

MARGINAL LIKELIHOOD

$$\Pi = \sigma_h^2 \mathbf{I}$$

$$p(y^d | z, \sigma_y^2) = \int p(y^d | z, h^d, \sigma_y^2) \underbrace{p(h^d)}_{p(h^d | z, \sigma_y^2)} dh^d$$

$$\propto \int \exp \left\{ -\frac{1}{2} [(h^d)^H \ (h^d)^T] \begin{bmatrix} \sigma_h^2 \mathbf{I} & 0 \\ 0 & \sigma_h^2 \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} h^d \\ (h^d)^* \end{bmatrix} \right\} \\ \times \exp \left\{ -\frac{1}{2} [(y^d - S h^d)^H \ (y^d - S h^d)^T] \begin{bmatrix} \sigma_y^2 \mathbf{I}_T & 0 \\ 0 & \sigma_y^2 \mathbf{I}_T \end{bmatrix}^{-1} \begin{bmatrix} y^d - S h^d \\ (y^d - S h^d)^* \end{bmatrix} \right\} dh^d$$

Dentro exponential:

$$(h^d)^H \Pi^{-1} h^d + (h^d)^T \Pi^{-1} (h^d)^* - (y^d)^H \sigma_y^{-2} S h^d - (h^d)^H S^H \sigma_y^{-2} y^d \\ + (h^d)^H S^H \sigma_y^{-2} S h^d + \underbrace{(y^d)^H \sigma_y^{-2} y^d} - (y^d)^T \sigma_y^{-2} S^* (h^d)^* - (h^d)^T S^T \sigma_y^{-2} (y^d)^* \\ + (h^d)^T S^T \sigma_y^{-2} S^* (h^d)^* + \underbrace{(y^d)^T \sigma_y^{-2} (y^d)^*} \\ = (y^d)^H \sigma_y^{-2} y^d + (y^d)^T \sigma_y^{-2} (y^d)^* + (h^d - \mu_h)^H \Pi_h^{-1} (h^d - \mu_h) + (h^d - \mu_h)^T \Pi_h^{-1} (h^d - \mu_h)^* \\ + C$$



TRANSITION PROBABILITIES FOR COLLAPSED GIBBS SAMPLING

• $p(s_{tm}=k | s_{\tau tm})$

$$A^m = \begin{bmatrix} a^m & 1-a^m \\ b^m & 1-b^m \end{bmatrix}$$

We assume binary states s_{tm} .

For non-binary states, $\text{prob}(s_{tm} = \text{ActiveState}(k) | s_{\tau tm}) = \frac{1}{\# \text{ActiveStates}} p(s_{tm} = \text{act} | s_{\tau tm})$

Prior $\begin{cases} a^m \sim \text{Beta}(1, \frac{\alpha}{m}) \\ b^m \sim \text{Beta}(\gamma_1, \gamma_2) \end{cases}$

Def. $\begin{cases} j = s_{(t-1)m} \\ l = s_{(t+1)m} \end{cases}$

For $k \neq j$

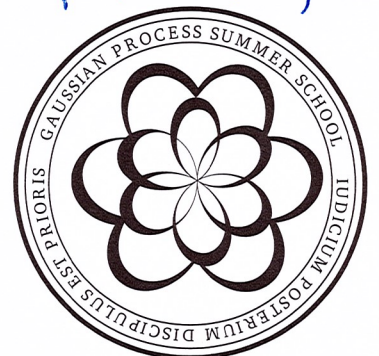
$$p(s_{tm}=k | s_{\tau tm}) \propto \int p(s_{tm}=k | a_j^m) p(a_j^m | \text{rest}) da_j^m \times \int p(s_{(t+1)m}=l | a_k^m) p(a_k^m | \text{rest}) da_k^m$$

③ For $j=0, k=1$ $\emptyset \rightarrow A \rightarrow x$

$$\begin{aligned} & \int (1-a^m) \times \text{Beta}(a^m | 1+n_{00}^m, n_{01}^m) da^m \times \int \overbrace{(b^m)^{l=0}}^{l=0} \overbrace{(1-b^m)^{l \neq 0}}^{l \neq 0} \times \text{Beta}(b^m | \gamma_1+n_{10}^m, \gamma_2+n_{11}^m) db^m \\ &= \frac{n_{01}^m}{1+n_{00}^m+n_{01}^m} \times \frac{\overbrace{(\gamma_1+n_{10}^m)^{l=0}}^{l=0} \overbrace{(\gamma_2+n_{11}^m)^{l \neq 0}}^{l \neq 0}}{\gamma_1+\gamma_2+n_{10}^m+n_{11}^m} \end{aligned}$$

② For $j \neq 0, k=0$: $A \rightarrow \emptyset \rightarrow x$

$$\begin{aligned} & \int b^m \times \text{Beta}(b^m | \gamma_1+n_{10}^m, \gamma_2+n_{11}^m) db^m \times \int \overbrace{(a^m)^{l=0}}^{l=0} \overbrace{(1-a^m)^{l \neq 0}}^{l \neq 0} \times \text{Beta}(a^m | 1+n_{00}^m, n_{01}^m) da^m \\ &= \frac{\gamma_1+n_{10}^m}{\gamma_1+\gamma_2+n_{10}^m+n_{11}^m} \times \frac{\overbrace{(1+n_{00}^m)^{l=0}}^{l=0} \overbrace{(n_{01}^m)^{l \neq 0}}^{l \neq 0}}{1+n_{00}^m+n_{01}^m} \end{aligned}$$



