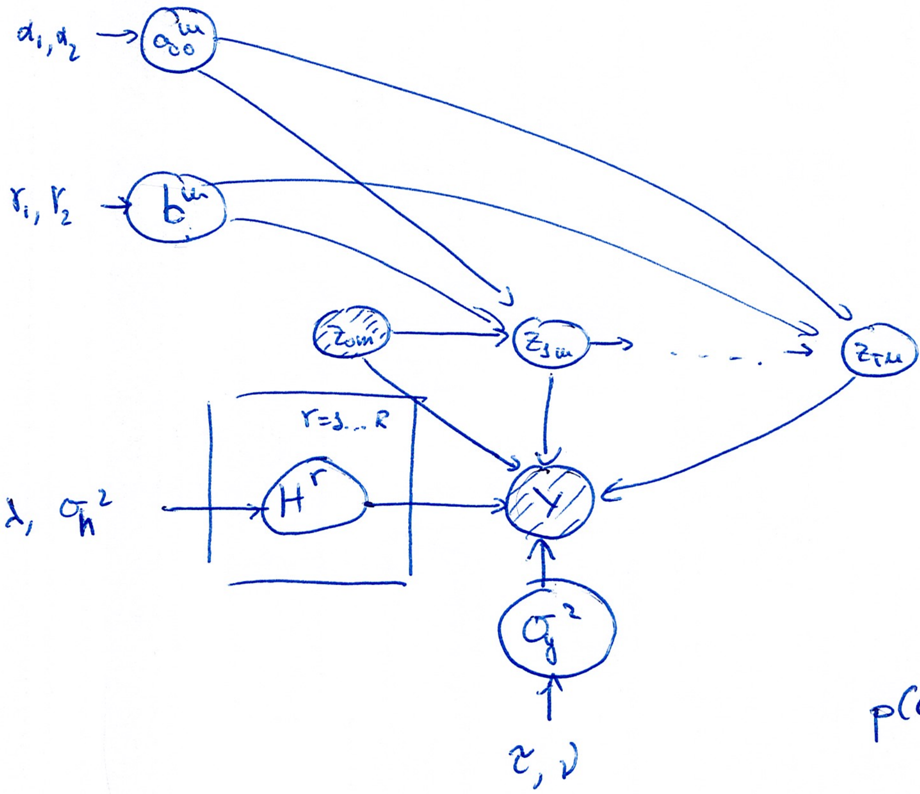
# MODELO MIMO




$$a_{00}^m \sim \text{Beta}\left(\alpha_1, \frac{\alpha_1 \alpha_2}{M}\right)$$

$$a_{0q} = \frac{1}{Q}\left(1 - a_{00}^m\right)$$

$$b_m \sim \text{Beta}\left(\gamma_1, \gamma_2\right)$$

$$a_{q0} = b_m$$

$$a_{qj} = \frac{1}{Q}\left(1 - b_m\right)$$

$$p(\sigma_y^2) = \text{I-Gamma}(\nu, \tau) = \frac{\tau^{\nu} (\sigma_y^2)^{-\nu-1} e^{-\tau/\sigma_y^2}}{\Gamma(\nu)}$$

- **Likelihood** :

$$y_t = \sum_{m=1}^{M} \sum_{r=1}^{R} z_{(t-r)m} \underline{h}_m^r + n_t$$

$$y_t \sim \mathcal{CN}\left(\sum_{m=1}^{M} \sum_{r=1}^{R} z_{(t-r)m} h_m^r, \sigma_y^2 I_D, C=0\right)$$

- **Prior over $h_m^r$** $(N_r \times 1)$

1) $$p(h_m^r) = \mathcal{CN}\left(0, \sigma_h^2 e^{-\lambda r} I_D, C=0\right)$$

2) $$p(\underset{\substack{\downarrow \\ r_x}}{h_{ml}^r}) = \pi_r \delta_0 + (1 - \pi_r)\, \mathcal{CN}\left(0, \sigma_h^2\right)$$

$$\pi_r \sim \text{Beta}\left(\underset{\substack{\downarrow \\ \gg 1}}{\omega_1}, f(\omega_2 r)\right) \begin{cases} f = \frac{\omega_2}{r} \\ f = \exp(-\omega_2 r) \end{cases}$$

INFERENCE

1) $Z(\{H^r\}, \{a_{00}, b_m\}_{m=1}^{M_+}, \sigma_y^2)$

- pMCMC
- EP
- ¿BCJR?

