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Decomposing Attitude Distributions to Quantify Mass Polarization in Europe

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Abstract

In recent years, political discourse and election results appear to be more polarized than in the years before. Empirical evidence for opinion polarization has been found regarding specific topics but is there a general trend in society? We compare various polarization measures and find that in empirical data most of them correlate strongly with the average attitude discrepancy between randomly selected pairs which we propose as a catch-all measure for polarization. In an exploratory data analysis of the European Social Survey, we analyze distributions of individual responses on the left-right political self-placements and several other attitudes. We find that distributions are typically not unimodal or bimodal, but show more structure with up to five modes.

We exploit this structure by fitting a new model to distributions of answers on an eleven-point attitude scale, and demonstrate that distributions can be apportioned into moderates, extremists, and centrists. We use the model to decompose general polarization into empirically meaningful components which we use to analyze the complete data set across topics, across countries, and with respect to time-trends establishing an overview and new perspectives on polarization in Europe.

Keywords: attitude polarization; Europe; ideological polarization; mass polarization; attitude landscape; moderates' polarization; extremists' polarization

Introduction

Pundits and public opinion see political polarization on the rise in Western democracies. Empirical studies of various kinds claim that polarization is increasing (Dettrey and Campbell, 2013), decreasing (Bauer and Munzert, 2013), or not really changing (DiMaggio et al, 1996; Fiorina and Abrams, 2008). These conflicting messages stem partly from the fact that different issues or different countries are treated, but an underlying problem is also that polarization is conceptualized based on many aspects with various measurement concepts (see Lelkes, 2016). A very general definition of polarization is the *accentuation of differences*. However, there are many ways how difference can become accentuated.

Psychology focuses on the individual or small groups and thus uses an individual-based definition of polarization, e.g., the increase of extremity through a move away from neutrality. Already Myers and Lamm (1976) point out in their review on the *group polarization* phenomenon that there are other more complex concepts: "This use of polarization in the relevant literature is somewhat specialized, as in general usage the term may also refer to a split within a group of people."[1] The latter is what current debates are about focusing on societies instead of groups. Therefore we call it *societal polarization*. Group polarization is the phenomenon that the average attitude of a group in, e.g., jury decisions, ethical decisions, judgments, person perceptions, negotiations, or risk taking shifts through discussion to become more extreme after group discussion. The shift happens in the direction the group's average already had before the discussion, in particular, if group members' initial positions are on this one side, only. Group polarization is assumed to be one main mechanism driving societal polarization (Sunstein, 2002) and has reached popular science in the more recent debate on polarization (Sunstein, 2009).

Political science distinguishes *elite polarization*, e.g., in parliaments and political media, from *mass polarization* in the general public (Fiorina and Abrams, 2008). Mass polarization is traditionally measured via responses in representative surveys. However, the particular focus on attitude distributions, polarization is considered a multidimensional phenomenon. DiMaggio et al (1996) identified four principles of societal polarization measured in distributions of social attitudes: The *dispersion* and the *bimodality* principle are based on the distribution of a single attitudinal variable. The *constraint* principle[2] is based on distributions of several attitudes and how much they are aligned. Finally, the *consolidation* principle relates attitude distributions of different groups, e.g., supporters of the two main political parties in the United States of America, for which between group potential for conflict should be measured to quantify polarization.

Since the study of DiMaggio et al (1996), the focus has been on the latter two principles. These two dimensions of polarization and their corresponding measures seem to increase in the United States as Abramowitz and Saunders (2008) emphasize for ideological alignment (constraint principle), or for the consolidation principle with survey data of trait evaluations of presidential candidates (Hetherington et al, 2016). Also a recent study by the Pew Research Center (2014) focuses on the constraint and consolidation principle by measuring partisanship and using scales constructed as the sum of several issues typically associated with a liberal and conservative ideological worldview (based on the concepts of Abramowitz and Saunders (1998)).

Baldassarri and Gelman (2008) call the constraint and the consolidation aspect of polarization *issue alignment* and *issue partisanship*. DellaPosta (2020) relates increasing issue alignment to a "pluralistic collapse" in the US. Bauer and Munzert (2013) applied similar methods for survey data from Germany, but found decreasing polarization.

Furthermore, Iyengar et al (2019) introduced *affective polarization* as another aspect of polarization. While all other concepts are in one way or another about differences in *ideology*, affective polarization is the degree of in-party love and out-party hate. It can be measured with "feeling thermometer" surveys on the individual level. Finkel et al (2020) propose a related construct on the individual level:

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[1] In line with the latter definition, Macrae and Kilpatrick (1959) defined polarization as the "separation of groups from one another in the relevant attitudes" in an early study on public opinion.

[2] With the name going back to Converse (2006) characterization ideologies as belief systems where views on different attitude interdependently constrain each other.

political sectarianism, the tendency to adopt a moralized identification with one political group and against another with the core ingredients of othering, aversion, and moralization.

Many of the mentioned studies focus on the United States. With their almost unique two party system this is a particularly easy case for the operationalization of polarization measurements based on exogenous groups. It is more difficult for Europe where a clear cut division of people into two camps does not exist. On the other hand, analyzing Europe provides other opportunities. The European Social Survey provides representative attitude data for many European countries with consistent questionnaires in biennial waves from 2002 to 2018 with many questions appearing repeatedly. This survey thus provides the opportunity to explore time trends, as well as cross-country and cross-topic comparison of single attitude polarization measures and test their empirical feasibility and practical relevance.

In the following we focus on societal polarization on a single attitude dimension. So, our focus is polarization concerning the dispersion principle, the bimodality principle and other aspect but not on ideological alignment and polarization based on attitudes in exogenously given groups. It is about ideological differences between individuals and not on individualized constructs like political sectarianism. We use data exploration of a large data set of eleven-point attitude distributions from the ESS. We pursue three research questions:

1. Which measurement concepts of societal polarization on a single attitude dimension are empirically relevant?
2. Can we model stylized characteristics of attitude distributions to improve the measurement?
3. Based on the answers to the former questions, what can we say about the questions:
   (a) Which topics are most polarized?
   (b) Which countries are most polarized?
   (c) Is polarization increasing?

Related Studies

Also within in the narrow focus of societal polarization on a single attitude dimension many concepts exist beyond the dispersion and the bimodality principle of DiMaggio et al (1996). Recently, Bramson et al (2016) distinguished and labeled nine different aspects of polarization. Two are based on the distribution of one attitude: *spread* (range of a sample, maximum minus minimum) and *dispersion* (measured by mean absolute deviation instead of variance). Two more are based on counting attitudes which haven't been taken by any individual (empty bins in a histogram): *coverage* (number of nonempty bins) and *regionalization* (number of gaps of empty bins). *Community fragmentation* is the number of different groups. This measure is of interest if groups are not given exogenously but have to be assessed endogenously from the shape of the distribution. Bramson et al (2016) proposed to count modes of the distribution, e.g., by the number of local minima plus one. The further four measures all rely on exogenously given, or already endogenously identified, groups. While all measures can deal with several groups, it suffices here to focus on two groups. This coincides with a common view of polarization as society splitting into two opposing camps. For groups Bramson et al (2016) propose to distinguish *distinctness*, the overlap of attitude distributions of two groups (Schmid and Schmidt, 2006), *group divergence* (the difference of the groups' averages), *group consensus* (increasing with low dispersion within groups), and *size parity* (being maximal when both groups are of equal size). The nine aspects provide useful terminology, but many measures cannot be used in a robust way to compare different distributions. For example, larger group divergence may become pointless when at the same time size parity approaches zero.

Another description of polarization is that the center looses people to the extremes. A simple measure of this for single attitude distributions and rating scales with a midpoint is the fraction of those who are not reporting that midpoint. This has been used, e.g., by Fiorina and Abrams (2008) and Dettrey and Campbell (2013). Most likely this measure has also been inspired by eyeballing the empirical attitude distribution in US surveys on the typical seven-point liberal-conservative scale. These always show a strong peak for moderates. Downey and Huffman (2001) further observed that attitude distributions often tend to become trimodal with one large central and two off-central peaks. This emphasizes that measurement concepts based on bimodality have to be used with care taking the patterns of attitude distributions into account.

Van der Eijk (2001) provides another route to measure polarization as the opposite of the measurement of agreement in ordered rating scales. The *coefficient of agreement* was developed to measure, how much respondents in a survey agree on the position of, e.g. a political party. The standard deviation is considered inappropriate here because it reflects not only dispersion but also skewness when used on a bounded rating scale. Consider the case where half of the respondents give random answers from a uniform distribution, while the other half perfectly agrees on the "correct" position. The standard deviation would be lower when the correct position were central and more if it were extreme. The agreement index is designed to deliver the same agreement in this class of examples.

