

Belief Network Analysis: A Relational Approach to Understanding the Structure of Attitudes¹

Andrei Boutyline
University of California, Berkeley

Stephen Vaisey
Duke University

Many accounts of political belief systems conceive of them as networks of interrelated opinions, in which some beliefs are central and others peripheral. This article formally shows how such structural features can be used to construct direct measures of belief centrality in a network of correlations. This method is applied to the 2000 ANES data, which have been used to argue that political beliefs are organized around parenting schemas. This structural approach instead yields results consistent with the central role of political identity, which individuals may use as the organizing heuristic to filter information from the political field. In light of recent accounts of belief system heterogeneity, a search for population heterogeneity in this organizing logic was undertaken first by comparing 44 demographic subpopulations and then using inductive techniques. Contra these recent accounts, the study finds that belief systems of different groups vary in the amount of organization but not in the logic that organizes them.

Theories of the structure of political beliefs typically conceive of them as networks of interrelated opinions, in which some beliefs are central and others are derived from these more fundamental positions (Converse 1964; Jost,

¹ This research was supported by fellowships from National Science Foundation Graduate Research Fellowship Program (NSF-GRFP) and Interdisciplinary Graduate Edu-

Federico, and Napier 2009).² There are many such center-periphery theories of political ideology, each of which places something different at the center (e.g., political identity, authoritarianism, moral relativism). Research has established the plausibility of such accounts using such distinct quantities as the reliability of survey responses or the ability of “central” opinions to predict peripheral items (e.g., Converse 1964; Barker and Tinnick 2006). Though the empirical examinations of these theories have yielded valuable findings, they have generally not made use of the rich structural features of the theoretical accounts they test. Following the intuition of sociological network analysis (Breiger 1974; Wellman 1988; Freeman 2004; Pachucki and Breiger 2010) and building on recent work in the sociology of culture (Baldassarri and Goldberg 2014), we demonstrate how such structural features can be used to construct direct measures of belief centrality in the network of correlations. These centrality measures, together with other network metrics, enable intuitive comparisons between many theories of political belief structure. In this article, we use this belief network analysis (BNA) approach to contrast several prominent accounts of belief structure and to further elaborate the account most supported by the comparisons. We further demonstrate that these results are robust to sampling error and model selection. We then conduct additional analyses to support key assumptions behind the model against competing claims about heterogeneity of belief structure.

As an orienting case, we first focus on Lakoff’s (2002) theory of “moral politics.” This theory posits that people reason about the complex domain of policy by metaphorically mapping it onto the domain of family and parenting. By this process, cultural schemas describing two common parenting styles, nurturant and strict, become the “deep structures” underlying the liberal and conservative political worldviews. Lakoff’s model is frequently cited in sociology (e.g., Jacobs and Carmichael 2002; Somers and Block 2005; Wuthnow 2007; Vaisey 2009; Gross, Medvetz, and Russell 2011; Edgell 2012; Hitlin and Vaisey 2013; DellaPosta, Shi, and Macy 2015) and has also

cation and Research Traineeship Program (NSF-IGERT). We thank Neil Fligstein, John Levi Martin, James Moody, Fabiana Silva, Paul Sniderman, and participants in the Berkeley Mathematical, Analytical and Experimental Sociology and Center for Culture, Organization and Politics workshops for feedback on the manuscript. We are also indebted to Grigori Nepomniachtchi for generous help with the article’s formal reasoning. Direct correspondence to Andrei Boutyline at University of California, Berkeley, Department of Sociology, Berkeley, California 94720. E-mail: boutyline@berkeley.edu

² In this article, we use the term “beliefs” as shorthand for what Converse (1964) calls “idea-elements” and Zaller (1992) calls “considerations”: that is, the various kinds of persistent mental content that make up a person’s political ideology, including both information and moral values. This is also the same usage as in Borhek and Curtis’s *A Sociology of Belief* (1975).

has been deeply influential outside the academy.³ However, we know of only a single peer-reviewed work that has lent it full support. This article (Barker and Tinnick 2006), published in the *American Political Science Review*, interpreted the ability of parenting variables to predict other opinions in the 2000 American National Elections Study data as evidence of their central role in the belief system. To enable comparisons between our method and existing techniques, we revisit this study with our BNA methodology.

Our analysis shows that parenting values in fact occupy a peripheral position in the observed network. We find that the center is instead occupied by ideological identity, which is broadly consistent with theories of social constraint (e.g., Campbell et al. 1960; Converse 1964; Zaller 1992; Mondak 1993; Goren, Federico, and Kittilson 2009). In such theories, actors begin with a rudimentary understanding of the institutional field of politics (Sniderman and Stiglitz 2012), and a political identity within this field, which they then use as a heuristic for selecting political views from mass media and other information sources. We find that this centrality of ideological identity is remarkably robust to both statistical noise and the specific choice of variables to include in the model. The low centrality of parenting attitudes is equally robust. Our results thus provide strong evidence in favor of the social constraint account and against the theory of moral politics.

Both of these theories—and our BNA technique—make the assumption that the organization of attitudes is driven primarily by a single dominant process in the population. This assumption is shared by many theories of belief organization (e.g., Jost et al. 2003). However, a number of recent sociological treatments (Boltanski and Thévenot 1999; van Eijck 1999; Achterberg and Houtman 2009; Thornton, Ocasio, and Lounsbury 2012; Baldassarri and Goldberg 2014) assume substantial heterogeneity in how attitudes are organized, arguing for the existence of many different “logics” of constraint. If overall patterns mask substantial heterogeneity, our method could lead to invalid conclusions. Thus, in the second part of the article, we partition the population along 16 key demographic dimensions and examine heterogeneity in belief organization between the 44 resulting subpopu-

³ A 2014 *Salon* profile called *Moral Politics* “a book that should have utterly transformed our understanding of politics” and continued with “and for many who read it, it certainly did” (Rosenberg 2014). Lakoff’s guides to political framing, which are based on this theory, carry endorsements from prominent figures like Howard Dean, Anthony Romero, and George Soros (e.g., Lakoff and Rockridge Institute 2006; Lakoff 2014). In 2005, the *New York Times* reported that one of these guides was “as ubiquitous among Democrats in the Capitol as Mao’s *Little Red Book* once was in the Forbidden City” and quoted Nancy Pelosi describing his framing advice as “perfect for us, because we were just arriving in an unscientific way at what Lakoff was arriving at in a scientific way” (Bai 2005). In the 2016 election cycle, Lakoff’s work has appeared in the popular media to explain Donald Trump’s rise within the Republican party (DeVega 2016; Williamson 2016).

lations. In the appendixes, we also search for potential heterogeneity with a novel information-theoretic method we introduce here. We then further investigate it using model comparison techniques from structural equation modeling. Contrary to Baldassarri and Goldberg's (2014) high-profile work and other recent sociological accounts, we find that heterogeneity in the organizational logic of political beliefs is the exception rather than the rule.

Among these 44 subpopulations, we find that all belief networks with substantial levels of organization are centered on political identity and feature similar patterns of pairwise constraint. For those groups further removed from the field of "mainstream" U.S. politics, we find that belief systems instead generally lack organization—a result in line with a substantial volume of older work that showed the belief systems of such populations to be low in constraint (e.g., Converse 1964). In a few key subpopulations, however, we find some tentative evidence of a different belief system—one centered on religious identity rather than political identity. But any potential alternate scheme of political belief organization would appear highly limited in scope, suggesting that religious identity may not generally provide a heuristic sufficient to organize the full range of beliefs usually deemed "political."

The rest of the article proceeds as follows. First, we develop a formal model of belief formation and introduce an empirical method that can adjudicate between competing center-periphery models of beliefs. Second, we use this method to contrast existing theories of belief formation with survey data from the 2000 American National Election Study (ANES). These analyses lead us to reject the theory of moral politics and to offer an elaboration of the theory of social constraint. Third, we empirically examine a key assumption about population heterogeneity made by our method through demographic comparisons between subgroups, as well as with information-theoretic and psychometric techniques. We again find results consistent with the social constraint account. More broadly, our overarching goal in this investigation of belief structures is to help students of culture better understand how some cultural elements can organize and structure others within a cultural system, which Swidler (2001, p. 206) has identified as "the biggest unanswered question in the sociology of culture."

BELIEF STRUCTURES AS NETWORKS

Most prominent accounts define ideology as "a learned knowledge structure consisting of an interrelated network of beliefs, opinions and values" (Jost et al. 2009, p. 310; but see Martin 2000). The network metaphor for belief systems fits well with both the definitions and the questions posed by the literature on ideology. A network, after all, is simply a system consisting of a finite set of identifiable entities called "nodes," as well as a set of defined re-

relationships between them called “edges” or “ties.” Converse’s (1964) classic description of a system of belief elements held together by pairwise constraint or functional interdependence fits this definition of a network. Lakoff’s (2002) account of belief generation, in which models of parenting are extended through metaphor and logical inference into full-fledged political belief systems, can also be rendered in network terms.

The benefit of the network language comes from the leverage it provides in succinctly describing the relational properties of such systems. Two key ideas that recur in many accounts of belief systems—the structural positions occupied by different beliefs and the degree of organization of the belief system—make use of network thinking to evoke such properties. We draw on social network analysis to show that the network understanding of belief systems need not stop at evocative metaphor (Breiger 1974; Wellman 1988; Freeman 2004; Pachucki and Breiger 2010). Like Baldassarri and Goldberg (2014), we interpret a set of survey responses as an empirical manifestation of the belief network, where the belief items are nodes and the associations between the beliefs are weighted ties. Going beyond existing work, we construct a formal network model of belief structure and use it to demonstrate that a particular set of network indices—shortest-path betweenness centrality and centralization—provide theoretically relevant measurements for such a system. We develop this approach, which we term “belief network analysis” (BNA), in the context of comparing two prominent theories of political belief structure: the theory of moral politics and the theory of social constraint. Both lend themselves well to a network conceptualization and rest on core concepts for which measures are readily available.

Moral Politics

We begin by describing the structural features of Lakoff’s theory of moral politics (Lakoff 2002). Lakoff’s model has roots in his earlier work on metaphor theory (Lakoff and Johnson 1980; Lakoff 1990). Lakoff proposes that conceptual systems are structured largely through metaphorical inference: metaphors project complex cognitive domains onto simpler ones. For example, two common metaphors used to understand anger are “anger is an opponent” (e.g., “he was struggling with his anger”; “his anger overpowered him”) and “anger is a fluid heated up in a container” (e.g., “he was boiling with rage”; “simmer down”). He argues that this word usage reflects deeper differences in conceptual structure: the person who speaks of anger as an opponent may thus decide that he should try his best to fight it, while someone who thinks of anger as a boiling fluid concludes he should let some of it out lest he explode (Lakoff 1990).

In *Moral Politics*, Lakoff (2002) argues that political cognition is also fundamentally metaphorical. He points to terms like “fatherland,” “Uncle

Sam,” “founding fathers,” and “big brother” to argue that the “nation is a family” metaphor is the key to understanding political differences. This metaphor maps the complex domain of government onto the more familiar domain of family, allowing people to use their intuitions about parenting to make judgments in the otherwise opaque domain of policy. Lakoff concludes that ideological divisions stem from the fact that “liberals and conservatives have different models of how to raise children” (2002, p. 337). The “strict father” model used by conservatives emphasizes authority, strict discipline, and “tough love” as ways to lead the child to self-reliance. The “nurturant parent” model used by liberals emphasizes caring, protection, and respect as the best ways to help children grow up to be fulfilled and happy adults. Liberals thus support environmental protection and generous welfare policies because they are metaphorically understood as forms of parental caring. Conservatives oppose abortion and support mandatory sentencing for drug possession because their morality stresses personal accountability.

The derivation of political beliefs in Lakoff’s (2002) account can be roughly broken down into three phases. A person starts with a simple, unelaborated model of parenting—strict or nurturant. In the first derivation phase, beliefs about how to parent lead to broader moral judgments involving parenting and family. In the second phase, these expanded moral claims are applied to the domain of government. Via the “nation is a family” metaphor, the government becomes the parent, citizens become children, and proper governance becomes proper parenting. Intuitions about family thus yield intuitions about government. In the final phase, these intuitions are used to develop specific policy stances. Parenting beliefs thus become “central” to the system of political views in the sense that they serve as the initial basis from which this system is formed.

Under this model, debates over parenting philosophy are an important form of partisan conflict. Lakoff argues that liberals benefit from the popularity of advice books promoting nurturant parenting, as it “means that there are plenty of parents and children who have an intuitive understanding of the basis of Nurturant Parent morality and liberal politics” (2002, p. 364). However, he proposes that the greater prominence of conservative parenting groups like Focus on the Family may give them an advantage in the longer term, as “the more children brought up with Strict Father values, the more future conservatives we will have” (2002, p. 424). These arguments are consistent with a dynamic model in which peripheral beliefs are recursively generated from a central belief.

To date, only two studies have attempted to provide empirical support for Lakoff’s model.⁴ McAdams and colleagues (2008) tested Lakoff’s asser-

⁴ *Moral Politics* itself contains little systematic support for Lakoff’s argument beyond its intuitive plausibility and consistency with some anecdotal evidence (see Lakoff 2002, p. 158).

tions by examining the authority figures appearing in 128 life-narrative interviews. They found only mixed support: while conservatives were more likely to have strict authority figures, liberals were not more likely to have nurturant ones. Another study (Barker and Tinnick 2006), published in the *American Political Science Review*, used data from the 2000 ANES to show that respondents' parenting values can predict many policy positions net of a large number of controls. Although the authors interpreted this as evidence for Lakoff's theory, the existence of net associations is not sufficient to make the structural claim that parenting morality is the element which "unifies the collections of liberal and conservative issue positions" (Lakoff 2002, p. 12). Below, we will use our network-analytic methodology to test this claim directly.

Social Constraint

The main alternative we consider to Lakoff's (2002) account comes from theories of social constraint. Beginning with the classic works of Campbell et al. (1960) and Converse (1964), this diverse body of theories has been unified by the claim that people use political identity as a heuristic for acquiring further political beliefs via the flow of information from opinion leaders, including politicians, journalists, and activists (Zaller 1992, p. 6). We draw our term for this theory from Converse (1964, p. 209), who characterized the belief systems produced by this process as "much less logical in the classical sense than they are psychological—and less psychological than social." Research on social constraint has also appeared under other titles, including "source cues," "elite theory," "partisan information processing," and "psychological" (as opposed to "rational") theories of partisan behavior (e.g., Zaller 1992; Mondak 1993; Lee 2002; Goren 2005). Since our primary interest is in the structural features of the belief systems described by these theories, we focus on prominent structural statements (Converse 1964; Zaller 1992) and elaborations of relevant mechanisms (e.g., Mondak 1993; Goren et al. 2009).

Social constraint theorists begin by highlighting the cognitive complexity of ideological reasoning (e.g., Converse 1964; Zaller 1992) and ask how one can come to a coherent or "constrained" political worldview. The difficulties in this task are multiple. Policy positions are not so well defined that consistent positions across multiple domains could be logically derived from some bounded set of principles. On the other hand, the empirical makeup of most policy issues is complex enough that fully considered judgments would require a prohibitive amount of information. Part of the solution may come from broadly applicable psychological principles such as cognitive heuristics or moral values, which can make it possible to arrive at judgments on such issues based on only a partial understanding (Mondak 1993; Lau and

Redlawsk 2001). However, to be mutually consistent, these principles themselves require systematization. Moreover, it may still be far from apparent which principle to apply to each issue, as most issues have many aspects and can often be judged using multiple conflicting principles (see, e.g., Feinberg and Willer 2013).

For these reasons, adopting an existing system of belief organization is vastly easier than creating such a system from scratch. Various kinds of political elites, such as politicians or television pundits, have much to gain by becoming a “cognitive authority” (Martin 2002) for their audiences. However, adopted views would likely be consistent only if they are received from elites that agree with each other. Ideological and partisan identity can solve this coordination task. Once a person acquires such an identity—by, for example, imitating their parents or following widely known cultural stereotypes (Green, Palmquist, and Schickler 2002)—he or she can replace the abstract question of “what should I believe?” with the social question “which team am I on?” Humans appear to be highly adept at this kind of social reasoning (Mondak 1993; Goren et al. 2009; Sniderman and Stiglitz 2012).

Ideological identity allows people to tune in to ideological information streams that contain relatively consistent stances on policy issues. They also convey broadly applicable ideological heuristics and stereotypical beliefs about the social world (Hurwitz and Peffley 1997; Kinder 1998; Zaller 1992; Petersen 2009; Martin and Desmond 2010)—for example, heuristics about which social groups require help and which punishment (e.g., drug addicts or the homeless), which potential threats are real and which overblown (e.g., global warming or voting by noncitizens), and which social domains are or are not “the government’s business” to regulate (e.g., gun control or abortion restrictions). Individuals can then deploy these principles to pass original judgments on newly encountered issues or to fill in the gaps in their knowledge about an issue with ideologically consistent stereotypes (Zaller 1992; Martin and Desmond 2010). Such beliefs can also give people positions within specific issue domains (e.g., “antiwar” or “prolife”), enabling segments of the population to know who is “on their side” even without direct references to ideological identity, and thus easing their acquisition of more finely differentiated domain-specific knowledge. This process yields an expanding and branching belief network that structurally resembles the one suggested by the moral politics account.

MODEL OF BELIEF FORMATION AND NETWORK STRUCTURE

Though the accounts reviewed above have obvious differences, the generative processes they depict share key structural features. We will now develop a formal model of belief formation and network structure that is consistent with these features, which we summarize as follows: (a) Individuals start

with a single central belief (parenting model or political identity). (b) This central belief is used to produce a number of broad stances (moral views or political heuristics), which are then used to stochastically produce further beliefs. (c) Newly added sets of beliefs then form the basis for yet newer and more specific beliefs, repeating recursively to yield a center-periphery structure.

This summary is clearly a simplification of both accounts and is not intended to capture the full psychological or social complexity of belief formation. For example, the model does not leave room for individuals to update their beliefs after they are created. Though prior work indeed suggests that individuals do not often change their political beliefs (Zaller 1992), even in the face of disconfirming information (Taber and Lodge 2006), it is unlikely that proponents of either account would argue that belief change never occurs. Other potential complexities are similarly elided, leaving a minimal model that captures only the main features of both accounts while remaining parsimonious enough to formalize and examine mathematically. We use the analytical leverage provided by this model to answer the following questions: Given a correlation network of survey responses from people who formed their beliefs in this way, is it possible to identify the original, central, belief? And if so, how?

We begin with the central belief, designated x_0 . New beliefs are recursively produced from older ones, beginning with x_0 's direct descendants. When one belief is created from another, each position on the older belief corresponds to some position on the new belief created from it. However, since the inference process is imperfect, the newer variables may assume values other than the ones implied by the central belief. We can formalize this generation process as

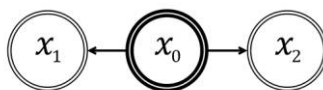
$$x_i = x_h + \phi_i.$$

That is, belief i is produced from belief h , with exogenous error ϕ_i which represents imperfections in the inference process. For example, if x_0 produces x_1 , and x_1 produces x_2 and x_3 , then $x_1 = x_0 + \phi_1$, $x_2 = x_1 + \phi_2$, and $x_3 = x_1 + \phi_3$ (and thus $x_2 = x_0 + \phi_1 + \phi_2$). We will refer to x_2 and x_3 as the "descendants" of x_1 and to all three of those beliefs as descendants of x_0 . We will assume that all the ϕ terms have a variance of ϵ and are independent of each other and x_0 , and that x_0 has a variance of 1.⁵

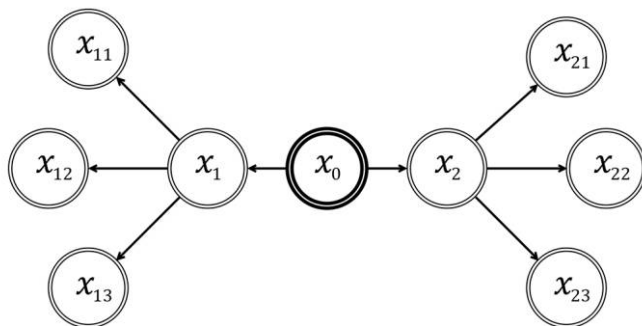
Let us now imagine a very simple belief system consisting of only the central belief x_0 and two derivative beliefs x_1 and x_2 , so that $x_1 = x_0 + \phi_1$ and $x_2 = x_0 + \phi_2$ (see first diagram in fig. 1). All three of these beliefs are positively correlated. Furthermore, it can be shown that $|\text{cor}(x_0, x_1)| =$

⁵ For a complete formal statement of these assumptions and other details of the model, see app. A.

(1) After 1st
generation



(2) After 2nd
generation



(3) Snippet of
a large belief
network.

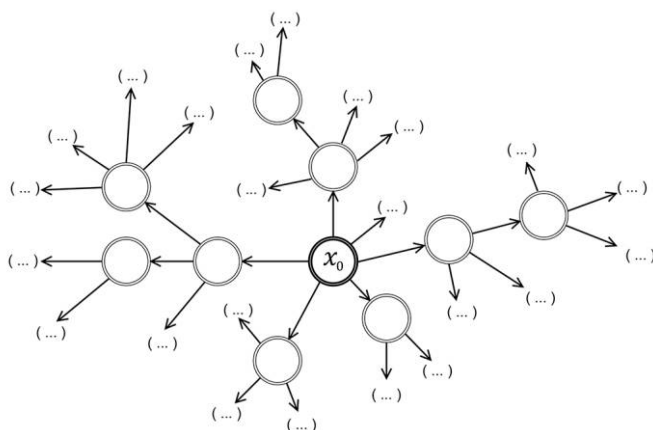


FIG. 1.—Belief network at different stages

$|\text{cor}(x_0, x_2)| = 1/\sqrt{1 + \epsilon}$, while $|\text{cor}(x_1, x_2)| = 1/(1 + \epsilon)$.⁶ Since $\epsilon > 0$, $|\text{cor}(x_0, x_1)|$ and $|\text{cor}(x_0, x_2)|$ are both greater than $|\text{cor}(x_1, x_2)|$. In this simple case then, the central belief can be discovered by simply examining the sum of all pairwise absolute correlations for each variable, which we will call the *total constraint* of those variables:

$$\text{totalcons}(x_i) = \sum_{j \neq i} |\text{cor}(x_i, x_j)|.$$

⁶ Both of these quantities can be derived from eq. (A6) in app. A by substitution ($P = 0, K = 0, S = 1$ in the first case, and $P = 0, K = 1, S = 1$ in the second).

The central belief will have the highest value of $\text{totalcons}(x_i)$. In empirical problems where the derivative beliefs are hypothesized to be closely related to the central belief, total constraint is the simplest and most intuitive centrality measure to examine. In fact, if the theory under examination proposes a simple one-step belief derivation from origin to outcome, this simple sum of absolute correlations would provide a tight methodological fit.

Now consider a slightly more complex case. Let us imagine that six more beliefs are added to the system: x_{11}, x_{12}, x_{13} and x_{21}, x_{22}, x_{23} are derived from x_1 and x_2 , respectively, so that $x_{1i} = x_1 + \phi_{1i} = x_0 + \phi_1 + \phi_{1i}$ and $x_{2i} = x_2 + \phi_{2i} = x_0 + \phi_2 + \phi_{2i}$ for $i = 1, 2, 3$ (see second diagram in fig. 1). Perhaps surprisingly, the central belief may no longer be the variable with the greatest total constraint. Some straightforward (if tedious) algebra can be used to show that $\text{totalcons}(x_0) > \text{totalcons}(x_1)$ only if ϵ is less than approximately 0.98. Thus, even with only two generations of derivative beliefs, the central belief may already not be the most highly correlated belief in the sample. The accumulation of error variance introduced by imperfect inference can “swamp out” the variance of the central belief. Put another way, total constraint is too local a feature of the belief network to correctly identify the central belief.

Fortunately, however, this same accumulation of error variance can be used to locate the central belief even in very spread out belief systems. Our method proceeds from a simple intuition. Many center-periphery accounts of ideology describe the central belief as being the “glue” (Converse 1964) that holds together the disparate parts of the belief system. That is, the central belief is what enables coherent stances to exist across the relatively disconnected domains like environmental protection and gay rights (Converse 1964; Lakoff 2002). By this logic, the center may not be the most constrained belief, but it should be the “broker” (Burt 2004) possessing the most unique and valuable pattern of constraint. Below we formally demonstrate that this intuition can be used to find the center of such a belief system.

