

# BAYESIAN NONPARAMETRICS FOR TIME SERIES MODELING

DOCTORAL THESIS

Francisco Jesús Rodríguez Ruiz



June 30th, 2015

# OUTLINE

## ① INTRODUCTION

## ② BAYESIAN NONPARAMETRICS

## ③ CONTRIBUTIONS

Infinite Factorial Unbounded-State HMM  
Infinite Factorial Finite State Machine

## ④ CONCLUSIONS

# OUTLINE

## ① INTRODUCTION

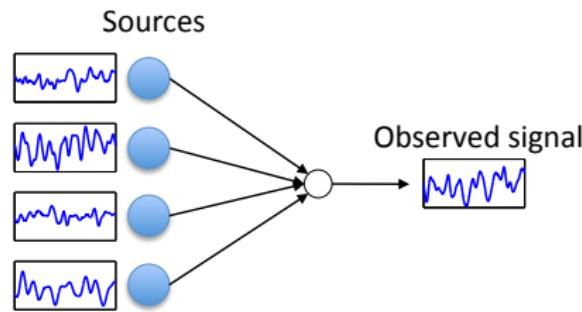
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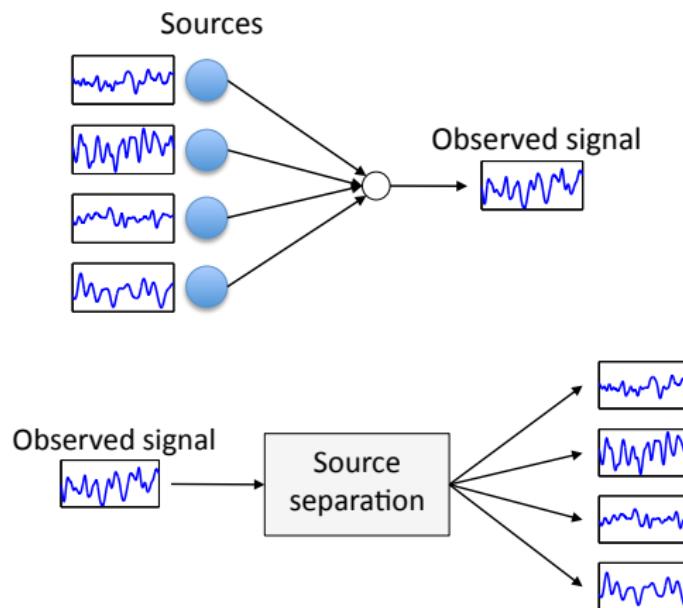
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Infinite Factorial Finite State Machine

## ④ CONCLUSIONS

# MOTIVATION



# MOTIVATION



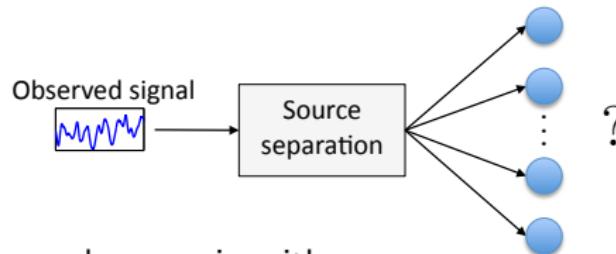
# MOTIVATION

## Applications:

- Power disaggregation.
- Multiuser communication systems.
- Speech separation.
- Multi-target tracking.
- Electroencephalography (EEG).
- ...

# MOTIVATION

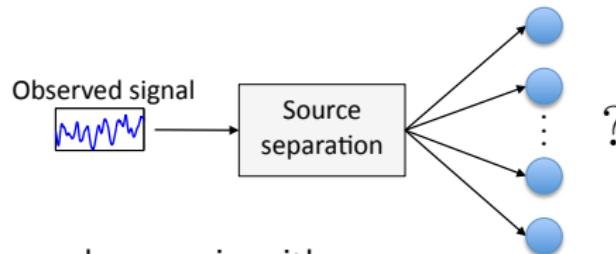
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- Classical approaches require either
  - known number of sources.
  - upper bound.
  - model selection.

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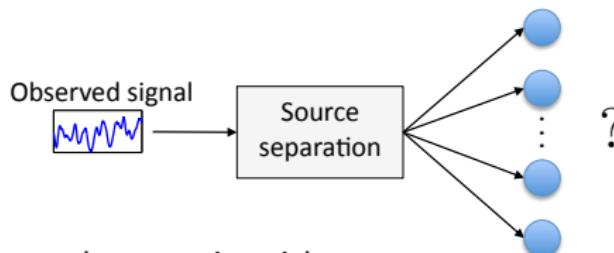
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- Bayesian nonparametrics can
  - infer the number of latent sources from the data.
  - avoid model selection.

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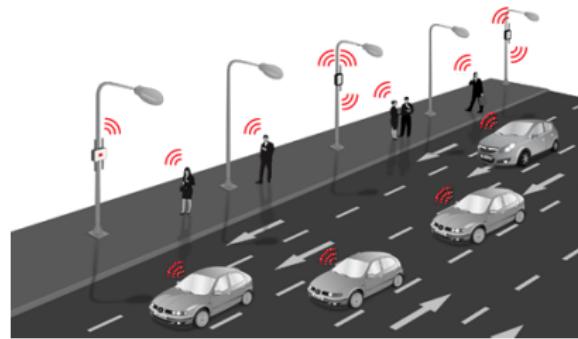
## OUR APPROACH

Bayesian nonparametric modeling of  
discrete-time series for source separation problems