Surprisingly, the econometric measurement concept of Esteban and Ray (1994) is largely ignored in political and sociological polarization studies although it is based on a solid axiomatization which distinguishes polarization from dispersion and from inequality in a common framework. Further on, it provides probabilistic interpretations based on the idea to assess the average attitudinal discrepancy in random pairwise encounters, which we find useful to exploit and adapt to polarization in real-world attitude landscapes.[3]

Bauer (2019) provides a review for the various attempts to quantify polarization. We conclude for this study that there is a demand for a practical measure of polarization in single attitude distributions which is adapted towards the empirical reality of attitude distributions. The four principles of DiMaggio et al (1996) are still worth to distinguish and the nine aspects of Bramson et al (2016) provide useful terminology. Nevertheless, we claim some integration is desirable. In the following, we approach the goal of integration by exploration of empirical data based on the theoretical measurement concepts provided in the literature.

[3] We neglect the related measures of Foster and Wolfson (2009) and Wang and Tsui (2000) which axioms are focused to measure polarization as a genuine bimodal phenomenon.

Data

We use data from the European Social Survey (ESS) (European Social Survey ERIC (ESS ERIC), 2016). Nine biennial ESS waves cover the years 2002 to 2018. Our data set includes data from 33 European countries (including Israel and Russia), but coverage of countries, topics, and years are not complete. Twelve countries have data in all waves, and 19 questions have been asked in all of these (with one question not included in the main ESS data files for Ireland 2002). In Appendix A Figure 12 we show the number of rounds a topic is covered in a country.

We focus on 33 variables measured on eleven point rating scales. The description of these variables are listed in Appendix A Table 4 including the verbal labels given to the extreme points zero and ten. The selected variables (ESS short-label indication in parentheses) from the core module include the catch-all political position on the left-right scale (LRSCALE, appearing in all waves), the position if European unification should go further or has gone too far (EUFTF, appearing in seven waves), attitudes about immigration (IM..., three questions, appearing in all waves), various satisfaction topics (STF..., HAPPY, seven questions, appearing in all waves), generalized trust (PPL..., three questions, appearing in all waves), trust in several institutions (TRST..., seven questions, appearing in all waves except for TRSTPRT appearing in eight waves) and the two questions about emotional attachment to the country and Europe (ATCH...) which were added to the core module in the last two waves 2016 and 2018 (Boehnke et al, 2016). Further on, we use questions from the rotating modules about fairness (..FR..., four questions, 2018) and climate change (CC..., or ...CC, four questions, 2016). All these topics have certain aspects of individual attitudes but some are not at the core of political discourse which is thought to be especially prone to polarization. We consider the three topics LRSCALE, EUFTF, and IMUECLT ("Does immigration undermine or enrich the culture of the country?") as the core political topics in our data set, and give special attention to them in the following.

The ESS is conducted to allow inferences about the general population of each country in each wave, which allows us to infer the degree of mass polarization. Its high standard of multilingualism allows the comparison between countries. Representativity for the general population is reached by probability sampling with an effective sample size of 1,500 (800 for small countries), that means, after discounting for design effects via design weights. We use the design weights of respondents to compute the distribution of responses on the 0-10 scale for every variable-country-year combination. We record the fractions of responses for each answering option and ignore all non-valid answers. Appendix A also provides information about response rates (fractions of individuals in the random sample who responded) for all country surveys. We call such a distribution an *attitude landscape*. In total, we have 4,155 attitude landscapes in our data set.

Our aim is to quantify the degree of polarization for these attitude landscapes. It will turn out, that this can be done in an empirically more meaningful way when we exploit characteristic properties of attitude landscapes using a model which assumes five endogenous groups in the population.

Measurement Methods

In the following, we discuss measurement concepts for polarization in attitude landscapes for the eleven point scale from zero to ten. The methods can be generalized to other discrete ordered rating scales which are bounded from both sides, but for our purpose it suffices to focus on this scale. We focus first on measurement concepts which do not rely on partial distributions of different groups. The common ground for measuring societal polarization is that almost all measurement concepts agree on which attitude landscapes are maximally and minimally polarized. Minimal polarization is reached when all respondents agree on one attitude value (similarly already discussed for measuring consensus by Leik (1966)). It does not matter where the consensual value lies on the attitude scale. Maximal polarization is achieved when the population is equally divided on both extremes of the scale. In our data, this would be the attitude landscapes, where 50% have attitude zero and the other 50% attitude ten. That way all polarization measures can be normalized to range from zero to one. This view on the extremes captures the dispersion as well as the bimodality principle of DiMaggio et al (1996). The conceptual problems in measuring polarization come when intermediate polarization is to be assessed.

Figure 1 demonstrates the conceptual measurement problem for polarization in attitude landscapes. It shows three stylized example landscapes of intermediate polarization which are not trivial to rank. The example "equal powers" shows two bins of equal size. Thus, there are two opposing groups with high internal consensus, but the discrepancy in attitude between these groups is not maximal as for the two extreme bins in the other two examples. So, these two could be seen as more polarized. The "maximal diversity" example shows no structure of opposing camps but a uniform distribution. In some sense, there is no accentuation of the differences as for the other two examples. The distribution "unequal extremes" shows the accentuation of two groups and maximal difference. However, 90% have a consensus and thus, in another sense it is much less polarized than the other two examples. So, for each of the three distributions, there are arguments that this is either the most or the least polarized one.

However, all three examples are stylized and in our sample of real-world attitude landscapes, we find none which comes close to these. Therefore, we explore how the most common measures of polarization rank these examples and how different they measure those attitude landscapes in our data.

For the following formal definitions of polarization measures, we call the attitude values of the rating scale to be $0, 1, \dots, n$ and the fraction of the population holding them $p_0, p_1, \dots, p_n$. We call the vector p, the *attitude landscape*. By definition it holds, $p_i \geq 0$ for all i and $\sum_i p_i = 1$. For the rating scale of the ESS it holds $n = 10$.

Esteban and Ray (1994) measure polarization as

$$\mathrm{Pol}_\alpha(p) = \frac{2^{1+\alpha}}{n} \sum_{i,j=0}^{n} p_i^{1+\alpha} p_j |i - j| \tag{1}$$

where α is called the *polarization sensitivity*. Following the theory of Esteban and Ray (1994), α should be larger than zero because for $\alpha = 0$ the measure would coincide with the Gini coefficient for the measurement of inequality. A note of caution:

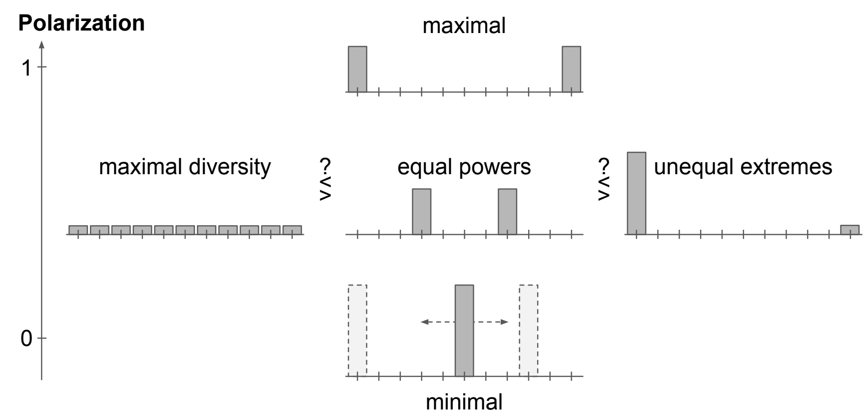

Figure 1 The conceptual problem of measuring polarization.

A common notation of the Gini coefficient is not equivalent to Pol_0. The common definition has a scaling factor which includes the average income $\mu = \sum_i p_i i$ and reads $\frac{1}{2\mu}\sum_{i,j=0}^{n} p_i p_j |i-j|$. In particular, the maximal polarization landscape in Figure 1 would have a lower Gini coefficent than the "unequal extremes" distribution. The "flipped" distribution where most people have attitude ten instead would have a much lower Gini coefficient.