First, we need to introduce the notions of tie length and path length. We define the length of tie T_{ij} to equal 0 if $i = j$, and otherwise to be:

$$|T_{ij}| = |T_{ji}| = \frac{1}{\text{cor}(x_i, x_j)^2}.$$

The network defined by such ties is a symmetric, complete weighted network of the kind that can be analyzed by many software packages.⁷ We will

⁷ The exponentiation of the correlation coefficient is referred to as “soft thresholding” (Zhang and Horvath 2005). It is a standard technique in the analysis of correlation networks and is used to dampen the effects of statistically insignificant correlations.

be interested in analyzing the paths between pairs of nodes in this network. A path Λ_{kl} between beliefs $x_k, x_l \neq k$ is an ordered set of connected ties. The length of the path is the sum of the tie lengths it contains: $|\Lambda| = \sum_{T_{ij} \in \Lambda} |T_{ij}|$. The shortest path between any two nodes x_i and x_j (i.e., the path with the lowest value of $|\Lambda|$) is termed their *geodesic*.⁸ We use the term *transverse* to describe ties or paths between two nodes that have x_0 as their only common ancestor. We assume that most geodesics in the belief system are transverse.⁹

Let us return to the belief structure depicted in diagram 2 of figure 1 and consider the transverse tie $T_{11,21}$. Due to accumulation of error, this tie will be relatively long: $|T_{11,21}| = (1 + 2\epsilon)^2$. The tie lengths $|T_{11,0}|$ and $|T_{0,21}|$, however, will be significantly shorter: in fact, $|T_{11,0}| = |T_{0,21}| = \sqrt{|T_{11,21}|}$. Thus, as long as $\epsilon > 0.5$, the direct path $\Lambda_1 = (T_{11,21})$ will be longer than the indirect path $\Lambda_2 = (T_{11,0}, T_{0,21})$, which may come as a surprise to those of us used to spending our lives in Euclidian space.¹⁰ This indirect path is indeed their geodesic.

In appendix A, we derive a general formal model of geodesics in such belief systems. In brief, we prove that, in general, these geodesics share a uniform structure (see theorems 3 and 4). We use this structure to derive an algebraic formula for their length, which lets us apply standard calculus optimization techniques to find the nodes these geodesics pass through (see theorem 5). We show that, in general, the transverse geodesics connecting any two k th generation beliefs will consist of more than one tie (i.e., be “non-trivial”) whenever $k > 1/\epsilon$ (see corollary 6A). In other words, after the number of generations in the belief system exceeds $1/\epsilon$, no generation of newly added transverse nodes will be connected by single-tie geodesics.

At the end of appendix A, we arrive at the key finding of this reasoning, which is the strong and persistent “central pull” of the belief system. Whenever transverse geodesics increase from a single tie to two or more ties, these intervening ties take them closer to the center of the system. Even though further generations of beliefs added to the system will grow less and less correlated with x_0 , the shortest paths connecting them will still pass close to the center. As we prove in theorem 6, every single nontrivial transverse geo-

⁸ Note that the length of a path is the sum of tie lengths that compose it, as opposed to the simple count of ties as would usually be the case in networks with unweighted ties. Since the observed tie lengths are continuous random variables, it is practically impossible that two distinct observed paths should have exactly equal lengths. This allows us to use a simpler notion of geodesic than is usual for networks of unweighted ties, which need to deal with the possibility that multiple equally short paths may exist between the same two nodes.

⁹ This is always the case unless a single first-generation node counts half or more of all the descendants of x_0 as its own descendants. Intuitively, this assumption can be understood as a prohibition against highly “lopsided” networks.

¹⁰ See corollary 2A in app. A for proof of this statement.

desic passes through either x_0 or through a node highly correlated with x_0 ($|\gamma| \geq 0.74$). Thus, in the absence of interference from highly correlated nodes (collinearity), every one of such geodesics will pass through the center. And, even if such highly correlated nodes exist, no single one of them will generally lay on enough geodesics to be confused for the center.¹¹

The formal proof thus confirms our informal intuitions. The center of a large belief system (see fig. 1,3) may be identified by finding which belief lies on the greatest portion of geodesics in the network of squared correlations—that is, the node with the highest “*shortest-path betweenness*” (Freeman 1978). If M is the total number of beliefs in the system and $\mathbf{1}(x) = 1$ if condition x is true and $\mathbf{1}(x) = 0$ otherwise, then the betweenness of node x_k is

$$\text{betweenness}(x_k) = \frac{\sum_{i \neq j} \sum_{T_{ab} \in \Gamma_{ij}} [\mathbf{1}(a = k) * \mathbf{1}(a \neq i)]}{(M - 1) * (M - 2) / 2}.$$

The numerator is the count of geodesics that pass through x_k , while the denominator is the number of pairs of beliefs not including x_k (see Wasserman and Faust 1994, pp. 189–91). As expected of a proportion, this quantity varies from 0 to 1.

By the same logic, Freeman’s (1978) index of betweenness centralization can be used to measure the extent to which the belief network as a whole possesses a single, well-defined center. The centralization of a network is the sum of pairwise differences between the centrality of the most central node and the centrality of each other node, all normalized by the maximum possible value such a sum could obtain in any network of M nodes (Wasserman and Faust 1994, p. 176). This index achieves its maximum of 1 when one belief in the network has a betweenness of 1, and every other belief has a betweenness of 0. It achieves its minimum of 0 when every belief has exactly the same betweenness centrality.

The centrality and centralization indexes introduced above can be used to determine which belief lies at the center of the system and how much more central it is than the rest of the network. However, they provide no basis to judge whether the difference in centrality is robust to statistical variation—that is, whether or not its position at the center of the network is “statistically significant.” We will use a nonparametric bootstrap to produce

¹¹ The geodesics that bypass x_0 can only do so via a transverse tie T_{ab} , where a and/or b are highly correlated with x_0 . However, two geodesics cannot use the same tie for their “shortcut” unless the endpoints of one geodesic are related to the endpoints of the other (see corollary 6B). In other words, while x_0 lies on geodesics between nodes in all branches of the system, any given “shortcut” node will lie only on geodesics between particular branches of the system. Thus, though the possibility of such shortcuts means that users of this method should exercise caution in the presence of multicollinearity, it does not appear likely to interfere with finding x_0 .

estimates of this statistical robustness. In each iteration of the bootstrap, we will resample the respondents (rows) in the survey data set with replacement, construct a correlation network for the resample, and finally recalculate the betweenness indexes for this network. We use these results to estimate the confidence intervals for each variable's centrality.

Our primary reason for constructing these confidence intervals is to determine whether any given node is reliably more central than others in the network. Since confidence intervals constructed from raw values can yield a misleading picture of this comparison, we will compare the distributions of relative rather than absolute betweenness centrality scores.¹² To calculate the relative centrality of the nodes in each bootstrap sample, we first calculate their betweenness centralities and then divide the centrality of each node by the maximum centrality for each sample. Thus, if a network of K beliefs contained three beliefs a , b , c with betweenness centralities of 0.80, 0.40, 0.08, and no node in the network had a centrality higher than 0.80, their relative centralities would equal 1, 0.5, and 0.1. A node with an absolute betweenness of 0 also has a relative betweenness of 0. To avoid confusion, we will adopt a convention of reporting absolute betweenness scores as proportions and relative betweenness scores as percentages.

DATA AND ANALYTIC STRATEGY

With this model in mind, we can test competing center-periphery accounts of ideology by applying shortest-path betweenness to a correlation network of survey responses. In order to ensure comparability with previous work, we construct the network from the same 2000 ANES data set Barker and Tinnick (2006) used to argue for the central role of parenting values using linear regression. This data set is also a good fit for our analyses because of the wide diversity of political attitude items it contains.

Barker and Tinnick use three items to measure parenting values. All of these items start with the stem "Although there are a number of qualities that people feel that children should have, every person thinks that some

¹² Imagine, e.g., that a belief network of K beliefs contained nodes a and b and that this network was resampled 200 times, with a having the highest centrality in the network in all 200 resamples. Further imagine that, in the first 100 resamples, a has a betweenness of 0.8 and b has a betweenness of 0.7. In the second 100, a has a betweenness of 0.9, and b has a betweenness of 0.81. Thus, in all 200 resamples, a is more central than b . However, the 95% confidence range for the raw centrality of a would then be [0.8, 0.9], and for b would be [0.7, 0.81], indicating that their centralities are not significantly different from each other. On the other hand, the relative centrality measure we introduce here would produce confidence intervals of [1, 1] for a and [0.875, 0.9] for b , thus capturing the fact that a has a reliably higher relative centrality than b .

are more important than others. I am going to read you pairs of desirable qualities. Please tell me which one you think is more important for a child to have." The first one offers the response options "independence" or "respect for elders," the second "curiosity" or "good manners," and the third "being considerate" or "being well-behaved." These items let Barker and Tinnick distinguish between respondents who prefer the independent, curious, and considerate child of a nurturant parent and those who favor the respectful, well-mannered, and well-behaved child of a strict parent. In their analyses, they demonstrate with 15 regression equations that these parenting values predict a variety of political attitudes (values and issue positions) net of roughly two dozen demographic and attitudinal covariates.

Because Barker and Tinnick use regression analysis, they must make some arbitrary decisions about which beliefs to treat as dependent variables and which to treat as controls. They ultimately classify 15 variables as outcomes and predict each of them using a separate regression model. Although the BNA method of course requires us to decide which variables to include in the network, we do not need to decide which beliefs are causes and which are effects. BNA also allows us to collapse the 15 distinct models used by Barker and Tinnick into one. To decide which of the resulting variables to retain for our model of subjective political beliefs, we followed the rule of thumb established by Alwin (2007): a question is "factual" (nonsubjective) if the answer can be verified against objective records. Thus demographic and behavioral questions tend to be factual, while beliefs, attitudes, values, and many self-descriptions are not (Alwin 2007). In addition to removing nonsubjective questions, we also dropped the three-variable "need for cognition" scale because the questions it contained did not pertain to politics. The remaining 46 variables are summarized in table 1.

We also made a number of different methodological choices from Barker and Tinnick. First, as Layman et al. (2007) also point out in their unpublished critique, roughly a third of the respondents volunteered the answer "both" to at least one of the parenting questions. Barker and Tinnick treat these responses as missing data and drop the respondents from the sample. Since doing so biases the sample toward those who have a strong opinion on parenting, this procedure may artificially inflate its apparent importance. Thus, instead of dropping these respondents, we treat these variables as ordinal instead of binary and code "both" as the middle of three values. We follow the same procedure with other variables where respondents frequently volunteer responses indicative of ambivalence or indifference. Finally, since our method uses only the pairwise correlations between attitudes, we deal with other missing values by pairwise deletion.

The second pertinent issue with Barker and Tinnick's analysis is their use of scales with low reliability scores. We found that many of their scales have Cronbach's alpha values below 0.6, and some far below 0.5 (estimated

TABLE 1
VARIABLE MNEMONICS, SURVEY ITEMS, AND VARIABLE TYPE
AND RANGE (Alphabetical by "Short Name")

Belief Variable	Short Name	Survey Item(s), with Variable Type and Range (in <i>Italics</i>)*
Abortion Legal	Abort. 1	Abortion be legal: Never; If clear need; If rape/incest/risk to life; Always? <i>Ordinal (4)</i>
Abortion for Teens	Abort. 2	Favor a law to require "girls under age 18 to receive her parent's permission before she could obtain an abortion?" <i>Ordinal (4)</i>
Affirmative Action	Affirm. act.	"Should companies that have discriminated against blacks have to have an affirmative action program?" <i>Ordinal (4)</i>
AIDS Spending	AIDS spend	Increase/decrease "federal spending on AIDS research"? <i>Ordinal (4)</i>
Biblical Literalism	Biblical lit.	Is the Bible the word of God? Yes, literally/Yes, but not literally/No, not the word of God. <i>Ordinal (3)</i>
Crime Spending	Crime spend.	Increase/decrease "federal spending on dealing with crime"? <i>Ordinal (4)</i>
Environmentalism 1	Envir. 1	"Toughen regulations to protect the environment" or regulations are "too much of a burden on business." <i>Ordinal (5)</i>
Environmentalism 2	Envir. 2	More important to protect the environment or maintain jobs and standard of living? <i>Ordinal (5)</i>
Environmentalism 3	Envir. 3	Increase/decrease "federal spending on environmental protection"? <i>Ordinal (4)</i>
Equal Chance	Eq. chance	"One of the big problems in this country is that we don't give everyone an equal chance." <i>Ordinal (5)</i>
Equal Opportunity	Eq. oppor.	"Our society should do whatever is necessary to make sure that everyone has an equal opportunity to succeed." <i>Ordinal (5)</i>
Equal Rights 1	Eq. rights 1	Agreement with [†] "We have gone too far in pushing equal rights in this country." <i>Ordinal (5)</i>
Equal Rights 2	Eq. rights 2	Agreement with "This country would be better off if we were less worried about how equal people are." <i>Ordinal (5)</i>
Equal Treatment	Equal treat	Agreement with "If people were treated more equally in this country we would have many fewer problems." <i>Ordinal (5)</i>
Foreign Aid	For'gn aid	Increase/decrease "federal spending on foreign aid"? <i>Ordinal (4)</i>
Gay Rights	Gay rights	Two-item scale: (1) Should "homosexuals . . . be allowed to serve in the [U.S.] Armed Forces"? (2) Should "[gay/lesbian/homosexual] couples be legally permitted to adopt children?" <i>Interval [1,4]</i>
Buying Guns	Gun rights	Should federal government make it "more difficult" or "easier for people to buy a gun"? <i>Ordinal (5)</i>
Death Penalty	Death penal.	"Do you favor or oppose the death penalty for persons convicted of murder?" <i>Ordinal (5)</i>

TABLE 1 (Continued)

Belief Variable	Short Name	Survey Item(s), with Variable Type and Range (in <i>Italics</i>)*
Ideological Identity	Ideol. id	Self-placement on ideological 7-point scale from "extremely liberal" to "extremely conservative." <i>Ordinal (7)</i>
Immigration 1	Immig. 1	Increase/decrease "number of immigrants from foreign countries who are permitted to come to the [United States] to live." <i>Ordinal (5)</i>
Immigration 2	Immig. 2	Increase/decrease "federal spending on tightening border security to prevent illegal immigration." <i>Ordinal (4)</i>
Immigration 3	Immig.	"Favor a law making English the official language of the United States?" <i>Ordinal (3)</i>
Individualism	Indiv.	Should be "cooperative person who works well with others" or "a self-reliant person able to take care of oneself?" <i>Ordinal (2)</i>
Inequality	Ineq.	Agreement with "It is not really that big a problem if some people have more of a chance in life than others." <i>Ordinal (5)</i>
Limited Government	Limit. gov't.	Three-item scale: (1) "There are more things that the government should be doing" vs. "the less government the better"; (2) "Need a strong gov't to handle today's complex economic problems" vs. "the free market can handle [them]"; (3) "Gov't bigger because it does things people should do for themselves" vs. "because we face bigger problems." <i>Interval [-1.1,1.5]</i>
Military Spending	Milit. spend	"Should the government decrease/increase defense spending?" <i>Ordinal (5)</i>
Military	Military	Feelings thermometer about the military, from not favorable/"cold" (0) to favorable/"warm" (100). <i>Interval [0,100]</i>
Moral Relativism	Moral rel.	"The world is always changing and we should adjust our view of moral behavior to those changes." <i>Ordinal (5)</i>
Newer Lifestyles	New lifest.	Agreement: "The newer lifestyles are contributing to the breakdown of our society." <i>Ordinal (5)</i>
Parenting 1	Parent. 1	Repeated stem: "Please tell me which one you think is more important for a child to have:" (1) . . . "independence, or respect for elders?" <i>Ordinal (3)</i>
Parenting 2	Parent. 2	(2) . . . "curiosity, or good manners?" <i>Ordinal (3)</i>
Parenting 3	Parent. 3	(3) . . . "being considerate, or well behaved?" <i>Ordinal (3)</i>
Party Identity	Party id	Self-placement on party-identification 7-point scale from "Strong Democrat" to "Strong Republican." <i>Ordinal (7)</i>
Antiblack Racism	Racism	Three-item scale. Rate blacks from (1) "hard working" to "lazy"; (2) "intelligent" to "unintelligent"; (3) "trustworthy" to "untrustworthy." <i>Interval [-2.5,2.7]</i>
Religiosity	Relig.	Do you consider religion to be an important part of your life or not? <i>Ordinal (2)</i>

TABLE 1 (Continued)

Belief Variable	Short Name	Survey Item(s), with Variable Type and Range (in Italics)*
Surplus Taxes	Taxes	Agreement: "Most of the expected federal budget surplus should be used to cut taxes." <i>Ordinal (2)</i>
Tolerance	Toler	Should be "tolerant of people who choose to live according to their own moral standards" even if different from ours. <i>Ordinal (5)</i>
Welfare Recipients	Welf're recip.	Feelings thermometer about "people on welfare," from not favorable/"cold" (0) to favorable/"warm" (100). <i>Interval [0,100]</i>
Welfare Spending	Welf're spend.	Two-item scale. (1) Increase/decrease "federal spending on welfare programs?" (2) Increase/decrease "federal spending on food stamps?" <i>Interval [1,4]</i>

NOTE.—The "short name" column contains the abbreviations we use in the network diagrams. Numeric items are coded so that lower values correspond to respondents providing the first response option. For example, for Antiracism, lower values correspond to "hard-working," "intelligent," and "trustworthy"; for Welfare Spending, they correspond to "increase" for both types of funding.

* Ordinal variable (with number of categories) or interval variable with range [from, to].
† "Agreement" items are coded from strongest agreement (lowest value) to strongest disagreement (highest value).

via polychoric correlations), which is substantially below accepted levels.¹³ For example, their scale for gun control/crime policy contains a question about gun ownership rights, a question about federal spending on crime prevention, and a question about support for the death penalty. This scale has a Cronbach's alpha of 0.41, suggesting that these items may not be closely related to a single underlying concept. The scales representing equal rights, equal opportunity, abortion, and defense spending also have alphas below 0.6.

Scale construction can be used to remove the variance that stems from the response error associated with individual items (Ansolabehere, Rodden, and Snyder 2008). However, scales constructed of weakly related items could instead remove large amounts of nonerror variance, producing composite variables that are instead more weakly correlated than the component variables. We thus retain only those scales where all the items have

¹³ Throughout this article, we always compute polychoric correlations between ordinal variables, polyserial correlations between numeric and ordinal, and Pearson's correlations between numeric variables. This includes the correlations we use to estimate all factor loadings. Note that correlation measures implicitly assume that the pairwise relationships between the latent and/or manifest variables in our data are predominantly linear in character. In app. C, we compare these correlations with nonparametric measures of nonindependence between pairs of variables based on entropy and mutual information. Our results confirm that this assumption is justified.

pairwise polychoric correlations of 0.6 or above.¹⁴ In those scales where some items are correlated above this threshold while others are correlated below, we construct the scale using only the strongly correlated items. We include all the remaining items as separate variables. This includes the three parenting items, which have pairwise polychoric correlations of 0.47, 0.10, and 0.44. We also replicate our primary analysis with these three items joined into one scale and find that it does not affect the substance of our results (see app. D).

We proceed with the analysis as follows. First, we examine the belief correlation network we constructed from the full 2000 ANES data set. To test the moral politics and social constraint accounts, we compare the relative centralities of parenting and ideological identity. We then investigate the robustness of our findings using bootstrapping. We resample the respondents of the survey data set to demonstrate the reliability of our findings to sampling error. As an extra robustness check, we then resample the variables to examine the robustness of our findings to the specifics of variable selection. After addressing these methodological concerns, we turn to the theoretical challenge presented by potential population heterogeneity. We create 16 partitions of the survey population along major demographic and cultural variables, yielding 44 subsamples corresponding to various social groups (e.g., women, middle-income respondents, etc.). We compare these across different dimensions of belief structure. Finally, we analyze three subgroup belief networks in more detail to search for exceptions to the primary pattern.

RESULTS

Belief Network for Full ANES Sample

We depict the belief network we constructed from the 2000 ANES data set in figure 2. We used darker and thicker lines to represent stronger correlations, omitting correlations below $|r| = 0.15$ for legibility.¹⁵ The force-directed plot reveals a network structure with a sparse periphery and relatively densely connected groups of nodes near the core. Visually, the densely connected core appears to contain two groups of variables, which we label *A* and *B* on the diagram.¹⁶ The bulk of the items that make up group *A* have

¹⁴ We replicated our primary analysis with scales constructed at other thresholds. The substantive findings remained the same when other thresholds were used.

¹⁵ The omission is for visual purposes only. All analyses are based on the full network with no ties omitted.

¹⁶ The node groupings we discuss here have a conceptual resemblance to partitions produced by modularity maximization. Modularity analyses also suggest that the belief groups we label *A* and *B* likely belong to two different modules. However, we found the modularity results for this network to be unreliable (see app. B). To avoid creating an undue impression of accuracy, we report this informal visual analysis of group structure instead.

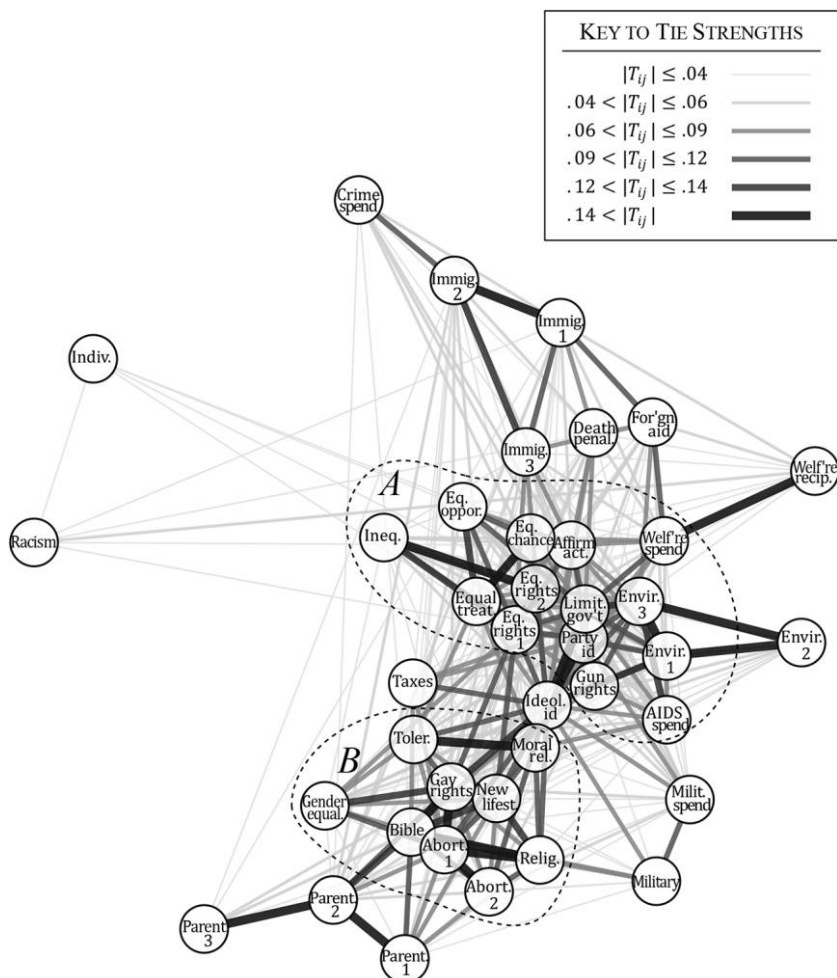


FIG. 2.—Correlation network for the full population sample (see table 1 for full node names). Tie strength $|T_{ij}| = \text{cor}^2(x_i, x_j)$ is represented by thickness and boldness (see inset). Correlations below $|r| = 0.15$ are not depicted. The force-directed layout places strongly connected nodes closer together and weakly connected and unconnected nodes further apart.

connections to the social welfare agenda that has divided the two major U.S. parties since the New Deal (Carmines and Layman 1997), including items on affirmative action and government efforts to redress inequality. It also contains questions on the size and scope of government activity (e.g., regulation of the environment, gun control). The group labeled *B*, on the other hand, contains many items that correspond to the “New Left”

issue agenda which became part of the mainstream political discourse in the late 1960s and early 1970s (Carmines and Layman 1997), such as items on abortion, gender equality, and gay rights. It also contains items concerning religious identity and moral worldview.

The first column of table 2 shows the centrality estimates for the individual variables. The “absolute” column shows that the centralities range from 0 for the least-central nodes to 0.35 for the most central (overall centralization: 0.33). The variables Parenting 1 and Parenting 3 both have centralities of 0, as do eight other nodes located near the periphery. The remaining parenting variable, Parenting 2, has a centrality of 0.07. Thus, the three parenting variables combined lie on only 7% of the geodesics.¹⁷ The network plot in figure 2 provides context for this low centrality. Except for their ties to each other, their strongest ties are to items concerning religious identity and abortion that make part of group *B*. All of their ties to the social welfare or limits of government items that make up group *A* were too weak to depict. They thus appear to occupy a peripheral position near the edge of group *B*, far from the network’s center.

Ideological identity, on the other hand, can be found near the middle of the plot in figure 2, between the groups we labeled *A* and *B*. Its position between the two relatively densely connected clusters appears intuitively central. The betweenness scores confirm this visual impression. Its betweenness centrality of 0.35 makes it the most central node in the network. It lies on five times as many geodesics as all three parenting variables combined.

