar exchange rate (i.e., the price of domestic currency in terms of dollars) so that its effective exchange rate remains unchanged; conversely, when pegging to the dollar would lead to a decrease in the effective exchange rate, it is necessary to increase its dollar exchange rate.

Similarly, for a country wishing to peg to a basket of currencies by maintaining a stable harmonic effective exchange rate EERH, it can be shown that its dollar exchange rate at time t should be defined by

$$(6) \qquad D_t = D_o [\sum_i w_i E_{io}/E_{it}] = D_o [\sum_i w_i F_{it}/F_{io}]$$

where F_{it} ($= 1/E_{it}$) is the price of the i^{th} currency in terms of dollars at time t. If a country wishes to maintain a constant geometric effective exchange rate EERG its appropriate dollar exchange rate is given by

$$D_t = D_o / [\prod_i (E_{it}/E_{io})^{w_i}]$$

An additional method of pegging to a basket is to define the value of a country's currency in terms of the sum of specified units of foreign currencies. Indeed, this is precisely the manner in which the SDR and the ECU are defined, as well as several national currencies[1].

While it would therefore appear that there exist four separate techniques for pegging a currency to a basket of foreign currencies, the "specified units" approach is equivalent to defining the peg by stabilizing a harmonic effective exchange rate index EERH[2]. This can be proved

[1] For example, Malta defines the value of its currency to be equivalent to the sum of specified units of nine major currencies: Deutsche mark, pound sterling, U.S. dollar, Italian lira, French franc, Dutch guilder, Belgian franc, Japanese yen and Swiss franc.

[2] This is apparently not generally appreciated; for example, Black [1976, p. 15f.] implies that the value of the SDR is determined by an effective exchange rate index EERA, with the weights continually adjusted so that the initial weights determine the period-to-period relative changes.

as follows: letting u_i represent the specified amount of the i^{th} currency in the basket, the dollar exchange rate of the home currency at time t is given by

$$(7) \quad D_t = \sum_i u_i F_{it}$$

Similarly, the dollar exchange rate at time o (i.e., the period in which the peg to the basket was begun) is

$$D_o = \sum_i u_i F_{io}$$

so that the implied weight of the i^{th} currency at time o can be defined by

$$(8) \quad w_i = u_i F_{io}/D_o$$

From (7) and (8) it follows that

$$D_t = \sum_i (w_i D_o/F_{io}) F_{it} = D_o [\sum_i w_i F_{it}/F_{io}]$$

which by comparison to equation (6) is seen to be precisely the dollar exchange rate derived by maintaining a constant harmonic effective exchange rate EERH with weights w_i.

For a given set of weights and base period, there are therefore three possible "basket" exchange rates corresponding to stabilization of EERA, EERH or EERG[1]. Let BA_t denote the dollar exchange rate (price of domestic currency in terms of dollars) at time t corresponding to a stable EERA, and define BH_t and BG_t in the analogous fashion. Making use of the fact that

$$\text{EERH} \leq \text{EERG} \leq \text{EERA} \quad \text{for all t}$$

we have the following reversed set of inequalities in terms of the three basket exchange rates:

$$BH_t \geq BG_t \geq BA_t \quad \text{for all t}$$

In other words, the dollar exchange rate which yields an unchanged EERH would lead to both EERG and EERA being greater than 100; hence, the corresponding "basket" dollar exchange rate for each of these two indices must be less than that for EERH.

A conclusion reached earlier in the paper was that the two arithmetic indices, EERA and EERH, are systematically biased with respect to the symmetric index EERG, with the extent of the bias generally in-

[1] In practice, it would appear that basket pegging is currently restricted to stabilizing — or managing — EERH (e.g., Malta, Finland, Sweden) or EERA (e.g., Australia, New Zea- and, Samoa). A number of countries pegging to baskets do not publish sufficient information lto allow a determination of the mathematical formulation employed.

creasing over time. From this it follows that the corresponding methods of pegging to a basket are also systematically biased: BA_t, which gives greater weight to "depreciating" currencies, is biased downwards over time, while BH_t, which gives greater weight to "appreciating" currencies, is biased upwards over time.