Esteban and Ray (1994) distinguish polarization from inequality by taking into account that the contribution of *antagonism* between people with different attitudes (reflected in the term $|i-j|$) is increasing with the fraction of people holding that attitude reflected in the factor p_i in $p_i^{1+\alpha} p_j$, which is called the *identification* of people with attitude i.[4]

In contrast to Esteban and Ray (1994), we will also call Pol_0 a polarization measure. Further on, α should be less than or approximate 1.6 because otherwise not all axioms of Esteban and Ray (1994) would be fulfilled. In the following we mostly use Pol_0 and Pol_1 for which the specified definitions are:

$$\mathrm{Pol}_0(p) = \frac{2}{n}\sum_{i,j=0}^{n} p_i p_j |i-j| \tag{2}$$

$$\mathrm{Pol}_1(p) = \frac{4}{n}\sum_{i,j=0}^{n} p_i^2 p_j |i-j| \tag{3}$$

Both have a probabilistic interpretation. Pol_0 is the expected attitude distance of a pair of individuals randomly sampled from the population. So, we could also call it the *average pair discrepancy*. For Pol_1 the probabilistic reasoning is about sampling triplets from the population and only regard those as contributing to polarization when two have the same attitude. Ignoring triplets with three different attitudes

[4] Esteban and Ray (1994) define the measure for a general discrete set of possible real-valued attributes, as attitudes of incomes, x_i. In our case it suffices to define $x_i = i$.

reflects that a pair's discrepancy is considered only important when one of them is supported by a third person. When all three sampled individuals have the same attitude there would be no discrepancy anyway.

Other common measures are the normalized *mean absolute deviation* from the mean (MAD)[5] and the normalized *standard deviation* (SD)

$$\mathrm{MAD}(p) = \frac{2}{n}\sum_{i=0}^{n} p_i |i - \bar{x}|, \tag{4}$$

$$\mathrm{SD}(p) = \frac{2}{n}\sqrt{\sum_{i=0}^{n} p_i (i - \bar{x})^2} \tag{5}$$

where $\bar{x} = \sum_i p_i i$ is the average attitude.

Furthermore, the agreement index (Van der Eijk, 2001) can be reversed and normalized to measure polarization (Ruedin, 2016) which we call *disagreement* ($\mathrm{Dis}(p)$) in the following.

MAD, Dis, and Pol_0 all measure the uniform distribution "maximal diversity" in Figure 1 as most polarized among the three intermediate landscapes, followed by "equal powers", and "unequal extremes" as least polarized. SD is similar but measures "unequal extremes" second and "equal powers" last because it weighs the large distance higher than the imbalance. The picture changes for Pol_α with increasing α. For $\alpha = 0.4$, "equal powers" is most polarized and "maximal diversity" only second. The effect that discrepancy of larger bins is weighted higher with increasing α kicks in. For $\alpha = 1$, "maximal diversity" even drops to be least polarized and "unbalanced extremes" becomes second. Finally, for $\alpha = 1.6$, the order of Pol_0 is completely reversed: "Unequal extremes" is most and "maximal diversity" least polarized. This demonstrates that very large α tends to measure high polarization in the presence of one large bin as long as there are some other bins. [6]

In the following, we analyze how these measures differ empirically. Table 1 shows that Pol_0, MAD, SD, and Dis are highly correlated in the empirically observed attitude landscapes. In particular, Pol_0 shows the strongest correlation to all the three other measures compared to their pairwise correlation. So, empirically all these measures measure essentially the same thing.[7] Further on, Table 1 shows

[5] A variant of this type of measure is the mean absolute deviation from the median (instead of the mean) which has been proposed by Leik (1966) as an appropriate measure of dispersion for ordered (and not interval) scales. Although theoretically not the same, we found that it is empirically in almost perfect correlation with MAD.

[6] A theoretical side note: For example, with $\alpha = 3$ the "unequal extremes" example would even have a polarization measure $\mathrm{Pol}_3 > 1$ while the "maximal" landscape with two equal-sized extreme bins would still have polarization one. That is one example showing that the measurement concept needs an upper bound on α as shown by Esteban and Ray (1994). Later, Esteban and Ray (2012) refined the axiomatization restricting $0.25 \leq \alpha \leq 1$.

[7] Of course, there are theoretical examples with meaningful differences, in particular with respect to the disagreement index, because the general concept is quite

Table 1 Correlation of polarization measures in the data set of 4,155 attitude landscapes from the European Social Survey showing that mean absolute deviation (MAD), standard deviation (SD), and disagreement index (Dis) are all highly correlated with the polarization measure Pol_0 or $\mathrm{Pol}_{0.4}$, while Pol_1 and $\mathrm{Pol}_{1.6}$ measure a different aspect of polarization.

		PearsonCorrelation	LRSCALE, EUFTF, IMUECLT only
Pol_0	MAD	0.99	0.98
Pol_0	SD	0.99	1.00
Pol_0	Dis	0.96	0.98
MAD	SD	0.98	0.97
MAD	Dis	0.93	0.97
SD	Dis	0.93	0.97
Pol_0	$\mathrm{Pol}_{0.4}$	0.97	0.98
Pol_0	Pol_1	0.39	0.36
Pol_0	$\mathrm{Pol}_{1.6}$	-0.23	-0.32

that the measurement concept of Esteban and Ray (1994) with varying α is empirically relevant. There is only a moderate correlation between Pol_0 and Pol_1 and the correlation between Pol_0 and $\mathrm{Pol}_{1.6}$ is even slightly negative. All these results also hold when focused on the three core political topics.

From the short theoretical and empirical exploration of the most common measures for polarization we conclude that Pol_0 captures almost all information from MAD, SD and Dis, while the increasing α blends to another aspect which is empirically relevant. This other aspect kicks in only for α larger than 0.4 and turns to become anti-correlated with Pol_α reaching its theoretical upper bound close to $\alpha = 1.6$. Anti-correlated polarization aspects are not desirable for practical purposes, therefore, we focus on Pol_0 and Pol_1 in the following.

Figure 2 shows scatter plots of Pol_0 and Pol_1 against the average attitude for all attitude landscapes in our data set.

Pentamodal Model of Attitude Landscapes

Although the measures Pol_0 and Pol_1 measure different aspects of polarization, what they distinguish is not easily interpreted in an empirically relevant way. For example, Figure 2B shows that the left-right self-placement in Norway is low on Pol_1 and comparably high on Pol_0 although it has clear peaks with supposedly high identification more than left-right self-placement in Switzerland which scores minimally higher on Pol_1 but not because of multiple peaks but mostly because the landscape is dominated by a large amount of neutral attitudes.

It is easy to see mathematically, that $\mathrm{Pol}_1 \leq \mathrm{Pol}_0$ for any attitude landscape. So, Pol_1 may be conceptualized as a component of Pol_0. Further on, antagonism (defined as difference to attitudes of others by Esteban and Ray (1994)) caused by a central bin which is much larger than neighboring bins may be to a large extent caused by a lack of interest of many of the neutral individuals. Thus, it would be desirable to distinguish this antagonism. The same would be of interest for antagonism caused by extreme attitudes when the corresponding bins largely exceed neighboring bins. This would enable to distinguish between polarization caused by extremists and those caused among moderates. In the following, we develop a theoretical model of the composition of an attitude landscape which we then use for such refinements.

different. Nevertheless, the differences do not seem to be empirically relevant to measure aspects of polarization in our sample.

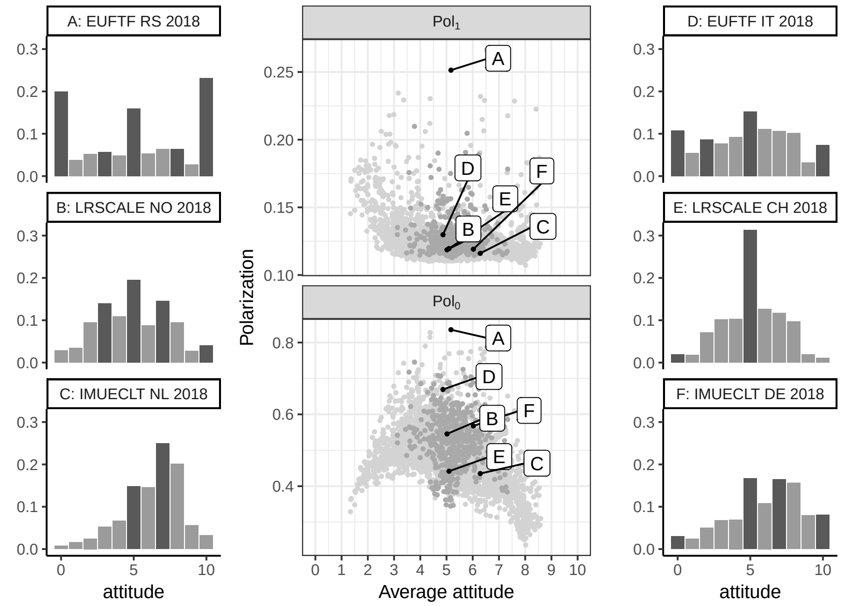

Figure 2 The polarization measures Pol_0 and Pol_1 of 4,155 attitude landscapes from the European Social Survey against their average attitude and six examples of attitude landscapes from core political topics in 2018. The dark bins in the attitude landscapes are peaks in the landscape (neighboring bins are smaller). Countries in the examples are Serbia, Norway, Netherlands, Italy, Switzerland and Germany.