# WHY BNP?

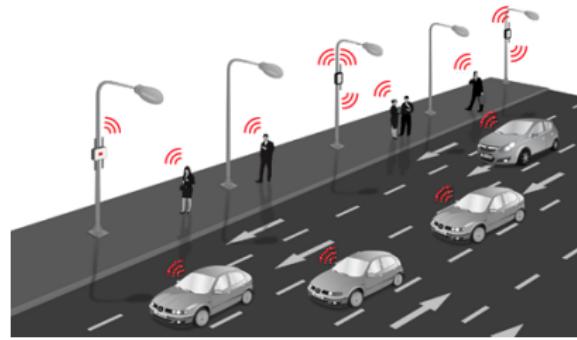


# WHY BNP?



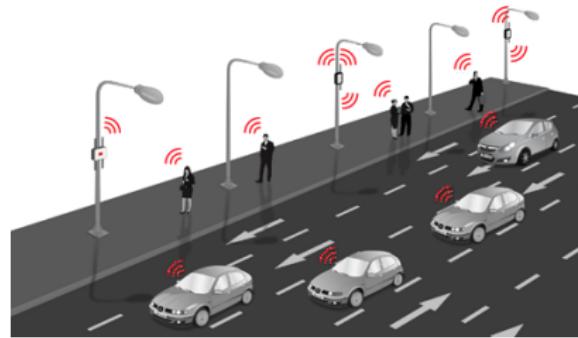
- Pick a large enough  $\#$ sources.

# WHY BNP?



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- Pick a large enough #sources.
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- BNP:
  - Model complexity grows with data size.
  - Unbounded #sources.

# STATE OF THE ART

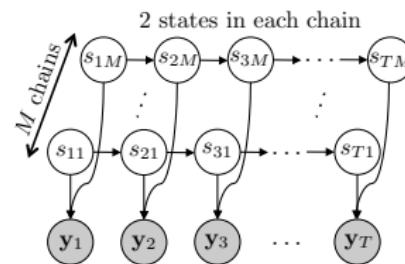
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  - e.g., infinite HMM.
- Not many BNP models for source separation.

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  - Infinite ICA.

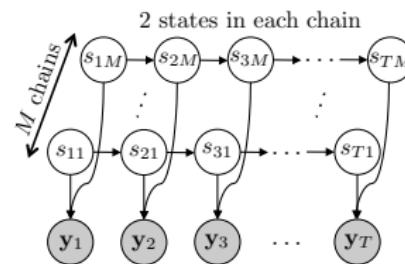
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- Many BNP models for discrete-time series.
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- ICA-IFHMM.

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Lack of BNP models for source separation:

- Infinite factorial HMM with **non-binary** hidden states.
  - e.g., power disaggregation.

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- Model that accounts for **multipath propagation**.
  - e.g., multiuser communications systems.

# STATE OF THE ART

Lack of BNP models for source separation:

- Infinite factorial HMM with **non-binary** hidden states.
  - e.g., power disaggregation.
- Model that accounts for **multipath propagation**.
  - e.g., multiuser communications systems.
- Model with continuous-valued states that captures **temporal dependencies**.
  - e.g., speech separation.

# CONTRIBUTIONS

## INFINITE FACTORIAL UNBOUNDED-STATE HMM

### ① Non-binary IFHMM.

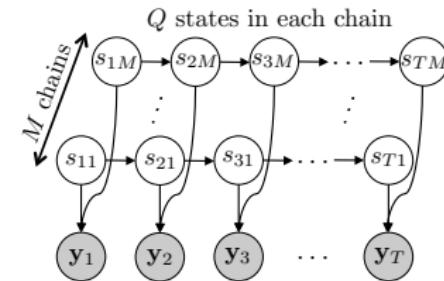
- Can infer the number of HMMs in a factorial model.

### ② IFUHMM.

- Can additionally infer the cardinality of the state space.

Applications:

- Power disaggregation.
- Multiuser communication systems.



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## INFINITE FACTORIAL UNBOUNDED-STATE HMM

- ① Non-binary IFHMM.
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Applications:

- Power disaggregation.
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## INFINITE FACTORIAL FINITE STATE MACHINE

- Can infer the number of FSMs in a factorial model.
- Naturally account for multipath, echo, ...

Applications:

- Multiuser communication systems.

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## ④ CONCLUSIONS

# BAYESIAN NONPARAMETRICS

- Bayesian framework for **model selection**.
- Prior over **infinite-dimensional** parameter space.
- Only a **finite subset** of the parameters is used for any finite dataset.
- The model complexity is allowed to grow with data size.
- Rely on **stochastic processes**:
  - Gaussian process.
  - Dirichlet process.
  - Beta process.
  - ...

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# INDIAN BUFFET PROCESS

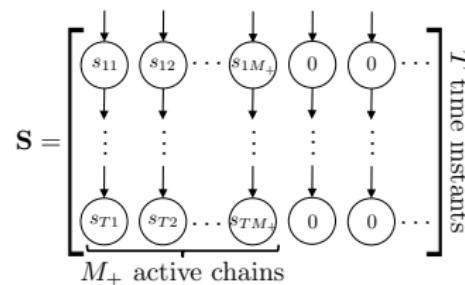
- Prior over binary matrices with infinite columns.
- Rows  $\equiv$  Data points. Columns  $\equiv$  Features.
- $\mathbf{S} \sim \text{IBP}(\alpha)$ .
- $\alpha$ : Concentration parameter.
- Each element  $s_{tm} \in \{0, 1\}$  indicates whether the  $m$ -th feature contributes to the  $t$ -th data point.
- Only a finite number of columns  $M_+$  active for any finite number of rows.

$$\mathbf{S} = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1M_+} & 0 & 0 & \cdots \\ s_{21} & s_{22} & \cdots & s_{2M_+} & 0 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots \\ s_{T1} & s_{T2} & \cdots & s_{TM_+} & 0 & 0 & \cdots \end{bmatrix}_{\substack{\text{$M_+$ non-zero columns} \\ \text{$M$ columns (features)} }}^{T \text{ observations}}$$

# MARKOV INDIAN BUFFET PROCESS

- Prior over binary matrices with infinite columns.
- Each column follows a Markov process.
- For any  $T$ , only  $M_+$  chains become active.
- The probability  $p(\mathbf{S})$  vanishes, but  $p([\mathbf{S}]) > 0$ .
  - $[\mathbf{S}]$ : set of matrices equivalent to  $\mathbf{S}$ .
- Useful to build a (binary) infinite factorial HMM.

