Regularized Logistic Regression

We can regularize logistic regression in a similar way that we regularize linear regression. As a result, we can avoid overfitting. The following image shows how the regularized function, displayed by the pink line, is less likely to overfit than the non-regularized function represented by the blue line:

Regularized logistic regression.

$$h_{\theta}(x) = g(\theta_{0} + \theta_{1}x_{1} + \theta_{2}x_{1}^{2} + \theta_{3}x_{1}^{2}x_{2} + \theta_{4}x_{1}^{2}x_{2}^{2} + \theta_{5}x_{1}^{2}x_{2}^{3} + \dots)$$

Cost function:

$$J(\theta) = -\left[\frac{1}{m} \sum_{i=1}^{m} y^{(i)} \log h_{\theta}(x^{(i)}) + (1 - y^{(i)}) \log(1 - h_{\theta}(x^{(i)}))\right]$$

$$+ \frac{\lambda}{2m} \sum_{j=1}^{n} \bigotimes_{j}^{2} \left(\bigcirc_{i}, \bigcirc_{i}, \dots, \bigcirc_{n} \right)$$

Cost Function

Recall that our cost function for logistic regression was:

$$J(heta) = -rac{1}{m} \sum_{i=1}^m [y^{(i)} \; \log(h_ heta(x^{(i)})) + (1-y^{(i)}) \; \log(1-h_ heta(x^{(i)}))]$$

We can regularize this equation by adding a term to the end:

$$J(heta) = -rac{1}{m} \sum_{i=1}^m [y^{(i)} \; \log(h_{ heta}(x^{(i)})) + (1-y^{(i)}) \; \log(1-h_{ heta}(x^{(i)}))] + rac{\lambda}{2m} \sum_{j=1}^n heta_j^2$$

The second sum, $\sum_{j=1}^n \theta_j^2$ means to explicitly exclude the bias term, θ_0 . I.e. the θ vector is indexed from 0 to n (holding n+1 values, θ_0 through θ_n), and this sum explicitly skips θ_0 , by running from 1 to n, skipping 0. Thus, when computing the equation, we should continuously update the two following equations:

Gradient descent



$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_0^{(i)}$$

$$\theta_{0} := \theta_{0} - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_{\theta}(x^{(i)}) - y^{(i)}) x_{0}^{(i)}$$

$$\theta_{j} := \theta_{j} - \alpha \left[\frac{1}{m} \sum_{i=1}^{m} (h_{\theta}(x^{(i)}) - y^{(i)}) x_{j}^{(i)} + \frac{\lambda}{m} \Theta_{j} \right]$$