It is difficult to measure the extent of these biases for individual countries which peg to a basket of currencies, however, since the precise weights used are generally not disclosed[1]. Nevertheless, the quantitative evidence presented earlier suggests that the problem is potentially a serious one, and that basket peggers should perhaps give consideration to stabilizing EERG. Indeed, the technique currently employed by several countries which peg to a basket — that of frequently "re-initializing" the weights — was shown earlier to be essentially equivalent to maintaining a constant EERG. Use of this "remedy" can create a problem of credibility, however, for one of the ostensible virtues of pegging to a basket is the "automaticity" of the procedure. Finally, it should be noted that there is one potential advantage in pegging to a basket of currencies via EERH — although only in cases in which the weights are publicly disclosed and remain fixed — namely, that traders and investors can, at least in principle, protect themselves against exchange risk by "simulating" the physical basket of currencies.

2. The SDR as a Basket of Currencies

Since 1 July 1974, the value of the SDR has been calculated as a basket of specified currencies, i.e., by stabilizing a harmonic effective exchange rate index with initial weights determined by the specified units as described above[2]. The sixteen currencies initially chosen to form the SDR basket were those of the IMF members having the largest exports of goods and services over the period 1968—1972, and the corresponding weights were determined roughly in proportion to each country's share in world exports[3]. The weights were first revised in July 1978, in order to take account of shifts in world trade patterns. In January 1981, the basket of currencies was reduced from sixteen to five, the aim

[1] Disclosure of the weights could lead to one-sided speculation in those cases in which the exchange rate (vis-à-vis a numéraire, generally the dollar) is adjusted at discrete intervals rather than continuously.

[2] From January 1970 to December 1971 the value of the SDR was defined to be precisely one U.S. dollar. Following the U.S. devaluations of December 1971 and February 1973 the SDR maintained its parity in terms of gold, i.e., increasing in value to $ 1.08571 in December 1971 and to $ 1.20635 in February 1973.

[3] This was done with the proviso that the weight of the U.S. dollar was set at 33 per cent to reflect its greater importance in financial transactions [IMF, d].

being to "unify and simplify... the currency baskets that determine the value of and the interest rate on the SDR"[1].

One means of assessing the "bias" in the SDR is to compare its level with the values which would have prevailed had the SDR been determined by stabilizing, instead of the harmonic, the corresponding geometric (EERG) effective exchange rate index. This alternative SDR will be denoted SDRG; the corresponding SDR based on stabilization of the arithmetic index EERA will be denoted SDRA. From the earlier discussion, at each point in time the following inequalities must hold

$$\text{SDR} \geq \text{SDRG} \geq \text{SDRA}$$

Taking SDRG as an "unbiased" measure, it can be shown that the biases of both the actual SDR and of SDRA have increased since 1974; as of 31 December 1979, for example, each was of the order of two per cent. These biases are small relative to those noted earlier, reflecting the fact that among the sixteen currencies relative exchange rate movements were of a moderate nature.

An alternative means of evaluating the potential bias in the calculation of the SDR is to analyze the evolution of the implied weights making up the SDR basket. From the previous discussion we know that, over time, the weights in the SDR of "appreciating" currencies will increase, while those of depreciating currencies will decrease. To quantify this, the implied weights as of 30 June 1978 (i.e., the day before the first revision of weights took effect) were compared to the initial weights which had been introduced four years earlier. The weights of the two strongest currencies over this period — the Deutsche mark and the Japanese yen — were found to have increased substantially: thus, while the initial currency units had been chosen so as to give these two currencies a combined weight of somewhat less than 20 per cent, by 30 June 1978 their combined share had risen by more than one quarter. Conversely, as a result of their general depreciation over this period, the implied weights of the pound sterling, the Italian lira, the Spanish peseta and the Australian dollar had each fallen by approximately one quarter.

