

# Route optimization for a fleet of vehicles with temporal constraints

Algorithmics Methods and Mathematical Models

Master in Research and Innovation - UPC

Juan Francisco Martínez Vera  
`juan.francisco.martinez@est.fib.upc.edu`

January 12, 2017

### **Abstract**

Optimization problems can appear in almost all situations on life. There are specially important those ones that appears on the industry because if we can achieve an optimal solution, we will improve the efficiency of the industrial process and then get lower manufacturing cost. This improvement of the costs have an effect on the competitiveness of the companies and on the final quality of their products and there are always good news for the customers. The challenging part is that those kind of problem have a really high computational complexity therefore there are several methods to face with them. By one hand we have the, let's say, always-optimality methods that obtain the optimal solution but without taking into account the huge amount of time that it could imply. e.g. ILP<sup>1</sup>. By the other hand we have those methods that concerns about the execution time and are looking for a tradeof between the exeution time and the quality of the solution<sup>2</sup>.

In this pages will be shown this two approaches in order to get the optimal routes for a fleet of vehicles taking into account temporal constraints.

---

<sup>1</sup>Integer Linear Programming

<sup>2</sup>The distance to the optimal

# Contents

<b>1</b>	<b>Introduction</b>	<b>3</b>
1.1	Document structure . . . . .	3
<b>2</b>	<b>Problem definition</b>	<b>1</b>
2.1	Description . . . . .	1
2.2	Formal definition . . . . .	1
2.2.1	A relaxed TSP with temporal constraints . . . . .	1
2.2.2	Input data . . . . .	2
2.2.3	Constratints definition . . . . .	3
2.2.4	Objective function definition . . . . .	3
<b>3</b>	<b>ILP model</b>	<b>4</b>
3.1	Input data . . . . .	4
3.2	Decission variables . . . . .	4
3.3	Constraints . . . . .	5
3.3.1	Graph constraints . . . . .	5
3.3.2	Time constraints . . . . .	6
<b>4</b>	<b>Meta-heuristics</b>	<b>7</b>
<b>5</b>	<b>Comparissons</b>	<b>8</b>
<b>6</b>	<b>Discussion</b>	<b>9</b>
<b>7</b>	<b>Conclusions</b>	<b>10</b>

# Chapter 1

## Introduction

Optimization problems can appear in almost all situations on life. There are specially important those ones that appears on the industry because if we can achieve an optimal solution, we will improve the efficiency of the industrial process and then get lower manufacturing cost. This improvement of the costs have an effect on the competitiveness of the companies and on the final quality of their products. The challenging part is that those kind of problem have a really high computational complexity therefore there are several methods to face with them. By one hand we have the, let's say, always-optimality methods that obtain the optimal solution but without taking into account the huge amount of time that it could imply. e.g. ILP<sup>1</sup>. By the other hand we have those methods that concerns about the execution time and are looking for a tradeof between the exeution time and the quality of the solution<sup>2</sup>.

In this project has been developed an Integer Linear Programming model in order to optimize the routes for a fleet of vehicles with temporal constraints derived from the distance between locations and from the tasks that has to be done for every location. Also have been developed different meta-heuristic strategies in order to get results more efficiently in terms of exeuction time even if there are not the optimal but close enough. Finnaly comparissons in term of execution time and solution quality have been done.

### 1.1 Document structure

The structure of this document is the following: The problem definition is done in chapter 2. Here it is explained the problem that is faced, which constraints is needed to take into account and what we want to optimize. At chapter 3 is explained the mathematical model that has been developed, i.e. the decision variables and also the constraints with a mathematical nomenclature. After this chapter, at chapter 4 is explained how it has been used the heuristics approach in order to face with the problem, two meta-heuristics has been used: GRASP<sup>3</sup> and BRKGA<sup>4</sup>. After perform several executions for those approaches, a comparisson in terms of time and quality of the result is done at chapter 5. Finnally a discussion about the results obtained is done at chapter 6 and the conclusions of the project are explained at chapter 7

---

<sup>1</sup>Integer Linear Programming

<sup>2</sup>The distance to the optimal

<sup>3</sup>Greedy Adaptative Search Procedure

<sup>4</sup>Biased Random-Key Genetic Algorithm

# Chapter 2

## Problem definition

### 2.1 Description

We have been asked to help one logistic company in the design of the daily routes of its fleet of vehicles. The workload for every day consist in some tasks that have to be done at a different locations, therefore, the problem is to identify the optimal routes for the minimum number of vehicles in order to perform all the work in time, i.e. starting at 8 a.m. and finalizing before 8 p.m.

