MARGINAL LIKELIHOOD

$\Pi = \sigma_h^2 I$

$$p(Y^d|Z, \sigma_y^2) = \cancel{\int p(Y|Z, H, \sigma_y^2)\, p(H)\, dH}$$

$$= \int p(Y^d|Z, H^d, \sigma_y^2)\, \overbrace{p(H^d|Z, \sigma_y^2)}^{p(H^d)}\, dH$$

$$\propto \int \exp\left\{-\frac{1}{2}\begin{bmatrix}(h^d)^H & (h^d)^T\end{bmatrix}\begin{bmatrix}\sigma_h^2 I & 0 \\ 0 & \sigma_h^2 I\end{bmatrix}^{-1}\begin{bmatrix}h^d \\ (h^d)^*\end{bmatrix}\right\}$$

$$\times \exp\left\{-\frac{1}{2}\begin{bmatrix}(y^d - Sh^d)^H & (y^d - Sh^d)^T\end{bmatrix}\begin{bmatrix}\sigma_y^2 I_T & 0 \\ 0 & \sigma_y^2 I_T\end{bmatrix}^{-1}\begin{bmatrix}y^d - Sh^d \\ (y^d - Sh^d)^*\end{bmatrix}\right\} dh^d$$

<u>Dentro exponencial:</u>

$$(h^d)^H \Pi^{-1} h^d + (h^d)^T \Pi^{-1} (h^d)^* - (y^d)^H \sigma_y^{-2} S h^d - (h^d)^H S^H \sigma_y^{-2} y^d$$

$$+ (h^d)^H S^H \sigma_y^{-2} S h^d + \underline{(y^d)^H \sigma_y^{-2} y^d} - (y^d)^T \sigma_y^{-2} S^* (h^d)^* - (h^d)^T S^T \sigma_y^{-2} (y^d)^*$$

$$+ (h^d)^T S^T \sigma_y^{-2} S^* (h^d)^* + \underline{(y^d)^T \sigma_y^{-2} (y^d)^*}$$

$$= (y^d)^H \sigma_y^{-2} y^d + (y^d)^T \sigma_y^{-2} (y^d)^* + (h^d - \mu_H)^H \Pi_H^{-1} (h^d - \mu_H) + (h^d - \mu_H)^T \Pi_H^{-1} (h^d - \mu_H)^*$$

$$+ C$$

MODELO MIMO

TRANSITION PROBABILITIES FOR COLLAPSED GIBBS SAMPLING

- $p\left(s_{tm}=k \mid S_{\neg tm}\right)$

$$A^m = \begin{bmatrix} a^m & 1-a^m \\ b^m & 1-b^m \end{bmatrix}$$

We assume binary states s_{tm}.

For non-binary states, $\text{prob}\left(s_{tm}=\text{ActiveState}(k) \mid S_{\neg tm}\right) = \frac{1}{\#\text{ActiveStates}}\, p\left(s_{tm}=\text{act} \mid S_{\neg tm}\right)$

$$\text{Prior} \begin{cases} a^m \sim \text{Beta}\left(1, \frac{\alpha}{M}\right) \\ b^m \sim \text{Beta}(\gamma_1, \gamma_2) \end{cases} \qquad \text{Def.} \begin{cases} j = s_{(t-1)m} \\ l = s_{(t+1)m} \end{cases}$$

$\boxed{\text{For } k \neq j}$

$$p\left(s_{tm}=k \mid S_{\neg tm}\right) \propto \int p\left(s_{tm}=k \mid a_j^m\right) p\left(a_j^m \mid \text{rest}\right) da_j^m \times \int p\left(s_{(t+1)m}=l \mid a_k^m\right) p\left(a_k^m \mid \text{rest}\right) da_k^m$$

(1) For $j=0,\ k=1$ $\qquad \emptyset \to A \to x$

$$\int (1-a^m) \times \text{Beta}\left(a^m \mid 1+n_{00}^m, n_{01}^m\right) da^m \times \int \overbrace{(b^m)}^{l=0}\overbrace{(1-b^m)}^{l\neq 0} \times \text{Beta}\left(b^m \mid \gamma_1+n_{10}^m, \gamma_2+n_{11}^m\right) db^m$$

$$= \frac{n_{01}^m}{1+n_{00}^m+n_{01}^m} \cdot \frac{\overbrace{(\gamma_1+n_{10}^m)}^{l=0}\ \overbrace{(\gamma_2+n_{11}^m)}^{l\neq 0}}{\gamma_1+\gamma_2+n_{10}^m+n_{11}^m}$$

(2) For $j\neq 0,\ k=0$: $\qquad A \to \emptyset \to x$

$$\int b^m \times \text{Beta}\left(b^m \mid \gamma_1+n_{10}^m, \gamma_2+n_{11}^m\right) db^m \times \int \overbrace{(a^m)}^{l=0}\overbrace{(1-a^m)}^{l\neq 0} \times \text{Beta}\left(a^m \mid 1+n_{00}^m, n_{01}^m\right) da^m$$

$$= \frac{\gamma_1+n_{10}^m}{\gamma_1+\gamma_2+n_{10}^m+n_{11}^m} \times \frac{\overbrace{(1+n_{00}^m)}^{l=0}\ \overbrace{(n_{01}^m)}^{l\neq 0}}{1+n_{00}^m+n_{01}^m}$$