The model is motivated based on an empirical exploration of typical characteristics of attitude landscapes, in particular of their peaking patterns.

The example attitude landscapes in Figure 2 show their peaking bins highlighted darker. Formally, a *peak* is a bin for which neighboring bins are smaller. It seems attitude landscapes are not simple distributions (e.g., uniform, bell-, or U-shaped), but have a multimodal structure. Besides these two observations, Lorenz (2017) quantified more stylized facts of attitude landscapes of left-right self-placements in the ESS: (i) The largest bin is almost always the central one usually exceeding the neighboring bins by far suggesting a discontinuous jump. (ii) Peaks appear often at the extremes (bins 0 and 10). Though often small in magnitude, still usually more people are extreme than close to extreme. (iii) The maximum amount of peaks is five while six are theoretically possible (iv) Moderate off-center peaks (at bins 2, 3, 7, and 8) are frequent while peaks directly next to the center or the extremes (at bins 1, 4, 6, and 9) are extremely rare. Moreover, the bins around these moderate off-center peaks usually give "smoother" impression suggesting an underlying bell shape. In our exploration of the ESS attitude landscapes we observed these stylized facts similarly for all topics with the only notable exception that the central bin is not always the largest in attitude landscapes when the mean attitude is far away from neutral, though it is still almost always a peak. Figure 13 in Appendix B shows more details of our exploration of peaks in ESS attitude landscapes.

We use these insights to construct a measurement model based on the idea that attitude landscapes are composed of five latent groups.

The Model

Attitude landscapes show properties of continuous distributions with smooth shapes, but these are contrasted by peaks at the extremes and in the center which often spike out. We translate this empirical duality into an assumption that there are two different types of individuals: Those who answer the question based on an underlying continuous valuation (being a real number) and those who answer the question based on an underlying discrete valuation like "yes", "undecided/neutral", or "no" (where "yes" and "no" are replaced by the extreme labels of the underlying question, e.g. "fully agree" and "fully disagree"). Central answer options in surveys often tend to peak, which Downey and Huffman (2001) explains as follows: "Respondents who place themselves at a semantic midpoint of a scale are usually assumed to indicate either true neutrality, or a sense of ambivalence regarding the choices, or even a lack of issue salience." Our model aims to distinguish the fraction of respondents who are truly neutral and those who are ambivalent or lack issue salience.

Often, we see two off-center peaks. Therefore, we assume that those who answer the question based on an underlying continuous valuation come from two different groups, and the attitudes of people in these groups are normally distributed.

Taken together, we postulate that an attitude landscape is composed of individuals from five endogenous groups: The Left Extremists (ExL), the Left Moderates (ModL), the Centrists (C), the Right Moderates (ModR) and the Right Extremists (ExR). The attitude distributions in these five groups are:

$$
\begin{aligned}
\pi^{\mathrm{ExL}} &= [1,0,0,0,0,0,0,0,0,0,0] \\
\pi^{\mathrm{ExR}} &= [0,0,0,0,0,0,0,0,0,0,1] \\
\pi^{\mathrm{C}} &= [0,0,0,0,0,1,0,0,0,0,0] \\
\pi^{\mathrm{ModL}} &= [\pi_0^{\mathrm{ModL}}, \pi_1^{\mathrm{ModL}}, \dots, \pi_9^{\mathrm{ModL}}, \pi_{10}^{\mathrm{ModL}}] \\
\pi^{\mathrm{ModR}} &= [\pi_0^{\mathrm{ModR}}, \pi_1^{\mathrm{ModR}}, \dots, \pi_9^{\mathrm{ModR}}, \pi_{10}^{\mathrm{ModR}}]
\end{aligned}
$$

We assume the attitude distributions of the moderate groups are discretized and confined normal distributions (see Lorenz (2009)). That means, we define

$$
\begin{aligned}
\pi_0^{\mathrm{ModL}} &= \int_{-\infty}^{0.5} \varphi(x;\mu_{\mathrm{L}},\sigma_{\mathrm{L}})dx = \Phi(0.5;\mu_{\mathrm{L}},\sigma_{\mathrm{L}}), \\
\pi_{10}^{\mathrm{ModL}} &= \int_{9.5}^{\infty} \varphi(x;\mu_{\mathrm{L}},\sigma_{\mathrm{L}})dx = 1-\Phi(9.5;\mu_{\mathrm{L}},\sigma_{\mathrm{L}}),
\end{aligned}
$$

and

$$
\pi_i^{\mathrm{ModL}} = \int_{i-0.5}^{i+0.5} \varphi(x;\mu_{\mathrm{L}},\sigma_{\mathrm{L}})dx = \Phi(i+0.5;\mu_{\mathrm{L}},\sigma_{\mathrm{L}}) - \Phi(i-0.5;\mu_{\mathrm{L}},\sigma_{\mathrm{L}})
$$

for $i = 1, \dots 9$, where $\varphi(\cdot;\mu,\sigma)$ is the probability density function of a normal distribution with mean μ and standard deviation σ, and Φ the corresponding cumulative distribution function. Analog, we define π_i^{ModR} with normal distribution parameters μ_{R} and σ_{R}.

The *Pentamodal Model* exposes that an attitude landscape p can be modeled as a *pentamodal distribution* π which is the weighted sum of the attitude distributions of the five groups

$$\pi = w_{\mathrm{ExL}}\pi^{\mathrm{ExL}} + w_{\mathrm{ExR}}\pi^{\mathrm{ExR}} + w_{\mathrm{C}}\pi^{\mathrm{C}} + w_{\mathrm{ModL}}\pi^{\mathrm{ModL}} + w_{\mathrm{ModR}}\pi^{\mathrm{ModR}} \quad (6)$$

where the weights sum up to one $w_{\mathrm{ExL}} + w_{\mathrm{ExR}} + w_{\mathrm{C}} + w_{\mathrm{ModL}} + w_{\mathrm{ModR}} = 1$.

A pentamodal distribution is completely defined by the nine parameters – the five population frequencies and the location and scale parameters of the two moderate groups. The effective number of parameters is eight because the five weight parameters must sum up to one.

Motivating the model by investigating distribution of voters of specific parties in combination with political interest is discussed in the Appendix C.

Parameter Estimation

Given a real-world attitude landscape p (e.g., from ESS data) we estimate the best-fitting pentamodal distribution $\pi = \pi(w_{\mathrm{ModL}}, \mu_{\mathrm{L}}, \sigma_{\mathrm{L}}, w_{\mathrm{ModR}}, \mu_{\mathrm{R}}, \sigma_{\mathrm{R}}, w_{\mathrm{ExL}}, w_{\mathrm{ExR}}, w_{\mathrm{C}})$ by fitting the nine parameters with a customized standard optimization algorithm. To that end, we solve the minimization problem

$$\min_{\theta} \left(\sum_{i=0}^{10} (p_i - \pi_i(\theta))^2 + \beta(\hat{w}_{\mathrm{ExL}}^2 + \hat{w}_{\mathrm{ExR}}^2 + \hat{w}_{\mathrm{C}}^2) \right) \quad (7)$$

where $\theta = [w_{\mathrm{ModL}}, \mu_{\mathrm{L}}, \sigma_{\mathrm{L}}, w_{\mathrm{ModR}}, \mu_{\mathrm{R}}, \sigma_{\mathrm{R}}, w_{\mathrm{ExL}}, w_{\mathrm{ExR}}, w_{\mathrm{C}}]$ is the vector of parameters, β a fitting parameter weighting the following three penalty terms which penalize negative weights $\hat{w}_{\mathrm{ExL}} = \min\{w_{\mathrm{ExL}}, 0\}$, analog for $\hat{w}_{\mathrm{ExR}}$ and $\hat{w}_{\mathrm{C}}$, subject to the constraints

$$0 < \mu_{\mathrm{L}}, \mu_{\mathrm{R}} < 10 \quad (8)$$

$$w_{\mathrm{ModL}}, w_{\mathrm{ModR}} \geq 0 \quad (9)$$

$$w_{\mathrm{ModL}} + w_{\mathrm{ModR}} + w_{\mathrm{ExL}} + w_{\mathrm{ExR}} + w_{\mathrm{C}} = 1 \quad (10)$$

using the 'minimize' function with the Sequential Least Squares Programming (SLSQP) algorithm in Python's scipy package.

The weights of extremists and centrists are allowed to be below zero. This is necessary, because some attitude landscapes have zero individuals answering zero or ten but any Pentamodal Model has a tiny positive fraction of extreme moderates due to the properties of the normal distribution. Small negative weights $w_{\mathrm{ExL}}, w_{\mathrm{ExR}}$ can compensate for this and forbidding negative weights in total would deliver very bad fits for some attitude landscapes. Nevertheless, negative weights are not desirable. Therefore, we introduced the penalty terms $\hat{w}_{\mathrm{ExL}}^2, \hat{w}_{\mathrm{ExR}}^2, \hat{w}_{\mathrm{C}}^2$.