The second- and third-most central nodes are Limited Government and Gay Rights. They are positioned near the visual centers of *A* and *B*, respectively. Their respective centralities of 0.12 and 0.10 indicate that they are only roughly one-third as central as ideological identity. They thus do not appear to occupy the same brokerage position as ideological identity. We therefore find that ideological identity is the clear center of this belief network. These results are consistent with the theory of social constraint, but not with the theory of moral politics.

Potential Sources of Error

Sampling variability.—Thus far, we have found that ideological identity is the most central node in this network. We now use the nonparametric bootstrap to establish the statistical significance of this finding. In each of the 1,000 iterations of the bootstrap, we drew a sample of $N = 1,543$ ANES respondents with replacement and followed the same BNA procedure as

¹⁷ An enumeration of these geodesics reveals that they all end in either Parenting 1 or Parenting 3, which is consistent with the visual intuition that Parenting 2 serves as a “gatekeeper” (Freeman 1980) for these further removed nodes and nothing else.

TABLE 2
CENTRALITY OF NODES IN BELIEF NETWORK FROM FULL 2000 ANES SAMPLE

ATTITUDE	BETWEENNESS CENTRALITY		
	Original Sample	Bootstrapped (1,000 Row Resamples)	
		Relative (Mean and 95% CI)	Absolute (Mean)
Ideological Identity352		.319
Limited Government119		.105
Gay Rights104		.082
Equal Rights 1074		.076
Welfare Spending089		.065
Parenting 2070		.061
Party Identity065		.048
Biblical Literalism020		.050
Environmentalism 3023		.041
Environmentalism 1046		.039
Immigration 2035		.037
Immigration 3036		.037
Abortion Legal030		.038
Affirmative Action024		.032
Immigration 1024		.029
Equal Rights 2023		.026
Newer Lifestyles016		.017
Buying Guns008		.016
Religiosity011		.017
Foreign Aid007		.011
Equal Chance008		.009
Moral Relativism005		.010
Welfare Recipients024		.015
Equal Treatment004		.006
Military Spending003		.006
Crime Spending004		.005
Tolerance001		.004
AIDS Spending003		.005
Parenting 1	0		.004
Equal Opportunity	0		.004
Abortion for Teens	0		.003
Environmentalism 2	0		.002
Death Penalty	0		.001
Military	0		.001
Surplus Taxes	0		.001
Inequality001		.001
Gender Equality	0		.001
Parenting 3	0		0
Individualism	0		0
Antblack Racism	0		0

above to create a set of betweenness estimates. We then used these scores to estimate the 95% confidence intervals for the relative betweenness centralities of each belief, which we report in the “Relative” column of table 2. The leftmost and rightmost endpoints of each error bar correspond to the 2.5th and 97.5th percentiles of the estimate, respectively, and the black circles represent the means (also printed numerically next to each error bar). Twenty-five of the 40 beliefs in this column have confidence intervals starting at 0%, indicating that their centralities are not statistically distinct from zero. Two of the parenting variables (1 and 3) are among these 25 low-centrality nodes. Their mean relative centralities are 1% and 0%, respectively. The remaining parenting node (2) has a mean relative centrality of 20%, indicating that, in an average iteration of the bootstrap, it is roughly one-fifth as central as the most central node.

The mean relative centrality of Limited Government is 34%, which indicates that, on the average, it lies on roughly one-third as many geodesics as the most central node. It is again the second most central node in the network. Gay Rights and Equal Rights 1 have relative centralities of 27% and 25%, respectively, which places both at roughly one-fourth of the centrality of the most central node and makes them the third and fourth most central nodes. The confidence intervals for these three nodes begin at 11%, 6%, and 7%, and extend up to 67%, 55%, and 54%, respectively. These confidence intervals are wide and overlapping, as are those belonging to most other nodes.

The reliably central position of Ideological Identity stands in stark contrast with the largely undifferentiated centralities of the other variables. Its mean relative centrality of 100% shows that, in the average run, it is the most central node. Moreover, the lower and upper ends of its confidence interval are also at 100%, which indicates that its relative centrality does not significantly deviate from the maximum. In fact, when we examined the full range of the 1,000 bootstrap iterations, we found that it was the most central node in every iteration. The dominant role played by ideological identity thus appears both substantively significant and remarkably robust to statistical variation ($P \approx 0$). Our analyses are thus consistent with the social constraint view that ideological identity lies at the core of the system of political attitudes.¹⁸ In contrast, they offer no support to the moral politics view that parenting attitudes play a central role in the system of political views.

¹⁸ As an additional robustness check, we repeated this bootstrapping analysis with the three parenting variables joined into a single scale. In 1,000 bootstraps of the resulting 38-variable network, we found that this parenting scale was on average no more central than the parenting variables were individually (see app. table D2). The average absolute centrality of the parenting scale was 0.001, which is lower than the absolute centrality Parenting 2 had in the main sample (0.06). The centrality of ideological identity remained unchanged.

Variable selection.—The analyses we describe above made use of a set of 40 belief variables constructed from the 2000 ANES data set. While this collection of political attitude items is large and seemingly comprehensive, its representativeness of the domain of politics as a whole cannot, of course, be guaranteed statistically. Given that it is impossible to enumerate the full set of political attitudes, we cannot rule out the possibility that the belief network we examine may feature too many beliefs from some parts of the unobserved belief network and too few from others. Since prior work has demonstrated that betweenness centrality estimates may be unstable to changes in the node set (Zemljic and Hlebec 2005), this raises the possibility that our findings may be skewed by particular details of the set of variables we included in our model.

To rule out this possibility, we again turn to resampling. In the preceding analyses, we resampled the rows (respondents) of the 2000 ANES data set to demonstrate that ideological identity is reliably central in the face of fluctuations in pairwise correlations (i.e., tie strengths). To examine reliability to fluctuations in the set of beliefs included in the analysis (i.e., the node set), we now resample its columns (items). Since including two copies of the same variable in one betweenness analysis would not yield meaningful results, we resampled the item set using an “ m out of n ” resampling scheme (Bickel, Götze, and Zwet 2012). In 2,000 additional resamples, we dropped different 12-belief subsets from the network and analyzed the networks consisting of the remaining 28 beliefs.¹⁹ Since the number of ties in a fully connected network is roughly proportional to the square of the number of nodes, each of these resamples retained only 351 out of the 780 ties (49%) that composed the original network. These resamples were thus highly distinct from the original network as well as from each other. To distinguish this procedure from the first set of bootstraps, we will call the first set “row bootstraps” and this second set “column bootstraps.”

We considered two distinct ways in which details of the node set could affect the results of our analysis. First, the apparent centrality of ideological identity could be exaggerated by eccentric features of the node set, such as the inclusion of structurally redundant nodes that dilute each other’s centrality. Second, the presence of ideological identity may mask the centrality of another node that, in its absence, would have occupied an equally central position. We used two different column resampling procedures to rule out these possibilities.

¹⁹ We also replicated this analysis with $k = 2, 3, 7, 10, 13$, and 15 nodes dropped from the network. As could be expected, lower values of k result in smaller confidence intervals for our results, and vice versa. However, the substantive content of our findings was unaffected by these changes.

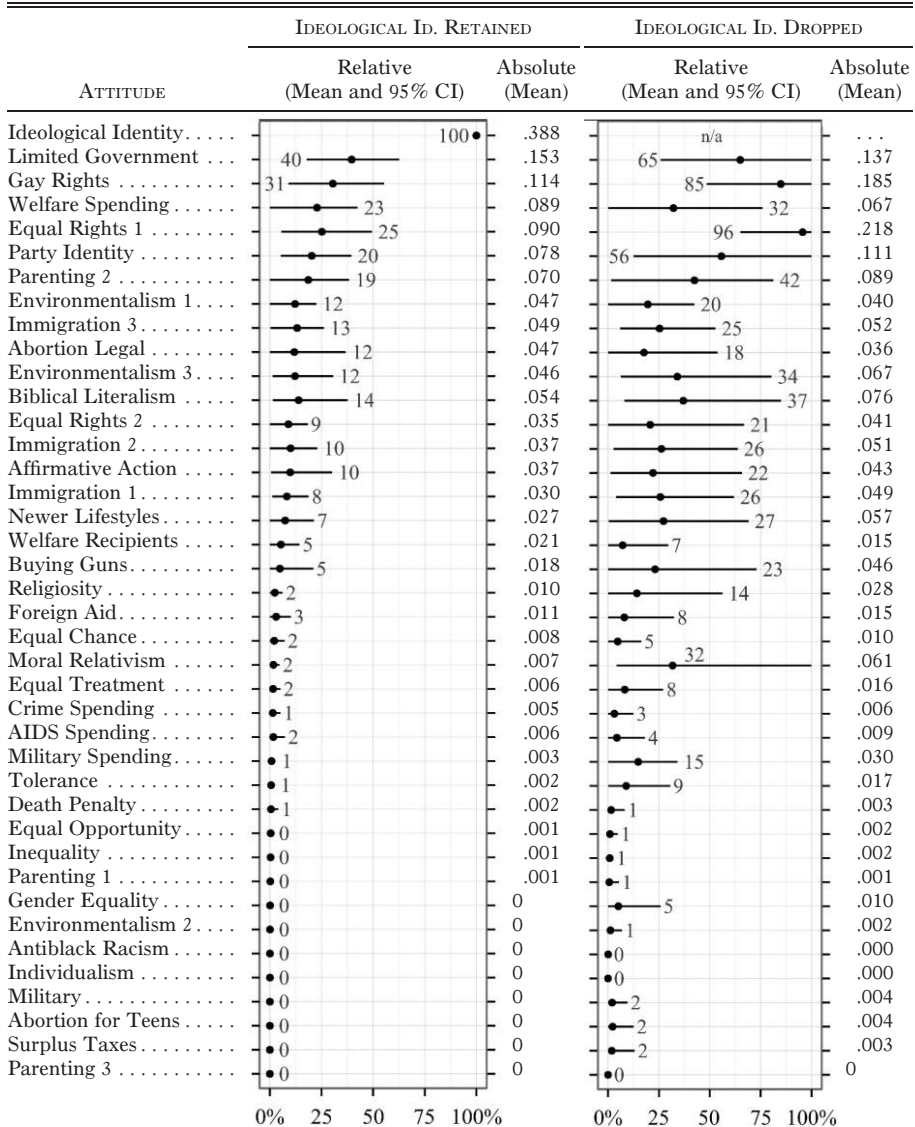
We first examined whether the centrality of ideological identity drops when subsets of variables are removed from the analysis. To do this, we drew 1,000 column resamples by first selecting ideological identity and then adding a uniform random sample of 27 other beliefs drawn without replacement out of the remaining 39 beliefs. This yielded 1,000 networks of 28 beliefs each. The relative betweenness centralities for this set of resamples can be found on the left side of table 3, under the title “Ideological Id. Retained.”²⁰ These centrality results strongly resemble the ones previously depicted in table 2. As before, the 95% confidence interval belonging to ideological identity never declines from the 100% mark, indicating that it reliably remained the most central node throughout the full range of belief subsets analyzed. All other beliefs have lower relative centralities with confidence intervals that overlap one another and remain reliably below that of ideological identity. Thus, the central position occupied by ideological identity appears robust even to dramatic changes in the set of beliefs we include in the analysis.

In the second column-resampling procedure, we instead omitted ideological identity from our belief set. We then drew 1,000 uniform random samples of 28 beliefs each, again sampling from the remaining 39 beliefs without replacement. We examined each of these resampled 28-belief networks to determine whether any other belief node comes to occupy a reliably central position when ideological identity is dropped. We report these results in the right column of table 3. The 95% confidence intervals in this set are noticeably wider than previously, indicating that the nodes’ relative centralities become substantially less stable when ideological identity is omitted. Moreover, in contrast with results in the left column of table 3, five distinct beliefs—limited government, gay rights, equal rights 1, party identification, and moral relativism—now have relative centralities statistically indistinguishable from 100%. Thus, in the absence of ideological identity, no other belief appears to occupy a robustly central position.

We also used our row (respondent) resampling procedure to examine the belief network that excludes ideological identity but includes all remaining 39 beliefs. To construct each of these additional resamples, we drew a uniform random sample of 1,543 rows with replacement from a data set that excluded ideological identity but retained all other columns. We then performed betweenness analyses on each of the 1,000 resulting 39-belief networks. We found that the resulting centrality distribution again consisted of overlapping confidence intervals with no clear central variable and overall resembled the results of the second set of column bootstraps we report above (see left column of app. table D2). These results are again consistent

²⁰ When we calculated the centrality distributions for any one node, we simply omitted all the cases for which this node was dropped from the analysis. Thus, those samples where a variable was absent have no effect on its centrality scores.

TABLE 3
STABILITY TO CHANGES IN VARIABLE SET: BETWEENNESS
CENTRALITY IN 1,000 COLUMN RESAMPLES



with ideological identity occupying a uniquely central position in the belief network.²¹

Population Heterogeneity

Our analyses thus far have produced substantial evidence consistent with the theory of social constraint and inconsistent with theories (like Lakoff's) that put a different concept at the center of a belief system. The method we used, however, rests on the assumption that the population-wide organization of attitudes is produced largely through a single dominant process that does not vary systematically for subgroups of the population. That is, it assumes a single network of which each person's beliefs are a noisy realization.

This single-network view is shared by both moral politics theory and social constraint theory, as well as many other center-periphery accounts of ideology (e.g., Jost et al. 2003). However, a number of recent sociological treatments (Boltanski and Thévenot 1999; van Eijck 1999; Achterberg and Houtman 2009; Thornton et al. 2012; Baldassarri and Goldberg 2014) instead assume substantial heterogeneity by arguing for the existence of many different "logics" that organize beliefs differently for different subgroups. We develop these contrasting views of heterogeneity in more detail below and then test them empirically on the ANES data set.

In the social constraint view, individuals construct their belief systems by acquiring pieces of political content from attitude producers, which they select using their political identity as a heuristic. However, individuals vary greatly in the extent to which they care about politics and attend to political information flows (Delli Carpini and Keeter 1996). Moreover, the taste for political communication, much as tastes for other cultural products, is highly socially patterned, with some social groups systematically further removed from the institutional field of organized politics than others (Bourdieu 1984). Those who consume enough informational flows may learn the partisan pseudo-logics which make certain positions on far-flung issues such as global warming and gay rights entail views on, for example, military policy and health-care spending—attitudes which may otherwise be mostly unconstrained. By this logic, different social groups should then vary in the *amount* of belief system organization they exhibit—a point that has been demonstrated in much empirical work (Converse 1964, 2000)—but they should not vary in the *logic* of organization of their beliefs.

²¹ We additionally reanalyzed all 2,000 28-node column bootstraps using multivariate linear regression, with individual simulation runs as observations, network centralization as the outcome variable, and 39 dummy variables indicating which variables were dropped as the predictors. As expected, we found that the presence or absence of ideological identity was by far the strongest predictor of centralization.

Baldassari and Goldberg propose a contrasting view of population heterogeneity, arguing that “the heterogeneity of political belief systems does not simply derive from differences in levels of political sophistication,” that is, amount of belief organization, “but in individuals’ social identities: people with different sociodemographic profiles structure their political preferences in systematically different ways” (2014, p. 78). They thus see demographic positions as lying at the root of differences in both the amount and the logic of belief organization. Under this view, sets of political positions that are perceived to be coherent from the perspective of one population may appear to be contradictory from the perspective of another. For example, though support for environmental regulations is generally negatively correlated with support for gun ownership in the population as a whole, we can imagine, say, a subpopulation of hunting enthusiasts where environmental protection and gun ownership rights go hand in hand. The practical implication of this argument is that attitudes that are positively correlated in one subgroup may be negatively correlated in another. And, if two such evenly sized populations are mixed together in a single sample, the two patterns may simply cancel out, yielding two variables that appear uncorrelated in the full sample.

To compare these views of heterogeneity empirically, we constructed separate belief networks for 44 different subpopulations, which we produced by partitioning the population 16 times along different demographic dimensions (see table 4 for descriptive statistics and app. E for details of variable coding). Prior work has found that various forms of social status are predictive of average belief constraint (i.e., mean absolute correlation between beliefs), with higher-status groups generally exhibiting a higher level of constraint than lower-status groups (see review in Gordon and Segura [1997]). For this reason, we examine nine dimensions that are associated with major social and economic cleavages in contemporary American society. These dimensions are respondent’s income bracket, occupational category, class self-identification, education, gender, age, race (black, not black), Hispanic status, and religious denomination.²²

We include six further dimensions to tap cultural cleavages. These measure whether the respondent’s parents are foreign or U.S. born, whether the respondent attends church, the type of populated place where the respondent resides (large city, rural area, etc.), and whether or not this location is in the southeastern United States. Because of the importance of child-

²² Each respondent is assigned to exactly one subpopulation of each demographic dimension. Aside from the interaction between religious attendance and income we discuss below, we do not intersect these dimensions. The respondents with missing demographic data along a dimension are not included in any of the groups in that dimension. (They are only omitted from the analyses of dimensions where they have missing data and are present for analyses of all other dimensions.)

rearing to the theory of moral politics, we also partition respondents by the number of children they have (zero, one, or more). We also include an index of the respondent's factual knowledge about politics, which is frequently used as a measure of the respondent's involvement with the field of organized mainstream politics (Delli Carpini and Keeter 1996).²³

In their recent study of belief heterogeneity, Baldassarri and Goldberg (2014) use an inductive partitioning approach that does not require subpopulations with different belief structures to lie on different sides of a demographic divide. The demographic positions of the belief systems they locate, however, are central to their interpretation of these results, as well as to arguing against their possible spuriousness.²⁴ They state that "sociodemographic characteristics—particularly class and religiosity—account for this divergence in political belief systems" (p. 47), and "nonreligious high earners and religious low earners . . . occupy social positions that push them to take ideological stances that are seemingly contradictory" (p. 69). They thus claim that, "if the overlap between people's class and religiosity has a bearing on how they combine their political preferences, then we should find that the interaction between the two explains how respondents combine their political beliefs" (p. 69). We use our demographic heterogeneity analysis to test the validity of these claims.

Empirically, Baldassarri and Goldberg operationalize this social position via church attendance and income, arguing that, while high-income church

²³ To measure the information levels of the respondents, we made use of the factual political information quiz included on the ANES (eight questions, 0 or 1 points each). See app. E for quiz questions. Since this quiz-based measure can confound knowledge with personality traits such as confidence and competitiveness (Mondak 2000, 2001), we also make use of the interviewers' subjective assessments how informed the respondent appeared (two items, 0–4 points each). We summed these scores into an index that ranges from 0 to 16, and labeled the bottom third of the range (0–5) "low information" and the top third (11–16) "high information."

²⁴ Inductive heterogeneity detection can yield positive results even in the absence of any true heterogeneity of belief structure. Consider two unrelated attitudes A (Yes/No) and B (Yes/No) with no logics connecting any position on A to a position on B, and with all response pairs (A = Yes, B = Yes), (No, No), (Yes, No), (No, Yes) equally likely. This population can then be partitioned into two groups, with (Yes, Yes) and (No, No) respondents assigned to one group, and (Yes, No) and (No, Yes) to the other. A and B would then be positively correlated in the first group, and negatively in the second. Since we know, however, that A and B are simply unconstrained, it is incorrect to interpret this as evidence of alternate systems of belief organization; the result is completely artefactual. Inductively located heterogeneity thus requires external validation. Baldassarri and Goldberg seek it partly in demographic position: "while [relational class analysis] allows us to identify groups of respondents that exhibit distinctive patterns of opinion, we cannot, with survey data alone, determine the underlying psychological processes that generate these patterns. Nevertheless, we can make reasonable assumptions about these causes and how they relate to people's location in sociodemographic space" (2014, p. 59). Below, we examine whether the stated demographics actually correspond to substantial differences in attitude structure and find that they do not. This raises questions about potential spuriousness.

TABLE 4
SIGNIFICANT CORRELATIONS BETWEEN PAIRS OF ATTITUDES THAT HAVE
OPPOSITE SIGNS WITHIN DIFFERENT SUBGROUPS (Percentage)

DEMOGRAPHIC SUBGROUPS (with Group Size)	COR'S WITH $P < .05$	% OPPOSITE SIGNS IF COMPARED TO SUBGROUP NUMBER				
		1	2	3	4	5
Gender:						
1. Male (671)	6494			
2. Female (872)	587	.4	...			
Class (self-identified):*						
1. Working class (699)	582	...	1.3			
2. Middle class (844)	639	1.3	...			
Parents foreign born:						
1. Parents foreign born (229)	449	...	1.0			
2. Not foreign born (1,310)	663	1.0	...			
Number of children:						
1. One or more children (1,124)	6282			
2. No children (403)	577	.2	...			
Black:						
1. Black (159)	226	...	10.5			
2. Not black (1,344)	683	10.5	...			
Hispanic:						
1. Hispanic (101)	243	...	2.6			
2. Not Hispanic (1,432)	676	2.6	...			
Age group:						
1. Under 40 (545)	577	...	0	1.4		
2. 40–55 (491)	589	0	...	1.3		
3. Over 55 (499)	517	1.4	1.3	...		
Education:						
1. High school or below (578)	464	...	1.8	4.0		
2. Associate or some college (467)	561	1.82		
3. Bachelor's or above (494)	636	4.0	.2	...		
Income:						
1. Under \$35,000/year (513)	5105	1.4		
2. \$35,000–\$65,000/year (385)	586	.54		
3. Over \$65,000/year (408)	576	1.4	.4	...		
Southeastern United States:						
1. Southern (550)	5844			
2. Not southern (993)	640	.4	...			
Religion:						
1. Catholic (396)	410	...	1.1	1.4	1.9	
2. Mainline Protestant (420)	574	1.1	...	2.0	.8	
3. Other Protestant (287)	423	1.4	2.0	...	2.5	
4. Other religion or none (323)	588	1.9	.8	2.5	...	
Occupational category:						
1. Manager (203)	4455	.3	0	1.4
2. Professional (326)	587	.55	.5	2.4
3. Routine nonmanual (354)	474	.3	.53	.9
4. Skilled or semi-skilled (241)	435	0	.5	.3	...	1.6
5. Unskilled or farm (276)	440	1.4	2.4	.9	1.6	...
Type of place:						
1. Larger city (282)	436	...	1.5	1.3	1.6	
2. Rural area (431)	488	1.5	...	1.2	1.8	

TABLE 4 (Continued)

DEMOGRAPHIC SUBGROUPS (with Group Size)	COR'S WITH $P < .05$	% OPPOSITE SIGNS IF COMPARED TO SUBGROUP NUMBER				
		1	2	3	4	5
3. Town or smaller city (507)	592	1.3	1.22	
4. Suburb (310)	569	1.6	1.8	.2	. . .	
Church attendance:						
1. Attends church (1,057)	6416			
2. Does not attend church (478)	604	.6	. . .			
Cross-pressures:						
1. Pressures aligned (471)	591	. . .	0	1.6		
2. Neither (385)	586	09		
3. Pressures crossed (448)	499	1.6	.9	. . .		
Political knowledge:						
1. Low (398)	372	. . .	4.2	12.6		
2. Medium (667)	543	4.2	. . .	1.6		
3. High (473)	700	12.6	1.6	. . .		

NOTE.—Each category is compared to each other category within the same demographic dimensions. The column numbers (1–5) index the categories in the current dimension (e.g., for the categories within Religion, subgroup number 1 is Catholics, 2 is mainline Protestants, etc.). The counts of statistically significant correlations are out of 780 possible. See app. E for detailed descriptions of the variables.

* Identification with other classes is rare; 97% of respondents either “thought of” themselves as middle or working class or identified with one of the two when prompted to do so explicitly.

attendees and lower-income nonattendees experience economic and moral pressures that are aligned, lower-income church attendees and high-income nonattendees face pressures that are at odds (p. 69). Thus, they conclude that the former two groups would have traditional belief systems with consistently conservative or liberal attitudes, while the latter would hold liberal positions on economic issues and conservative positions on moral ones, or vice versa. They do not, however, test this hypothesis directly. To carry out this test, we constructed an Economic and Moral Pressures stratifying variable. We labeled higher-income church attendees and lower-income nonattendees Aligned Pressures, lower-income attendees and higher-income nonattendees Cross Pressures, and all middle-income respondents Neither.

Heterogeneous logics.—To examine whether different demographic groups display distinctive logics of beliefs organization, we contrasted the way the same pairs of beliefs are correlated with each other in different groups. For each subpopulation, we constructed a matrix consisting of polychoric, polyserial, and Pearson’s correlations, as appropriate. Since statistical noise can make weak correlations fluctuate around zero, we first took each belief network and removed from it all the correlations that were not statistically different from 0 at $P < .05$. Out of the 780 correlations in each network, this left a median of 577, or roughly three-quarters (see first numerical column in table 4).