For $k=j$

$$p(s_{tm}=k | s_{7tm}) \propto \int p(s_{t+s)m=l, s_{tm}=k | a_k^m) p(a_k^m | \text{rest}) da_k^m$$

③ For $j=k=0$ $\emptyset \rightarrow \emptyset \rightarrow x$

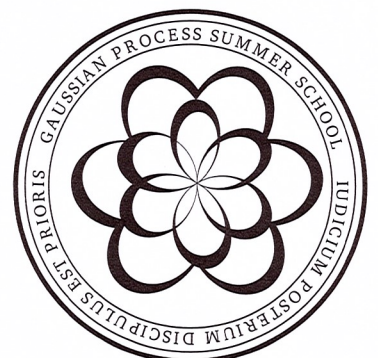
$$\begin{aligned} & \int a^m \cdot \overbrace{(a^m)}^{l=0} \overbrace{(1-a^m)}^{l \neq 0} \times \text{Beta}(a^m | 1+n_{00}^m, n_{01}^m) da^m \\ &= \frac{\overbrace{(2+n_{00}^m)}^{l=0} \overbrace{(1+n_{00}^m)}^{l \neq 0}}{(2+n_{00}^m+n_{01}^m) (1+n_{00}^m+n_{01}^m)} \end{aligned}$$

④ For $j=k \neq 0$ $A \rightarrow A \rightarrow x$

$$\begin{aligned} & \int (1-b^m) \cdot \overbrace{(b^m)}^{l=0} \overbrace{(1-b^m)}^{l \neq 0} \times \text{Beta}(b^m | \gamma_3+n_{30}^m, \gamma_2+n_{31}^m) db^m \\ &= \frac{\overbrace{(\gamma_3+n_{30}^m)}^{l=0} \overbrace{(\gamma_2+n_{31}^m)}^{l \neq 0}}{(\gamma_3+\gamma_2+n_{30}^m+n_{31}^m+1) (\gamma_3+\gamma_2+n_{30}^m+n_{31}^m)} \end{aligned}$$

To ADD NEW CHAINS

The prior is $\text{Poisson}(\frac{\alpha}{T})$.



For this to be equal:

$$\begin{cases} \mu_H = \Pi_H S^H \sigma_y^{-2} y^d \\ \Pi_H^{-1} = \Pi^{-1} + S^H \sigma_y^{-2} S \end{cases}$$

The term

$$\begin{aligned} & \cancel{(h^d)^H \Pi^{-1} h^d} + \cancel{(h^d)^T \Pi^{-1} (h^d)^*} - \cancel{(y^d)^H \sigma_y^{-2} S h^d} - \cancel{(h^d)^H S^H \sigma_y^{-2} y^d} \\ & + \cancel{(h^d)^H S^H \sigma_y^{-2} S h^d} - \cancel{(y^d)^T \sigma_y^{-2} S^* (h^d)^*} - \cancel{(h^d)^T S^T \sigma_y^{-2} (y^d)^*} \\ & + \cancel{(h^d)^T S^T \sigma_y^{-2} S^* (h^d)^*} \\ \triangleq & \cancel{(h^d)^H \Pi^{-1} h^d} + (h^d)^H S^H \sigma_y^{-2} S h^d + (y^d)^H \sigma_y^{-2} S \Pi_H^H \cancel{\Pi_H^{-1}} \cancel{\Pi_H} S^H \sigma_y^{-2} y^d \\ & - \cancel{(y^d)^H \sigma_y^{-2} S \Pi_H^H \Pi_H^{-1} h^d} - \cancel{(h^d)^H \Pi_H^{-1} \cancel{\Pi_H} S^H \sigma_y^{-2} y^d} \\ & + \cancel{(h^d)^T \Pi^{-1} (h^d)^*} + (h^d)^T S^H \sigma_y^{-2} S (h^d)^* + (y^d)^T \sigma_y^{-2} S^* \Pi_H^* S^T \sigma_y^{-2} (y^d)^* \\ & - \cancel{(y^d)^T \sigma_y^{-2} S^* (h^d)^*} - \cancel{(h^d)^T S^T \sigma_y^{-2} (y^d)^*} + C \end{aligned}$$

$$\Rightarrow C = -(y^d)^H \sigma_y^{-2} S \Pi_H^H S^H \sigma_y^{-2} (y^d) - (y^d)^T \sigma_y^{-2} S^* \Pi_H^* S^T \sigma_y^{-2} (y^d)^*$$

Therefore:

$$p(y^d | z, \sigma_y^2) = \mathcal{CN}(y^d | \mu_y, \Pi_y, C_y = 0)$$

where:

$$\begin{aligned} \mu_y &= 0 \\ \Pi_y &= \left(\frac{1}{\sigma_y^2} \mathbf{I}_T - \frac{1}{\sigma_y^4} S \Pi_H^H S^H \right)^{-1} \\ C_y &= 0 \end{aligned}$$

with

$$\Pi_H = \left(\Pi^{-1} + \underset{\sigma_h^2 \mathbf{I}}{S^H \sigma_y^{-2} S} \right)^{-1}$$