$$\mathbf{S} = \left[ \begin{array}{cccccc} s_{11} & s_{12} & \cdots & s_{1M_+} & 0 & 0 & \cdots \\ s_{21} & s_{22} & \cdots & s_{2M_+} & 0 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots \\ s_{T1} & s_{T2} & \cdots & s_{TM_+} & 0 & 0 & \cdots \end{array} \right] \begin{matrix} T \text{ time instants} \\ M \text{ columns (chains)} \\ \underbrace{\hspace{10em}}_{M_+ \text{ non-zero columns}} \end{matrix}$$



# MARKOV INDIAN BUFFET PROCESS

$$\mathbf{S} \sim \text{MIBP}(\alpha, \beta_0, \beta_1)$$

- Can be obtained by defining the transition probabilities

$$\mathbf{A}^m = \begin{bmatrix} a^m & 1 - a^m \\ b^m & 1 - b^m \end{bmatrix} \quad \begin{aligned} a^m &= p(s_{tm} = 0 | s_{(t-1)m} = 0) \\ b^m &= p(s_{tm} = 0 | s_{(t-1)m} = 1) \end{aligned}$$

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- ... with priors

$$a^m \sim \text{Beta}\left(1, \frac{\alpha}{M}\right) \quad b^m \sim \text{Beta}(\beta_0, \beta_1)$$

- ... and let  $M \rightarrow \infty$

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- ... and let  $M \rightarrow \infty$
- After integrating out  $a^m$  and  $b^m$ :

$$\lim_{M \rightarrow \infty} p([\mathbf{S}]) = \frac{\alpha^{M_+}}{\prod_{h=1}^{2T} M_h!} e^{-\alpha H_T} \prod_{m=1}^{M_+} \frac{(n_{01}^m - 1)! (n_{00}^m)! \Gamma(\beta_0 + \beta_1) \Gamma(\beta_0 + n_{10}^m) \Gamma(\beta_1 + n_{11}^m)}{(n_{00}^m + n_{01})! \Gamma(\beta_0) \Gamma(\beta_1) \Gamma(\beta_0 + \beta_1 + n_{10}^m + n_{11}^m)}$$

- Markov exchangeable in the rows.

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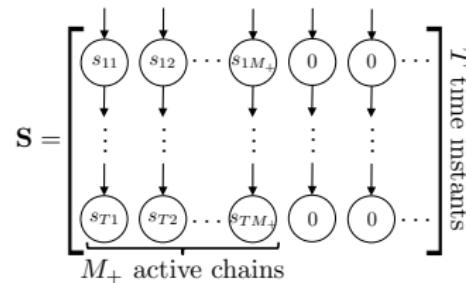
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## ④ CONCLUSIONS

# NON-BINARY INFINITE FACTORIAL HMM

- Generalization of the MIBP for non-binary matrices.
- Each state  $s_{tm} \in \{0, 1, \dots, Q - 1\}$ .
- Inactive state ( $s_{tm} = 0$ ).

$$\mathbf{S} = \left[ \begin{array}{cccccc} s_{11} & s_{12} & \cdots & s_{1M_+} & 0 & 0 & \cdots \\ s_{21} & s_{22} & \cdots & s_{2M_+} & 0 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots \\ s_{T1} & s_{T2} & \cdots & s_{TM_+} & 0 & 0 & \cdots \end{array} \right] \begin{matrix} T \text{ time instants} \\ M \text{ columns (chains)} \\ M_+ \text{ non-zero columns} \end{matrix}$$



# NON-BINARY INFINITE FACTORIAL HMM

- Can be obtained by defining the transition probabilities

$$\mathbf{A}^m = \begin{bmatrix} a_{00}^m & a_{01}^m & \cdots & a_{0(Q-1)}^m \\ a_{10}^m & a_{11}^m & \cdots & a_{1(Q-1)}^m \\ \vdots & \vdots & \ddots & \vdots \\ a_{(Q-1)0}^m & a_{(Q-1)1}^m & \cdots & a_{(Q-1)(Q-1)}^m \end{bmatrix}$$

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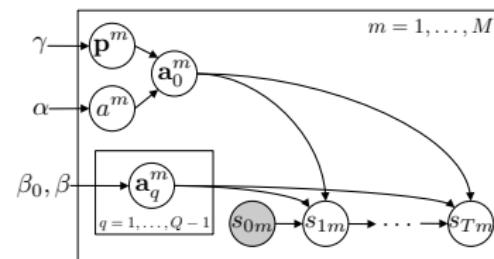
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- Prior distribution:

$$a^m \sim \text{Beta}\left(1, \frac{\alpha}{M}\right) \quad \mathbf{p}^m \sim \text{Dirichlet}(\gamma)$$

$$\mathbf{a}_0^m = [a^m \quad (1 - a^m)\mathbf{p}^m]$$

$$\mathbf{a}_q^m \sim \text{Dirichlet}(\beta_0, \beta, \dots, \beta), \quad q = 1, \dots, Q - 1$$



# LIMIT OF $p([\mathbf{S}])$

$$\begin{aligned}
 \lim_{M \rightarrow \infty} p([\mathbf{S}]) &= \frac{(Q-1)!}{(Q-N_Q)!N_f} \frac{\alpha^{M_+}}{\prod_{h=1}^{Q^T-1} M_h!} e^{-\alpha H_T} \\
 &\times \prod_{m=1}^{M_+} \left[ \frac{\Gamma(n_{00}^m + 1) \Gamma \left( \sum_{i=1}^{Q-1} n_{0i}^m \right)}{\Gamma(n_{0\bullet}^m + 1)} \frac{\Gamma((Q-1)\gamma) \prod_{i=1}^{Q-1} \Gamma(n_{0i}^m + \gamma)}{\Gamma \left( \sum_{i=1}^{Q-1} (n_{0i}^m + \gamma) \right) (\Gamma(\gamma))^{Q-1}} \right. \\
 &\quad \left. \times \prod_{q=1}^{Q-1} \left( \frac{\Gamma(\beta_0 + (Q-1)\beta)}{\Gamma(\beta_0) (\Gamma(\beta))^{Q-1}} \frac{\Gamma(n_{q0}^m + \beta_0) \prod_{i=1}^{Q-1} \Gamma(n_{qi}^m + \beta)}{\Gamma(n_{q\bullet}^m + \beta_0 + (Q-1)\beta)} \right) \right].
 \end{aligned}$$