For that purpose we are given a set of locations and a start location  $l_s$  where all the vehicles will start. For our point of view, the locations are not needed at all but we need the distance in terms of time between all of them. Because that we will be provided by the  $dist_{l_1, l_2}$  input data.

As is already mentioned, in every location (but not in  $l_s$ ) there is a task that has to be done, then, also the information about how many time is needed for every task is provided as input data. We are talking about  $task_l$ .

Finally one more constraint is imposed by this logistic company. Every task has to be done but can not be started at any hour of the day. There is a temporal windows for every task that describes when an specific task can be started. This windows consists on a lower boundary  $min_l$  and on a upper boundary  $max_l$ . Then the starting time can not be before  $min_l$  and after  $max_l$ .

Considering that the company has an unlimited number of vehicles, the goal of the project is to find the minimum number of vehicles needed to visit all locations and perform all tasks, and define their routes. Given two solutions with the same number of vehicles it is preferred the one in which the latest vehicle arrives at its final destination sooner.

### 2.2 Formal definition

In this section is defined how the problem will be managed, also the input variables, the constraints and the objective function. This is a formal definition, therefore is implementation independant. These definitions have been used both for ILP and for meta-heuristics.

#### 2.2.1 A relaxed TSP with temporal constraints

This problem has been formally defined as a relaxed TSP<sup>1</sup> with temporal constraints. It is relaxed because allows more than one cycle, i.e. more than one travel salesman (vehicle), and it is with temporal constraints because in addition to the usual weighted edges (routes) in the original TSP, we have temporal windows and tasks times for every node (location). The key point is that all the cycles in the graph, i.e. all routes, **must imply the  $l_s$** .

This point of view allows to manage this problem without taking into account vehicles. The number of vehicles will be, in fact, the number of cycles in the solution.

As in TSP we can represent the towns and routes as a directed weighted complete graph, then, we can describe a route of every vehicle by mean of a set of edges that describes a cycle. Remember that the cycle must implice the  $l_s$  node.

---

<sup>1</sup>Travel Salesman Problem

The definition of the graph is defined by the below expression. Is important to mention that the cardinal of the  $A$  set is like that because the distance of the travel from location  $a$  to location  $b$  could be different than the travel from  $b$  to  $a$  because the roads or whatever else.

$$G = (V, A) \quad \text{where} \quad V: \text{set of nodes, } A: \text{set of directed edges}$$

$$|V| = n, \text{ being } n = \text{number of locations} \quad |A| = n^2 - n$$

Now, let's define the set the whole possible cycles in the graph as  $C$  where every cycle is defined as a subset of edges that fulfill all the conditions to be a cycle. Then, the set of all possible vehicles  $VH \subset C$  are all the cycles that implies the  $l_s$  as is described in the next expression:

$$vh \in VH \Leftrightarrow \exists a \in vh \text{ s.t. } a_{source} = l_s \vee a_{destination} = l_s$$

We can describe a set of potential solutions for the problem  $PS \subset VH$  as all of subsets of  $VH$  that cover all the locations without repeat any one of them. Those are potential solutions because for the moment we have not taked into account the temporal constraints. Finally, the set of feasible solutions will be  $S \subseteq PS$ . And the challenging point is to get the optimal solution from this set.

## 2.2.2 Input data

According the definition, this variables are provided and describes a single instance of this problem. The following list describe all the input data that is provided. In order to improve the understanding, the input data have been split into two subsets. By one hand all data related to the graph stuff, and by the other hand the input data related to temporal restrictions.

### Graph input data

This input data is used in order to build up the graph that represents the problem and has been described several lines above. On next lines this input data is showed and explained.

- **Number of locations** that the vehicles have visit. From here we can define the  $V$  set that is the nodes set of the graph that are in fact the set of locations.

$$n \in \mathbb{N}$$

- **Starting location** where all vehicles are placed at the beginning of times.

$$l_s \in V$$

- **Distances** in minutes from one locations to the other. This data form the  $A$  set because it is the weights for the edges for our completed directed and weighted graph. Note that when  $a = b$ , then  $dist_{l_a, l_b} = 0$

$$dist_{l_a, l_b} \in A \quad \forall l_a, l_b \in V$$

Could be said that this is a temporal data but as this data is also needed to build up the graph, and is present on the original TSP as well, is preferred to be here.

### Time constraints input data

This input data is used in order to build up the time constraints needed for the problem definition. On next lines this input data is shoed and explained.

