SIMULATED TEMPERING FOR IFFSM MODEL

(Págs 210-211 Jun S. Liu)

We have:

Target distribution $\pi(x)$

By varying a single parameter, we can construct

$$\pi_i(x) \propto \exp\left(-\frac{h(x)}{T_i}\right) \qquad i=1,\dots,L$$

such that the original target belongs to this family:

$$\pi(x) = \pi_L(x) \propto \exp\left(-\frac{h(x)}{T_L}\right)$$

Define the new target:

$$\pi_{st}(x,i) \propto c_i \exp\left(-\frac{h(x)}{T_i}\right)$$

Algorithm:

$$\left(x^{(t)}, i^{(t)}\right) = (x, i)$$

1.- Draw $u \sim$ Uniform$[0,1]$

2.- If $u < \alpha_0$, $i^{(t+1)} = i$

$x^{(t+1)} \sim T_i\left(x, x^{(t+1)}\right)$

3.- If $u > \alpha_0$, $x^{(t+1)} = x$

Propose a movement $i \to i' \sim \alpha(i,i')$

Accept w.p. $\min\left(1, \dfrac{\alpha(i',i)\,\pi_{i'}(x)\,c_{i'}}{\alpha(i,i')\,\pi_i(x)\,c_i}\right)$

$\boxed{\text{In our case}}$

Target: $p\left(\{a^m, b^m, \sigma_\ell^2, \underline{h}^\ell_m, x_{tm}, s_{tm}\} \mid \{y_t\}\right)$

$$\propto \overbrace{p\left(\{y_t\} \mid \text{all}\right)}^{\sigma_y^2 \text{ appears here}} \overbrace{p(\text{all})}^{\sigma_y^2 \text{ doesn't appear here}}$$

$$\propto p(\text{all}) \cdot \frac{1}{(2\pi\sigma_y^2)^{TD/2}} e^{-\frac{1}{2\sigma_y^2}(\ )^T \cdot (\ )}$$

$$\propto p(\text{all}) \cdot e^{-\frac{1}{2\sigma_y^2}(\ )^T \cdot (\ )}$$

We define the family of distributions:

$$\pi_i(\text{all}) \propto p(\text{all})\, e^{-\frac{1}{2T_i}(\cdot)^T(\cdot)}$$

The target belongs to this family for $T_1 = \sigma_y^2$.

This will be the lowest temperature, i.e.,

$$T_i \geq \sigma_y^2 \quad \text{for all } i.$$

For simulated annealing,

1) $u \sim \text{Uniform}[0,1]$

2) - If $u < d_0$, continue with T_i

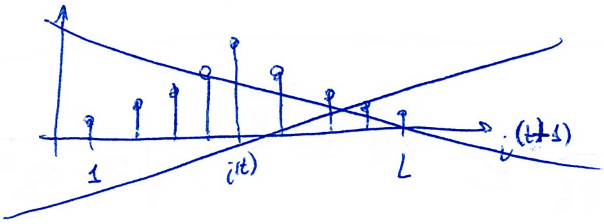
$$\text{all}^{(t+1)} \sim \text{proposal}(\text{all} \mid \text{all}^{(t)})$$

- If $u > d_0$, propose changing T_i:

$$i^{(t+1)} \sim \text{proposal}(i \mid i^{(t)}) = r(i \mid i^{(t)})$$

Accept w.p.

$$\min\left(1, \frac{r(i^{(t)} \mid i^{(t+1)})\, p(Y \mid \text{all}, T_{i^{(t+1)}})}{r(i^{(t+1)} \mid i^{(t)})\, p(Y \mid \text{all}, T_{i^{(t)}})}\right)$$



$$r\left(i^{(t+1)} \mid i^{(t)}\right) = \begin{cases} p_1, & \text{if } i^{(t+1)} = i^{(t)} + 1 \\ p_2 = 1 - p_1, & \text{if } i^{(t+1)} = i^{(t)} - 1 \\ 0, & \text{otherwise.} \end{cases}$$