We first compared each pair of mutually exclusive subpopulations. These are the populations that come from partitions along the same demographic dimension. For example, for the dimension "age," such groups are "under 40," "40–55," and "over 55," which yields three unique comparisons: "under 40" versus "40–55," "under 40" versus "over 55," and "40–55" versus "over 55." Within each of these 45 subpopulation pairs, we compared the directions (signs) of all the correlations that were significant for both groups. For example, out of the 780 unique belief pairs, 649 attained statistical significance in the male sample, 587 in the female sample, and 535 in both the male and the female subsample. If different groups indeed use substantially different logics to organize their political views, we should observe that these signs frequently point in different directions.²⁵

The results of these comparisons can be found in table 4. For example, out of the 535 correlations that were significant in both the male and female samples, only two correlations (0.4%) had different signs for males and females, with the remaining 533 pointing in the same direction. The table shows that the same basic finding occurs for all comparisons on class, parents' nativity, age, education, income, region, religion, occupation, and type of place. It also holds for Economic and Moral Pressures, where 98.4% of the significant correlations retained the same sign between the Cross Pressures and Aligned Pressures groups. Overall, in 43 of these 45 group comparisons, 95% or more of the significant correlations pointed in the same direction. Even in the two comparisons that recorded the most extreme differences—between blacks and nonblacks and between high- and low-information respondents—89.5% and 87.4% of the correlations still retained the same sign.²⁶ Overall, among all the 45 comparisons we carried out, 98.7% (median) of the correlations retained the same sign for both groups, with only 1.3% switching directions (IQR: 0.5%–1.6%).

We then extended this analysis to each unique pairing of the 44 demographic subgroups, independent of the dimension used to create them.²⁷ There are 946 such group pairs. In 901 of the resulting comparisons (95.2%), 95% or more of the correlations had the same sign, and in 941 comparisons (99.5%), this same-sign proportion exceeded 90%.²⁸ Among all the group pairs, 99.6% (median) of the correlations had the same sign, with 0.4% switching di-

²⁵ We discuss the theoretical meaning of these sign comparisons in more detail in app. F.

²⁶ We examine these groups in detail in the following section.

²⁷ For example, while the previous analysis compared the category "Male" only to "Female," this analysis also compares "Male" to "Black," "Under 40," etc.

²⁸ All five of the remaining group pairs, for which between 87.3% and 89.8% of the correlations retained the same sign, again involved either low-information or African-American respondents.

rections (IQR: 0%–1.2%). Given the much-bemoaned statistical noisiness of survey data, the constancy of sign is strikingly robust.

Our results therefore provide no evidence to support the assertion that different groups typically organize their beliefs according to different logics. Rather, we find that, even in the most contrasting of groups, the overwhelming majority of political attitudes “go together” in the same way. The issue positions that go together for one group—at least to a statistically significant extent—are very rarely opposed for another group. Heterogeneity in the organizing logic of political beliefs thus appears to be the exception rather than the rule.

In addition to this analysis of heterogeneity across demographic dimensions, we also conducted a more general heterogeneity analysis using mutual information (see app. C). Normalized mutual information (\hat{I}_{ij}) and squared correlation yield similar estimates of pairwise relationships between variables when these relationships are strong and linear. However, while correlation captures only linear relationships, mutual information is a general nonparametric measure of nonindependence. As we demonstrate in appendix C, \hat{I}_{ij} can detect relationships between variables even in the presence of two subpopulations where the variables obtain opposite linear relationships. The same heterogeneity would cause the linear relationships to cancel each other out, yielding an overall correlation of zero. Thus, in the presence of such heterogeneity, \hat{I}_{ij} and squared correlation should diverge. However, when we apply both \hat{I}_{ij} and squared correlation to our data, we find that the two measures are instead mutually correlated at $r = 0.91$, indicating that they overwhelmingly vary in unison. These supplementary analyses thus also find no evidence of heterogeneous logics of belief organization.

Amount of organization.—We next examine whether the belief networks exhibit a heterogeneity in their amount of organization, as suggested by the theory of social constraint. There are two senses in which beliefs can vary in the degree of organization between groups. First, in different groups, pairs of beliefs (node dyads) can “hold together” to a greater or lesser extent. We have previously referred to this quantity as “mean constraint,” which we operationalize as the mean of the absolute correlations between pairs of beliefs. Second, whole networks can vary in how much structure they exhibit. For social constraint and other center-periphery accounts, this network-wide property can be captured by betweenness centralization.²⁹

We present an overview of these measures in figure 3. Here, each of the 44 subgroup belief networks is represented with a point, the coordinates of

²⁹ We can also measure it using row bootstrapping, as the portion of resamples in which the node with the highest overall centrality across the resamples occupies the most central position. We take this approach later in the analysis.

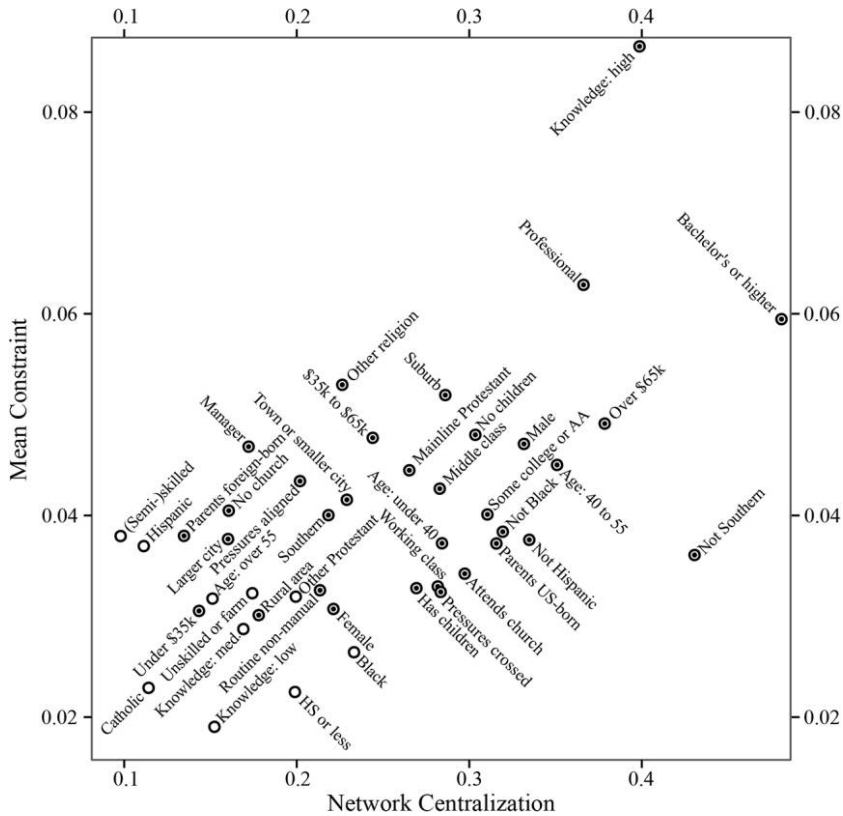


FIG. 3.—Degree of organization in belief networks of 44 demographic subgroups. Filled circles indicate networks where the most central node was ideological identity.

which correspond to its centralization (x) and mean constraint (y). The figure reveals significant differences in the amount of belief system organization between different populations. The subgroups range from 0.02 to 0.09 in constraint ($\sigma = 0.01$), a difference of a factor of 4. They also range from 0.10 to 0.48 in centralization ($\sigma = 0.09$), or roughly a factor of 5. A detailed analysis of which properties of demographic groups predict greater or lesser belief constraint is outside the scope of this article. However, we note that, consistent with prior work on group differences in political knowledge (Delli Carpini and Keeter 1996), higher status groups appear to generally possess higher belief constraint than the lower status groups on the same dimension. Network centralization follows a similar pattern.

We previously drew on the social constraint account to predict that belief networks would either be centered on ideological identity or have no dis-

cernible center at all. In figure 3, the networks where ideological identity is the most central node are marked with filled circles, while those with a different central variable are indicated with hollow circles. All of the hollow circles are clustered near the lower-left corner of the plot, indicating that all networks with high constraint or high centralization have ideological identity at their center. The relationship between network centralization and the centrality of ideological identity is plotted in figure 4. The Pearson's correlation between these two quantities among the 44 networks is $r = 0.93$. Thus, as expected, the bulk of the variance in network centralization ($R^2 = 0.86$) can be explained by the centrality of ideological identity. Taken together, the results reported in this section show that although demographic groups vary in the degree to which they are organized, they do not vary in the way in which they are organized.

Comparison to existing work.—Baldassarri and Goldberg (2014) have previously analyzed ANES data to claim support for the existence of different logics of belief organization—a claim contrary to ours. In appendix F, we re-examine some key evidence they offer in support of their argument to clarify

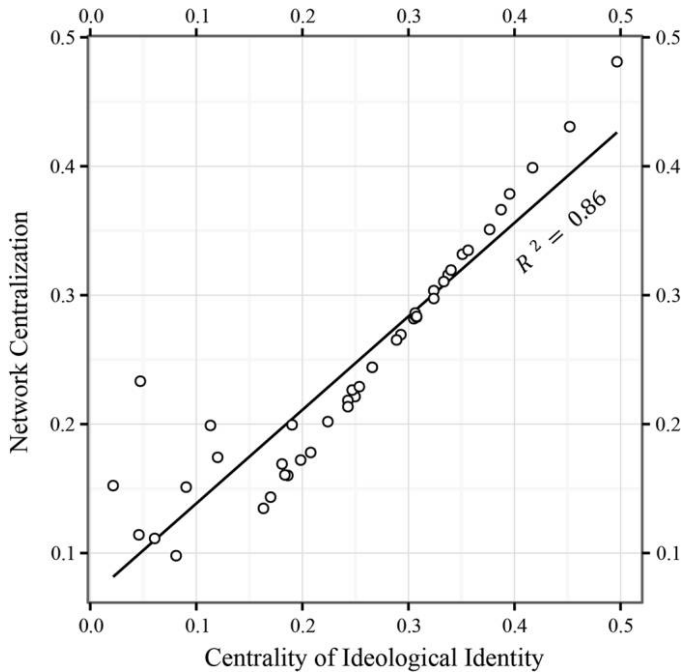


FIG. 4.—Network centralization by centrality of ideological identity in 44 demographic subgroups, with line of best fit.

this disagreement. Their heterogeneity analyses of the eight ANES years each partitioned respondents into three groups, which they termed “ideologues,” “agnostics,” and “alternatives.” They propose that agnostics follow the same logic of belief organization as ideologues, albeit to a lesser extent, whereas alternatives employ a wholly different logic. Their results show that, for ideologues, the average cross-domain beliefs correlations are strong and positive, while for agnostics they are indeed either positive but weaker, or are insignificant. However, even for alternatives, who are supposed to follow a different logic, only three out of the 48 average cross-domain correlations are actually negative—and, even in those rare cases, the average negative correlations are weak. The remaining correlations are generally either still positive but weaker than for ideologues or are insignificant. The belief system of alternatives thus appears to overwhelmingly be a subset of the belief system of ideologues. Much like agnostics, alternatives follow some of this system’s logics to their full extent and weaken or omit other logics but very rarely actually introduce unique logics of their own. Thus, contra their interpretation, we argue that Baldassarri and Goldberg’s results support our view that belief systems generally differ in the extent of organization but not in its logic (see app. F for more details).

Given that agnostics, alternatives, and ideologues thus appear to overwhelmingly follow the same logic of belief organization, it may seem surprising that the relational class analysis (RCA) algorithm used by Baldassarri and Goldberg identified them as separate groups. RCA is, after all, designed to find distinct patterns of correlation within the population. However, as we point out in appendix G, RCA does not currently provide a goodness-of-fit statistic to indicate whether the groups it located actually differ significantly from one another in their belief organization. Furthermore, existing work indicates that the modularity maximization partitioning technique used by RCA may have a substantial bias toward detecting heterogeneity in the data even when none exists (see discussion in app. B), making this absence of a goodness-of-fit statistic especially problematic. In appendix G, we show how multiple group analysis in structural equation modeling can be adapted to provide such a goodness-of-fit statistic for RCA. We applied this technique to the RCA results for the 2000 ANES, comparing the heterogeneity model with respondents partitioned into the three RCA-detected groups to a no-heterogeneity model where all respondents kept in a single group. Both the Akaike information criterion (AIC) and Bayesian information criterion (BIC) goodness-of-fit indices greatly preferred the no-heterogeneity model over the model with RCA-based partitions. This fits with our argument that the RCA-identified classes do not actually follow different logics of organization (app. F) and again supports our view of heterogeneity over Baldassarri and Goldberg’s (see app. G for more details.).

Belief Systems by Political Information and Race

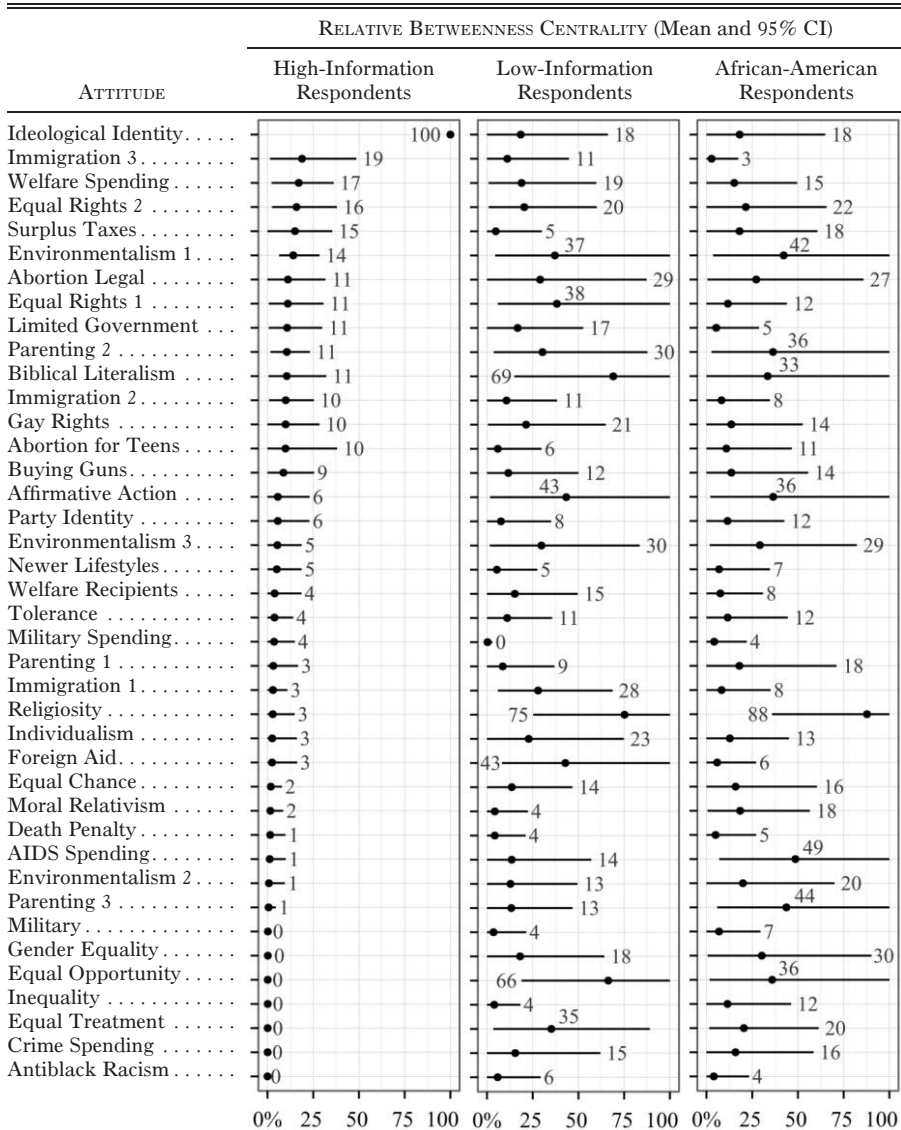
When we contrasted the pairwise relationships between beliefs for different demographic subpopulations, we found that the direction of association does not typically vary, with 95% or more of the significant correlations having the same sign in 43 of the 45 subgroup comparisons we conducted. The remaining two comparisons were those between black and nonblack respondents, and between respondents with high and low levels of political information. Black and nonblack respondents differed on 10.5% of the correlation signs, and high- and low-information respondents differed on 12.6% of the signs. Though these differences are not large, these subgroups are nonetheless the most likely to contain evidence of an alternate basis of political belief organization from the mainstream. To explore this possibility, we examine whether an alternate belief occupies the central position in any of these subgroups.

Table 5 shows the relative betweenness centralities for high-information, low-information, and black subsamples. We omit the nonblack subsample from this table because it contains 90% of all ANES respondents, and its pattern of relative centralities is nearly identical to the full population's. Following our row bootstrapping procedure, we resampled each of these three subgroups 1,000 times to determine the distributions of relative centralities in each.

We plotted the belief network for the high-information subsample in figure 5. The average constraint of this network (0.26) is visibly higher than that of the full population sample (0.16). Ideological identity again has a mean relative centrality of 100%, with the other beliefs occupying significantly less central positions (left column of table 5). In fact, the gap between ideological identity and the other variables has grown. The runner-up belief now has a mean centrality of 19%, as compared to 40% in the full sample. Of the remaining 39 beliefs, only 10 have confidence intervals that do not include zero, as compared to 15 in the full sample. The belief network for the high-information sample thus in many ways appears to be an exaggerated version of the same structure we observed in the full sample, with ideological identity as an even more clearly defined central node.

Since the high-information subsample follows the same general pattern of organization as the overall sample, we turn to the low-information group to search for evidence of an alternate structure (fig. 6). Indeed, religiosity (75%) and biblical literalism (69%) have the highest centralities in the network, whereas ideological identity (18%) occupies a relatively peripheral role. This leads to the intriguing suggestion that religion may play a more important structuring role among respondents further removed from the institutional field of mainstream politics—a topic we return to in the discussion. However, the central column of table 5 is composed almost entirely of

TABLE 5
BELIEF CENTRALITIES FOR DIFFERENT SUBPOPULATIONS



NOTE.—Each of the three sets of betweenness estimates is based on 1,000 row-wise bootstraps.

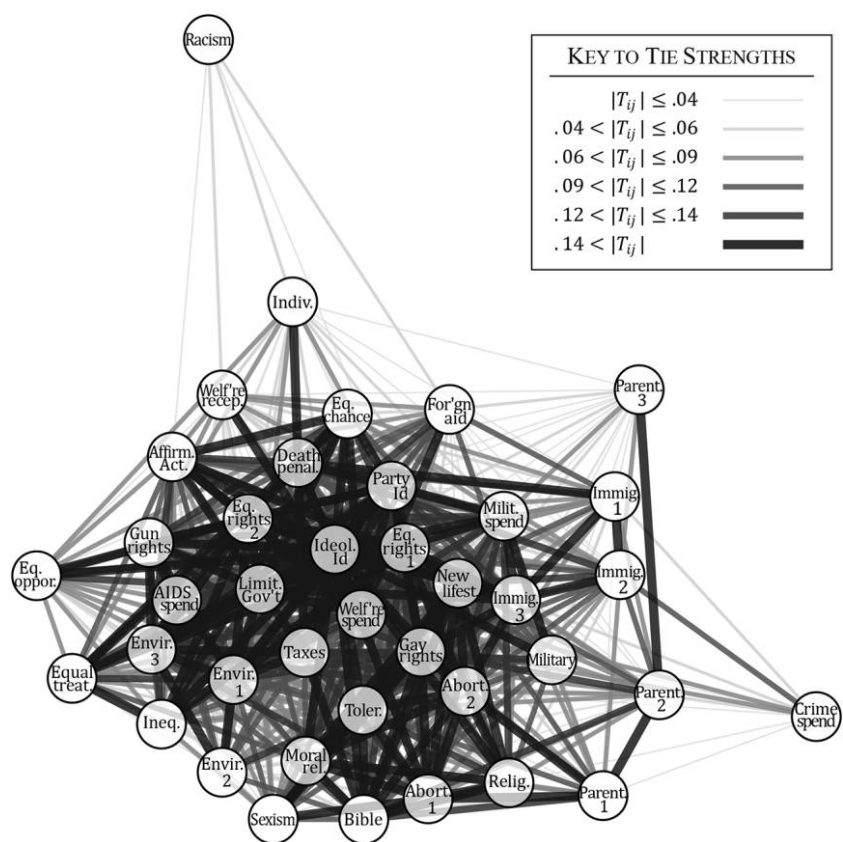


FIG. 5.—Correlation network for the high-information subsample (see table 1 for full node names). Tie strength is represented by thickness and boldness (see inset). Correlations below $|r| = 0.15$ are not depicted. Force-directed layout.

wide confidence intervals, indicating that this network exhibits little of the way of stable centrality structure. Every one of the seven confidence intervals extending to 100% also dips to 25% or below, indicating that the same nodes which occupy central positions in some iterations also occupy peripheral positions in others. This leaves the centralities of religiosity and biblical literalism statistically indistinct from those of 37 of the remaining 38 nodes.

The results for the black subsample follow a similar pattern (last column of table 5), with religiosity (88%) occupying the most central position. As with the low-information network, however, the centrality distribution is characterized by wide confidence intervals that leave this centrality statistically indistinct from that of most other nodes. Thus, our analyses of the sub-

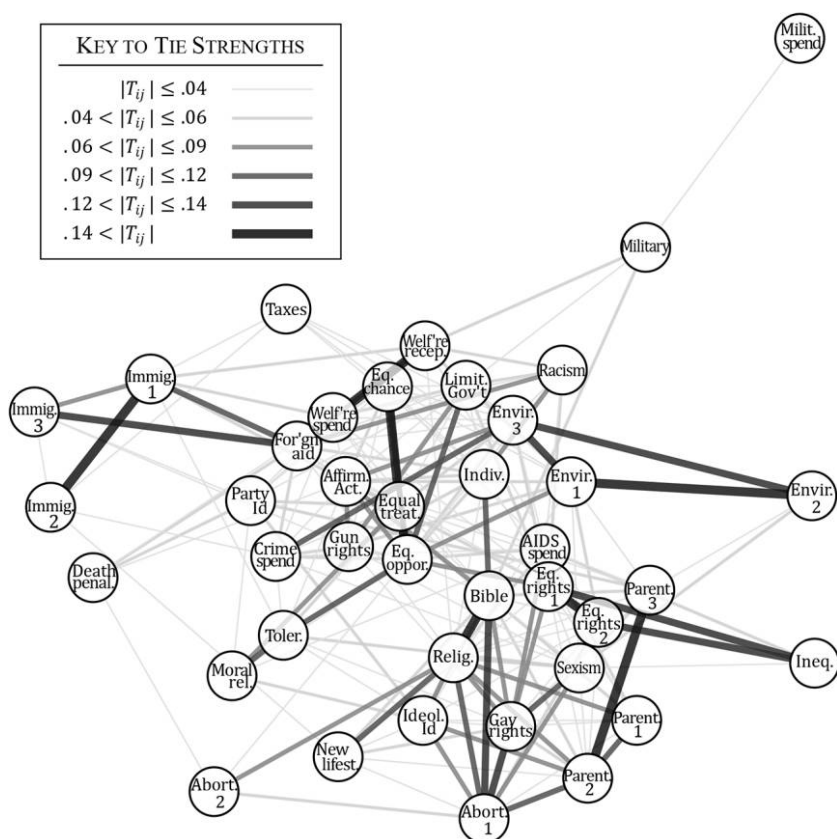


FIG. 6.—Correlation network for the low-information subsample (see table 1 for full node names). Tie strength is represented by thickness and boldness (see inset). Correlations below $|r| = 0.15$ are not depicted. Force-directed layout.

populations that appeared the most likely to provide evidence for alternate systems of organization instead better support the conclusion that the political attitudes in these populations have no reliable center. To examine whether this result holds more broadly, we extended this analysis to encompass all 44 subpopulations.³⁰ We found that 11 of the 44 networks had statistically reliable centers at $P < .05$. In all 11 of these networks, the central position was occupied by ideological identity. Relaxing the reliability cutoff to $P < .10$ or even an unusually lax $P < .25$ increased the number of qualifying networks to 15 and 22, respectively. Nonetheless, all of these networks still

³⁰ We estimated the relative centrality confidence intervals for each of the 41 remaining subgroups by drawing 250 row resamples of each (10,250 resamples total) and then performing the same betweenness analyses as above.

had ideological identity as their most central node. We thus found no evidence of subgroup networks reliably centered on any node other than ideological identity.

DISCUSSION

In this article, we developed BNA, a novel correlation network-based method for examining the structure of beliefs. We used this method to compare different theoretical accounts of belief structure. To focus our analysis, we used the same ANES data as Barker and Tinnick (2006), who employed regression analyses to argue that parenting models play a central role in structuring political beliefs (Lakoff 2002). We found no evidence to support this claim and found instead that ideological identity (liberal/conservative) is most likely to provide that organization. Since regression analyses of these data have, to our knowledge, been the only quantitative work to offer support for Lakoff's theory of moral politics, our results suggest significant skepticism toward this popular theory.

