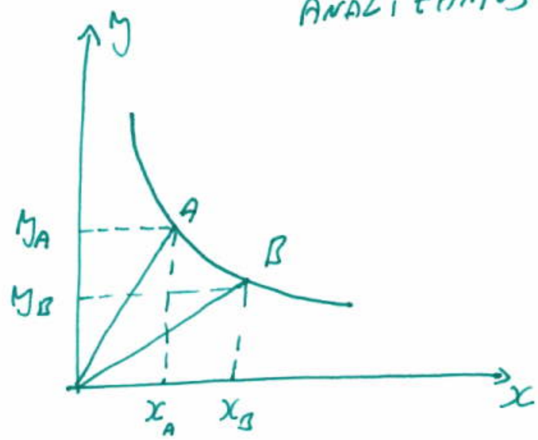
ELASTICIDAD DE SUBSTITUCIÓN

1.- INTUICIÓN GRÁFICA

ANALIZAMOS: $u(x,y) = c$

COMPOSICIÓN DE LA CANASTA:



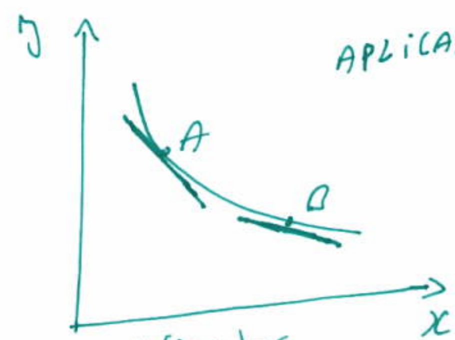
DE $A \to B$

$$\frac{y_A}{x_A} \longrightarrow \frac{y_B}{x_B} \qquad \Delta\left(\frac{y}{x}\right) = \left(\frac{y_B}{x_B} - \frac{y_A}{x_A}\right)$$

EN TÉRMINOS PROPORCIONALES:

$$\frac{\Delta\left(\frac{y}{x}\right)}{\frac{y}{x}}$$

PENDIENTES DE LA CURVA DE INDIFERENCIA:



APLICAMOS DERIVADA IMPLÍCITA:

$$\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = 0$$

$$\frac{dy}{dx} = -\frac{\partial u/\partial x}{\partial u/\partial y}$$

AL DECIR: $u(x,y)=c$

TENEMOS UNA FUNCIÓN IMPLÍCITA

$y = \varphi(x)$

OJO: NECESITAMOS

$\partial u/\partial y \neq 0$

$$\Delta\left(-\frac{\partial u/\partial x}{\partial u/\partial y}\right) = \Delta(\text{PENDIENTE}) = -\frac{\partial u/\partial x}{\partial u/\partial y}\bigg|_B - \left(-\frac{\partial u/\partial x}{\partial u/\partial y}\right)\bigg|_A$$

ESTAS DERIVADAS SON EVALUADAS EN B Y A

EN FORMA PROPORCIONAL:

$$\frac{\Delta\left(\frac{\partial u/\partial x}{\partial u/\partial y}\right)}{\frac{\partial u/\partial x}{\partial u/\partial y}}$$

ELASTICIDAD DE SUBSTITUCIÓN:

$$S = \frac{\frac{\Delta(y/x)}{y/x}}{\cdots}$$

RECORDEMOS:

$$d\ln z = \frac{dz}{z} \approx \frac{\Delta z}{z}$$

APLICAMOS A NUESTRA DEFINICIÓN DE S:

$$\frac{\Delta(y/x)}{y/x} = \partial \ln(y/x)$$

$$\frac{\Delta\left(\frac{\partial u/\partial x}{\partial u/\partial y}\right)}{\frac{\partial u/\partial x}{\partial u/\partial y}} = \partial \ln\left(\frac{\partial u/\partial x}{\partial u/\partial y}\right)$$

ENTONCES:

$$S = \frac{\partial \ln(y/x)}{\partial \ln\left(\frac{\partial u/\partial x}{\partial u/\partial y}\right)}$$

2.- CALCULAR S PARA LA CES.

$$u(x,y) = \left(a x^{\rho} + (1-a) y^{\rho}\right)^{1/\rho}$$

PACITO A PACITO:
$$\frac{\partial u}{\partial x} = \frac{1}{\rho}\left(a x^{\rho} + (1-a) y^{\rho}\right)^{\frac{1}{\rho}-1} a \rho x^{\rho-1}$$

$$\frac{\partial u}{\partial y} = \frac{1}{\rho}\left(a x^{\rho} + (1-a) y^{\rho}\right)^{\frac{1}{\rho}-1} (1-a) \rho y^{\rho-1}$$

$$\frac{\partial u}{\partial x} \Big/ \frac{\partial u}{\partial y} = \frac{a}{1-a} \cdot \left(\frac{x}{y}\right)^{\rho-1} = \frac{a}{1-a}\left(\frac{y}{x}\right)^{1-\rho}$$

$$\ln\left(\frac{\partial u/\partial x}{\partial u/\partial y}\right) = \ln\frac{a}{1-a} + (1-\rho)\ln\left(\frac{y}{x}\right)$$

$$\frac{\partial \ln\left(\frac{\partial u/\partial x}{\partial u/\partial y}\right)}{\partial \ln(y/x)} = (1-\rho)$$

(PIENSALO ASÍ: $z = \dfrac{\partial u/\partial x}{\partial u/\partial y}$ $\quad w = y/x$

$$\ln z = \ln\left(\frac{a}{1-a}\right) + (1-\rho)\ln w$$

$$H = \ln z$$
$$G = \ln w$$

$$H = \ln\frac{a}{1-a} + (1-\rho)G$$

$$\frac{\partial H}{\partial G} = 1-\rho$$)

ENTONCES: $s = \dfrac{1}{1-\rho}$

RECORDAR: $\ln(a \cdot b) = \ln a + \ln b$

$$\ln\left(\frac{1}{a}\right) = -\ln a$$

$$\ln a^b = b\ln a$$