# CULINARY METAPHOR

$Q = 3$  states (1 inactive + 2 active)

$t = 1$



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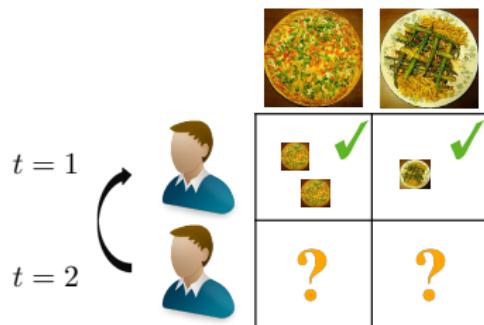
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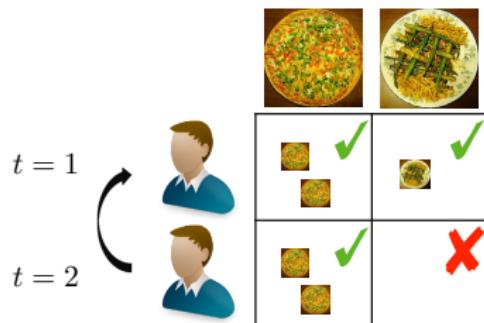
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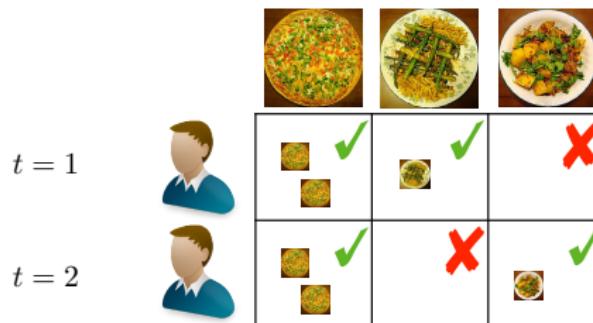
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$t = 1$						
$t = 2$						
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$t = T - 1$						

# CULINARY METAPHOR

$Q = 3$  states (1 inactive + 2 active)

	Pizza	Salad	Stir-fry	Dessert	Dessert
$t = 1$	?	✓	✓	X	X
$t = 2$	?	✓	X	✓	X
⋮	⋮	⋮	⋮	⋮	⋮
$t = T - 1$	?	✓	X	X	?
$t = T$	?				

$$p(s_{T1} = 0) \propto \beta_0 + n_{20}^1$$

$$p(s_{T1} = 1) \propto \beta + n_{21}^1$$

$$p(s_{T1} = 2) \propto \beta + n_{22}^1$$

# CULINARY METAPHOR

$Q = 3$  states (1 inactive + 2 active)

	Pizza	Salad	Stir-fry	...	Dessert
$t = 1$				...	
$t = 2$					...
$\vdots$					...
$t = T - 1$					...
$t = T$					

$p(s_{T2} = 0) \propto 1 + n_{00}^2$   
 $p(s_{T2} \neq 0) \propto n_{01}^2 + n_{02}^2$

# CULINARY METAPHOR

$Q = 3$  states (1 inactive + 2 active)

				...	
$t = 1$					
$t = 2$					
$\vdots$					
$t = T - 1$					
$t = T$					

A curved arrow points from the row  $t = T - 1$  to the row  $t = T$ , indicating a transition or final state.

$$p(s_{T2} = 1) \propto \gamma + n_{01}^2$$

$$p(s_{T2} = 2) \propto \gamma + n_{02}^2$$

# CULINARY METAPHOR

$Q = 3$  states (1 inactive + 2 active)

	1	2	3	...	5
$t = 1$					
$t = 2$					
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$t = T - 1$					
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# CULINARY METAPHOR

$Q = 3$  states (1 inactive + 2 active)

				...			
$t = 1$		✓		✓	X	...	X
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⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
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$t = T$		✓		✓	X	...	

# CULINARY METAPHOR

$Q = 3$  states (1 inactive + 2 active)

							
$t = 1$		 ✓	 ✓	 ✗	...	 ✗	 ✗
$t = 2$		 ✓	 ✗	 ✓	...	 ✗	 ✗
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$t = T - 1$		 ✓	 ✗	 ✗	...	 ✓	 ✗
$t = T$		 ✓	 ✓	 ✗	...	 ✓	 ✓

# CULINARY METAPHOR

$Q = 3$  states (1 inactive + 2 active)

						
$t = 1$	2	1	0	...	0	0
$t = 2$	2	0	1	...	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮
$t = T - 1$	2	0	0	...	1	0
$t = T$	1	1	0	...	1	2

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$t = 1$		0	2	0	...
$t = 2$		1	2	0	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$t = T - 1$		0	2	0	...
$t = T$		0	1	2	...

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$Q = 3$  states (1 inactive + 2 active)



$t = 1$		0	1	0	...
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$\vdots$		$\vdots$	$\vdots$	$\vdots$	$\vdots$
$t = T - 1$		0	1	0	...
$t = T$		0	2	1	...

# INFERENCE

## FIXED $Q$

### ① MCMC:

- Sample from the posterior.
- Blocked sampling approach.
- Slice sampling →  
Stick-breaking construction.
- FFBS for each Markov  
chain.

### ② Variational:

- Approximate the posterior.
- Structured approach.
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- Prior over the number of states:

$$Q = 2 + Q', \quad Q' \sim \text{Poisson}(\lambda)$$

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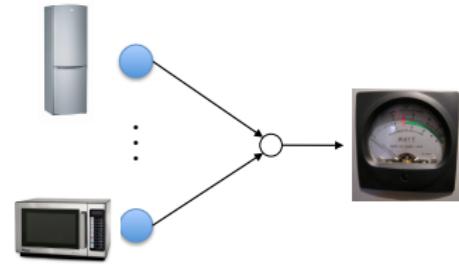
## UNKNOWN $Q$

### ① MCMC:

- Based on reversible jump MCMC.
- Integrate out dimension-changing variables.
- Updating variables:
  - $Q$ : Split/merge, birth/death.
  - $M_+$ : Slice sampling.