After fitting the parameters θ we computed R^2 as a goodness-of-fit measure as

$$R^2 = 1 - \frac{\sum_{i=0}^{10} (p_i - \pi_i(\theta))^2}{\sum_{i=0}^{10} (p_i - \frac{1}{11})^2} \quad (11)$$

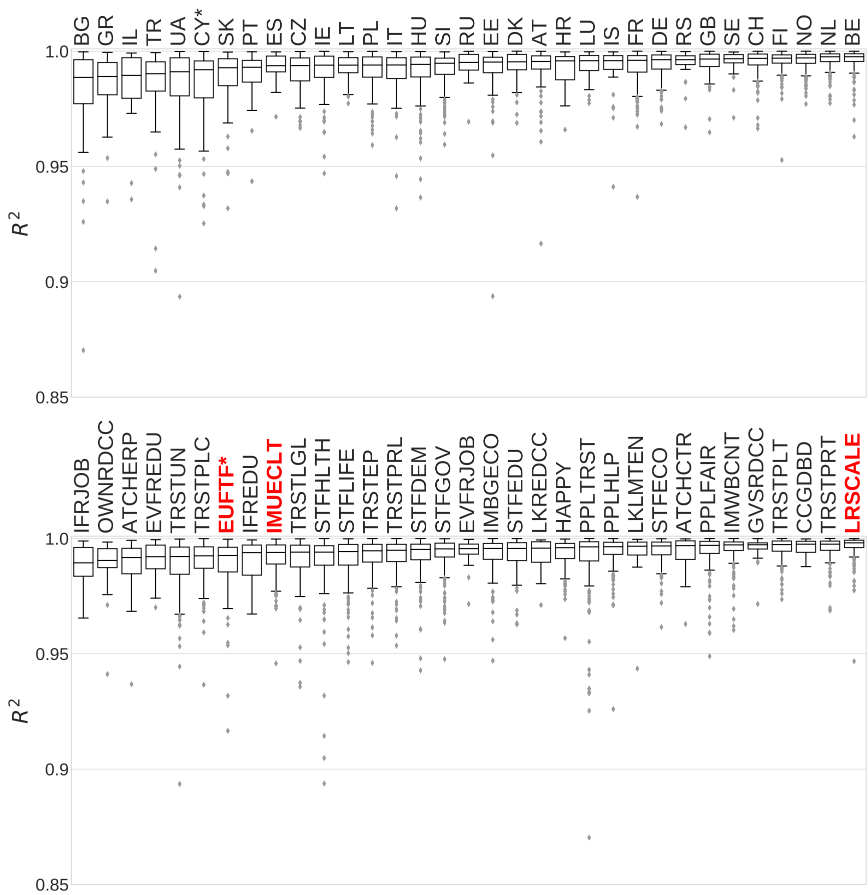

Figure 3 R^2 for each country and each topic. Ordered from lowest to highest median R^2 value. Core political topics marked as red (* Outlier CY EUFTF is not plotted $(R^2 = 0.76)$)

The term $\frac{1}{11}$ in the denominator results from the null hypothesis that the landscape has a uniform distribution and R^2 specifies what proportion of the deviation from the uniform distribution is explained by the Pentamodal Model. The R^2 values are summarized per topic and per country in Figure 3. Almost all are very close to perfect, which is not too surprising with an effective number of eight parameters to fit eleven values. In Appendix D, we show that median R^2 for different values of the weight β in the penalty term (Figure 18). These explorations lead us to use $\beta = 20$ for the results presented in the following. Furthermore, Appendix D shows an analysis of the residuum (Figure 17) between fitted and empirical distribution. These show only small variation around 0 with the worst fit being answer 8 over the ESS data set. Further on, Appendix D shows the distribution of estimated parameters for all attitude landscapes and the four attitude landscapes with the worst R^2 including a discussion why the model performs bad in these cases.

Figure 4 shows an example of the Pentamodal Model π as blue crosses together with the bins of the underlying attitude landscape p and a visual representation of the two Gaussian distributions of the moderates from the fitted Pentamodal Model. The upper part of the extreme and central bins matches the parameters $w_{\mathrm{ExL}}, w_{\mathrm{ExR}}$

and w_C closely but is a result of a decomposition of the attitude landscape based on the Pentamodal Model which we explain next.

Decomposition of Empirical Attitude Landscapes

Despite good model fit, a pentamodal distribution never completely coincides with the underlying empirical attitude landscape. Nevertheless, we can use a best-fit Pentamodal Model π to decompose the underlying attitude landscape p into three groups: moderates p^Mod, extremists p^Ex, and residual centrists p^resC.[8]

$$p = p^\mathrm{Mod} + p^\mathrm{Ex} + p^\mathrm{resC} \tag{12}$$

To that end, we define:

$$\begin{aligned} p_i^\mathrm{Mod} &= \min\{p_i\ ,\ w_\mathrm{ModL}\pi_i^\mathrm{ModL} + w_\mathrm{ModR}\pi_i^\mathrm{ModR}\} \\ p^\mathrm{Ex} &= [p_0 - p_0^\mathrm{Mod}, 0, .., 0, p_{10} - p_{10}^\mathrm{Mod}] \\ p^\mathrm{resC} &= p - p^\mathrm{Mod} - p^\mathrm{Ex} \end{aligned}$$

The landscape of the moderates p^Mod is the pentamodal distribution of the moderates only capped by the empirical landscape if necessary. Note, that this distribution also has positive population at the extremes and in the center. There might even be a peak at the extreme when a larger part of the moderate normal distribution exceeds zero or ten, e.g. when the fitted mean is close to extreme or the fitted standard deviation is very large. The landscape of the extremes p^Ex has positive values only at zero and ten. It captures all population on these bins which are not covered in the moderate population. When the Pentamodal fit is very close to the empirical landscape then $p^\mathrm{Ex} \approx [w_\mathrm{ExL}, 0, \dots, 0, w_\mathrm{ExR}]$. Finally, p^resC covers the residual population which is neither moderate nor extreme. A good Pentamodal fit makes $p^\mathrm{resC} \approx [0, \dots, 0, w_\mathrm{C}, 0, \dots, 0]$.

Figure 4 shows the decomposition of attitude landscapes through stacked bins with the moderates at the bottom. The figure highlights a descriptive value of the Pentamodal Model: It estimates the fraction in the central bin which potentially chose five based on a moderate continuous attitude and those who chose five as a discrete choice of neutrality, e.g. because of a lack of knowledge or interest.

Decomposition of Pol_0

We can use the decomposition of an attitude landscape into the moderates, extremists and residual centrists based on the Pentamodal Model to also decompose the polarization measure Pol_0 into meaningful components.

To that end, we first define the *partial polarization measure* for a *partial attitude landscape* $0 \le q \le p$ (the inequality is meant entrywise) as

$$\mathrm{Pol}_\alpha(q, p) = \frac{2^{1+\alpha}}{n} \sum_{i,j=0}^{n} q_i^{1+\alpha} p_j |i - j|. \tag{13}$$

[8] Of course we could also define a decomposition into five groups, but the chosen groups suffice for the our purposes.

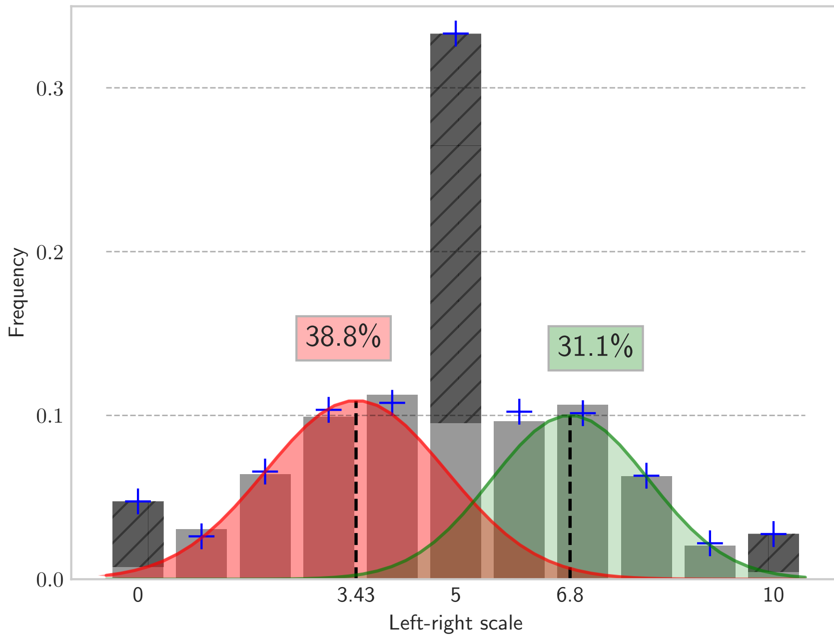

Figure 4 Distribution and including model results of France (top) left-right self-placements (LRSCALE) 2018. The underlying continuous distribution of moderates are displayed in red and green, the best-fit Pentamodal Model is shown with blue crosses. The hatched bars represent the extremists and centrists which are not from the moderate groups.