2) ~~[illegible]~~

$$H = \underbrace{\left[ \underset{\substack{\uparrow \\ m}}{\overset{L \times D}{\overbrace{H_1^T}}},\ H_2^T \ \ldots\ldots,\ H_{M_t}^T \right]^T}_{(N_t L) \times D}$$

$$h^{(d)}_{(N_t L \times 1)} \sim \mathcal{CN}\left(\underline{0},\ \Gamma,\ \underset{(N_t L \times N_t L)}{C = 0}\right)$$

$$W = \begin{bmatrix} \sigma_h^2 & & \emptyset \\ & \sigma_h^2 e^{-\lambda} & \\ \emptyset & & \ddots \\ & & & \sigma_h e^{-\lambda(L-1)} \end{bmatrix}_{L \times L}$$

$$\Gamma = I_{N_t} \otimes W$$

$$p(h^d) \propto \exp\left\{ -\frac{1}{2} \left[ (h^d)^H \ (h^d)^T \right] \begin{pmatrix} \Gamma & \emptyset \\ \emptyset & \Gamma \end{pmatrix}^{-1} \begin{bmatrix} h^d \\ (h^d)^* \end{bmatrix} \right\}$$

$$p(h^d \mid y^d, z) \propto p(h^d)\ p(y^d \mid z, h^d)$$

$$p(\underset{(T \times 1)}{y^d} \mid h^d, z) \propto \exp\left\{ -\frac{1}{2} \left[ (y^d - Sh^d)^H \ (y^d - Sh^d)^T \right] \begin{pmatrix} \sigma_y^2 I_T & 0 \\ 0 & \sigma_y^2 I_T \end{pmatrix}^{-1} \begin{bmatrix} (y^d - Sh^d) \\ (y^d - Sh^d)^* \end{bmatrix} \right\}$$

donde $\underset{T \times N_t L}{S} = [S_1, S_2, \ldots S_{M_t}]$

$$(S_1)_{T \times L} = \begin{bmatrix} z_{1m} & 0 & 0 & \cdots & 0 \\ z_{2m} & z_{1m} & 0 & \cdots & 0 \\ \vdots & \vdots & z_{1m} & & \vdots \\ \vdots & \vdots & \vdots & & \\ z_{Tm} & z_{(T-1)m} & z_{(T-2)m} & \cdots & z_{(T-L+1)m} \end{bmatrix}$$

$$p(h^d \mid y^d, z) \propto p(h^d)\, p(y^d \mid z, h^d)$$

$$\propto \exp\left\{ -\frac{1}{2} \left[ (h^d)^H \Gamma^{-1} \quad (h^d)^T \Gamma^{-1} \right] \begin{bmatrix} h^d \\ (h^d)^* \end{bmatrix} \right\}$$

$$\times \exp\left\{ -\frac{1}{2} \left[ \left( (y^d - Sh^d)^H \sigma_y^{-2} I_T \right) \quad \left( (y^d - Sh^d)^T \sigma_y^{-2} I \right) \right] \begin{bmatrix} y^d - Sh^d \\ (y^d - Sh^d)^* \end{bmatrix} \right\}$$

$$= \exp\left\{ -\frac{1}{2} \left[ (h^d)^H \Gamma^{-1} h^d + (h^d)^T \Gamma^{-1} (h^d)^* + (y^d - Sh^d)^H \sigma_y^{-2} I_T (y^d - Sh^d) \right.\right.$$

$$\left.\left. + (y^d - Sh^d)^T \sigma_y^{-2} I_T (y^d - Sh^d)^* \right] \right\}$$