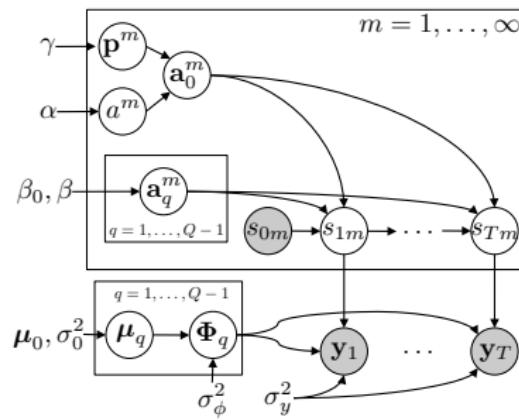
# POWER DISAGGREGATION

- Estimate the power consumption of each device.
- Non-invasive measurements.
  - Improve efficiency of consumers.
  - Detect faulty equipment.
- Two datasets.
  - REDD (1 day, 5 houses, 6 devices).
  - AMP (2 days, 1 house, 8 devices).



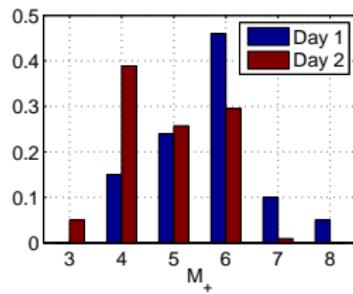
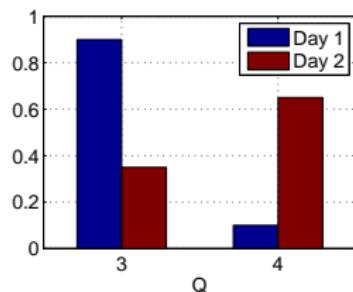
# POWER DISAGGREGATION

## Gaussian observation model



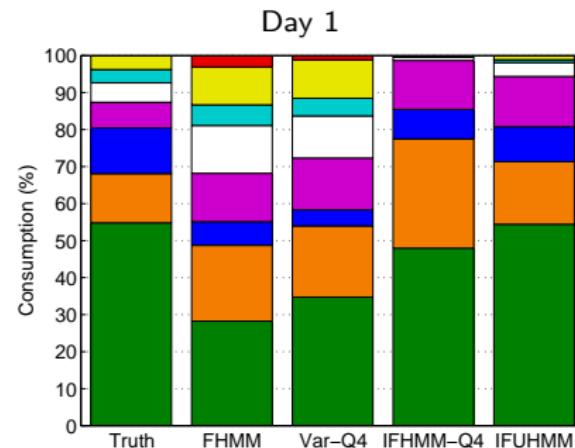
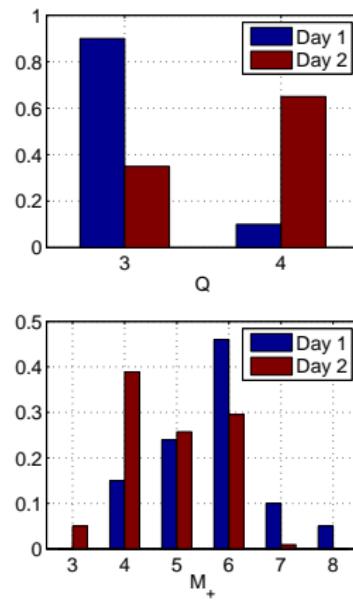
# POWER DISAGGREGATION

Results for the AMP database (2 days, 8 devices):



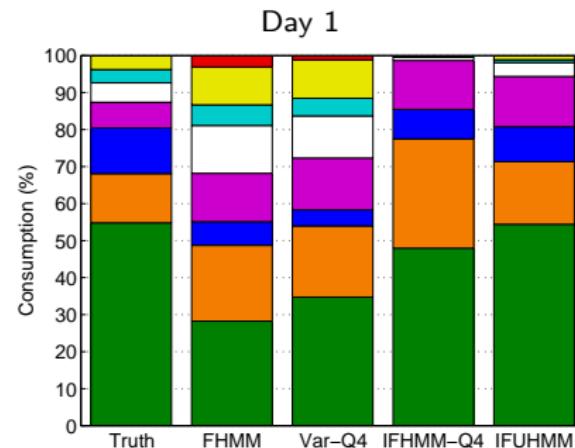
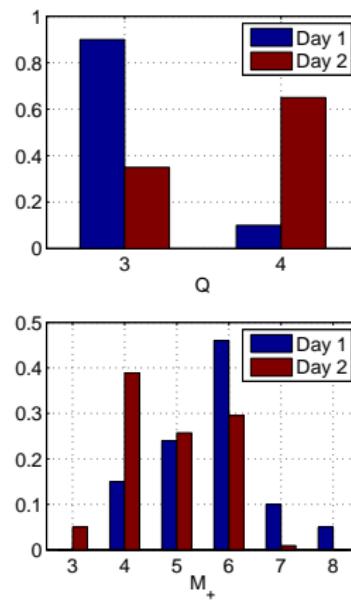
# POWER DISAGGREGATION

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# POWER DISAGGREGATION

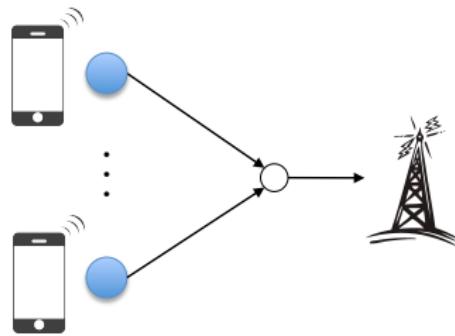
Results for the AMP database (2 days, 8 devices):



$$\text{accuracy} = 1 - \frac{\sum_{t=1}^T \sum_{m=1}^M |y_t^{(m)} - \hat{y}_t^{(m)}|}{2 \sum_{t=1}^T \sum_{m=1}^M y_t^{(m)}}$$

