If $s_{tm}=1$ but prior = Gauss:

$$p(\tilde{y}_t \mid s_{tm}=1) = \int \frac{1}{(2\pi\sigma_y^2)^{D/2}} e^{-\frac{1}{2\sigma_y^2}(\tilde{y}_t - x_{tm}\underline{w}^m)^T(\tilde{y}_t - x_{tm}\underline{w}^m)} \frac{1}{(2\pi\sigma_x^2)} e^{-\frac{1}{2\sigma_x^2}x_{tm}^2}\, dx_{tm}$$

$$\cancel{= \frac{1}{\left(\sqrt{2\pi\sigma_y^2}\sqrt{2\pi\sigma_x^2}\right)^D} e^{\frac{1}{2\sigma_y^2}\tilde{y}_t^T\tilde{y}_t}}$$

$$= \frac{1}{(2\pi\sigma_y^2)^{D/2}\sqrt{2\pi\sigma_x^2}} \int e^{-\frac{1}{2\sigma_y^2}\tilde{y}_t^T\tilde{y}_t} e^{-\frac{1}{2\sigma_x^2}x^2 - \frac{x^2}{2\sigma_y^2}(\underline{w}^m)^T\underline{w}^m} e^{+\frac{1}{2\sigma_y^2}\left(x(\underline{w}^m)^T\underline{\tilde{y}}_t + x\,\underline{\tilde{y}}_t^T\underline{w}^m\right)} dx$$

$$\boxed{\sigma_p = \sqrt{\frac{1}{\frac{1}{\sigma_x^2} + \frac{(\underline{w}^m)^T\underline{w}^m}{\sigma_y^2}}}} \quad \boxed{\mu_p = \frac{\sigma_p^2}{\sigma_y^2}(\underline{w}^m)^T\underline{\tilde{y}}_t}$$

$$= \frac{1}{(2\pi\sigma_y^2)^{D/2}\sqrt{2\pi\sigma_x^2}} e^{-\frac{1}{2\sigma_y^2}\underline{\tilde{y}}_t^T\underline{\tilde{y}}_t} \int e^{-\frac{1}{2\sigma_p^2}(x-\mu_p)^2} e^{+\frac{1}{2\sigma_p^2}\mu_p^2}\, dx$$

$$= \frac{1}{(2\pi\sigma_y^2)^{D/2}\sqrt{\cancel{2\pi}\sigma_x^2}} e^{-\frac{1}{2\sigma_y^2}\underline{\tilde{y}}_t^T\underline{\tilde{y}}_t + \frac{1}{2\sigma_p^2}\mu_p^2} \sqrt{\cancel{2\pi}\sigma_p^2}$$

$$= \frac{1}{(2\pi\sigma_y^2)^{D/2}} \cdot \sqrt{\frac{\sigma_p^2}{\sigma_x^2}} \cdot e^{\frac{1}{2\sigma_p^2}\mu_p^2 - \frac{1}{2\sigma_y^2}\underline{\tilde{y}}_t^T\underline{\tilde{y}}_t}$$