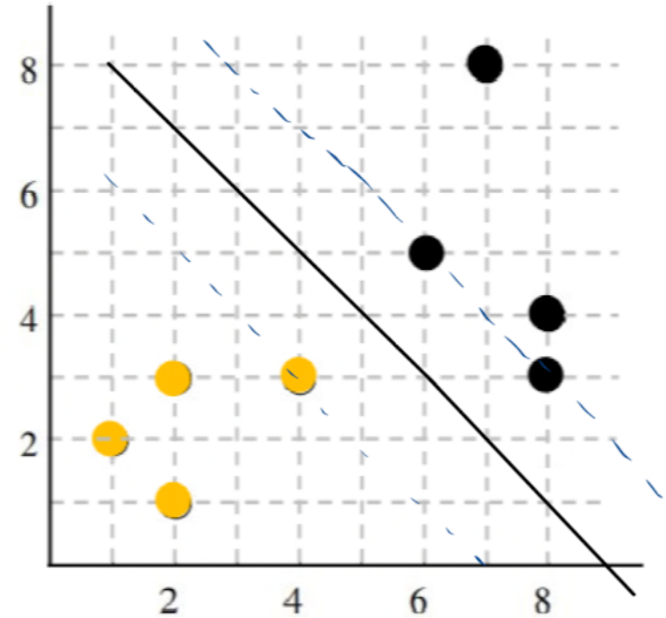
Assignment 3:

① a)

Draw the decision boundary for the SVM:

Boundary



Support vectors, SV = {<4,3>, <6,5>, <8,3>}

b. If a black circle is added as a training sample in the position (7,5), does this affect the previously learned decision boundary? Explain why

It would not affect the previously learned decision boundary because the new point sits above the parallel hyperplane that is above the decision boundary. The new point would not be a candidate for a support vector, only support vectors alter the decision boundary.

c. If a yellow circle is added as a training sample in the position (4,2), does this affect the previously learned decision boundary? Explain why.

It would not affect the previously learned decision boundary because the new point sits below the parallel hyperplane that is below the decision boundary. The new point would not be a candidate for a support vector, only support vectors alter the decision boundary.

d. If a black circle is added as a test sample in the position (7,5), will this sample be classified correctly according to the previously learned decision boundary? Explain why.

For some test sample, it will be classified as a black circle if w.z+b>0 and a yellow circle if w.z+b<0. Our decision boundary is defined as w.x+b=0, therefore any point that is above our decision boundary will get classified as a black circle and any point below the decision boundary will be classified as a yellow circle. Our test sample (7,5) is above our decision boundary, therefore it will be classified correctly.

e. If a black circle is added as a test sample in the position (6,4), will this sample be classified correctly according to the previously learned decision boundary? Explain why.

For some test sample, it will be classified as a black circle if w.z+b>0 and a yellow circle if w.z+b<0. Our decision boundary is defined as w.x+b=0, therefore any point that is above our decision boundary will get classified as a black circle and any point below the decision boundary will be classified as a yellow circle. Our test sample (6,4) is above our decision boundary, therefore it will be classified correctly.

f. If a yellow circle is added as a test sample in the position (4,2), will this sample be classified correctly according to the previously learned decision boundary? Explain why.

For some test sample, it will be classified as a black circle if w.z+b>0 and a yellow circle if w.z+b<0. Our decision boundary is defined as w.x+b=0, therefore any point that is above our decision boundary will get classified as a black circle and any point below the decision boundary will be classified as a yellow circle. Our test sample (4,2) is below our decision boundary, therefore it will be classified correctly.

g. If a yellow circle is added as a test sample in the position (5,3), will this sample be classified correctly according to the previously learned decision boundary? Explain why.

For some test sample, it will be classified as a black circle if w.z+b>0 and a yellow circle if w.z+b<0. Our decision boundary is defined as w.x+b=0, therefore any point that is above our decision boundary will get classified as a black circle and any point below the decision boundary will be classified as a yellow circle. Our test sample (5,3) is below our decision boundary, therefore it will be classified correctly. One interesting note is that it falls within the margin, but on the right side of it.

h. If a black circle is added as a test sample in the position (5,3), will this sample be classified correctly according to the previously learned decision boundary? Explain why.

For some test sample, it will be classified as a black circle if w.z+b>0 and a yellow circle if w.z+b<0. Our decision boundary is defined as w.x+b=0, therefore any point that is above our decision boundary will get classified as a black circle and any point below the decision boundary will be classified as a yellow circle. Our test sample (5,3) is below our decision boundary, therefore it will be classified incorrectly as a yellow circle. It falls within the margin but on the wrong side of the boundary.

i.

If a yellow circle is added as a test sample in the position (6,4), will this sample be classified correctly according to the previously learned decision boundary? Explain why

For some test sample, it will be classified as a black circle if w.z+b>0 and a yellow circle if w.z+b<0. Our decision boundary is defined as w.x+b=0, therefore any point that is above our decision boundary will get classified as a black circle and any point below the decision boundary will be classified as a yellow circle. Our test sample (6,4) is above our decision boundary, therefore it will be classified incorrectly as a black circle. It falls within the margin but on the wrong side of the boundary.

J . If a black circle is added as a training sample in the position (4,4), how this will affect the decision boundary if C = 1 and C = ∞? Consider the soft margin formulation.