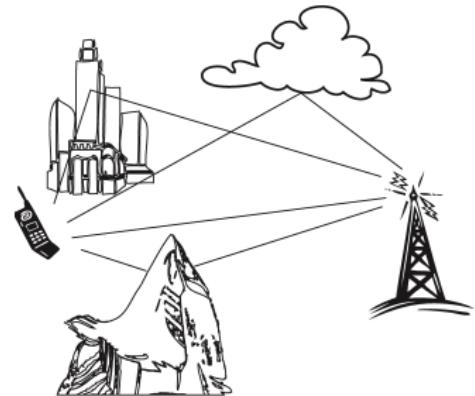
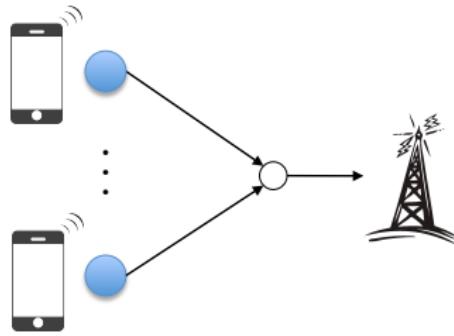
FHMM	Var-Q4	IFHMM-Q4	IFUHMM
$0.36 \pm 0.05$	$0.48 \pm 0.06$	$0.58 \pm 0.11$	$0.69 \pm 0.10$

# MULTIUSER COMMUNICATION SYSTEM



# MULTIUSER COMMUNICATION SYSTEM

Multipath propagation



# OUTLINE

## ① INTRODUCTION

## ② BAYESIAN NONPARAMETRICS

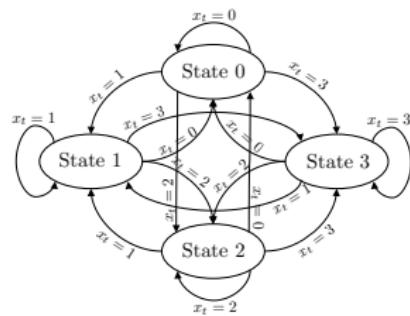
## ③ CONTRIBUTIONS

Infinite Factorial Unbounded-State HMM  
Infinite Factorial Finite State Machine

## ④ CONCLUSIONS

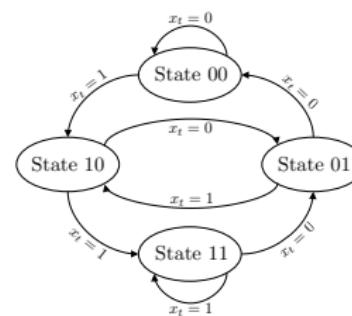
# FINITE-MEMORY FINITE STATE MACHINE

Finite-Memory FSM: The state depends on the last  $L$  inputs  $x_t$ .



HMM with  $Q = 4$  states.

Dense transition probability matrix.

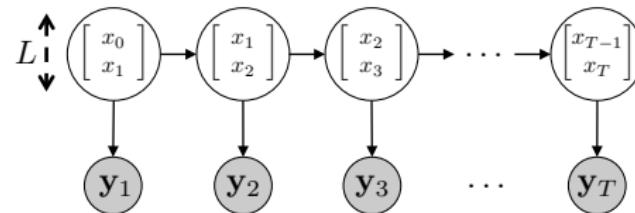


FSM with memory length  $L = 2$ .

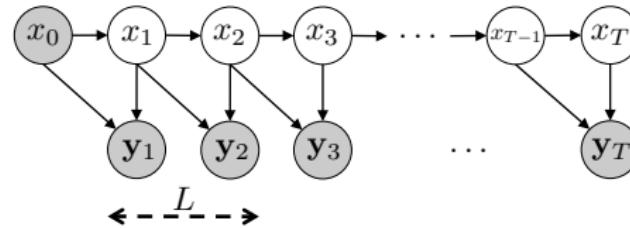
Sparse transition probability matrix.

# INFINITE FACTORIAL FINITE STATE MACHINE

- HMM representation of an FSM:

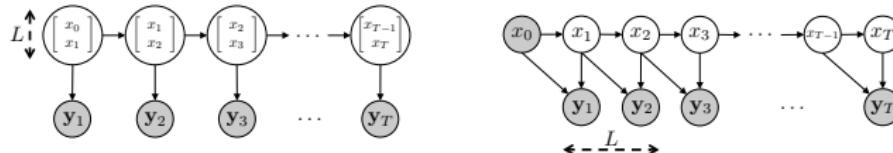


- Alternative representation (likelihood accounts for the memory):



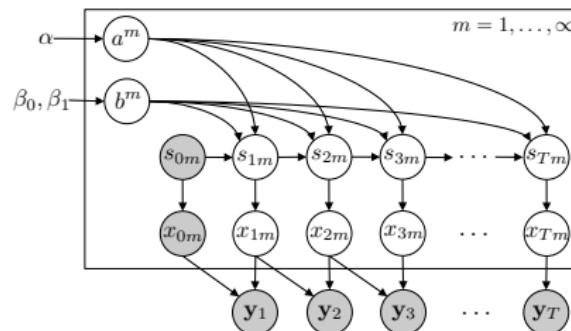
# INFINITE FACTORIAL FINITE STATE MACHINE

- The likelihood accounts for the memory.



- Infinite Factorial FSM:

- $M \rightarrow \infty$  parallel FSMs.
- $\mathbf{S} \sim \text{MIBP}(\alpha, \beta_0, \beta_1)$ .
- Auxiliary variables  $s_{tm}$  indicate activity/inactivity.
- $x_{tm} = 0$  if  $s_{tm} = 0$  and  $x_{tm} \in \mathcal{A}$  otherwise.