In the revised set of weights which took effect on 1 July 1978, the weights of both the Deutsche mark and the Japanese yen were reduced considerably, while those of a number of the "depreciating" currencies — including the four noted above — were substantially increased. The

[1] According to IMF Press Release No. 80/66, issued 18 September 1980. The five countries constituting the new SDR basket are: United States, Federal Republic of Germany, Japan, France and the United Kingdom. The interest rate paid to SDR holders (and charged to SDR users) had been based since 1974 on a weighted average of interest rates in these five countries.

pattern of these revisions was such as to effectively restore the weights of most of these currencies to their initial levels. This would seem to suggest that, in addition to the stated reason of incorporating changes in world trade patterns, an important motivation may also have been to make "corrections" for the biases which had arisen from the method of arithmetic averaging underlying the calculation of the SDR. One might well argue that this modification in the weights introduced a certain degree of discontinuity in the rate of change of the SDR, for the influence of changes in the exchange rates of each of these six currencies on the value of the SDR differed considerably on 1 July 1978 from the level of the previous day. To a large extent, moreover, the altered significance of these currencies reflected not a change in their perceived importance but rather an attempt to correct the mathematical imperfections of the relevant exchange rate index.

V. Conclusion

The debate in recent years concerning the appropriate manner of defining effective exchange rates has largely ignored the question of the proper mathematical formulation to employ. As a result, in the literature the choice of mathematical formulation has often been a haphazard one, governed far more by custom than by a particular belief in the merit of the index chosen[1]. Moreover, in many cases, the formulation employed has not been specified, apparently in the belief that the term "trade-weighted (effective) exchange rate" is sufficiently precise.

The quantitative results of this paper strongly suggest, however, that the issue merits far more attention than it is generally given. The most commonly employed measures of effective exchange rates — those based on arithmetic averages — were shown to be subject to potentially large biases arising from their asymmetric treatment of depreciating and appreciating currencies. Moreover, unlike those for geometric indices of effective exchange rates, the results for arithmetic indices are extremely sensitive to the arbitrary choice between two equally plausible definitions of "the" exchange rate.

[1] For example, among central banks the general practice has been for those in countries where the exchange rate is customarily quoted in terms of the price of domestic currency (e.g., Australia and New Zealand) to utilize EERA, while most of the remaining central banks have chosen EERH. However, in several cases this custom has been outweighed by the preference for using EERA in conjunction with export-weighted indices (e.g., Switzerland) and EERH in conjunction with import-weighted indices (e.g., Ireland). Finally, for reasons similar to the arguments advanced in this paper, in recent years at least three central banks (those in the Federal Republic of Germany, the U.K. and the U.S.) have switched to geometric indices (see, for example, the discussion in Deutsche Bundesbank [1979]).

These findings were seen to have important implications for analyses of both the direction and magnitude of exchange rate movements as well as for the large number of countries which are seeking exchange rate stability by means of pegging to a basket of currencies. Indeed, the term "pegging to a basket" was seen to be an ambiguous one, for it depends in a fundamental way on the mathematical formulation of effective exchange rate adopted. The two most frequently used methods of pegging to a basket suffer from inherent biases, reflecting the fact that the weights employed in calculating period-to-period changes differ progressively over time from the "optimal" weights initially chosen to define the basket.

Finally, it should be noted that in recent years a number of studies have attempted to evaluate the impact of the present exchange rate regime on developing countries. In general, such studies have focused on the extent to which the instability in effective exchange rates of individual countries has increased in the post-Bretton-Woods era[1]. The results of the present paper would suggest, however, that policy conclusions drawn from such analyses should be interpreted with caution in those cases in which the formulation of effective exchange rate employed has not been specified.

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