- Dentro exponencial:

$$(h^d)^H \Gamma^{-1} h^d + (h^d)^T \Gamma^{-1} (h^d)^* - (y^d)^H \sigma_y^{-2} S h^d - (h^d)^H S^H \sigma_y^{-2} y^d$$

$$+ (h^d)^H S^H \sigma_y^{-2} S h^d - (y^d)^T \sigma_y^{-2} S^* (h^d)^* - (h^d)^T S^T \sigma_y^{-2} (y^d)^*$$

$$+ (h^d)^T S^T \sigma_y^{-2} S^* (h_d)^*$$

$$= (h^d)^H \left( \Gamma^{-1} + S^H \sigma_y^{-2} S \right) h^d + (h^d)^T \left( \Gamma^{-1} + S^T \sigma_y^{-2} S^* \right) (h^d)^*$$

$$- (y^d)^H \sigma_y^{-2} S h^d - (h^d)^H S^H \sigma_y^{-2} y^d - (y^d)^T \sigma_y^{-2} S^* (h^d)^*$$

$$- (h^d)^T S^T \sigma_y^{-2} (y^d)^*$$

$$\Gamma_H = \left(\Pi^{-1} + S^H \sigma_y^{-2} S\right)^{-1}$$

$$\mu_H = \Gamma_H S^H \sigma_y^{-2} y^d$$

- POSTERIOR OVER H:

$$p(h^d \mid \text{rest}) = \mathcal{CN}\left(\Gamma_H S^H \sigma_y^{-2} y^d,\ \Gamma_H,\ C=0\right)$$

$$\Gamma_H = \left(\Pi^{-1} + S^H \sigma_y^{-2} S\right)^{-1} \quad \text{Size: } (N_t L) \times (N_t L)$$

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- FOR $\sigma_y^2$:

$$p(\sigma_y^2) = \text{InvGamma}(\nu, \tau) = \frac{\tau^{\nu} (\sigma_y^2)^{-\nu-1} e^{-\tau/\sigma_y^2}}{\Gamma(\nu)}$$

- POSTERIOR:

$$p(\sigma_y^2 \mid \text{rest}) \propto p(\sigma_y^2)\, p(Y \mid H, Z, \sigma_y^2)$$

$$\propto (\sigma_y^2)^{-\nu-1} e^{-\tau/\sigma_y^2} \times \prod_{d=1}^{N_r} \mathcal{CN}\left(y^d \mid S h^d,\ \sigma_y^2 I_T,\ C=0\right)$$

$$\propto (\sigma_y^2)^{-\nu-1} e^{-\tau/\sigma_y^2} \prod_{d=1}^{N_r} \frac{1}{\sqrt{(\sigma_y^2)^T (\sigma_y^2)^T}} \exp\left\{-\frac{1}{2\sigma_y^2} \times \right\}$$

$$\left[(y^d - Sh^d)^H, (y^d - Sh^d)^T\right] \begin{bmatrix} y^d - Sh^d \\ (y^d - Sh^d)^* \end{bmatrix}$$

$$\propto (\sigma_y^2)^{-\nu-1-TN_r} \exp\left\{-\frac{1}{\sigma_y^2}\left(\tau + \sum_{d=1}^{N_r} \frac{1}{2}\left[(y^d - Sh^d)^H (y^d - Sh^d) + (y^d - Sh^d)^T (y^d - Sh^d)^*\right]\right)\right.$$

$$\propto \text{Inv-Gamma}\left(\nu + TN_r,\ \tau + \frac{1}{2}\sum_{d=1}^{N_r} \|y^d - Sh^d\|^2\right)$$