- **Lower boundary** for the temporal windows for every location. It indicates from when a task can be executed. If one vehicles arrives before this time, it must wait. The temporal unit are the minutes. This minimum windows can be at most 720 that is the journey time in minutes ( $8h * 60min/h = 720min$ )

$$min_l \in [0, 720] \quad \forall l \in V$$

- **Upper boundary** for the temporal windows for every location. It indicates until when a task can be executed. Take into account that does not have sense that  $max_l < min_l$

$$max_l \in [min_l, 720] \quad \forall l \in V$$

- **Time** that a task spends for every location.

$$task_l \in [0, 720], \forall l \in V$$

### 2.2.3 Constratints definition

In this section are explained the fundamental constraints that are implementation independant. As well as with input data on the previous section, in order to improve the understanding, the constraints are splitted into two subsets, the graph constraint and the time constraints.

#### Graph constraints

These constraints are derived from the format description of the problem as a directed weighted graph. All the solutions that exists in set  $VH$  (section 2.2.1) fullfil the graph constraints. Just for summing up, those constraints are

- Every location has to be visited exactly one time. It means that all cycles in a solution must cover all nodes and can not share any node.
- All cycles in a solution must involve the starting location.

#### Time constraints

These constraints have to guarantee that the time restrictions are fullfild. Those constraints are:

- The arrival time to location  $l$  must be less or equal  $max_l$ .
- The arrival time to location  $l$  must be  $max(arrivalTime, min_l)$ . It means that if one vehicle arrive before  $min_l$  it will wait.
- The total time of a cycle must be less than 720 minutes that is 8 hours.

Note that the arrival time to a location is the sumation of the previous location arrival time plus the task of the previous location plus the time spent in the path (the weight of the edge).

### 2.2.4 Objective function definition

In order to get the optimal solution we need to clasify every solution by a number that can tell us the quality of every one of them. This number is provided by the **objective function**.

As the description of the problem said, we want to minimize the number of vehicles and when two solutions have the same number of vehicles, we want the solution that end the whole job before. We will need two variables for this purpose, by one hand, the number of vehicles, lets say  $nVehicles$  and by the other hand the time when the last vehicle ends its job, lets say  $lastArrival$ . How to get the correct value for these two variable is implementation dependant and therefore will be explained on the next chapters.

The objective functions is described as:

$$Minimize \quad nVehicles * M + lastArrival$$

In this expression, the  $M$  is a big enough number. It is needed in order to prioritize the solutions with less value for  $nVehicle$ . When two solutions have the same number of vehicles, then the important value is the  $lastArrival$ .

# Chapter 3

## ILP model

ILP<sup>1</sup> is the first of the two methods that has been used in this project. This method is always concerned on having the optimal solution without taking into account the amount of computational resources needed. This model is developed in CPLEX and the model implemented is described at the next sections.

### 3.1 Input data

From chapter 2 we already know which is the data that is provided. Now we need them in an specific shape in order to deal with them in CPLEX.

- **Number of locations**  $n$  is declared as an integer as well as **starting location**

$n \in \mathbb{N}$	<code>int nLocations=...;</code>
$l_s \in \mathbb{N}$ s.t. $1 \leq l_s \leq n$	<code>int startLocation=...;</code>

- The **distances** are modeled as two-dimensional matrix of integers that have  $n^2$  positions. If you take into account that the values on the diagonal of the matrix will not be taken into account, the total number of distances is  $n^2 - n$  as we have described in the previous chapter. The distance is expressed in minutes.

$dist_{a,b} \in \mathbb{N} \quad \forall a, b \in [0, n) \text{ s.t. } a \neq b$	<code>range N=1..nLocations;</code>
	<code>int distances[n in N, n2 in N]=...;</code>

- The remaining data fields are related to the locations, because that they are defined as a vector on  $n$  positions. Those data fields are **lower boundary** and **upper boundary** of the temporal windows and the time spent in a **task**. All of them are expressed in minutes.

$task_i \in \mathbb{N} \quad \forall i \in [0, n) \text{ s.t. } i \neq l_s$	<code>int task[n in N]=...;</code>
$minw_i \in \mathbb{N} \quad \forall i \in [0, n) \text{ s.t. } i \neq l_s$	<code>int minW[n in N]=...;</code>
$maxw_i \in \mathbb{N} \quad \forall i \in [0, n) \text{ s.t. } i \neq l_s$	<code>int maxW[n in N]=...;</code>

### 3.2 Decision variables

The solution of the problem will be encoded in a certain way in a decision variable. The decision variables are not only used for get the final result but also for enforcing some situation with constraints. Those variables unlike input data ones, are not statics and can change their value in order to fulfil the constraints (section 3.3), as is just said, and in the same time in order to maximize or minimize the objective function (section 2.2.4). Therefore, even if the important numbers for the solution are two. The number of used vehicles and the time when the last vehicle done the work is needed a sort of other decision variables. In the next list is presented a relation of the defined decision variable for model.