Our results are also not consistent with accounts that emphasize the heterogeneity of belief structures across different social groups (e.g., Baldassarri and Goldberg 2014). An analysis of 44 demographic subgroups showed that, at least in the domain of politics, there appears to be a single dominant logic of belief organization. On different sides of the major demographic divisions we examined—even those that past work suggested delineate alternative ways of organizing political attitudes—we found little reliable evidence of beliefs fitting together in opposite ways. If “support issue A” implied “support issue B” in one subpopulation, either it generally implied “support B” in the other population, or else support for A had little relationship with support for B. Across all the subpopulations we examined, the cases where “support A” implied “oppose B” in the other subpopulation were very rare, with only 1.3% (median) of the significant correlations between beliefs switching signs in a typical comparison. We thus concluded that groups generally differ in the extent to which their attitudes are organized at all, but not in the logic around which they are organized. Additional information-theoretic analyses and reexaminations of results from prior work also supported this conclusion.

Our centrality and heterogeneity analyses were therefore both consistent with the view that political identity serves as a key heuristic in structuring political beliefs (Converse 1964; Zaller 1992; Mondak 1993; Goren et al. 2009). Such social constraint accounts hold that individuals acquire their attitudes via attention to information flows from political elites, which they select by using their political identity as a filter. In support of the social constraint view, we found that ideological identity occupied the most central position in the overall population—a result that was extremely robust to a va-

riety of changes in the model. Our resampling analyses also showed that, in the absence of ideological identity from the model, the belief network simply appears uncentralized, which further highlights the unique structural position of ideological identity and is consistent with its role as the dominant organizing heuristic.

In our heterogeneity analyses, ideological identity also occupied the central position in every subpopulation whose belief system had a stable center, leaving us with no reliable evidence of belief systems organized around anything but ideological identity. While low constraint between beliefs and high noise in attitude measurement mean that practically any possible combination of attitudes can be empirically observed, we found no reason to believe that these combinations represent an alternate organized belief system. Across subpopulations, the centrality of ideological identity and the overall level of organization in the belief system were closely correlated—that is, belief systems appeared organized to the extent that ideological identity served as their center. Consistent with prior work, it was the demographic groups with greater participation in the field of organized partisan politics (Delli Caprini and Keeter 1996) that had belief systems organized around ideological identity. However, for groups further away, no other organizing principle appeared to step in and fill the gap. Taken together, these diverse findings fit with the argument that the logic of the political field may be the dominant organizing principle holding together the attitudes we term “political,” and that political identity—that is, a position within this field—is the main conduit via which individuals acquire this organization.

To search for possible exceptions, we closely examined low-information and African-American respondents, who showed some evidence of using religious beliefs to structure their political views. This examination, however, produced no statistically reliable evidence of alternate organization. Though in these populations, religiosity-related variables were somewhat more central, and political identities were less central, the overall belief networks lacked any clear center. And, even in these two groups, more than 85% of the statistically significant belief correlations still retained the same sign as in the comparison group. Thus, even if these results were to be interpreted as evidence of an alternate, religion-based system of organization, such a system would appear to provide alternate organization for only a small subset of political attitudes.

One possible explanation for this finding comes from the partial endogeneity of the concept of “politics” to the political field. As some critical scholars have argued, attitudes may come to be classified as “political” when they concern issues that have become the subject of competition between political parties, social movements, or other recognizably political actors (Lee 2002). By this reasoning, political attitudes may be exactly those attitudes that actors in the political field wish to influence in the general population

and thus weave into their competing belief systems. By contrast, since issues of special interest to other fields (like religion) do not automatically come to be classified as political, the belief systems they produce may then systematize political attitudes only to the extent that their interests intersect with those of the political field. Viewed as political belief systems, they would likely appear incomplete.

Although our results agree with the basic tenets of the social constraint view, they deviate from existing accounts on the relative importance of different political identities. All social constraint accounts posit that individuals acquire their political views from elite opinion leaders. To most scholars of American politics, this has specifically meant political parties (e.g., Campbell et al. 1960; Carmines and Wagner 2006; Goren et al. 2009; Sniderman and Stiglitz 2012).³¹ However, there may be theoretical reasons to doubt this focus on parties and politicians to the exclusion of other political identities. Many popular political commentators appear to flaunt their independence from the major political parties and to affiliate themselves with ideological rather than partisan labels (e.g., “Although I am a ‘conservative,’ I’m not a ‘Republican,’ and there’s a big difference” [Glenn Beck (2008)]). If identity is indeed primarily useful as the heuristic individuals use to evaluate political communication flows, then, *ceteris paribus*, the most relevant identities should be those used by the most visible communicators. In our analyses, we repeatedly found ideological identity to be more central than party identity, which supports this alternate account. It thus merits investigating whether existing work underestimates the importance of political identification in attitude formation.

Limitations and Future Directions

Belief network analysis allowed us to detect interesting patterns in survey data and to use these patterns to compare competing theories. The technique produced centrality and centralization scores that lent themselves to intuitively clear interpretations in terms of brokerage within belief structure, and this interpretation was further aided by network diagrams. The bootstrapping-based confidence intervals also provided clear measures of statistical significance and sensitivity to variable selection and in our case indicated that the primary centrality result is extremely robust to both sampling error and changes in the model. In addition to political beliefs, our approach can be applied directly to other cultural domains that can be reasonably approximated by the

³¹ Thus, for example, Goren and colleagues (2009) open their work with the claim that “party identification represents the most stable and influential political predisposition in the belief systems of ordinary citizens” (p. 805), while Legee and colleagues (2009 p. 252) argue that voter preferences are “largely the products of ambitious politicians seeking issues that will carry them to victory.”

center-periphery model. The development of cultural tastes may be one such area for investigation.

However, like all data analysis techniques, BNA has limitations. Most important, it gains its leverage from making some simplifying assumptions about its target domain. We focus our investigation on theoretical accounts of belief structure in which some beliefs are central and others are produced from them through a noisy inferential process. The logic of our method derives from the fact that this process of belief derivation should leave behind a correlation network where central items act as brokers uniting otherwise disparate parts of the system—a consequence that we prove formally in appendix A. Our proof, however, rests on a stylized model of the belief acquisition process that strips away complexity in order to leave a parsimonious structure suitable for formal investigation. Future work should examine the behavior of the model when assumptions are relaxed and complexities reintroduced. Such work could potentially greatly increase the applicability of the method we described here.

While the belief-generation process we capture in our model is in line with many theoretical accounts of belief structure, other accounts disagree. For example, some accounts assume broad heterogeneity in the belief structures of different subpopulations, while others envision deeply nonlinear relationships between the belief variables. We examined both of these conflicting accounts in this article and provided evidence that supported our model over these alternate accounts in the political domain (see also apps. C and G). We hope that, by clearly laying out our assumptions and formally deriving our method from them, we ease the task of potential challengers wishing to dispute or extend this model of belief structure. We include our full formal reasoning in appendix A so that others can build on this work, perhaps extending it to domains where the current model's assumptions do not hold.³²

Another current limitation of our method is that it is a test of structure rather than causality. Though we present many diverse pieces of evidence consistent with belief system generation by social constraint, we do not uniquely identify this causal process. More broadly, while many other approaches to belief structure aim to ascertain the causal precedence of some beliefs over others, our analysis uses a cross-sectional data set and is thus

³² Future work can also extend BNA to examine other structural features of belief systems. For example, while we have focused on centrality and centralization, many of the theoretical accounts we reviewed here also suggest that some beliefs may cluster into densely tied subgroups or “communities” that are relatively weakly connected to the rest of the network. These structural features can be examined using a network partitioning algorithm. While Newman's (2006) modularity maximization is the conventional approach to detecting this kind of community structure, our analyses documented some apparent problems with the method that preclude us from recommending its use here (see app. B).

unsuitable for such questions. Instead, our analysis focuses on the structural significance of beliefs within the system. Since theoretical accounts of belief systems make both causal and structural claims, both approaches are necessary. We show that, among the beliefs we analyze, ideological identity is unique in occupying a structural position at the center of the network; even in its absence, no other belief comes to occupy a reliably central network position. We thus rule out accounts that would place parental values, limited government, symbolic, and/or explicit racism, or any of the culture wars issues at the center of the belief structure. In our heterogeneity analyses, we also show that it is not the case that different populations have belief systems centered on different attitudes, which points against accounts that conceive of different subpopulations as achieving attitude organization through drastically different processes.

Our structural results provide a complement rather than a replacement to those analyses that have used experimental or longitudinal data to show that, for example, changes in political identification can cause changes to other items in the political belief system, that partisan source cues play an outsized role in how individuals reason about political information, and that identity changes lead to changes in political values but not vice versa (e.g., Zaller 1992; Mondak 1993; Bartels 2002; Cohen 2003; Mondak et al. 2004; Goren 2005; Goren et al. 2009; but see Johnston 2006). We rest on these existing findings to argue that political identity is not simply a post hoc label attached to constellations of beliefs acquired through another unknown process—an alternate mechanism that could, under some circumstances, place ideological identity at the center of the belief network and that we cannot ourselves rule out. In the future, it may be possible to combine some of the strengths of these approaches by using panel data to construct a network based on within-person belief changes. Since BNA is based on the well-developed foundation of correlation and network analysis, it may be able to benefit from the wealth of knowledge developed in these domains to quickly make these and other methodological advancements.

CONCLUSION

We believe that BNA represents a novel and important contribution to the study of belief systems. Building a belief network out of correlations enabled us to draw on the rich methodological and theoretical toolkit of network analysis to construct intuitive measures of structural features theorized in the literature on belief systems. Our specific empirical results were broadly consistent with the conception of political identity as the dominant heuristic for acquiring attitudes. They also provided considerable evidence against Lakoff's theory of moral politics and suggest the need for a degree of skepticism toward sociological accounts that assume substantial social het-

erogeneity in logics of belief organization, at least in the political domain. While this analysis focused on political attitudes, the techniques offered here are general and can be applied to other domains. Since no single methodological approach alone can provide sufficient understanding of culture's complex structures, our hope is that BNA will be joined by other inventive methodologies in a renewed effort to tackle "the biggest unanswered question in the sociology of culture" (Swidler 2001, p. 206)—how some cultural elements structure others.

APPENDIX A

Formal Proofs

In this appendix, we will prove a number of theorems about center-periphery belief networks produced by the derivation scheme described. In theorems 1 and 2, we derive basic formulas that serve as the foundation for the other proofs. In theorems 3 and 4, we show that all geodesics in such networks follow a simple topology. In theorem 5, we use this to derive a formula for lengths of transverse geodesics, which compose the majority of geodesics in the system. Finally, in theorem 6, we draw on the preceding theorems to demonstrate what we term the "central pull" of the belief network, which is the key result of this formal study. We prove that nontrivial transverse geodesics always tend toward the origin of the system, passing either directly through it or through a highly correlated node. This leads us to recommend shortest-path betweenness as the tool for identifying the belief at the origin of such networks.

Model, Definitions, and Notation

All the previously defined terms retain their original definitions, though we restate some of these below with greater precision. We also adopt a more expressive variable naming scheme. We use QED to indicate ends of proofs; $A \equiv B$ to mean "A is equal by definition to B." We also occasionally use the notation $X = (\text{reason}) = Y$ or $X \Leftrightarrow (\text{reason}) \Leftrightarrow Y$ to add concise explanations of why X is equivalent to Y .

Nodes

We will use the letters a, b, c, d, f, g , and x to refer to nodes (beliefs). Node a_K will be called the *ancestor* of node a_S , and a_S the *descendant* of node a_K , if and only if $a_S = a_K + \sum_{i=1}^n \phi_i$. Node a_K will be called the *parent* of node a_S , and a_S the *child* of a_K , if and only if $a_S = a_K + \phi_i$. The central belief x_0 is the ancestor of all other nodes, so any node b can be expressed as $b = x_0 + \sum_{i=1}^m \phi_i$. We assume that all ϕ_i are independent of x_0 , and of all ϕ_j unless

$i = j$. We also assume x_0 and ϕ_i have variances $\text{Var}(x_0) = 1$ and $\text{Var}(\phi_i) = \epsilon$, and finite means.³³

Node b 's *generation* $\eta(b)$ is then equal to m , which is the number of ϕ_i terms added to x_0 to produce b . Node x_0 is the only zeroth-generation node in the system. We will use subscript indexes to refer to a node's generation.³⁴ For example, a_1 is a first-generation node, and a_K and b_K are two nodes of the same generation $\eta(a_K) = \eta(b_K) = K$. If a_K is the parent of a_S , then $S = K + 1$. In cases when we need to use a node's index to indicate anything other than the node's generation, we will use a superscript instead: for example, a^K or a_3^K .

We will refer to a node and its full set of ancestors as the node's *ancestry* $\pi(a)$. The node c_i in $\pi(a) \cap \pi(b)$ that has the largest value of $\eta(c_i)$ will be called the *lowest common ancestor* of a and b . We will call two nodes a and b *strangers* if and only if (i) neither node is x_0 , and (ii) x_0 is their only common ancestor. The tie $T(a, b)$ would then be a *stranger* tie. We assume that the majority of node pairs in the network are strangers. We make no further assumptions about the network's topology.

The belief network $\mathcal{G} = \{\mathcal{N}, \mathcal{T}\}$ is the set of all nodes \mathcal{N} and all ties \mathcal{T} , which contains one tie for every pair of nodes in \mathcal{N} (i.e., \mathcal{G} is fully connected). We define the length of tie connecting any pair of nodes a and b as

$$|T(a, b)| \equiv 1/\text{cor}^2(a, b). \quad (\text{A1})$$

Paths

We will use capital Greek letters to refer to network paths. A path is an ordered sequence of adjacent ties or, equivalently, of the nodes that are the endpoints of these ties. We will use summation of ties or paths to indicate concatenation into paths. The length of a path is the sum of all the tie lengths composing it: for example, if $\Lambda = T(a, b) + T(b, c)$, then $|\Lambda| = |T(a, b)| + |T(b, c)|$.

If a path between a and b consists of S ties, we will call it an *S-path*. The distinction between number of ties in a path and path length is important: since path length is the sum of tie lengths as opposed to the count of ties, different *S*-paths will generally have different lengths. We will use superscript indexes and the function $\tau(\Lambda)$ to specify the number of ties in a path: $(\tau(\Lambda) = S) \equiv (\Lambda \text{ is a path with } S \text{ ties}) \equiv (\Lambda = \Lambda^S)$. Since there is only one one-path connecting any pair of nodes a and b , $\Lambda^1(a, b) = T(a, b)$. We will refer to one-paths as *trivial* paths. *Central* paths are paths containing x_0 . *Transverse* paths are those between strangers.

³³ The means must be finite for correlation to be defined. However, the means do not have to be the same, and the variables do not have to be identically distributed.

³⁴ Please note that this indexing convention applies to nodes only.

We will use the letter Γ to refer to geodesics. The path $\Gamma(a, b)$ is the shortest path connecting a and b . Note again that this refers to the path with the lowest sum of tie lengths, which is frequently not the same as the path with the fewest ties. We will also occasionally refer to shortest paths between pairs of nodes as *absolute* geodesics to distinguish them from *S-geodesics*, which are the shortest among all S -paths connecting the same pair of nodes. We denote S -geodesics as $\Gamma^S(a, b)$. If Θ^S is the set of all S -paths connecting a and b , then

$$A(a, b) = \Gamma^S(a, b) \text{ if and only if } A \in \Theta^S \text{ and} \quad (\text{A2})$$

$$|A| \leq |\Theta^S(a, b)| \quad \forall \theta^S \in \Theta^S.$$

Theorem 1: General Formula for Length of Ties

THEOREM.—For any two nodes $a = a_K$ and $b = b_S$,

$$|T(a, b)| = \frac{(1 + K*\epsilon)(1 + S*\epsilon)}{(1 + P*\epsilon)^2}, \quad (\text{A3})$$

where P is the generation of the lowest common ancestor of a and b .

Proof.—We begin with the variances of a and b . $\text{Var}(a) = \text{Var}(x_0 + \sum_{i=1}^K \phi_i) = (\text{due to independence of } x_0 \text{ and } \phi_i) = \text{Var}(x_0) + \sum_{i=1}^K \text{Var}(\phi_i) = 1 + K*\epsilon \equiv \sigma^2(a)$. By analogy,

$$\text{Var}(b) = 1 + S*\epsilon \equiv \sigma^2(b). \quad (\text{A4})$$

We now calculate $\text{Cov}(a, b) = \text{Cov}(x_0 + \sum_{i=1}^K \phi_i, x_0 + \sum_{j=1}^S \phi_j) = \text{Cov}(x_0, x_0) + \text{Cov}(x_0, \sum_{j=1}^S \phi_j) + \text{Cov}(\sum_{i=1}^K \phi_i, x_0) + \text{Cov}(\sum_{i=1}^K \phi_i, \sum_{j=1}^S \phi_j) = 1 + \sum_{j=1}^S \text{Cov}(x_0, \phi_j) + \sum_{i=1}^K \text{Cov}(\phi_i, x_0) + \sum_{i=1}^K \sum_{j=1}^S \text{Cov}(\phi_i, \phi_j)$. Thus, due to independence of x_0 and ϕ_i , and independence of ϕ_i and ϕ_j unless $i = j$,

$$\text{Cov}(a, b) = 1 + P*\epsilon. \quad (\text{A5})$$

Using (A4), we now get

$$\text{Cor}(a, b) = [1 + P*\epsilon] / \sqrt{(1 + K*\epsilon)*(1 + S*\epsilon)}, \quad (\text{A6})$$

and finally,

$$|T(a, b)| = \frac{1}{\text{Cor}^2(a, b)} = \frac{(1 + K*\epsilon)(1 + S*\epsilon)}{(1 + P*\epsilon)^2}.$$

QED

Theorem 2: Lengths of Ties to Strangers, Ancestors, and the Center

THEOREM.—If $a = a_K$ and $b = b_S$ are strangers, then

$$|T(a, b)| = (1 + K*\epsilon)*(1 + S*\epsilon). \quad (\text{A7})$$

If $b = b_s$ is an ancestor of $a = a_K$, then

$$|T(a, b)| = \frac{1 + K*\epsilon}{1 + S*\epsilon}. \quad (\text{A8})$$

If

$$b = x_0,$$

then

$$|T(a, b)| = (1 + K*\epsilon). \quad (\text{A9})$$

Proof.—To prove each of these three statements, substitute appropriate values into equation (A3): $P = 0$ (for [A7]), $P = S$ (for [A8]), and $S = 0$ (for [A9]). QED

COROLLARY 2A.—For any two strangers of the same generation, a_K and b_K , the central two-path $\Lambda^2 = T(a_K, x_0) + T(x_0, b_K)$ is longer than the one-path $\Lambda^1 = T(a_K, b_K)$ if and only if $K*\epsilon < 1$.

Proof of corollary.—The path Λ^2 has length $|\Lambda^2| = |T(a_K, x_0)| + |T(x_0, b_K)|$. By (A9), this equals $(1 + K*\epsilon) + (1 + K*\epsilon) = 2*(1 + K*\epsilon)$. On the other hand, by (A7), Λ^1 has length $|\Lambda^1| = (1 + K*\epsilon)^2$. Therefore $|\Lambda^2| > |\Lambda^1| \Leftrightarrow 2*(1 + K*\epsilon) > (1 + K*\epsilon)^2 \Leftrightarrow 2 > (1 + K*\epsilon) \Leftrightarrow 1 > K*\epsilon$. QED

Application to Figure 1B

In our discussion of figure 1,2, we claimed that, if $\epsilon > 0.5$, the path $\Lambda^1 = (T_{11,21})$ will be longer than the path $\Lambda^2 = (T_{11,0}, T_{0,21})$. Translating this to the more detailed notation we use in this appendix, path $\Lambda^1 = T(x^{11}, x^{21})$, whereas $\Lambda^2 = T(x^{11}, x_0) + T(x_0, x^{21})$. The generation K of x^{11} and x^{21} is 2. Substituting this into corollary 2A, we see that $|\Lambda^2| > |\Lambda^1|$ if and only if $2*\epsilon < 1$, or equivalently, if $\epsilon < 0.5$.

Theorem 3: S-Geodesics Contain at Most One Stranger Tie

For the remainder of this appendix, we will focus on S -geodesics (see [A2]), which we use as an analytical tool for studying absolute geodesics. Each absolute geodesic $\Gamma(a, b)$ can also be seen as an S -geodesic $\Gamma^S(a, b)$, with $S = \tau(\Gamma)$. Additionally, $|\Gamma(a, b)| = \min\{|\Gamma^S(a, b)| : 1 \leq S < \infty\}$. The crucial advantage of S -geodesics is that we can express their length analytically and thus study them with optimization via partial derivatives. Any statement proven to hold for all S -geodesics by extension also holds for absolute geodesics.

THEOREM.—If an S -path Ω between g and f contains two or more stranger ties, Ω is not the S -geodesic between g and f .

Proof.—Let us traverse path Ω from g to f . Let $T(a, b)$ be the first of two stranger ties we encounter, and $T(c, d)$ the second of these ties. There are only three possibilities about how $T(a, b)$ and $T(c, d)$ are related (see fig. A1):

- i. $T(a, b)$ and $T(c, d)$ are immediately adjacent, that is, $b = c$.
- ii. c is a descendant of b .
- iii. c is an ancestor of b .

In each of these cases we will show that some two-tie segment $\Theta \subset \Omega$ can be replaced by a shorter two-tie segment Θ_0 , which passes through x_0 . This contradicts the definition of an S -geodesic.

In case i, we will show that $\Theta = T(a, b) + T(b, d)$ is longer than $\Theta_0 = T(a, x_0) + T(x_0, d)$. By (A7) and (A9), $|\Theta_0| = (1 + \eta(a)*\epsilon) + (1 + \eta(d)*\epsilon)$ and $|\Theta| = (1 + \eta(a)*\epsilon)*(1 + \eta(b)*\epsilon) + (1 + \eta(c)*\epsilon)*(1 + \eta(d)*\epsilon)$. Since $\eta(b)$, $\eta(d)$, and $\epsilon > 0$, $|\Theta| > |\Theta_0|$.

In case ii, we will assume for simplicity that b is the parent of c (otherwise, the same proof holds if $T(b, c)$ is replaced with the tie from c 's parent to c). We will show that $\Theta = T(b, c) + T(c, d)$ is longer than $\Theta_0 = T(b, x_0) + T(x_0, d)$. By (A7)–(A9),

$$\begin{aligned}
 |\Theta| &= \frac{1 + \eta(c)*\epsilon}{1 + \eta(b)*\epsilon} + \\
 &(1 + \eta(c)*\epsilon)*(1 + \eta(d)*\epsilon) > 1 + (1 + \eta(c)*\epsilon)*(1 + \eta(d)*\epsilon) \\
 &> 1 + (1 + \eta(b)*\epsilon)*
 \end{aligned}$$

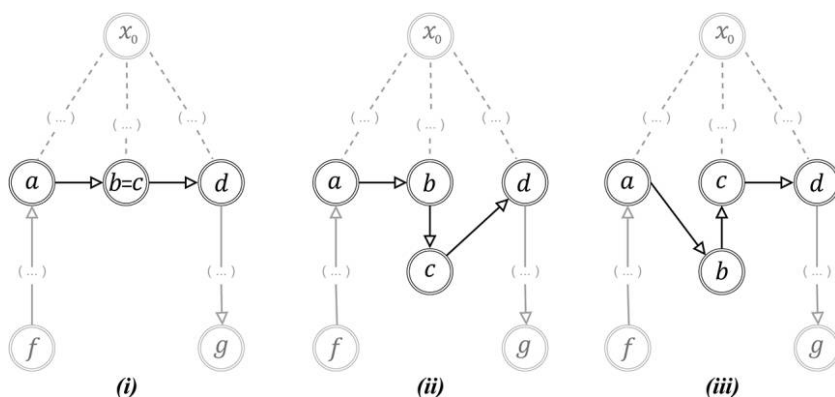


FIG. A1.—There are three possible relative locations for stranger ties $T(a, b)$ and $T(c, d)$ in a path $\Lambda(f, g)$: (i) the two ties could be immediately adjacent, i.e., $b = c$; (ii) c could be a descendant of b ; (iii) c could be an ancestor of b . Solid lines indicate ties, whereas dashed lines indicate ancestry. Possible omitted network segments. Note that the proof does not require f and g to be strangers.

$$\begin{aligned}
 (1 + \eta(d)*\epsilon) &= 1 + (1 + \eta(b)*\epsilon) + \eta(d)*\epsilon + \eta(b)*\eta(d)*\epsilon^2 \\
 &> (1 + \eta(b)*\epsilon) + \\
 (1 + \eta(d)*\epsilon) &\Leftrightarrow |\Theta| > |\Theta_0|.
 \end{aligned}$$

Since the tie lengths are symmetric, case iii can be made identical to ii by simply reversing the order in which the nodes in Ω are traversed and re-naming them accordingly. QED

Theorem 4: S -Geodesics Pass Only through Ancestors

THEOREM.—An S -geodesic $\Lambda = \Gamma^S(a, b)$ passes only through nodes that are ancestors of a or b :

$$\Gamma^S(a, b) \subseteq \pi(a) \cup \pi(b). \quad (\text{A10})$$

Proof.—Let us assume that (A10) is not the case, and Λ contains nodes not in $\pi(a)$ or $\pi(b)$. The ties connecting such nodes to those in $\pi(a)$ and $\pi(b)$ will then be stranger ties. Thus, Λ then contains at least two stranger ties, which contradicts theorem 3. QED

Theorem 5: Length of Transverse S -Geodesics

In this theorem, we derive a formula for the length of transverse S -geodesics by addressing it as a minimization problem over all possible S -paths. In order to perform this optimization task using partial derivatives, from this point on we let all N_i and K_j assume continuous values. It can be shown that this solution is asymptotically equivalent to the discrete case when $\max(N, K) \rightarrow \infty$.³⁵

THEOREM.—For any two strangers $a = a_K$ and $b = b_N$, if the path Λ is an S -geodesic $\Gamma^S(a, b)$, then its length satisfies the following equation:

$$|\Lambda(a, b)| = S*[(1 + K*\epsilon)*(1 + N*\epsilon)]^{1/S}. \quad (\text{A11})$$

Additionally, path Λ either passes through the center x_0 or through a pair of nodes a^1 and b^1 which satisfy the following equation:

$$(1 + K_1*\epsilon)*(1 + N_1*\epsilon) = ((1 + K*\epsilon)*(1 + N*\epsilon))^{1/S}, \quad (\text{A12})$$

where $K_1 = \eta(a^1)$ and $N_1 = \eta(b^1)$.