# INFERENCE

## INFERENCE

MCMC inference algorithm:

- ① Propose new parallel FSMs.
  - Slice sampling.
  - Stick-breaking construction.
- ② Update hidden states  $x_{tm}, s_{tm}$ .
  - Particle Gibbs with ancestor sampling.
- ③ Remove inactive FSMs.
- ④ Sample global variables.

# INFERENCE

## INFERENCE

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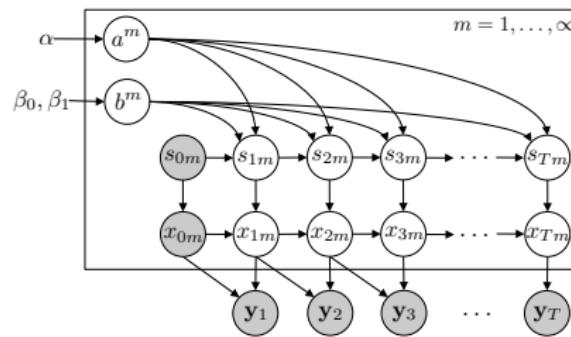
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  - Stick-breaking construction.
- ② Update hidden states  $x_{tm}, s_{tm}$ .
  - Particle Gibbs with ancestor sampling.
- ③ Remove inactive FSMs.
- ④ Sample global variables.

## PARTICLE GIBBS WITH ANCESTOR SAMPLING

- Combines MCMC and SMC.
- Better mixing properties than FFBS.
- Outperforms FFBS:
  - Quadratic complexity with memory  $L$ .
  - Can handle more general models.

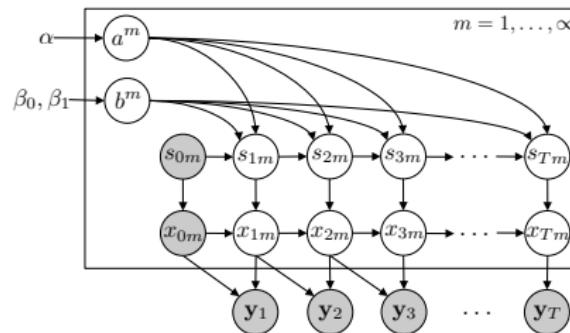
# GENERALIZATION OF THE MODEL

- Extensions that we can easily handle:
  - States  $x_{tm}$  do not necessarily belong to finite set.
  - The state  $x_{tm}$  depends on  $x_{(t-1)m}$ .



# GENERALIZATION OF THE MODEL

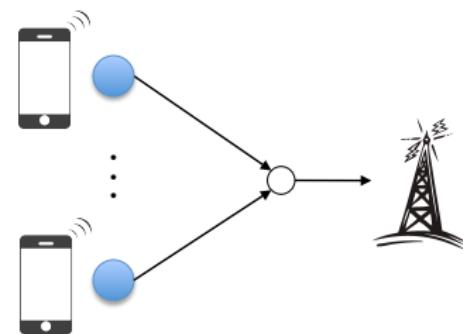
- Extensions that we can easily handle:
  - States  $x_{tm}$  do not necessarily belong to finite set.
  - The state  $x_{tm}$  depends on  $x_{(t-1)m}$ .



- Applications:
  - Multi-target tracking.
  - Speech separation.
  - ...

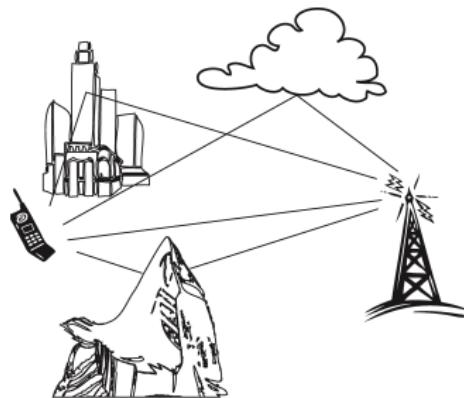
# MULTIUSER COMMUNICATION SYSTEM

- Estimate the number of users and the transmitted symbols.
- Machine-to-machine communications:
  - Transmitters switching on and off asynchronously.
  - Short bursts of symbols.
  - Reduce message overhead.
  - 5G systems.



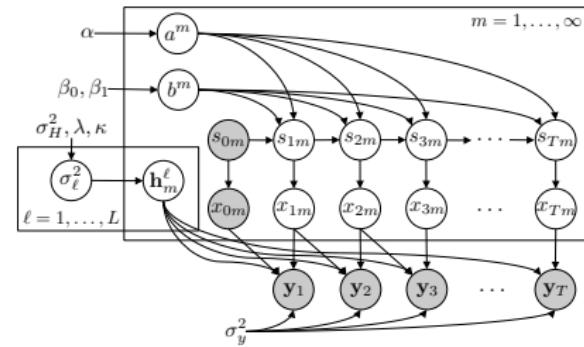
# MULTIUSER COMMUNICATION SYSTEM

Multipath propagation



Gaussian observation model

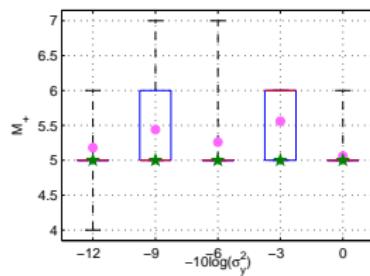
$$\mathbf{y}_t = \sum_{m=1}^{M_+} \sum_{\ell=1}^L \mathbf{h}_m^\ell x_{(t-\ell+1)m} + \mathbf{n}_t$$



# MULTIUSER COMMUNICATION SYSTEM

Synthetic experiment with 5 transmitters and 20 receivers.