---

<sup>1</sup>Integer Linear Programming



- The **tracked** variable. It is a  $n \times n$  matrix and keep information about whether a path from location  $n$  to location  $n2$  is taken, i.e. keep information about which edges are on the solution. It is needed in order to control the number of input and output edges for every location, i.e. to control the correctness of the cycles and for calculate the time spent in a route by determining which paths have been taken and which ones not.

$$tracked_{a,b} \in \mathbb{B} \quad \forall a, b \in [0, n) \text{ s.t. } a \neq b \quad \text{dvar bool tracked}[n \text{ in } N, n2 \text{ in } N];$$

- In order to build the temporal constraints that we have at section 2.2.3 we need information about when a vehicle arrives to a location. Because that the **arrivingTime** decision variable has been defined. It is an  $n$ -vector that keeps the arriving time for all locations (nodes) in minutes.

$$arriving_i \in \mathbb{N} \quad \forall i \in [0, n) \text{ s.t. } i \neq l_s \quad \text{dvar int arrivingTime}[n \text{ in } N];$$

The starting location will not have arriving time, i.e. it will be always 0. The reasons are explained at section 3.3.

- One of the most important decision variables is the numbers of vehicles needed. **nVehicles** keep the number of vehicles in the solution.

$$nVehicles \in \mathbb{N} \text{ s.t. } 1 \leq nVehicles \leq n \quad \text{dvar int nVehicles};$$

On the best case, we only will need one vehicle but in the worst one we will need as much vehicles as locations.

- The other really important value for our result is the end time of the last vehicle. **lastDone** have the time in minutes of the last arrival vehicle.

$$lastDone \in \mathbb{N} \quad \text{dvar int lastDone};$$

### 3.3 Constraints

For an ILP model the constraints part is the most important part, or at least the most tricky part. They are defining the solution space and because that we need to be careful in order to try to limit as much as possible but without prune some solutions and forbidding CPLEX to find a feasible solution when it exists.

The constraints below, as in section 2.2.3 have been separated into two sets. By one hand, the graph constraints and by the other hand, the time constraints.

#### 3.3.1 Graph constraints

This constraints are the ILP version of the constraints defined at section 2.2.2.

- The first constraint is worried about the fact that every location has to be visited exactly one time. For this purpose the *tracked* decision variable is used. There is one special case where the number of visits for a location must be one or more. This special case will be the starting location because we want that all cycles involve it, therefore it will be visited at least one time.

$$\begin{array}{l} \sum_{j \in V} tracked_{i,j} = 1 \quad \forall i \in V \text{ s.t. } i, j \neq l_s \\ \sum_{i \in V} tracked_{i,l_s} \geq 1 \quad \text{s.t. } i \neq l_s \end{array} \quad \left| \begin{array}{l} \text{forall } (n1 \text{ in } N) \{ \\ \quad \text{if } (n1 == \text{startLocation}) \\ \quad \hookrightarrow \text{sum}(n2 \text{ in } N) tracked[n1, \\ \quad \hookrightarrow n2] \geq 1; \\ \quad \text{else sum}(n2 \text{ in } N) tracked[n1, \\ \quad \hookrightarrow n2] == 1; \\ \} \end{array} \right.$$

The key point in this constraint is that we are counting the ingoing edges for all locations.

- From the previous constraint, we can ensure that every location just will have one ingoing edges but we are not ensuring anything about the outgoing edges. For cycles we want to ensure that there is only one outgoing edge for all location except for the starting one, that must be same outgoing edges as ingoing ones. Then, the sumation of outgoing edges must be equal as the ingoing edges for all locations.

$$\sum_{j \in V} tracked_{i,j} = \sum_{k \in V} tracked_{k,i} \quad \forall i \in V$$

forall (n in N) {
sum(n2 in N) tracked[n, n2]
↪ == sum(n3 in N)
↪ tracked[n3, n];
}

- One desirable constraint is that we want to avoid the cycles that just imply one location. The point for this constraint is that we are enforcing 0 on the diagonal of the tracked matrix.

$$\sum_{i \in V} tracked_{i,i} = 0$$

forall(n in N) {
tracked[n,n] == 0;
}

The constraints above are ensuring that our solution will contain cycles, and those cycles will visit all locations. Also is ensuring that all cycles are independant between them. The important thing that those constraints are not ensuring is that all cycles implies the starting location. This requirement will be fulfilled by time constraints as you will see on the next section.

### 3.3.2 Time constraints

Hola manola

## Chapter 4

# Meta-heuristics

## Chapter 5

# Comparisons

## Chapter 6

# Discussion

## Chapter 7

# Conclusions