The original measure appears as the special case $\mathrm{Pol}_\alpha(p) = \mathrm{Pol}_\alpha(p, p)$.

Reconsidering the probabilistic interpretation of $\mathrm{Pol}_0(p)$ as average pair discrepancy, the partial polarization $\mathrm{Pol}_0(q, p)$ of the group with partial attitude landscape q is the average pair discrepancy individuals of the group represented by q perceive when the other person is selected from the whole population weighted by the total population of the group represented by q.

It is easy to see that $\mathrm{Pol}_\alpha(q, p) \leq \mathrm{Pol}_\alpha(p)$ for any q and $\mathrm{Pol}_\alpha(q, p) \leq \mathrm{Pol}_{\hat{\alpha}}(q, p)$ for any $\alpha \leq \hat{\alpha}$. These properties and the additive nature of the equation are the basis for the decomposition

$$\mathrm{Pol}_0(p) = \underbrace{\mathrm{Pol}_0(p^{\mathrm{Ex}}, p)}_{\text{(i)}} + \underbrace{\mathrm{Pol}_0(p^{\mathrm{resC}}, p)}_{\text{(ii)}} + \underbrace{\mathrm{Pol}_1(p^{\mathrm{Mod}}, p)}_{\text{(iii)}} + \underbrace{\mathrm{resPol}_0(p)}_{\text{(iv)}} \tag{14}$$

where $\mathrm{resPol}_0(p) = \mathrm{Pol}_0(p) - \mathrm{Pol}_0(p^{\mathrm{Ex}}, p) - \mathrm{Pol}_0(p^{\mathrm{resC}}, p) - \mathrm{Pol}_1(p^{\mathrm{Mod}}, p)$ is the residual part of Pol_0 without the parts of the extremists and the residual centrists and without Pol_1 of the moderates.

In the following we call the decomposed components of polarization in an attitude landscape

(i) extremists' polarization ($\mathbf{Pol_0^{Ex}}$),
(ii) centrists' polarization, ($\mathbf{Pol_0^{resC}}$)
(iii) moderates' identification-weighted polarization ($\mathbf{Pol_1^{Mod}}$), and
(iv) moderates' residual polarization ($\mathbf{resPol_0^{Mod}}$).

Findings

Figure 5 shows the four components for all attitude landscapes in our data set as scatter plots against the average attitude analogously to Figure 2. Further on, the example landscapes from core political topics in 2018 are decomposed into the extremist p^{Ex}, the centrists p^{resC}, and the moderates p^{Mod}. The landscape of moderates is split into a part which is identification-weighted $2(p^{\mathrm{Mod}})^2$ and the major remaining part $p^{\mathrm{Mod}} - 2(p^{\mathrm{Mod}})^2$ (exponentiation is meant entry-wise). The vector of identification weighted moderates appears as part of the definition of $\mathrm{Pol}_1^{\mathrm{Mod}}$.

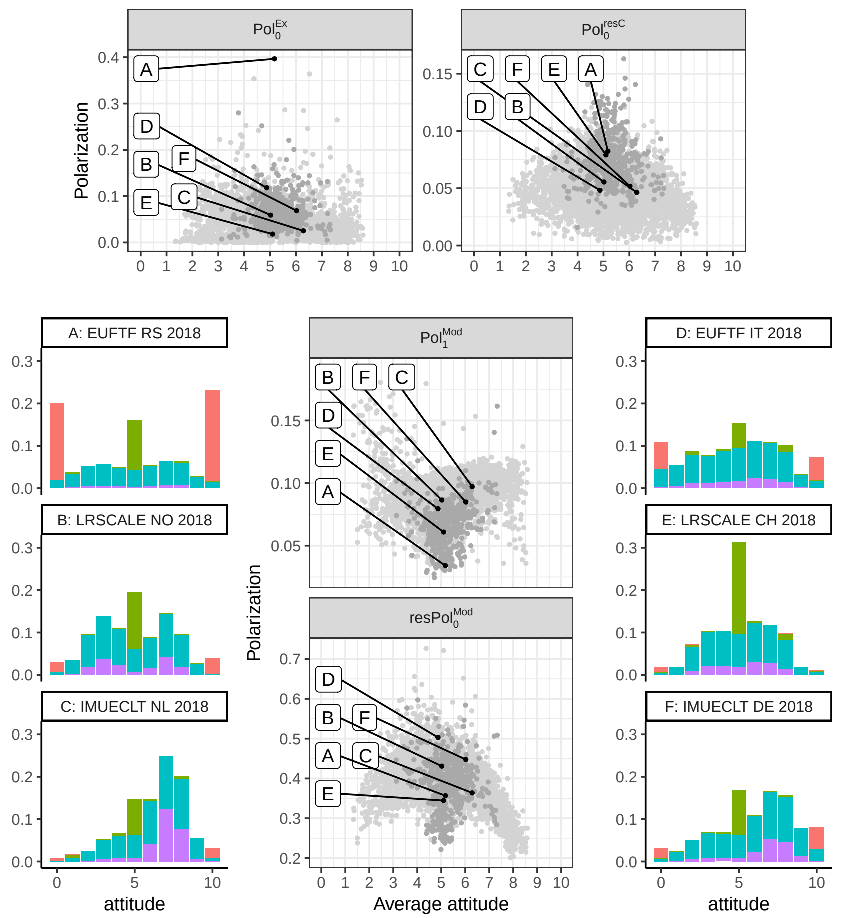

Figure 5 The four components of the decomposition of Pol_0 from Equation (14) based on the Pentamodal Model. The examples are the same as in Figure 2. The color code for examples A-F is red = extremists p^{Ex}, green = residual centrists p^{resC}, purple = moderates identification weighted $2(p^{\mathrm{Mod}})^2$, and cyan = remaining part of moderates $p^{\mathrm{Mod}} - 2(p^{\mathrm{Mod}})^2$.

The decomposition of Pol_0 into the four parts gives a more meaningful interpretation of the importance of the different aspects of polarization. For example, the European unification appears as most polarized in Serbia with respect to Pol_0 and Pol_1 (see Figure 2), however, Figure 5A shows that polarization is driven by extremists and to a lower degree by centrists compared to other topics and countries. The polarization of moderates is low in particular when identification-weighted. Instead, the identification-weighted polarization of moderates is high in the left-right

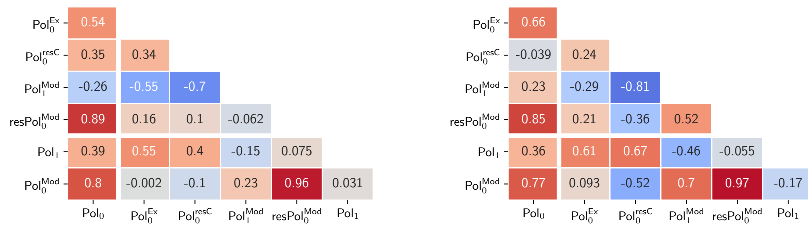

Figure 6 Pearson correlation coefficients of Pol_0, its four components, and $\mathrm{Pol}_0^{\mathrm{Mod}}$ and Pol_1 for comparison. Full data set on the left and reduction to the core political topics (LRSCALE, EUFTF, and IMUECLT) on the right.

self-placement in Norway (Figure 5B) and even more in the assessment of the cultural impact of migration in the Netherlands (Figure 5C). The moderates in B show a bimodal distribution, while in C the right moderates dominate. This difference is reflected in B having higher values of the residual polarization $\mathrm{resPol}_0^{\mathrm{Mod}}$. The polarization about European unification in Italy (Figure 5D) also shows a similar degree of identification-weighted moderate polarization as Norway on left-right self-placement but an even higher residual polarization. This reflects that polarization is driven by moderates though not by one or two peaks but rather by the broad distributions of the moderates. For the left-right polarization in Switzerland (Figure 5E) our set of indicators clearly show that the generally low level of polarization is driven by the centrists to a comparably high degree. Finally, the polarization about the cultural impact of immigration in Germany (Figure 5F) is in between of distributions B and C.

Switching from examples to the full data set, Figure 6 shows the Pearson correlation of Pol_0, its four components, and the non-decomposed polarization of moderates $\mathrm{Pol}_0^{\mathrm{Mod}} = \mathrm{Pol}_0(p^{\mathrm{Mod}}, p)$ and Pol_1 for the full attitude landscape. The correlation coefficients are based on all 4,155 attitude landscapes in our sample (see Table 1 for a similar correlation analysis).