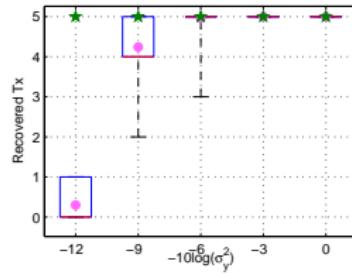
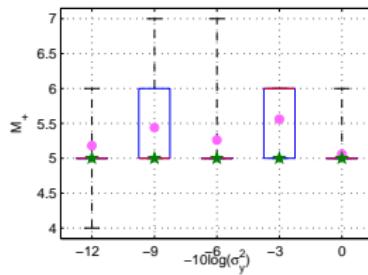
$$L = 1$$



# MULTIUSER COMMUNICATION SYSTEM

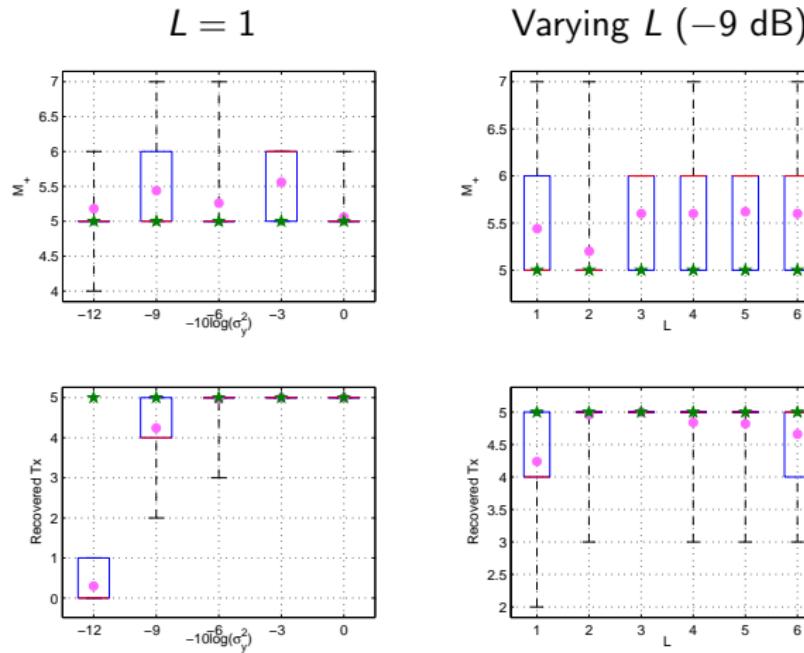
Synthetic experiment with 5 transmitters and 20 receivers.

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# MULTIUSER COMMUNICATION SYSTEM

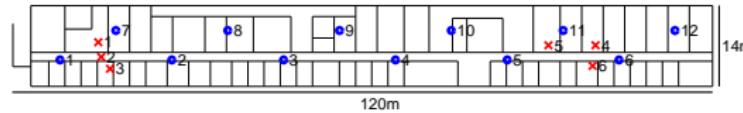
Synthetic experiment with 5 transmitters and 20 receivers.



# MULTIUSER COMMUNICATION SYSTEM

Wi-Fi experiment:

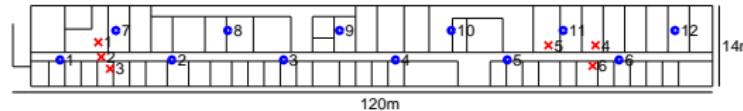
- Ray-tracing software (WISE).
- 6 transmitters, 12 receivers.
- Office at Bell Labs Crawford Hill.



# MULTIUSER COMMUNICATION SYSTEM

Wi-Fi experiment:

- Ray-tracing software (WISE).
- 6 transmitters, 12 receivers.
- Office at Bell Labs Crawford Hill.



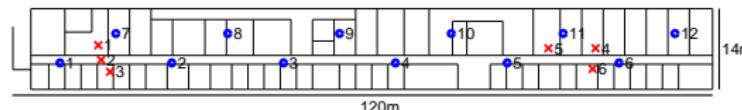
- Recovered transmitters / Inferred  $M_+$ :

Algorithm	$L$				
	1	2	3	4	5
<b>PGAS</b>	6/6	6/6	6/6	6/6	6/6
<b>FFBS</b>	3/11	3/11	3/8	1/10	—

# MULTIUSER COMMUNICATION SYSTEM

Wi-Fi experiment:

- Ray-tracing software (WISE).
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- Office at Bell Labs Crawford Hill.



- Recovered transmitters / Inferred  $M_+$ :

Algorithm	$L$				
	1	2	3	4	5
<b>PGAS</b>	6/6	6/6	6/6	6/6	6/6
<b>FFBS</b>	3/11	3/11	3/8	1/10	—

- MSE ( $\times 10^{-6}$ ) of the first channel tap ( $\ell = 1$ ):

Algorithm	$L$				
	1	2	3	4	5
<b>PGAS</b>	2.58	2.51	0.80	0.30	0.16
<b>FFBS</b>	2.79	1.38	5.53	1.90	—

(noise variance is  $\sim 10^{-8}$ )

# OUTLINE

## ① INTRODUCTION

## ② BAYESIAN NONPARAMETRICS

## ③ CONTRIBUTIONS

Infinite Factorial Unbounded-State HMM  
Infinite Factorial Finite State Machine

## ④ CONCLUSIONS

# CONCLUSIONS

## CONTRIBUTIONS

- ① Non-Binary Infinite Factorial HMM.
  - MCMC inference.
  - Variational inference.
- ② Infinite Factorial Unbounded-State HMM.
  - MCMC inference.
- ③ Infinite Factorial Finite State Machine.
  - Particle MCMC inference.

# CONCLUSIONS

## CONTRIBUTIONS

- ① Non-Binary Infinite Factorial HMM.
  - MCMC inference.
  - Variational inference.
- ② Infinite Factorial Unbounded-State HMM.
  - MCMC inference.
- ③ Infinite Factorial Finite State Machine.
  - Particle MCMC inference.

## FUTURE WORK

- Doubly nonparametric IFHMM.
- Semi-Markov approaches.
- Inference:
  - Scalability.
  - Mixing of MCMC.
  - Online.
- Other applications.
- Time-varying channels.

Thanks for your attention!



# BINARY IFHMM FOR POWER DISAGGREGATION

- REDD dataset (5 houses, 1 day, 6 devices).
- Binary IFHMM ( $Q = 2$ ).
- Histogram of inferred  $M_+$ :