³⁵ Their equivalence is in fact stronger than asymptotic, as when both the discrete and continuous solutions to the minimization problem approach ∞ , the absolute difference between the two remains less than a fixed value.

Proof.—First let $\Lambda(a, b)$ be a noncentral S -geodesic. According to theorems 3 and 4, all nodes in Λ belong to $\pi(a) \cup \pi(b)$, and Λ contains a single stranger tie $T(a^1, b^1)$ which connects a node in $\pi(a)$ to a node in $\pi(b)$. We will refer to $T(a^1, b^1)$ as the *bridge*. We will number the nodes beginning with a^1 and b^1 and counting outward toward a and b (see right side of fig. A2). Then, by theorem 4 and definition of noncentral S -geodesics, Λ can be represented as:

$$\Lambda = (a = a^T, a^{T-1}, \dots, a^2, a^1, b^1, b^2, \dots, b^{S-T+1} = b). \quad (\text{A13})$$

There are $T - 1$ ties between nodes in $\pi(a)$ and $S - T$ ties between nodes in $\pi(b)$, so that the total number of ties in Λ is $(T - 1) + 1 + (S - T) = S$. We will denote $K_i = \eta(a^i)$ and $N_j = \eta(b^j)$. The following are true by definition:

$$0 < K_1 < K_2 < \dots < K_{T-1} < K_T \text{ and}$$

$$0 < N_1 < N_2 < \dots < N_{S-T} < N_{S-T+1}, \quad (\text{A14})$$

$$K_T = K = \eta(a_K), \quad (\text{A15})$$

$$N_{S-T+1} = N = \eta(b_N). \quad (\text{A16})$$

By (A7) and (A8), the length of Λ can be expressed as the following function L , which adds up the lengths of all the ties that lie within $\pi(a)$ and $\pi(b)$, as well as of the bridge:

$$L = \sum_{i=1}^{T-1} \frac{1 + K_{i+1} * \epsilon}{1 + K_i * \epsilon} + (1 + K_1 * \epsilon) * (1 + N_1 * \epsilon) + \sum_{j=1}^{S-T} \frac{1 + N_{j+1} * \epsilon}{1 + N_j * \epsilon}. \quad (\text{A17})$$

Since a and b are fixed, L is a function of $S - 1$ independent variables:

$$L = L(K_1, \dots, K_{T-1}, N_1, \dots, N_{S-T}). \quad (\text{A18})$$

By the definition of S -geodesics, $|\Lambda(a, b)|$ is the minimum value of L over all the possible combinations of intermediate nodes from within the appropriate ancestries. To find the length of the S -geodesic, we thus minimize L using the partial derivatives $\partial L / \partial K_i$ for $1 \leq i \leq T - 1$ and $\partial L / \partial N_j$ for $1 \leq j \leq S - T$. Note that each K_i term except K_1 appears in exactly two elements of the first summation in (A17), and different K_i terms are not functions of each other. The same situation also holds for N_j . Thus:

$$\frac{\partial L}{\partial K_i} = \frac{\partial}{\partial K_i} \left[\frac{1 + K_{i+1} * \epsilon}{1 + K_i * \epsilon} + \frac{1 + K_i * \epsilon}{1 + K_{i-1} * \epsilon} \right] \text{ for } 2 \leq i \leq (T - 1), \quad (\text{A19})$$

$$\frac{\partial L}{\partial N_j} = \frac{\partial}{\partial N_j} \left[\frac{1 + N_{j+1} * \epsilon}{1 + N_j * \epsilon} + \frac{1 + N_j * \epsilon}{1 + N_{j-1} * \epsilon} \right] \text{ for } 2 \leq j \leq (S - T). \quad (\text{A20})$$

Terms K_1 and N_1 appear once in their respective summations and once in the bridge term. Thus:

$$\frac{\partial L}{\partial K_1} = \frac{\partial}{\partial K_1} \left[\frac{1 + K_2 * \epsilon}{1 + K_1 * \epsilon} + (1 + K_1 * \epsilon) * (1 + N_1 * \epsilon) \right], \quad (\text{A21})$$

$$\frac{\partial L}{\partial N_1} = \frac{\partial}{\partial N_1} \left[\frac{1 + N_2 * \epsilon}{1 + N_1 * \epsilon} + (1 + K_1 * \epsilon) * (1 + N_1 * \epsilon) \right]. \quad (\text{A22})$$

Performing the differentiations in (A19)–(A22) and setting each partial derivative to zero, we get the following system of equations:

$$\begin{aligned} \text{from (A19): } (1 + K_{i+1} * \epsilon) * (1 + K_{i-1} * \epsilon) &= (1 + K_i * \epsilon)^2 \\ \text{for } 2 \leq i \leq (T - 1), \end{aligned} \quad (\text{A23})$$

$$\begin{aligned} \text{from (A20): } (1 + N_{j+1} * \epsilon) * (1 + N_{j-1} * \epsilon) &= (1 + N_j * \epsilon)^2 \\ \text{for } 2 \leq j \leq (S - T), \end{aligned} \quad (\text{A24})$$

$$\text{from (A21): } (1 + K_2 * \epsilon) = (1 + K_1 * \epsilon)^2 * (1 + N_1 * \epsilon), \quad (\text{A25})$$

$$\text{from (A22): } (1 + N_2 * \epsilon) = (1 + N_1 * \epsilon)^2 * (1 + K_1 * \epsilon). \quad (\text{A26})$$

We note that equations (A23) and (A24) are geometric progressions. Thus we will search for a solution to (A23)–(A26), as well as (A15)–(A16), that satisfies the following two equations with five unknown parameters $r, \alpha, \beta, \gamma, \delta$, which define such geometric progressions:

$$1 + K_i * \epsilon = r^{\alpha * i + \beta}, 1 \leq i \leq T; \quad (\text{A27})$$

$$1 + N_j * \epsilon = r^{\gamma * j + \delta}, 1 \leq j \leq S - T + 1. \quad (\text{A28})$$

As can be shown by substitution, (A27)–(A28) satisfy (A23) and (A24) for any values of r, α, β, γ , and δ . To determine which values satisfy the remaining equations, we substitute the following into (A25): $(1 + K_2 * \epsilon) = r^{\alpha * 2 + \beta}$, $(1 + K_1 * \epsilon)^2 = r^{2 * (\alpha * 1 + \beta)} = r^{2\alpha + 2\beta}$, and $(1 + N_1 * \epsilon) = r^{\gamma * 1 + \delta}$. This yields

$$r^{2\alpha + \beta} = r^{2\alpha + 2\beta} * r^{\gamma + \delta} \Leftrightarrow 2\alpha + \beta = 2\alpha + 2\beta + \gamma + \delta \Leftrightarrow 0 = \beta + \gamma + \delta. \quad (\text{A29})$$

Analogous substitutions into (A26) yield

$$r^{2\gamma+\delta} = r^{2\gamma+2\delta} * r^{\alpha+\beta} \Leftrightarrow 0 = \beta + \alpha + \delta. \quad (\text{A30})$$

Subtracting (A29) from (A30) produces $\beta + \alpha + \delta - (\beta + \gamma + \delta) = 0 \Leftrightarrow \alpha = \gamma$. Since the values of r , β , and δ can be adjusted to make α and γ equal any constant, we assume that

$$\alpha = \gamma = 1. \quad (\text{A31})$$

Substituting this into (A29) or (A30) yields

$$\beta = -1 - \delta. \quad (\text{A32})$$

Substituting (A31) and (A32) into (A27) and (A28) now yields:

$$1 + K_i * \epsilon = r^{i-1-\delta}, 1 \leq i \leq T, \quad (\text{A33})$$

$$1 + N_j * \epsilon = r^{j+\delta}, 1 \leq j \leq S - T + 1. \quad (\text{A34})$$

To find the value of r , we now examine the end nodes A_K and B_N . By (A15) and (A16),

$$1 + K * \epsilon = 1 + K_T * \epsilon = (\text{by [A33]}) = r^{T-1-\delta}, \quad (\text{A35})$$

$$1 + N * \epsilon = 1 + N_{S-T+1} * \epsilon = (\text{by [A34]}) = r^{S-T+1+\delta}. \quad (\text{A36})$$

Multiplying both sides of (A35) by (A36) produces:

$$\begin{aligned} (1 + K * \epsilon) * (1 + N * \epsilon) &= r^{T-1-\delta} * r^{S-T+1+\delta} = r^S \Leftrightarrow \\ r &= [(1 + K * \epsilon) * (1 + N * \epsilon)]^{1/S}. \end{aligned} \quad (\text{A37})$$

Now, we substitute (A33)–(A34) into each term of (A17), beginning with the first summation:

$$\begin{aligned} \sum_{i=1}^{T-1} \frac{1 + K_{i+1} * \epsilon}{1 + K_i * \epsilon} &= \sum_{i=1}^{T-1} \frac{r^{i+1-1-\delta}}{r^{i-1-\delta}} = \sum_{i=1}^{T-1} r = (T-1) * r, \\ (1 + K_1 * \epsilon) * (1 + N_1 * \epsilon) &= r^{1-1-\delta} * r^{1+\delta} = r, \\ \sum_{j=1}^{S-T} \frac{1 + N_{j+1} * \epsilon}{1 + N_j * \epsilon} &= \sum_{j=1}^{S-T} \frac{r^{j+1+\delta}}{r^{j+\delta}} = \sum_{j=1}^{S-T} r = (S-T) * r. \end{aligned}$$

Therefore, the minimum value of L over all real values of $K_1, \dots, K_{T-1}, N_1, \dots, N_{S-T}$ is

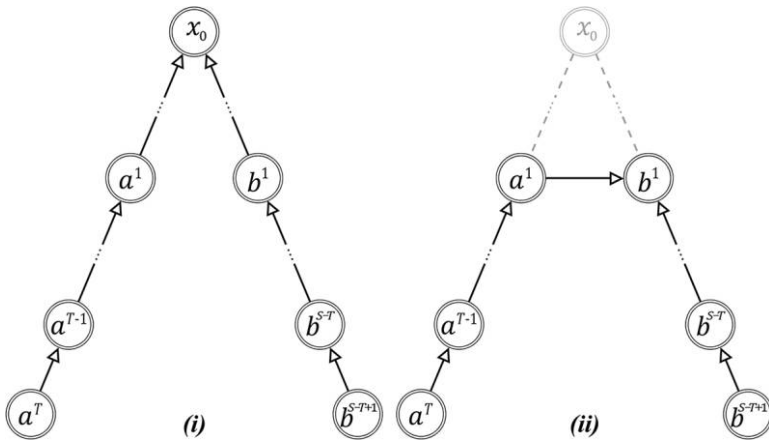


FIG. A2.—Geodesics between stranger nodes can follow one of two topologies: (i) central and (ii) noncentral.

$$(T - 1)*r + r + (S - T)*r = S*r = (\text{by [A37]})$$

$$= S*[(1 + K*\epsilon)*(1 + N*\epsilon)]^{1/S},$$

which proves (A11) for noncentral S -geodesics.

If $\Lambda^*(a, b)$ is a central geodesic, it will have the form $\Lambda^* = (a = a^T, a^{T-1}, \dots, a^2, a^1, x_0, b^1, b^2, \dots, b^{S-T+1} = b)$, as depicted on the left side of figure A2. Since Λ^* does not contain a bridge tie, its length is simply

$$L^* = \sum_{i=1}^T \frac{1 + K_{i+1}*\epsilon}{1 + K_i*\epsilon} + \sum_{j=1}^{S-T} \frac{1 + N_{j+1}*\epsilon}{1 + N_j*\epsilon}.$$

The proof of (A11) for the central case is otherwise nearly identical to the noncentral case above, and we omit it for reasons of space.

Finally, to prove statement (A12), we combine (A37), (A33), and (A34) when $i = j = 1$:

$$(1 + K_1*\epsilon)*(1 + N_1*\epsilon) = r^{1-\delta}*r^{1+\delta} = r = ((1 + K*\epsilon)*(1 + N*\epsilon))^{1/S}.$$

This concludes the proof of theorem 5.QED

Theorem 6: The “Central Pull” of the System

Finally, we will prove that nontrivial transverse geodesics—which are the majority of geodesics in the system—pass through x_0 directly or through a

closely correlated node x' . This “central pull” of the system suggests shortest-path betweenness as the tool for identifying the center of such systems. The two corollaries further support the use of betweenness centrality. Corollary 6A shows that trivial geodesics occur only near the center of the system—or, in other words, that a large-enough system will have many nontrivial geodesics. This is a useful property, as betweenness centrality relies on nontrivial geodesics to identify the center. Since some geodesics may pass through a closely correlated node x' instead of x_0 , empirical researchers should exercise caution in the presence of multicollinearity. However, corollary 6B shows that, if two transverse geodesics connecting unrelated pairs of nodes both bypass x_0 , they will do so via different nodes. In other words, while there is only one x_0 , there are many possible nodes that can play the role of x' . This lessens the inferential threat presented by collinearity, as it makes it less likely that any one x' should lie on enough geodesics to be mistaken for the center.

THEOREM.—All nontrivial transverse geodesics $\Gamma(a_K, b_N)$ pass through either x_0 or through node x' whose absolute Pearson’s correlation with x_0 exceeds $(2/3)^{3/4} \approx 0.74$.

COROLLARY 6A.—If a transverse geodesic $\Gamma(a_K, b_N)$ is trivial, then $\eta(a) \leq 1/\epsilon$ or $\eta(b) \leq 1/\epsilon$.

COROLLARY 6B.—Assume $\Gamma(a, b)$ and $\Omega(c, d)$ are two noncentral, nontrivial transverse geodesics. Let x' be the node in $\Gamma(a, b)$ that is closest to x_0 and x'' be the node in $\Omega(c, d)$ that is closest to x_0 . If a, b, c , and d are strangers, then $x' \neq x''$.

Proof of theorem.—By theorem 3, $\Gamma(a, b)$ can contain either no stranger ties or one stranger tie. In either case,

$$\Gamma(a, b) \text{ must contain a tie } T(f, g) \text{ uniting a node in } \pi(a) \quad (\text{A38}) \\ \text{with a node in } \pi(b).$$

If $\Gamma(a, b)$ contains no stranger ties, f or g must be in $\pi(a) \cap \pi(b) = \{x_0\}$. Thus, without stranger ties, $\Gamma(a, b)$ must pass through the origin, which satisfies the condition of this theorem.

We now turn to cases where Γ contains one stranger tie. If $S = \tau(\Gamma)$, by (A11),

$$|\Gamma(a, b)| = S * [(1 + K * \epsilon) * (1 + N * \epsilon)]^{1/S}. \quad (\text{A39})$$

Substituting $C \equiv (1 + K * \epsilon) * (1 + N * \epsilon)$ into (A39) transforms it to

$$|\Gamma(a, b)| = S * C^{1/S}. \quad (\text{A40})$$

Furthermore, since Γ is the absolute geodesic between a and b , it by definition cannot be longer than the S -geodesic $\Gamma^{S+1}(a, b)$. Thus, by (A11),

$$|\Gamma(a, b)| \leq (S + 1) * C^{1/(S+1)}. \quad (\text{A41})$$

Combining (A40) and (A41) and transforming the result yields

$$\begin{aligned} S * C^{1/S} &\leq (S + 1) * C^{1/(S+1)} \Leftrightarrow \\ C^{\frac{1}{S} - \frac{1}{S+1}} &\leq \frac{S + 1}{S} \Leftrightarrow C^{\frac{1}{S*(S+1)}} \leq \frac{S + 1}{S} \Leftrightarrow C \leq \left(\frac{S + 1}{S}\right)^{S*(S+1)} \Leftrightarrow \\ C^{-1} &\geq \left(\frac{S}{S + 1}\right)^{S*(S+1)}. \end{aligned} \quad (\text{A42})$$

By (A12), there are two nodes in $\Gamma = \Gamma^S(a, b)$ with generations K_1 and N_1 , respectively, which satisfy the equation $(1 + K_1 * \epsilon) * (1 + N_1 * \epsilon) = [(1 + K * \epsilon) * (1 + N * \epsilon)]^{1/S}$. Thus, by definition of C ,

$$(1 + K_1 * \epsilon) * (1 + N_1 * \epsilon) = C^{1/S}. \quad (\text{A43})$$

We will consider the case when $K_1 \leq N_1$ (the opposite case is nearly identical):

$$\begin{aligned} (1 + K_1 * \epsilon) &\leq (1 + K_1 * \epsilon) * (1 + N_1 * \epsilon) \Leftrightarrow (1 + K_1 * \epsilon)^2 \leq C^{1/S} \\ &\Leftrightarrow (1 + K_1 * \epsilon) \leq C^{1/(2*S)} \Leftrightarrow 1/(1 + K_1 * \epsilon)^{1/2} \geq C^{-1/(4*S)}. \end{aligned} \quad (\text{A44})$$

By (A1), $\text{Cor}^2(K_1, x_0) = 1/(1 + K_1 * \epsilon) \Leftrightarrow \text{Cor}(K_1, x_0) = 1/(1 + K_1 * \epsilon)^{1/2}$. Thus, by (A44), $\text{Cor}(K_1, x_0) \geq C^{-1/(4*S)}$, and by (A42),

$$\text{Cor}(K_1, x_0) \geq \left(\frac{S}{S + 1}\right)^{(S+1)/4}. \quad (\text{A45})$$

We will now consider the natural logarithm of the function $R(S) = [S/(S + 1)]^{(S+1)/4}$:

$$\ln R(S) = \frac{S + 1}{4} * \ln\left(\frac{S}{S + 1}\right) = \frac{S + 1}{4} * \ln\left(1 - \frac{1}{S + 1}\right).$$

Since both factors increase with S , $\ln R(S)$ increases monotonically, and thus so does $R(S)$. Since Γ is nontrivial, $\min(S) = 2$. Substituting this into (A45) yields $\text{Cor}(K_1, x_0) \geq (2/3)^{3/4} \approx 0.74$. QED

Proof of corollary 6A.—For trivial geodesics, $S = 1$. Substituting $S = 1$ and $\text{Cor}(K_1, x_0) = 1/(1 + K_1 * \epsilon)^{1/2}$ into (A45) yields

$$\frac{1}{\sqrt{1 + K_1 * \epsilon}} > \left(\frac{1}{2}\right)^{1/2} \Leftrightarrow 1 + K_1 * \epsilon < 2 \Leftrightarrow K_1 * \epsilon < 1 \Leftrightarrow K_1 < 1/\epsilon.$$

Since in a trivial geodesic $K_1 = \min(\eta(a), \eta(b))$, this implies $\eta(a) < 1/\epsilon$ or $\eta(b) < 1/\epsilon$. QED

Proof of corollary 6B.—Let us assume that 6B is not the case and that $x' = x''$. Then, by theorem 4, $x' \in (\pi(a) \cup \pi(b))$ and $x' = x'' \in (\pi(c) \cup \pi(d))$. Thus $x' \in [\pi(a) \cup \pi(b)] \cap [\pi(c) \cup \pi(d)] = ([\pi(a) \cup \pi(b)] \cap \pi(c)) \cup ([\pi(a) \cup \pi(b)] \cap \pi(d)) \Leftrightarrow$

$$x' \in [\pi(a) \cap \pi(c)] \cup [\pi(b) \cap \pi(c)] \cup [\pi(a) \cap \pi(d)] \cup [\pi(b) \cap \pi(d)]. \quad (\text{A46})$$

If two nodes f and g are strangers, then by definition $\pi(f) \cap \pi(g) = \{x_0\}$. Thus (A46) equals

$$x' \in \{x_0\} \cup \{x_0\} \cup \{x_0\} \cup \{x_0\} \Leftrightarrow x' = x_0.$$

This contradicts the definition of noncentral geodesics. QED

APPENDIX B

Network Partitioning

The moral politics and social constraint perspectives both suggest that belief networks may contain subgroups of beliefs that exist in relative independence from the rest of the network. For example, in Zaller's (1992) account, antiwar beliefs exhibit a partial decoupling because over-time changes in party positions on this issue cause different followers of the same party to acquire different messages. In Lakoff's (2002) account, beliefs about gender roles are similarly decoupled because they share a common foundation that varies independently from other beliefs. Both accounts suggest that beliefs may form subgroups with ties that are stronger within groups than they are between them. Since the average strength of belief correlations varies between different populations (Converse 1964) and depends on properties of the survey instrument (Martin 1999), the specific criteria for within- and between-group tie strengths cannot be determined a priori. A method for locating community structure in belief networks should thus instead do so based on the observed distribution of tie strengths. Newman's (2006) modularity maximization technique is commonly used for this kind of partitioning. However, recent methodological work raises concerns about that method's accuracy and validity in many empirical settings. In this appendix, we review this work and examine whether the accuracy problems apply to our empirical case.

Newman's approach uses an objective function called "modularity." The modularity of any partition of a given network into mutually exclusive sub-

groups measures the degree to which that partition results in stronger-than-chance ties within each subgroup and weaker-than-chance ties between subgroups. Modularity maximization algorithms search the space of possible partitions of a given network for the one that yields the greatest modularity. This technique thus tries to locate partitions defined by “statistically surprising” (Newman 2006) arrangements of ties. However, two well-documented problems often compromise its ability to do so.

First, the modularity function suffers from a “resolution limit” that can bias it against (i) detecting small modules in large networks, as well as (ii) large modules in small networks, even when the existence of such modules appears intuitively clear and can be easily determined by other methods (Fortunato and Barthélemy 2007; Good, de Montjoye, and Clauset 2010; Lancichinetti and Fortunato 2011). Though more recent versions of the algorithm introduce a tuning parameter that can make either bias i or bias ii less likely, it is practically impossible to avoid both biases simultaneously (Lancichinetti and Fortunato 2011). The two biases make it less likely that modularity maximization would determine that all the nodes in the network belong to only a single module, or that every node belongs in its own single-node module—that is, that there is no subgroup structure. This renders it a poor fit for testing structural hypotheses like those suggested by the moral politics and social constraint accounts. As we discuss in appendix G, the RCA technique used by Baldassarri and Goldberg (2014) to claim evidence for heterogeneous logics of belief organization also relies on modularity maximization. We demonstrate problems with RCA results that are consistent with this bias (see app. G).

Second, the modularity function exhibits degeneracies that greatly compromise its reliability in many empirical settings (Good et al. 2010; Rubinov and Sporns 2011). Rather than a unique maximum that clearly recommends one optimal partition of the network, the modularity function often has multiple near-maxima corresponding to distinct partitions with approximately equal modularities. This leaves its results unstable to small changes in the network. To determine whether this occurs with our data, we analyzed the modularity function for the full population, low- and high-information, and African-American belief networks. For each, we compared the modularity scores across different partitions produced by the hierarchical “*fastgreedy*” maximization algorithm (in separate analyses, we compared *fastgreedy* to an exhaustive search and found that *fastgreedy* generally located the optimal or nearly optimal belief network partitions). Since these are only a small portion of all possible partitions of each network, this search for degeneracies was conservative. Nonetheless, all four distributions exhibited these near-maxima, with five to eight distinct partitions having modularity scores within 5% of the maximum in each network. A bootstrapping analysis confirmed that the resulting subgroup assignments were unreliable.

Because of these problems, we do not report the modularity maximization results in our analyses and cannot recommend the algorithm's use with belief networks. Future work should explore the reliability and theoretical fitness of other network partitioning methods and may instead need to develop a new method that is better tailored to this kind of data. Such work should also compare these partitioning schemes to more traditional factor-analytic techniques for grouping variables, both in terms of reliability and of theoretical fit.³⁶

APPENDIX C

Bivariate Linearity

The BNA methodology we develop in this article analyzes weighted networks constructed from squared correlations between pairs of belief variables. We use polychoric correlations between ordinal variables, polyserial correlations between numeric and ordinal, and Pearson's correlations between numeric variables. Our model and method thus carry an assumption that correlation can fully capture the pairwise relationships between beliefs—or, equivalently, that the relationships between these beliefs are linear in character.³⁷ This assumption is commonly made in the literature on belief structures (e.g., Converse 1964; Jennings 1992). Nonetheless, it may be possible that relationships between beliefs are more complex, due to either nonlinear functional form or to unobserved heterogeneity. For example, if two beliefs could be captured by integer-valued variables $x \in \{-2, \dots, 2\}$ and y , the true relationship between them may take the form of a parabola $y = x^2$. Even though x and y are deterministically interrelated, their Pearson's correlation $r(x, y = x^2)$ would equal 0. In the presence of such nonlinearity, our reliance on correlation could thus lead us to overlook relationships between variables.