If c=1 then we might have a similar boundary as our hard margin boundary because of the lower penalty for misclassifications during training. It's hard to say if it will be the exact same boundary, but it will likely be similar with perhaps a lower b value to bring the boundary downwards. If c=∞ then the boundary would sit tightly with a small margin between the support vectors (4,4) and (4,3), it would mimic a hard margin with those as the support vectors.

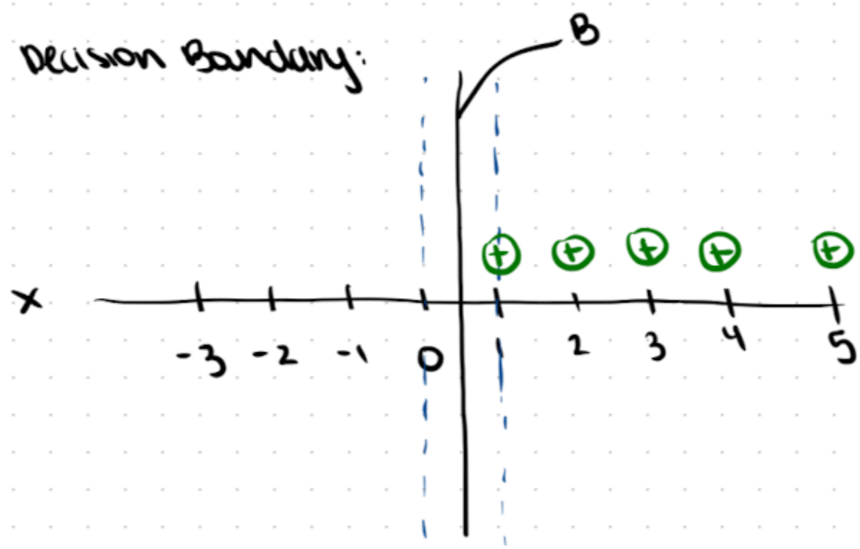
2.

Consider the following 1-dimensional data with two classes:

x	-3	0	1	2	3	4	5
Class	$-$	$-$	$+$	$+$	$+$	$+$	$+$

a. Find the decision boundary of a linear SVM on this data (hard-margin formulation) and identify the support vectors (write the x coordinate to provide your answer).

The support vectors are x=0 and x=1.
Decsion boundary at x=0.5



b.

b.) Find the sol for param $\vec{w}$ and b for the SVM. Also find the width of margin.

using s.v. $x=0$

$$y_i(w.x_i+b)=1 \quad \forall \text{ s.v}$$
$$y_i(w.0+b)=1$$
$$y_i(b)=1$$
$$-1(b)=1$$
$$\Rightarrow b=-1$$

$$y_i(w.x_i+b)=1$$
$$y_i(w.(1)+-1)=1$$
$$1(w-1)=1+1$$
$$w=2$$

$$\text{margin} = \frac{2}{\|w\|} = \frac{2}{\sqrt{(2)^2}} = \frac{2}{2} = 1 \checkmark$$

$$\boxed{w=2,\ b=-1,\ \text{margin}=1}$$

c. Our decision boundary is denoted by w.x+b=0. So any point above our boundary, x=0.5, will be classified as (+) and any point below the boundary will be classified as (-). For the test point (-1.5), since -1.5<0.5 then it will be classified as (-). For the test point (1.5), since 1.5>0.5 then it will be classified as (+).

d.

d.)

x	-3	0	[illegible]	2	3	4	5
Class	$-$	$-$	[illegible]	$+$	$+$	$+$	$+$

our s.v. are $x=0$ and $x=2$

our decision boundary is $x=1$.

$y_i(w \cdot x_i + b) = 1$

$x = 0$:

$$-1(w \cdot 0 + b) = 1$$
$$-b = 1$$
$$\Rightarrow b = -1$$

$x = 2$:

$$1(w \cdot 2 + (-\overset{+1}{1})) = 1 + 1$$
$$\tfrac{1}{2}: \; 2w = 2/2$$
$$\Rightarrow w = 1$$

margin: $\dfrac{2}{\|w\|} = \dfrac{2}{\sqrt{(1)^2}} = 2$

$w = 1$

$b = -1$

3.

a.) $\phi(A) = ((1)^2, (2)^2, \sqrt{2}(1)(2), (\sqrt{2})1, (\sqrt{2})2, 1)$

$= (1, 4, 2\sqrt{2}, \sqrt{2}, 2\sqrt{2}, 1)$

$B = (2,4)$

b.) $\phi(B) = ((2)^2, (4)^2, \sqrt{2}(2)(4), \sqrt{2} \cdot 2, 4\sqrt{2}, 1)$

$= (4, 16, 8\sqrt{2}, 2\sqrt{2}, 4\sqrt{2}, 1)$

c.) $\phi(A) \cdot \phi(B) = (1, 4, 2\sqrt{2}, \sqrt{2}, 2\sqrt{2}, 1) \cdot (4, 16, 8\sqrt{2}, 2\sqrt{2}, 4\sqrt{2}, 1)$

$= 4 + (4)(16) + 2(2)(8) + 2(2) + 4(2)(2) + 1$

$= 4 + 64 + 32 + 4 + 16 + 1$

$= 121$

d.) $K(A, B) = (x \cdot y + 1)^2$

$= ((1 \cdot 2 + 2 \cdot 4) + 1)^2$

$= ((2 + 8) + 1)^2 = 11^2 = 121$

4. https://github.com/franserr99/cs4210/blob/main/a3/svm.py

5. https://github.com/franserr99/cs4210/blob/main/a3/q5.xlsx

NOTE: both parts are in there, switch sheets for either part