$\boxed{\text{FOR } k=j}$

$$p\left(s_{tm}=k \mid s_{\neg tm}\right) \propto \int p\left(s_{(t+1)m}=l,\, s_{tm}=k \mid a_k^m\right) p\left(a_k^m \mid \text{rest}\right) da_k^m$$

③ For $j=k=0$ $\quad \emptyset \to \emptyset \to x$

$$\int a^m \cdot \overbrace{(a^m)}^{l=0} \overbrace{(1-a^m)}^{l\neq 0} \times \mathrm{Beta}\left(a^m \mid 1+n_{00}^m,\, n_{01}^m\right) da^m$$

$$= \frac{\overbrace{(2+n_{00}^m)(1+n_{00}^m)}^{l=0} \; \overbrace{(1+n_{00}^m)(n_{01}^m)}^{l\neq 0}}{(2+n_{00}^m+n_{01}^m)(1+n_{00}^m+n_{01}^m)}$$

④ For $j=k\neq 0$ $\quad A \to A \to x$

$$\int (1-b^m) \cdot \overbrace{(b^m)}^{l=0} \overbrace{(1-b^m)}^{l\neq 0} \times \mathrm{Beta}\left(b^m \mid \gamma_1+n_{10}^m,\, \gamma_2+n_{11}^m\right) db^m$$

$$= \frac{\overbrace{(\gamma_1+n_{10}^m)(\gamma_2+n_{11}^m)}^{l=0} \; \overbrace{(\gamma_2+n_{11}^m+1)(\gamma_2+n_{11}^m)}^{l\neq 0}}{(\gamma_1+\gamma_2+n_{10}^m+n_{11}^m+1)(\gamma_1+\gamma_2+n_{10}^m+n_{11}^m)}$$

TO ADD NEW CHAINS

The prior is $\mathrm{Poisson}\left(\frac{\alpha}{T}\right)$.

For this to be equal:

$$\begin{cases} \mu_H = \Gamma_H S^H \sigma_y^{-2} y^d \\ \Gamma_H^{-1} = \Gamma^{-1} + S^H \sigma_y^{-2} S \end{cases}$$

The term

$$\cancel{(h^d)^H \Gamma^{-1} h^d} + \cancel{(h^d)^T \Gamma^{-1} (h^d)^*} - \cancel{(y^d)^H \sigma_y^{-2} S h^d} - \cancel{(h^d)^H S^H \sigma_y^{-2} y^d}$$
$$+ \cancel{(h^d)^H S^H \sigma_y^{-2} S h^d} - \cancel{(y^d)^T \sigma_y^{-2} S^* (h^d)^*} - \cancel{(h^d)^T S^T \sigma_y^{-2} (y^d)^*}$$
$$+ \cancel{(h^d)^T S^T \sigma_y^{-2} S^* (h^d)^*}$$

$$\triangleq \cancel{(h^d)^H \Gamma^{-1} h^d} + \cancel{(h^d)^H S^H \sigma_y^{-2} S h^d} + (y^d)^H \sigma_y^{-2} S \Gamma_H^H \cancel{\Gamma_H^{-1}} \cancel{\Gamma_H} S^H \sigma_y^{-2} y^d$$
$$- \cancel{(y^d)^H \sigma_y^{-2} S \Gamma_H^H \Gamma_H^{-1} h^d} - \cancel{(h^d)^H \Gamma_H^{-1} \Gamma_H S^H \sigma_y^{-2} y^d}$$
$$+ \cancel{(h^d)^T \Gamma^{-1} (h^d)^*} + \cancel{(h^d)^T S^H \sigma_y^{-2} S (h^d)^*} + (y^d)^T \sigma_y^{-2} S^* \Gamma_H^* S^T \sigma_y^{-2} (y^d)^*$$
$$- \cancel{(y^d)^T \sigma_y^{-2} S^* (h^d)^*} - \cancel{(h^d)^T S^T \sigma_y^{-2} (y^d)^*} + C$$

$$\Rightarrow \quad C = -(y^d)^H \sigma_y^{-2} S \Gamma_H^H S^H \sigma_y^{-2} (y^d) - (y^d)^T \sigma_y^{-2} S^* \Gamma_H^* S^T \sigma_y^{-2} (y^d)^*$$

Therefore:

$$p(y^d \mid z, \sigma_y^2) = \mathcal{CN}\left(y^d \mid \mu_y, \Gamma_y, C_y = 0\right)$$

where:

$$\boxed{\begin{aligned} \mu_y &= 0 \\ \Gamma_y &= \left(\frac{1}{\sigma_y^2} I_T - \frac{1}{\sigma_y^4} S \Gamma_H^H S^H\right)^{-1} \\ C_y &= 0 \end{aligned}}$$

with

$$\Gamma_H = \left(\Gamma^{-1} + S^H \sigma_y^{-2} S\right)^{-1}$$

$$\uparrow$$

$$\sigma_h^2 I$$