To examine this possibility, we build on Martin's (1999, 2002) work on entropic measures of constraint to develop an information-theoretic test for whether such nonlinearity is present in our data. We base this test on mutual information, which is a general purpose nonparametric measure of association between discrete variables. Unlike linear relationship measures like Pearson's and polychoric correlation, mutual information quantifies the amount of nonindependence between x and y without any assumptions

³⁶ Older work has in fact used factor analyses of the adjacency matrix to partition social networks into subgroups, though Wasserman and Faust (1994, p. 290) note that this approach may not yield subgroups with theoretically desirable properties.

³⁷ In the case of our formal model, this is a consequence of our assumed belief derivation process (app. A) rather than a separate assumption in the formal sense of the term. For ordinal variables, the assumption concerns latent beliefs rather than manifest indicators.

about the functional form of their relationship or about the relative ordering of each variable's levels. The mutual information $I(x, y) \in [0, +\infty)$ equals zero if and only if x and y are fully independent. It otherwise quantifies the extent to which knowing the value of one variable would reduce uncertainty regarding the value of the other.

The relationship between mutual information and marginal (univariate) entropies $H(x)$ and $H(y)$ is roughly analogous to that between covariance and variance. To yield a correlation-like measure of pairwise association between x and y , mutual information can be normalized to the $[0, 1]$ range via $\hat{I}(x, y) = I(x, y) / \min\{H(x), H(y)\}$ (Kvalseth 1987; Yao 2003). If either of the variables fully describes all the variation in the other, $\hat{I}(x, y)$ achieves its maximum of 1. Since in the above example, each value of y maps onto exactly one value of x , $\hat{I}(x, y) = 1$. In the presence of random noise, this relationship will decrease but will remain above zero as long as the noise does not “overwhelm” y to the point that x and y are independent (we return to issues of noise later in the appendix).

Mutual information can also be used to detect a relationship between a pair of variables x and z even in the presence of unobserved heterogeneity, as long as the heterogeneity does not render x and z independent (in the latter case, no method could possibly detect the latent relationship between x and z without introducing further assumptions). For example, consider a further pair of integer-valued belief variables x and z , where $x \in \{-2, -1, 1, 2\}$. Belief x may have a positive linear relationship with belief $z = (z_1; z_2)$ in one half of the population, where $z = z_1 = 2x$, and a symmetrical negative relationship in the other half, where $z = z_2 = -2x$. The resulting relationship between x and z is an “ \times ” shape centered at the origin. Due to the absence of a linear relationship, their Pearson's correlation is $r(x, z) = 0$. However, x and z are not independent, as each value of z maps onto only two values of x , and vice versa. Correspondingly, their normalized mutual information is above zero: $\hat{I}(x, z) = 0.5$.

In general, if the population consists of two subgroups where $z = z_1 = ax + b$ in the first subgroup and $z = z_2 = cx + d$ in the second, and the variable x assumes at least three different values,³⁸ each value of x would correspond to only a proper subset of values of z . Therefore, even in the presence of such heterogeneity, x and z would still not be independent, and their \hat{I} would be above zero. The same can clearly not be said of correlation, which equals zero if $a = -c$.

With this consideration in mind, we calculated the mutual informations \hat{I}_{ij} between each pair (i, j) of the 46 variables in our ANES data, placing them in the matrix $\hat{\mathbf{I}} = [\hat{I}_{ij}]$. To calculate \hat{I} , we used the empirical estimators in the *R* package “*entropy*” (Hausser and Strimmer 2013). Since mutual in-

³⁸ Forty-one of the 46 variables in our data (89%) assume three or more values.

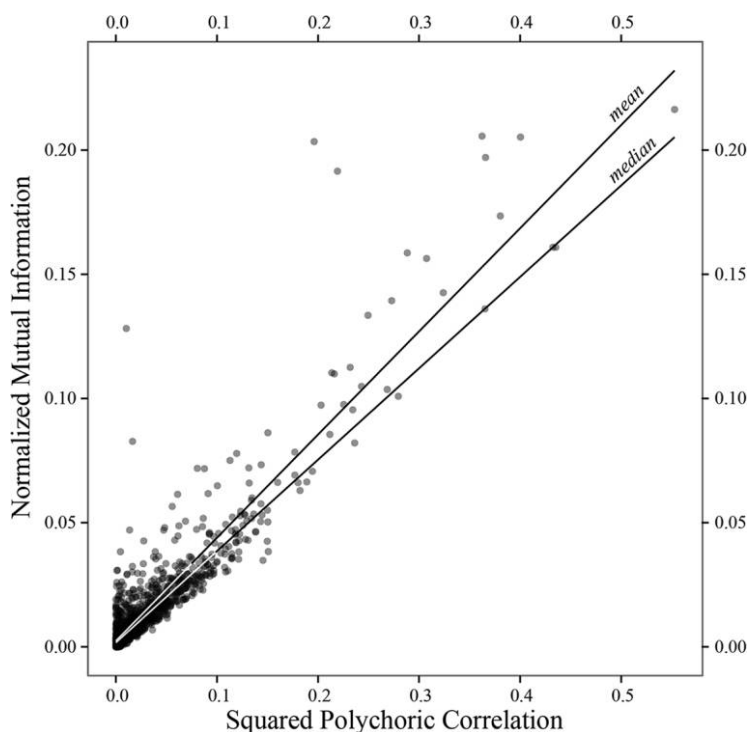


FIG. C1.—Normalized mutual information by squared polychoric correlation. Each point represents the relationship between a pair of belief variables. The lines of best fit are from OLS (mean) and quantile (median) regressions.

formation only exhibits the properties described above when the variables are discrete, we used the original survey items from the Limited Government, Gay Rights, Welfare, and Racism scales in place of the factored scores. This yielded 46 belief variables in total.³⁹ For comparison, we also constructed a correlation matrix $\mathbf{C} = [C_{ij}]$, where each cell C_{ij} contains the squared polychoric correlation between the same beliefs i and j . We plot the values of C_{ij} and \hat{I}_{ij} for all variable pairs $i \neq j$ in the data (fig. C1).

Consider the element-wise Pearson's correlation between these two matrices, $r(\mathbf{C}, \hat{\mathbf{I}})$. This correlation would only be high if N and C generally varied together, suggesting that mutual information and correlation are registering the same relative rates of interrelatedness between each of the pairs of variables.⁴⁰ On the other hand, $r(\mathbf{C}, \hat{\mathbf{I}})$ would be low if C and $\hat{\mathbf{I}}$ frequently

³⁹ We make this matrix available in an online appendix.

⁴⁰ Of course, since correlation reflects only simultaneous deviations from each matrix's mean, $r(C, N)$ could also be high if C always missed the same amount of the pairwise re-

varied independently of one other, or if their simultaneous variances were related in a nonlinear fashion. This makes $r(\mathbf{C}, \hat{\mathbf{I}})$ a relatively conservative measure of whether or not polychoric correlation is missing any substantial nonindependence between the variables.

When we calculated $r(\mathbf{C}, \hat{\mathbf{I}})$ for our observed ANES data, we found that $r(\mathbf{C}, \hat{\mathbf{I}}) = 0.91$, or 0.97 if the matrix diagonals are included in the comparison. This high value of $r(\mathbf{C}, \hat{\mathbf{I}})$ shows that polychoric correlations can account for the majority of the pairwise nonindependence between variables in this data set. It is thus inconsistent with the presence of substantial nonlinear relationships in our data.

It is possible, however, that measurement error in survey data may conceal some heterogeneity or complexity that is present in the population. Since survey data are not perfectly reliable, this error could conceivably distort a truly nonlinear or heterogeneous relationship between variables to the point that only a single linear trend remains observable. This could lead $r(\mathbf{C}, \hat{\mathbf{I}})$ to underestimate the true amount of heterogeneity or nonlinearity in the relationship. Conversely, it is also possible that measurement error may distort linear trends, creating the appearance of complexity when the true relationship between two variables is simply linear. If this were the case, $r(\mathbf{C}, \hat{\mathbf{I}})$ could instead overestimate the amount of heterogeneity or nonlinearity, thus leading to an even more conservative test.

To adjudicate between these two possibilities, we make use of the reliability estimates Alwin (2007) calculated for the 1990 ANES panel, which included 18 of the same questions we analyzed here. We subset our data to these 18 variables and examine the corresponding 18×18 matrixes (excluding diagonals). Within this subset, $r(\mathbf{C}, \hat{\mathbf{I}})$ equals 0.90 , which is close to the value it obtained for the full set of 46 belief variables we examined above. With all the matrix cells ($i, j \neq i$) as the units of analysis, we regress the normalized mutual informations \hat{I}_{ij} on the reliabilities of the two variables i and j , controlling for their squared polychoric correlations C_{ij} :

$$\hat{I}_{ij} = \beta_0 + \beta_1 \times \text{reliability}_i + \beta_2 \times \text{reliability}_j + \beta_3 \times C_{ij} + \varepsilon_{ij}.$$

relationships between all pairs of variables, that is, if the degree of nonlinearity in the pairwise relationships between variables was constant. If we include the diagonal elements of the matrixes in this comparison, this becomes impossible: the diagonal cells of each matrix measure the relationship of each belief to itself, that is, $C_{ii} = N_{ii} = 1$, and thus cannot possibly contain any nonlinearity. On the other hand, since the diagonal elements of the two matrixes are both at their maximum, their presence can lead us to overestimate the similarity between the two matrixes. We thus report $r(\mathbf{C}, \hat{\mathbf{I}})$ with and without the diagonal elements. Additionally, we note that high pairwise correlations between the variables that formerly made up Limited Government, Gay Rights, Welfare, and Racism scales also leave little room for substantial deviations from linearity.

The standardized coefficient for C_{ij} equals 0.93 ($t(302) = 40.39, P < .001$), which indicates that correlation retains a strong relationship with mutual information in the presence of these controls. Turning to the reliability variables, the standardized coefficients for both equal -0.13 ($t(302) = -5.75, P < .001$), indicating that an increase in reliability is accompanied by a moderate but highly significant decrease in the mutual information between the two variables, net of the amount of mutual information predicted by correlation. This result is not consistent with the idea that noise in the survey data leads us to underestimate the potential nonlinearity or heterogeneity in the bivariate relationships. Rather, it shows that noise may lead us to overestimate these possible complexities and that the true relationships between the variables may thus be even more homogeneous and linear than the already-high correlation $r(\mathbf{C}, \hat{\mathbf{I}}) = 0.91$ suggests. We therefore conclude that our choice of using correlation as the building block for the networks we examine in this article is justified.

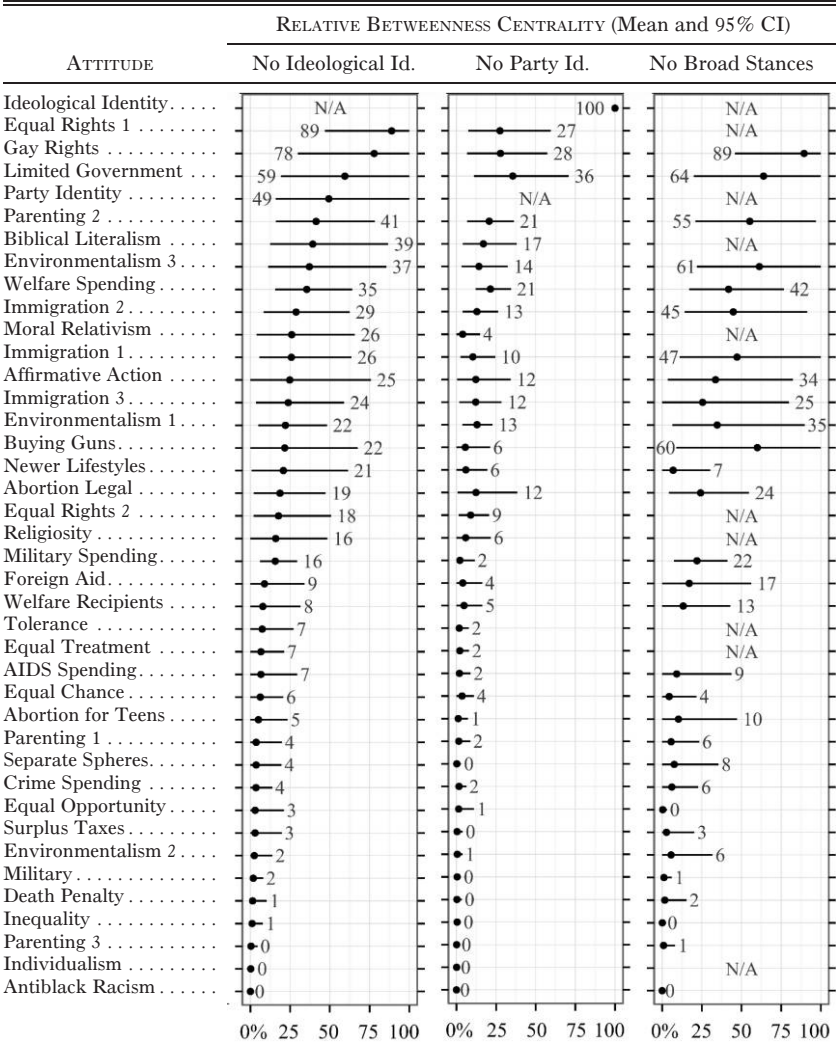
APPENDIX D

Additional Centrality Analyses

As further robustness checks, we used row bootstrapping to examine belief centralities in the 2000 ANES network with some key variables omitted or differently coded. The first column of table D1 contains the results from 1,000 bootstraps of the 39-belief network that excludes ideological identity. These results, like those in the right column of table 3, show that the network without ideological identity has no clear center. The second column of table D1 contains the results of 1,000 further bootstraps of the 39-belief network that retains ideological identity (Conservative-Liberal) but omits party identity (Democrat-Republican). These results retain much the same centrality distribution as the first set of relative centrality scores we reported in table 1, with ideological identity as the reliably most central variable. Finally, the last column of table D1 contains results from 1,000 bootstraps of the network that omits all broad identities and moral stances and retains only the more narrow domain-specific beliefs. Like other analyses that excluded ideological identity, these yielded wide overlapping confidence intervals with no clear center.

We also investigated whether our results are affected by the ways the key variables were coded. Our ideological identity variable contains seven levels, while each parenting variable contains only three. It may thus be possible that the greater observed centrality of ideological identity is due simply to better variable quality. To rule out this possibility, we examined the results of joining the three parenting items into a single variable via summation (which yields a seven-category scale). We also examined the results of

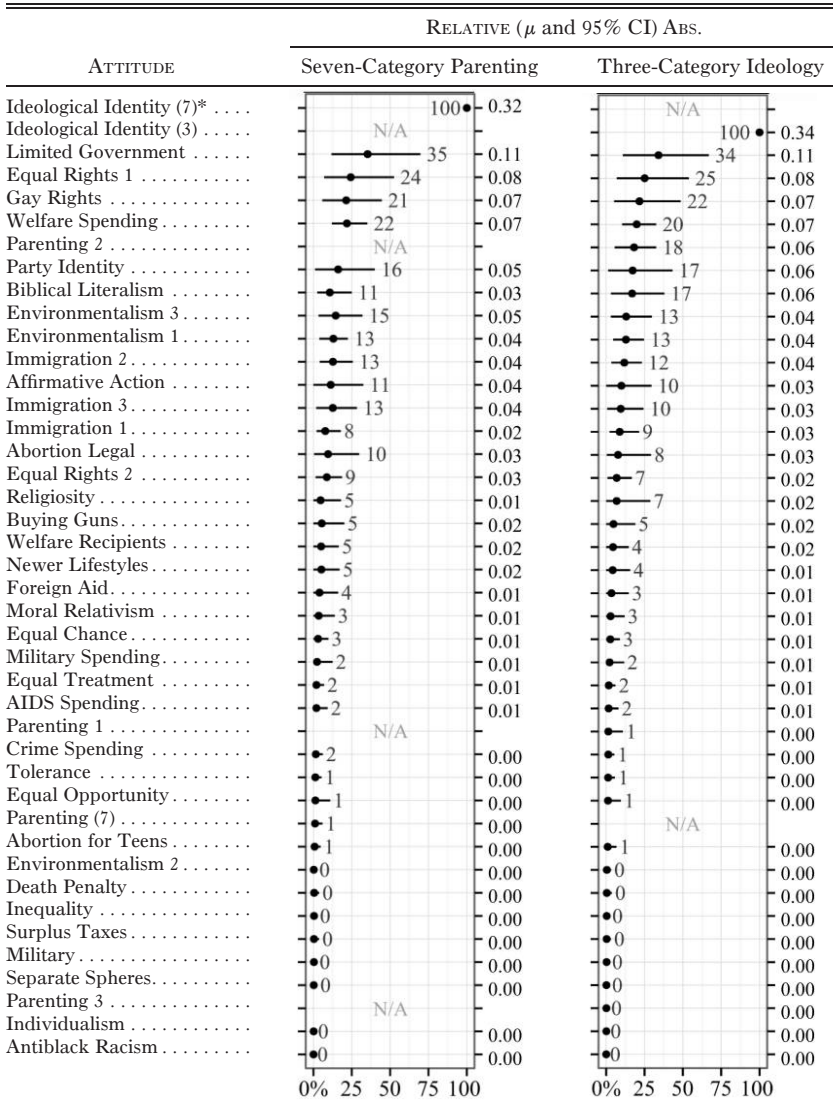
TABLE D1
BETWEENNESS CENTRALITIES FOR ALTERNATE SELECTIONS OF BELIEFS



NOTE.—Each of the three sets of betweenness estimates is based on 1,000 row-wise bootstraps.

reducing the number of levels in the ideological identity variable by collapsing the top two, middle three, and bottom two levels of this variable into single levels. This yielded a three-category ideological identity variable. As table D2 shows, neither change affected our results.

TABLE D2
BETWEENNESS CENTRALITIES FOR ALTERNATE VARIABLE FORMATS



NOTE.—Each of the three sets of betweenness estimates is based on 1,000 row-wise bootstraps.

* The numbers in parentheses indicate the number of categories in the variable.

APPENDIX E

Stratifying Variables

This appendix describes how we constructed the demographic variables we used in our heterogeneity analyses.

Parents foreign born.—Respondent answered no to “Were both of your parents born in this country?”

Class (self-identified).—Coded from branching question: “Most people say they belong either to the middle class or the working class. Do you ever think of yourself as belonging in one of these classes? (IF YES:) Which one? (IF NO:) Well, if you had to make a choice, would you call yourself middle class or working class?”

Black.—Coded from the primary racial group self-description variable. Respondents who identified as “Black” or its synonyms were the only ones coded as “Black.” Responses without a clear racial designation (e.g., “American” or “None”) were coded as missing. The remaining respondents were coded as nonblack.

Hispanic.—Respondent was coded “Hispanic” if (i) the respondent’s self-identified ethnicity was Mexican, Central American, South American, Cuban, Puerto Rican, or Spanish or if (ii) respondent answered yes to the question “Are you of Spanish or Hispanic origin or descent?”

Southeastern United States.—Interview location in the “South” region of the U.S. census.

Religion.—Coded from the religion/denomination variable. “Catholics” were self-identified. We identified “Mainline Protestants” using the list of Mainline Protestant denominations in Steensland et al. (2000). We coded remaining Protestants “Protestant (Other).” Due to small *N*s, we merged the remaining religious and nonreligious respondents into “Other Religion or none.”

Occupational category.—Coded from 14-level occupational category variable. We coded “Executive, Administrative and Managerial” as “Managerial”; “Professional Specialty Occupations” as “Professional”; “Technicians and Related Support Occupations,” “Precision Production, Craft and Repair,” and “Machine Operators, Assemblers and Inspectors” as “Skilled or Semi-Skilled”; “Sales Occupation” and “Administrative Support” as “Routine Nonmanual”; and “Private Household,” “Protective Service,” “Service except Protective and Household,” “Farming, Forestry and Fishing,” “Transportation and Material Moving,” and “Handlers, Equipment Cleaners, Helpers” as “Unskilled or Farm” (the remaining occupational categories had no respondents).

Type of place.—Coded from response to “Please tell me which category best describes where you were mostly brought up?” We coded “on a farm” and “in the country, not on a farm” as “Rural”; city/town of up to 100,000

residents as "Smaller City"; city of over 100,000 residents as "Larger City"; and suburb of any size of city as "Suburban."

Cross pressures.—Constructed by interacting the three-category income variable with the two-category church attendance variable. Lower-income respondents (under \$35,000 a year) were coded "Pressures Crossed" if they attended church and "Pressures Aligned" if they did not. Higher-income respondents (over \$65,000 a year) were coded "Pressures Crossed" if they did not attend church and "Pressures Aligned" if they did. Middle-income respondents (\$35,000–\$65,000 a year) were coded "Neither."

Political information.—Constructed from an eight-item quiz and two interviewer assessments. The eight-item quiz began with the stem "Now we have a set of questions concerning various public figures. We want to see how much information about them gets out to the public from television, newspapers and the like." The first question was (1) "The first name is TRENT LOTT. What job or political office does he NOW hold?" The next three questions had the same format and asked about (2) "WILLIAM REHNQUIST," (3) "TONY BLAIR," (4) "JANET RENO." The remaining four questions were (5) "What U.S. state does George W. Bush live in now?" (6) "What U.S. state is Al Gore from originally?" (7) "Do you happen to know which party had the most members in the House of Representatives in Washington BEFORE the election (this/last) month?" and (8) "Do you happen to know which party had the most members in the U.S. Senate BEFORE the election (this/last) month?" The quiz score was the count of responses that were identified as correct in ANES data. Interviewers were also asked to subjectively assess each respondent's "general level of information about politics and public affairs" both before and after the November 2000 elections. These responses were on 5-point Likert scales, which we coded "Very High" (4) to "Very Low" (0) and summed together with the quiz score. Finally, we coded total scores from 0 to 5 points "Low," 6–10 points as "Medium," and 11–16 as "High."

The remaining variables (Gender, Number of children, Age group, Education, Income, and Church attendance) were coded from the appropriate ANES variables and are unambiguous.

APPENDIX F

Comparison to Baldassarri and Goldberg (2014)

We have argued that social groups differ in the extent to which their belief systems are organized but rarely in the logic that organizes them. Baldassarri and Goldberg (2014) instead claim that differences in both the amount and the logic of organization are frequently encountered. In this appendix, we examine some key evidence they present in favor of their argument and

argue that their results support our view of heterogeneity and contradict theirs.

In their theoretical reasoning, Baldassarri and Goldberg note that “it is important to make an analytical distinction between differences that are the result of weak opinion constraint and those that present an alternative, internally coherent belief system” (2014, p. 55). In our terminology, the differences that result from constraint strength are differences in the amount of organization. If X and Y are two subgroups that differ in amount of belief organization, and in group X the belief “ P is true” strongly implies “ Q is true,” then in group Y “ P is true” should imply “ Q is true” more weakly. So, if A and B are a pair of belief variables, and $\text{cor}_X(A, B)$ and $\text{cor}_Y(A, B)$ are the correlations observed between these beliefs in subgroups X and Y , respectively, then we can express this kind of difference formally as:

$$\begin{aligned} \text{if } \text{cor}_X(A, B) > 0, \text{ then } \text{cor}_X(A, B) > \text{cor}_Y(A, B) \geq 0; \\ \text{if } \text{cor}_X(A, B) < 0, \text{ then } \text{cor}_X(A, B) < \text{cor}_Y(A, B) \leq 0. \end{aligned} \quad (\text{F1})$$

Baldassarri and Goldberg propose that the difference between ideologues and agnostics can be accounted for by amount of organization: for ideologues, all belief domains have strong positive mutual associations, while for agnostics, many of these associations are weakly positive or nonexistent. Thus, if we compare pairwise correlations between ideologues and agnostics, we should frequently see relationships of type (F1).

We refer to differences that stem from the presence of “alternative, internally coherent” belief systems as differences in the logic of belief organization. If there are two internally coherent belief systems, the differences between them should take the form of contrasting entailments: if in population X the belief “ A is true” implies “ B is true,” in population Y it should imply “ B is false.” Thus, the same pairs of beliefs must often exhibit opposite correlations in the two groups, so that there exist a substantial number of beliefs A and B such that

$$\begin{aligned} \text{if } \text{cor}_X(A, B) > 0, \text{ then } \text{cor}_Y(A, B) < 0; \\ \text{if } \text{cor}_X(A, B) < 0, \text{ then } \text{cor}_Y(A, B) > 0. \end{aligned} \quad (\text{F2})$$

This appears to be the distinction that Baldassarri and Goldberg propose between ideologues and alternatives. As Baldassarri and Goldberg note, “if alternatives are inherently different from ideologues and agnostics, we should find that issue domains correlate differently with one another in this group” (2014, p. 64). The term “correlate differently” is ambiguous. However, in order for the contrast between alternatives and ideologues to be “inherently different” from the contrast between agnostics and ideologues, we

reason that the differences in correlations cannot be again due to weak opinion constraint, and should thus be the differences of entailments as in situation (F2) above.