The correlation analysis shows that the moderate residual polarization is highly correlated with the overall polarization, while the correlation to the other three components is comparably close to zero. This reflects our idea to isolate different independent aspects of polarization with the extremist, centrist, and identification-weighted moderate polarization. The only strongly negative correlations between the four components are between identification-weighted moderate polarization and extremist and centrist polarization. This underpins that a comparison of identification-weighted moderate polarization is most informative in comparison with the moderates residual polarization. Further on, the identification-weighted moderate polarization is not correlated with the overall identification-weighted polarization Pol_1 which shows that the decomposition and focus to the moderates delivers different information. We think, the overall identification-weighted polarization is difficult to interpret in isolation because it lumps together the identification of extremists, centrists, and moderates. High values could be caused by any of those groups and thus say little about the character of polarization.

In the following we show how the Pentamodal Model and the decomposition of polarization can be used for cross-topic and cross-country comparisons and for the analysis of time trends.

Analysis of Cross-Topic Data

We evaluate how polarized different topics are by grouping attitude landscapes by topic. That means for each topic we have the attitude landscapes of all available country year combinations. Figure 7 shows box plots of Pol_0 ordered by the median from most to least polarized. We highlight the three core topics left-right self-placement, European unification, and cultural impact of migration in red. Topic explanations are shown in Table 4 in Appendix A.

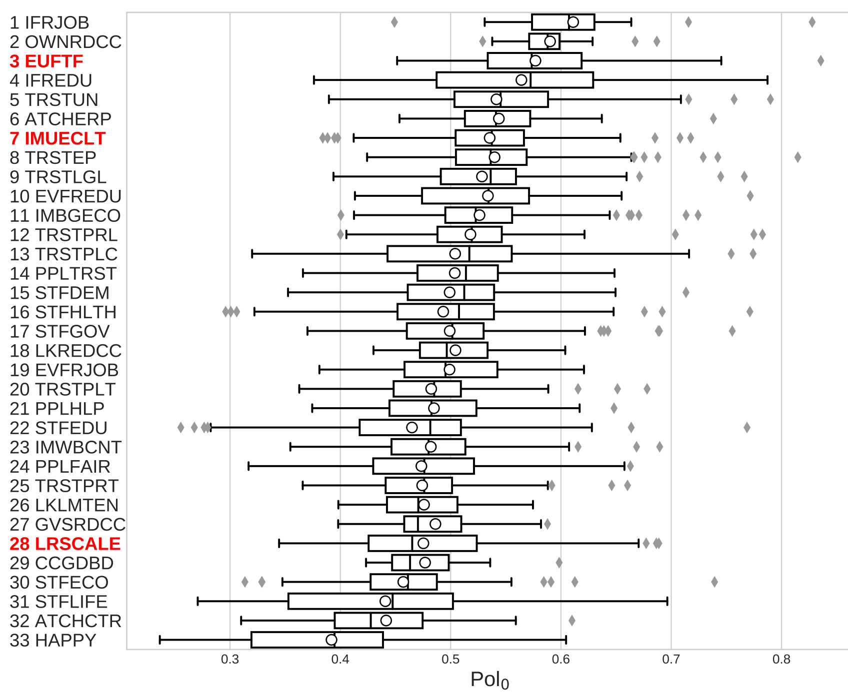

Figure 7 Box plot of the complete data set sorted by the median of Pol_0. The mean is indicated by a circle.

The rankings by median of the decomposition of Pol_0 are shown in Table 2

The question about fair job chances for everyone in the country shows the largest polarization with a median of $\mathrm{Pol}_0 \approx 0.6$. The second most polarized topic is how people assess if limiting their own energy use could reduce climate change. Small whiskers on both topics indicate that polarization is consistently high across countries and years. The third most polarized topic is European unification but less consistently. Individual happiness has the lowest polarization with $\mathrm{Pol}_0 \approx 0.4$.

From the three core topics the left-right self-placement has the lowest median polarization. All three core topics show comparably large variation in polarization. Furthermore, all are high on polarization of residual centrists $\mathrm{Pol}_0^{\mathrm{resC}}$ and low on identification-weighted moderate polarization $\mathrm{Pol}_1^{\mathrm{Mod}}$. This reflects that many political topics show large peaks of undecided or neutral centrists. The low level of

	Pol_0	Pol_0^{Ex}	$\text{Pol}_0^{\text{resC}}$	$\text{Pol}_1^{\text{Mod}}$	$\text{resPol}_0^{\text{Mod}}$
1	IFRJOB	IFREDU	**LRSCALE**	STFHLTH	OWNRDCC
2	OWNRDCC	IFRJOB	IMWBCNT	STFEDU	IFRJOB
3	**EUFTF**	ATCHCTR	**EUFTF**	OWNRDCC	TRSTUN
4	IFREDU	**EUFTF**	IMBGECO	HAPPY	TRSTLGL
5	TRSTUN	ATCHERP	**IMUECLT**	STFECO	TRSTEP
6	ATCHERP	CCGDBD	PPLFAIR	TRSTPLC	**EUFTF**
7	**IMUECLT**	EVFREDU	PPLTRST	STFLIFE	EVFREDU
8	TRSTEP	TRSTPRL	IFRJOB	TRSTLGL	STFHLTH
9	TRSTLGL	**IMUECLT**	PPLHLP	STFDEM	**IMUECLT**
10	EVFREDU	LKREDCC	CCGDBD	GVSRDCC	STFDEM
11	IMBGECO	IMBGECO	TRSTUN	STFGOV	ATCHERP
12	TRSTPRL	TRSTEP	TRSTEP	LKLMTEN	IMBGECO
13	TRSTPLC	TRSTPLT	TRSTPRL	LKREDCC	TRSTPRL
14	PPLTRST	OWNRDCC	ATCHERP	EVFRJOB	STFGOV
15	STFDEM	EVFRJOB	IFREDU	EVFREDU	TRSTPLC
16	STFHLTH	**LRSCALE**	TRSTPLC	TRSTPLT	LKREDCC
	⋮	⋮	⋮	⋮	⋮
27	GVSRDCC	STFECO	EVFRJOB	**IMUECLT**	STFECO
28	**LRSCALE**	STFDEM	STFGOV	IMBGECO	IMWBCNT
29	CCGDBD	HAPPY	LKLMTEN	ATCHCTR	CCGDBD
30	STFECO	PPLFAIR	STFEDU	IFREDU	**LRSCALE**
31	STFLIFE	STFEDU	GVSRDCC	**EUFTF**	STFLIFE
32	ATCHCTR	STFHLTH	STFECO	IMWBCNT	ATCHCTR
33	HAPPY	PPLHLP	STFHLTH	**LRSCALE**	HAPPY

Table 2 Ranking from the highest to the least polarized topics in the ESS (ordered by the median). The core political topics are marked as red.

$\text{Pol}_1^{\text{Mod}}$ is not surprising because the measure is anti-correlated with $\text{Pol}_0^{\text{resC}}$. European unification differs from immigration mostly on extremist's polarization which is much stronger for European unification. The low polarization of left-right self-placement is mostly due to much lower residual moderate polarization compared to the two other core topics.

Strong differences in polarization are also visible for the two topics about emotional attachment. While the emotional attachment to Europe (ATCHERP) is the sixth most polarized topic, the attachment to the own country (ATCHCTR) has the second lowest polarization. However, both are high on polarization of extremists, emotional attachment to the own country even slightly higher than emotional attachment to Europe.

Cross-Country Comparison

Figure 8 shows the country rankings for European unification in 2018. The table is sorted by Pol_0 but also shows all four components of its decomposition. In all columns, we color cells with largest values in deep red and lowest in deep blue to allow an easy assessment of rankings in the four component measures and an assessment which of the four components is over- or under represented in a country compared to the other countries.

Further on, the Figure includes as examples six attitude landscapes each including a representation of the underlying Pentamodal Model as in Figure 4 which may help further interpreting the results.