Our disagreement with Baldassarri and Goldberg concerns how to interpret the situation where $|\text{cor}_X(A, B)| > 0$, and $\text{cor}_Y(A, B)$ is not significantly different from zero (i.e., $\text{cor}_Y(A, B) \approx 0$). This situation occurs when “P is true” implies “Q is true” for population X, but “P is true” does not imply anything about Q for population Y. In the typology we offer above, this closely resembles condition (F1): “if $\text{cor}_X(A, B) > 0$, then $\text{cor}_X(A, B) > \text{cor}_Y(A, B) \geq 0$.” The weaker the overall level of belief constraint in population Y, the more often we will see pairs of beliefs A and B where $\text{cor}_Y(A, B) \approx 0$. At the most extreme end, if population Y displays absolutely no belief organization, this situation would apply to all belief pairs. We thus interpret it as difference in the amount of belief organization. On the other hand, Baldassarri and Goldberg interpret such nonsignificant relationships as a tendency to “dissociate” (62) or “decouple” (66) between pairs of attitudes, which they appear to treat as evidence of belief system difference that is *not* simply due to “weak opinion constraint.” Since insignificant correlations correspond to the weakest of the weak opinion constraints, this choice appears indefensible to us.

With this in mind, we examine the results Baldassarri and Goldberg report for the 1984, 1986, 1988, 1992, 1994, 1996, 2000, and 2004 ANES in their figure 3 (2014, p. 63). For each of the survey years, the authors report the average correlation between economic and moral attitudes for members of the ideologue, agnostic, and alternative groups. Of these eight average correlations between economic and moral attitudes presented for ideologues, all eight are positive and significant. Of the eight presented for agnostics, two are not significant, and the remainder are weaker than for ideologues. This is a clear example of situation (F1) and is consistent with their argument that the difference between ideologues and agnostics can be attributed to difference in constraint. We agree with both this interpretation of the data and the broader theoretical claim.

We now turn to the results they present for alternatives. Since the average correlations between economic and moral attitudes for ideologues were positive, the correlations for alternatives would need to frequently be negative to provide evidence for alternate belief systems, as in situation (F2) above. However, the results indicate that, for alternatives, five correlations are not significant, two are weakly positive, and only one is negative. Thus only a single ANES year analyzed by Baldassarri and Goldberg actually fits under condition (F2)—the same year (2004) they highlight in detail in their paper. For the other seven years, alternatives exhibit weakly positively constrained or unconstrained attitudes between these domains. As we argued above, this is a difference in the amount of belief organization, not in its

logic. This is consistent with our claims but not with those made by Baldassarri and Goldberg.

Overall, out of all the 48 average cross-domain correlations between economic, civil, moral, and foreign attitudes reported by Baldassarri and Goldberg in their figure 3, all 48 are positive and significant for ideologues. For agnostics, 35 are positive and significant and the rest are insignificant. For alternatives, 26 are positive and significant, 19 are insignificant, and only three are negative (all of them weakly).⁴¹ Thus, for both alternatives and agnostics, beliefs overwhelmingly follow pattern (F1) but not pattern (F2): if support for A implies support for B for ideologues, it only very rarely implies opposition to B in either of the two groups. In fact, though agnostics are supposedly the group defined by “weak associations among political beliefs” (2014, p. 60), alternatives appear to actually have weaker cross-domain belief associations than agnostics. Across all the years, the “alternatives” identified by Baldassarri and Goldberg thus appear to actually be primarily “agnostics,” who generally lack belief constraint as compared to the “ideologues.” This is consistent with our argument that groups overwhelmingly vary in the amount of their belief organization rather than in its logic.

As we noted above, Baldassarri and Goldberg provide a different interpretation of these results by treating the alternatives’ insignificant correlations between economic and moral attitudes as evidence of an alternate belief system. We think this choice is theoretically unjustified. Moreover, it contradicts how Baldassarri and Goldberg reason about this population elsewhere in the paper. In the abstract, they describe alternatives as “morally conservative but economically liberal or vice versa”—a group that they later propose consists of “free-market supporters [who are] culturally and socially progressive” (2014, p. 77) and free-market opponents who are culturally and socially conservative. But since the average correlations between economic and moral attitudes for this group are generally insignificant rather than negative, this group likely also contains as many individuals who are both morally and economically conservative, or both morally and economically liberal. And, given the well-documented relationship between low constraint and political moderation, it may also consist of those who are simply moderate across all the issues. Contra Baldassarri and Goldberg’s description, many of these would not be individuals for whom “selecting one party over the other necessarily entails suppressing one ideological orientation in favor of another” (69). The latter would be the case if and only if their moral and economic attitudes were correlated negatively—that is, if the evidence for alternatives having an alternate belief system fulfilled condition (F2) defined above.

⁴¹ Given the large number of comparisons, these three average correlations may be attributable to noise alone.

APPENDIX G

Goodness-of-Fit Test for Inductively Detected Heterogeneity

In this article, we have argued that social groups generally vary in the extent of their belief organization and not in their organizing logic. This contradicts Baldassarri and Goldberg's (2014) claim that substantial portions of the population use conflicting logics to organize their attitudes. We provided evidence for our claim by comparing belief networks between different demographic groups and demonstrating that they overwhelmingly arrange their beliefs according to the same logic. We also used an information-theoretic technique to demonstrate that linearity accounts for most of the relationship between variables, thus making heterogeneity unlikely (app. C). Additionally, we revisited the ANES time series results reported by Baldassarri and Goldberg and argued that they are more consistent with our view of heterogeneity than with theirs (app. F). In light of this evidence, it may seem surprising that the RCA technique used by Baldassarri and Goldberg detected multiple logics of belief organization in the population. In this appendix, we apply RCA to our 2000 ANES data and use structural equation modeling to examine whether the groups it identified follow different logics of belief organization.

Relational class analysis (Goldberg 2011) is a technique for identifying multiple logics of belief organization. Its goal is to locate respondent subgroups where the same pairs of beliefs correlate differently with one another. Thus, in essence, it is a technique for using individual-level data to partition a single sample correlation matrix into multiple subsample correlation matrices that are mixed to generate the overall sample matrix. The method represents the survey data set as a network, with individuals as nodes and their estimated response pattern similarities as ties, and then uses a modularity maximization algorithm to partition the network into groups. However, recent work demonstrates that modularity maximization can be strongly biased against correctly detecting situations where the network contains no partitions, dividing the population into groups even when a single-group solution would appear intuitively more correct (see discussion in app. B). Relational class analysis likely inherits this potential for bias, which means that it may indicate the existence of multiple logics of belief organization even when none truly exist. Since RCA provides no goodness-of-fit statistic to describe how distinct the logics it locates truly are, it is difficult to determine whether the partitions it produced are spurious. Relational class analysis is thus best thought of as an exploratory technique that detects multiple logics of organization but only under the untested assumption that multiple logics indeed exist in the data.

In order to test whether multiple logics are needed to interpret the ANES data or whether, as we argue, there is only one dominant logic, we use a well-

known multiple group testing technique from structural equation modeling (SEM). This technique tests whether using the separate correlation matrices of a proposed partition improves model fit over using one matrix alone. This approach is widely used in psychology to test whether, say, factor loadings in a confirmatory factor analysis are the same for different genders or ethnic groups (Bollen 1989; for a review, see MacCallum and Austin [2000]). We hope that this approach will be more widely used in future research to evaluate the partitions suggested by RCA and related techniques.⁴²

We used RCA to examine the 2,000 ANES items we studied in our primary analyses, which are largely the same items that Baldassarri and Goldberg used in their analyses of the 2000 data. Like Baldassarri and Goldberg (2014, p. 87), we list-wise deleted all respondents with one or more missing responses, leaving $N = 727$ respondents. We then rescaled each of the belief variables to the $[0, 1]$ range. The RCA software (Goldberg and Stein 2016) detected three groups of respondents, as it did in the Baldassarri and Goldberg (2014) analyses. It also left four respondents unassigned to any group, perhaps because their dissimilarity from other respondents left them as isolates in the network. We dropped these isolates from our analyses.

We use SEM to examine the fit of the three-group partition detected by RCA by comparing models in which the full data set is produced by a single correlation matrix to a model in which the data are produced separately for each RCA-detected group via its own correlation matrix. Of course, whenever any extra parameters are added to a model, the likelihood of the model will almost always increase and can never decrease. To assess whether this increase in model fit is meaningful, AIC and BIC can be used to examine it in light of the added complexity of the model (Raftery 1995). The single-matrix model has a log-likelihood of $-45,836$ (1,127 df), and the three-matrix model has a log-likelihood of $-43,697$ (3,381 df). Using both AIC and BIC as measures of model fit, the single-matrix model is overwhelmingly preferred ($\Delta \text{AIC} = 235$; $\Delta \text{BIC} = 10,578$). Consistent with our reasoning, the population homogeneity model thus offers a better description of the data than the model of heterogeneity detected by RCA. Thus even when we apply RCA directly to examine heterogeneity, we find no evidence for it in our data.

REFERENCES

- Achterberg, Peter, and Dick Houtman. 2009. "Ideologically Illogical? Why Do the Lower-Educated Dutch Display So Little Value Coherence?" *Social Forces* 87 (3): 1649–70.
- Alwin, Duane. 2007. *Margins of Error: A Study of Reliability in Survey Measurement*. 1st ed. Hoboken, N.J.: Wiley-Interscience.

⁴² It is worth noting here that this confirmatory SEM approach does provide evidence for different logics in other data sets (see Boutyline, forthcoming).

- Ansolabehere, Stephen, Jonathan Rodden, and James M. Snyder. 2008. "The Strength of Issues: Using Multiple Measures to Gauge Preference Stability, Ideological Constraint, and Issue Voting." *American Political Science Review* 102 (2): 215–32.
- Bai, Matt. 2005. "The Framing Wars." *New York Times*, July 17.
- Baldassarri, Delia, and Amir Goldberg. 2014. "Neither Ideologues nor Agnostics: Alternative Voters' Belief System in an Age of Partisan Politics." *American Journal of Sociology* 120 (1): 45–95.
- Barker, David C., and James D. Tinnick. 2006. "Competing Visions of Parental Roles and Ideological Constraint." *American Political Science Review* 100 (2): 249–63.
- Bartels, Larry M. 2002. "Beyond the Running Tally: Partisan Bias in Political Perceptions." *Political Behavior* 24 (2): 117–50.
- Beck, Glenn. 2008. "Commentary: Obama No, McCain Maybe." *CNN.com*, June 25.
- Bickel, Peter, Friedrich Götze, and Willem van Zwet. 2012. "Resampling Fewer than N Observations: Gains, Losses, and Remedies for Losses." Pp. 267–97 in *Selected Works of Willem van Zwet*. Selected Works in Probability and Statistics, edited by Sara van de Geer and Marten Wegkamp. New York: Springer.
- Bollen, Kenneth. 1989. *Structural Equations with Latent Variables*. New York: John Wiley & Sons.
- Boltanski, Luc, and Laurent Thévenot. 1999. "The Sociology of Critical Capacity." *European Journal of Social Theory* 2 (3): 359–77.
- Borhek, James, and Richard Curtis. 1975. *A Sociology of Belief*. New York: Wiley.
- Bourdieu, Pierre. 1984. *Distinction: A Social Critique of the Judgement of Taste*. Cambridge, Mass.: Harvard University Press.
- Boutyline, Andrei. Forthcoming. "Improving the Measurement of Shared Cultural Schemas with Correlational Class Analysis." *Sociological Science*.
- Breiger, Ronald L. 1974. "The Duality of Persons and Groups." *Social Forces* 53 (2): 181–90.
- Burt, Ronald S. 2004. "Structural Holes and Good Ideas." *American Journal of Sociology* 110 (2): 349–99.
- Campbell, Angus, Philip Converse, Warren Miller, and Donald Stokes. 1960. *The American Voter*. Oxford: John Wiley.
- Carmines, Edward, and Geoffrey Layman. 1997. "Issue Evolution in Postwar American Politics." Pp. 89–134 in *Present Discontents: American Politics in the Very Late Twentieth Century*, edited by Byron E. Shafer. Chatham, N.J.: Chatham House.
- Carmines, Edward, and Michael Wagner. 2006. "Political Issues and Party Alignments: Assessing the Issue Evolution Perspective." *Annual Review of Political Science* 9:67–81.
- Cohen, Geoffrey. 2003. "Party over Policy: The Dominating Impact of Group Influence on Political Beliefs." *Journal of Personality and Social Psychology* 85 (5): 808–22.
- Converse, Philip. 1964. "The Nature of Belief Systems in Mass Publics." In *Ideology and Discontent, International Yearbook of Political Behavior Research*, vol. 5. Edited by David Apter. Free Press.
- . 2000. "Assessing the Capacity of Mass Electorates." *Annual Review of Political Science* 3 (1): 331–53.
- DellaPosta, Daniel, Yongren Shi, and Michael Macy. 2015. "Why Do Liberals Drink Lattes?" *American Journal of Sociology* 120 (5): 1473–1511.
- Delli Carpini, Michael X., and Scott Keeter. 1996. *What Americans Know about Politics and Why It Matters*. New Haven, Conn.: Yale University Press.
- DeVega, Chauncey. 2016. "The GOP's Gross Adam Sandler Primary." *Salon*, March 9.
- Edgell, Penny. 2012. "A Cultural Sociology of Religion: New Directions." *Annual Review of Sociology* 38 (1): 247–65.
- Feinberg, Matthew, and Robb Willer. 2013. "The Moral Roots of Environmental Attitudes." *Psychological Science* 24 (1): 56–62.
- Freeman, Linton C. 1978. "Centrality in Social Networks Conceptual Clarification." *Social Networks* 1 (3): 215–39.

- . 1980. "The Gatekeeper, Pair-Dependency and Structural Centrality." *Quality and Quantity* 14 (4): 585–92.
- . 2004. *The Development of Social Network Analysis: A Study in the Sociology of Science*. Vancouver: Empirical Press.
- Fortunato, Santo, and Marc Barthélemy. 2007. "Resolution Limit in Community Detection." *Proceedings of the National Academy of Sciences* 104 (1): 36–41.
- Goldberg, Amir. 2011. "Mapping Shared Understandings Using Relational Class Analysis: The Case of the Cultural Omnivore Reexamined." *American Journal of Sociology* 116 (5): 1397–1436.
- Goldberg, Amir, and Sarah K. Stein. 2016. *RCA: Relational Class Analysis*. Retrieved on June 15, 2016. (<https://cran.r-project.org/web/packages/RCA/index.html>).
- Good, Benjamin, Yves-Alexandre de Montjoye, and Aaron Clauset. 2010. "Performance of Modularity Maximization in Practical Contexts." *Physical Review E* 81 (4): 046106.
- Gordon, Stacy, and Gary Segura. 1997. "Cross-National Variation in the Political Sophistication of Individuals: Capability or Choice?" *Journal of Politics* 59 (1): 126–47.
- Goren, Paul. 2005. "Party Identification and Core Political Values." *American Journal of Political Science* 49 (4): 881–96.
- Goren, Paul, Christopher Federico, and Miki Kittilson. 2009. "Source Cues, Partisan Identities, and Political Value Expression." *American Journal of Political Science* 53 (4): 805–20.
- Green, Donald, Bradley Palmquist, and Eric Schickler. 2002. *Partisan Hearts and Minds: Political Parties and the Social Identity of Voters*. 1st ed. New Haven, Conn.: Yale University Press.
- Gross, Neil, Thomas Medvetz, and Rupert Russell. 2011. "The Contemporary American Conservative Movement." *Annual Review of Sociology* 37 (1): 325–54.
- Hausser, Jean, and Korbinian Strimmer. 2013. "Entropy: Estimation of Entropy, Mutual Information and Related Quantities." <http://CRAN.R-project.org/package=entropy>. Retrieved on April 25, 2015.
- Hitlin, Steven, and Stephen Vaisey. 2013. "The New Sociology of Morality." *Annual Review of Sociology* 39 (1): 51–68.
- Hurwitz, Jon, and Mark Peffley. 1997. "Public Perceptions of Race and Crime: The Role of Racial Stereotypes." *American Journal of Political Science* 41 (2): 375–401.
- Jacobs, David, and Jason T. Carmichael. 2002. "The Political Sociology of the Death Penalty: A Pooled Time-Series Analysis." *American Sociological Review* 67 (1): 109–31.
- Jennings, M. Kent. 1992. "Ideological Thinking among Mass Publics and Political Elites." *Public Opinion Quarterly* 56 (4): 419–41.
- Johnston, Richard. 2006. "Party Identification: Unmoved Mover or Sum of Preferences?" *Annual Review of Political Science* 9 (1): 329–51.
- Jost, John, Christopher Federico, and Jaime Napier. 2009. "Political Ideology: Its Structure, Functions, and Elective Affinities." *Annual Review of Psychology* 60 (1): 307–37.
- Jost, John, Jack Glaser, Arie Kruglanski, and Frank Sulloway. 2003. "Political Conservatism as Motivated Social Cognition." *Psychological Bulletin* 129 (3): 339–75.
- Kinder, Donald R. 1998. "Communication and Opinion." *Annual Review of Political Science* 1 (1): 167–97.
- Kvalseth, Tarald O. 1987. "Entropy and Correlation: Some Comments." *IEEE Transactions on Systems, Man and Cybernetics* 17 (3): 517–19.
- Lakoff, George. 1990. *Women, Fire, and Dangerous Things*. Chicago: University of Chicago Press.
- . 2002. *Moral Politics: How Liberals and Conservatives Think*. Chicago: University of Chicago Press.
- . 2014. *The ALL NEW Don't Think of an Elephant!: Know Your Values and Frame the Debate*. 10th anniversary ed. White River Junction, Vt.: Chelsea Green.
- Lakoff, George, and Mark Johnson. 1980. *Metaphors We Live By*. Chicago: University of Chicago Press.

- Lakoff, George, and Rockridge Institute. 2006. *Thinking Points: Communicating Our American Values and Vision: A Progressive's Handbook*. New York: Farrar, Straus & Giroux.
- Lancichinetti, Andrea, and Santo Fortunato. 2011. "Limits of Modularity Maximization in Community Detection." *Physical Review E* 84 (6): 066122.
- Lau, Richard, and David Redlawsk. 2001. "Advantages and Disadvantages of Cognitive Heuristics in Political Decision Making." *American Journal of Political Science* 45 (4): 951–71.
- Layman, Geoffrey, John McTague, Shanna Pearson-Merkowitz, and Michael Spivey. 2007. "Which Values Divide?" Paper presented at the annual meeting of the Southern Political Science Association, New Orleans, January 3.
- Lee, Taeku. 2002. "The Sovereign Status of Survey Data." In *Navigating Public Opinion: Polls, Policy, and the Future of American Democracy*. Oxford: Oxford University Press.
- Leege, David C., Kenneth D. Wald, Brian S. Krueger, and Paul D. Mueller. 2009. *The Politics of Cultural Differences: Social Change and Voter Mobilization Strategies in the Post–New Deal Period*. Princeton, N.J.: Princeton University Press.
- Martin, John Levi. 1999. "Entropic Measures of Belief System Constraint." *Social Science Research* 28 (1): 111–34.
- . 2000. "The Relation of Aggregate Statistics on Beliefs to Culture and Cognition." *Poetics* 28 (1): 5–20.
- . 2002. "Power, Authority, and the Constraint of Belief Systems." *American Journal of Sociology* 107 (4): 861–904.
- Martin, John Levi, and Matthew Desmond. 2010. "Political Position and Social Knowledge." *Sociological Forum* 25 (1): 1–26.
- MacCallum, Robert, and James Austin. 2000. "Applications of Structural Equation Modeling in Psychological Research." *Annual Review of Psychology* 51 (1): 201–26.
- McAdams, Dan, et al. 2008. "Family Metaphors and Moral Intuitions: How Conservatives and Liberals Narrate Their Lives." *Journal of Personality and Social Psychology* 95 (4): 978–90.
- Mondak, Jeffery. 1993. "Public Opinion and Heuristic Processing of Source Cues." *Political Behavior* 15 (2): 167–92.
- . 2000. "Reconsidering the Measurement of Political Knowledge." *Political Analysis* 8 (1): 57–82.
- Mondak, Jeffrey J. 2001. "Developing Valid Knowledge Scales." *American Journal of Political Science* 45 (1): 224–38.
- Mondak, Jeffery, Christopher Lewis, Jason Sides, Joohyun Kang, and J. Olyn Long. 2004. "Presidential Source Cues and Policy Appraisals, 1981–2000." *American Politics Research* 32 (2): 219–35.
- Newman, Mark E. J. 2006. "Modularity and Community Structure in Networks." *Proceedings of the National Academy of Sciences* 103 (23): 8577–82.
- Pachucki, Mark, and Ronald Breiger. 2010. "Cultural Holes: Beyond Relationality in Social Networks and Culture." *Annual Review of Sociology* 36 (1): 205–24.
- Petersen, Michael Bang. 2009. "Public Opinion and Evolved Heuristics: The Role of Category-Based Inference." *Journal of Cognition and Culture* 9 (3): 367–89.
- Raftery, Adrian E. 1995. "Bayesian Model Selection in Social Research." *Sociological Methodology* 25:111–63.
- Rosenberg, Paul. 2014. "This Is Why Conservatives Win: George Lakoff Explains the Importance of Framing—and What Democrats Need to Learn." *Salon*. http://www.salon.com/2014/11/22/this_is_why_conservatives_win_george_lakoff_explains_the_importance_of_framing_and_what_democrats_need_to_learn/, accessed April 10, 2015).
- Rubinov, Mikail, and Olaf Sporns. 2011. "Weight-Conserving Characterization of Complex Functional Brain Networks." *NeuroImage* 56 (4): 2068–79.

- Sniderman, Paul M., and Edward H. Stiglitz. 2012. *The Reputational Premium: A Theory of Party Identification and Policy Reasoning*. Princeton, N.J.: Princeton University Press.
- Somers, Margaret R., and Fred Block. 2005. "From Poverty to Perversity: Ideas, Markets, and Institutions over 200 Years of Welfare Debate." *American Sociological Review* 70 (2): 260–87.
- Steensland, Brian et al. 2000. "The Measure of American Religion: Toward Improving the State of the Art." *Social Forces* 79 (1): 291–318.
- Swidler, Ann. 2001. *Talk of Love: How Culture Matters*. Chicago: University of Chicago Press.
- Taber, Charles S., and Milton Lodge. 2006. "Motivated Skepticism in the Evaluation of Political Beliefs." *American Journal of Political Science* 50 (3): 755–69.
- Thornton, Patricia H., William Ocasio, and Michael Lounsbury. 2012. *The Institutional Logics Perspective: A New Approach to Culture, Structure and Process*. 1st ed. Oxford: Oxford University Press.
- Vaisey, Stephen. 2009. "Motivation and Justification: Toward a Dual-Process Theory of Culture in Action." *American Journal of Sociology* 114 (6): 1675–1715.
- Van Eijck, Koen. 1999. "Socialization, Education, and Lifestyle: How Social Mobility Increases the Cultural Heterogeneity of Status Groups." *Poetics* 26 (5–6): 309–28.
- Wasserman, Stanley, and Katherine Faust. 1994. *Social Network Analysis: Methods and Applications*. 1st ed. Cambridge: Cambridge University Press.
- Wellman, Barry. 1988. "Structural Analysis: From Method and Metaphor to Theory and Substance." Pp. 19–61 in *Social Structures: A Network Approach*. Structural Analysis in the Social Sciences, vol. 2. Edited by B. Wellman and S. D. Berkowitz. New York: Cambridge University Press.
- Williamson, Kevin D. 2016. "Chaos in the Family, Chaos in the State: The White Working Class's Dysfunction." *National Review*, March 17.
- Wuthnow, Robert. 2007. "Cognition and Religion." *Sociology of Religion* 68 (4): 341–60.
- Yao, Y. Y. 2003. "Information-Theoretic Measures for Knowledge Discovery and Data Mining." Pp. 115–36 In *Entropy Measures, Maximum Entropy Principle and Emerging Applications, Studies in Fuzziness and Soft Computing*, edited by Professor Karmeshu. Berlin, Heidelberg: Springer.
- Zaller, John R. 1992. *The Nature and Origins of Mass Opinion*. Cambridge: Cambridge University Press.
- Zemljic, Barbara, and Valentina Hlebec. 2005. "Reliability of Measures of Centrality and Prominence." *Social Networks* 27 (1): 73–88.
- Zhang, Bin, and Steve Horvath. 2005. "A General Framework for Weighted Gene Co-Expression Network Analysis." *Statistical Applications in Genetics and Molecular Biology* 4: article17.