As already discussed with respect to Figure 5, the polarization on European unification in Serbia results from extremist polarization and, consequently, a small number of only 48% moderates. The landscape is essentially trimodal in shape due

Country	Pol_0	$\mathrm{Pol}_0^{\mathrm{Ex}}$	$\mathrm{Pol}_0^{\mathrm{resC}}$	$\mathrm{Pol}_1^{\mathrm{Mod}}$	$\mathrm{resPol}_0^{\mathrm{Mod}}$
RS	0.836	0.396	0.083	0.034	0.323
AT	0.673	0.05	0.074	0.092	0.457
IT	0.669	0.118	0.049	0.079	0.423
BG	0.63	0.036	0.073	0.079	0.442
EE	0.621	0.109	0.088	0.06	0.364
DE	0.611	0.129	0.05	0.077	0.357
CY	0.605	0.116	0.1	0.057	0.331
HU	0.597	0.093	0.06	0.076	0.368
CZ	0.587	0.049	0.046	0.085	0.408
PL	0.584	0.123	0.068	0.067	0.326
FR	0.579	0.083	0.078	0.065	0.352
GB	0.569	0.084	0.09	0.055	0.34
SI	0.552	0.073	0.062	0.073	0.343
CH	0.532	0.044	0.083	0.062	0.343
BE	0.532	0.065	0.071	0.071	0.325
IE	0.527	0.04	0.077	0.066	0.344
NL	0.486	0.034	0.064	0.078	0.308
FI	0.475	0.045	0.055	0.076	0.298
NO	0.458	0.048	0.092	0.047	0.27

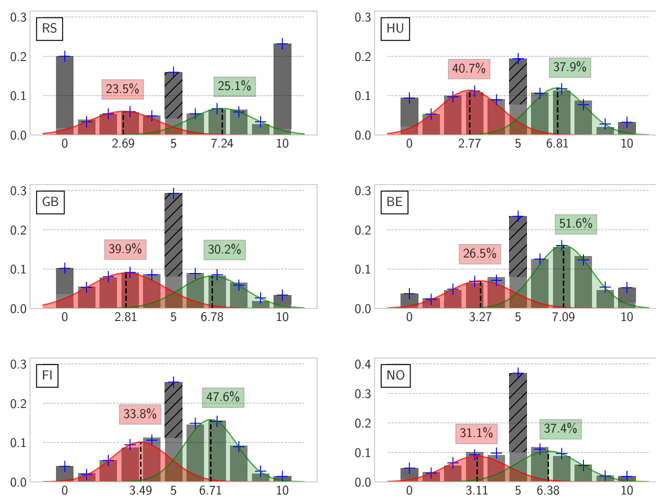

Figure 8 Polarization ranking of countries on the topic of European unification (EUFTF) in 2018. Table and histograms are sorted by Pol_0. Histograms from top left to bottom right

to the lack of strong moderate peaks. The ranking points out that this type of polarization about European unification is unique to Serbia with extremist and centrist polarization being much lower in other countries. We skip the analysis of Austria because the fit of the Pentamodal Model is bad resulting from p_1 being unusually high. Further explanation are given in Appendix D.

The lowest polarization on European unification appears in Norway with a low amount of extremists, means of the moderates close to five, and a particular broad consensus on the centrist's position.

For the countries in between both extremes the distributions have various structures. The Czech Republic, Hungary, and Italy have comparable structure and similar moderates identification-weighted polarization $\mathrm{Pol}_1^{\mathrm{Mod}}$. The Czech Republic has the highest $\mathrm{Pol}_1^{\mathrm{Mod}}$ (neglecting Austria) due to the greater moderate weights especially for the moderate left (for which European unification has gone too far). The overall higher polarization in Italy results from larger extremism and centrist polarization. Hungary and the Czech Republic share a similar polarization (Pol_0), here, the Pentamodal Model illustrates the minor difference: More distinctly bimodal moderates but a greater amount of extremists in Hungary. At the lower ranks of polarization, Belgium highlights a country with more approval for European unification and double the weights of moderates right versus left. With over 75% of moderates, identification-weighted moderate polarization is relatively high due to the overall weights. For comparison, Finland has about 80 % moderates with more similar sized weights for the opposing moderates. Interestingly, the Brexit referendum in 2016 has not brought the United Kingdom towards strong polarization on European unification compared to other countries. However, polarization in the United Kingdom is more driven by polarization perceived by centrists than in most other countries.

The country ranking for left-right self-placement and migration in 2018 are shown in the same way in Figures 20 and 21 in Appendix E.

Time Trend Analysis

Increasing polarization is a big lament in public media and science. However, such claimed trends do not show off as often in survey data. The Pentamodal Model and the decomposition of Pol_0 can refine the picture. We demonstrate this with three examples for the core political topics in three different countries.

Figure 9 shows five distributions and the corresponding Pentamodal Model of the left-right self-placement in Denmark from 2002 to 2014, in the panels on the left-hand side. The right hand side, shows a stacked area plot of all four measures of which Pol_0 is composed. Below are all components independently including a linear trend approximation with a $\pm 1\sigma$ confidence area. The overall left-right polarization Pol_0 in Denmark was increasing from 2002 to 2014. The increase can be attributed to a simultaneous increase of both the identification-weighted residual ($\mathrm{Pol}_1^{\mathrm{Mod}}$) and moderates polarization ($\mathrm{resPol}_0^{\mathrm{Mod}}$), while extremists' polarization fluctuates without a clear trend, and centrists' polarization declined. The moderate left was initially large, close to centrist, and widely dispersed while the moderate right were smaller, less centrist, and more condensed. Until 2010 this picture changed with the right moderates becoming more, moving a bit closer to the center, and more dispersed, while the left becoming less, more extreme and more condensed. Overall, the group moderates on both sides became more and the residual centrists became less. In 2014, it looks a bit like a reversion to the 2002 situation with larger and more dispersed left moderates. So, the main driving force behind increasing polarization is the shrinking of the groups of residual centrists in favor of moderates on both

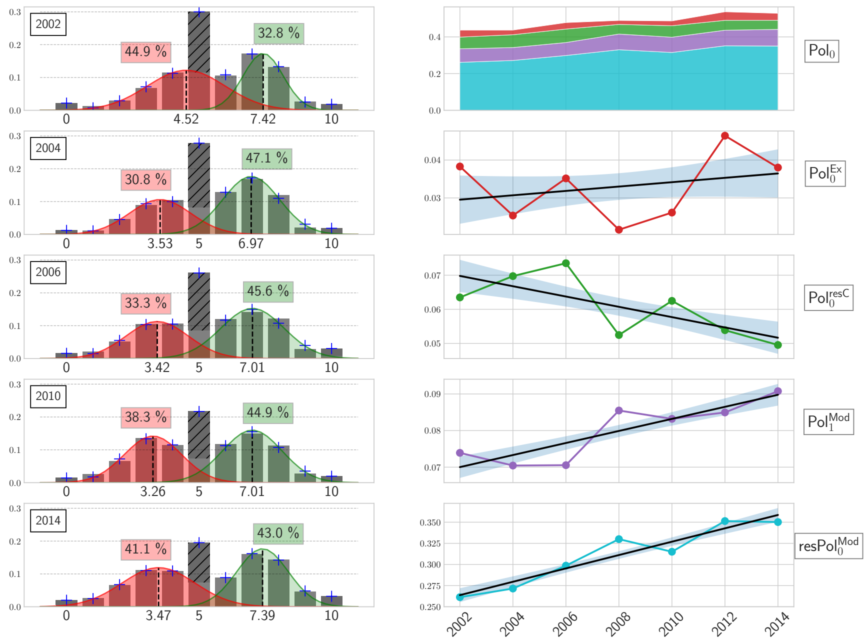

Figure 9 Analysis of the left-right self-placement in Denmark from 2002 until 2014. Left: Distributions and results of the Pentamodal Model including the proportion of moderate left and right and their according means. Right: Pol_0 and the decomposed components of polarization over time.

sides. This is also the main reason for the overall decline of centrists' polarization. We end the analysis in 2014 because there is no data for Denmark in the years 2016 and 2018.

The polarization about European unification in the United Kingdom in Figure 10 shows that the Pentamodal Model can reveal trends in different aspects of polarization which remain covered when trends in total polarization are mostly constant. From 2004 to 2014 the ratio between the moderate left and moderate right increased in favor for the opinion that European unification has gone too far. This shows a clear trend towards the Brexit referendum's decision in 2016. The ratio became much more balanced after the Brexit decision and is becoming equally sized in the year 2018 again. The distribution of all moderates combined (the light grey distribution without the hatched parts) became more uniform until 2018 and the total share of moderates was decreasing from 81% to 70% in favor of centrists as well as extremists. This explains the increase of centrists' and extremists' polarization and the decrease of identification-weighted moderates' polarization.

The example of Hungary in Figure 11 shows an opinion shift in the attitudes about immigration's influence on the country's culture. With the European migrant crisis in 2015 the Hungarian parliament decided to enforce an anti-immigration policy (Thorleifsson, 2017), particularly "*protecting their Christian roots and culture*" (Viktor Orban). This shift is also represented in the society with a spike in left extremists and moderate left in 2016. "Left" stands here for people who think that immigration undermines the Hungarian culture. Simultaneously, the centrist polarization drops, pointing towards a politicisation of the society in favor of anti-migration policies. Furthermore, the extremist polarization and residual moderates'

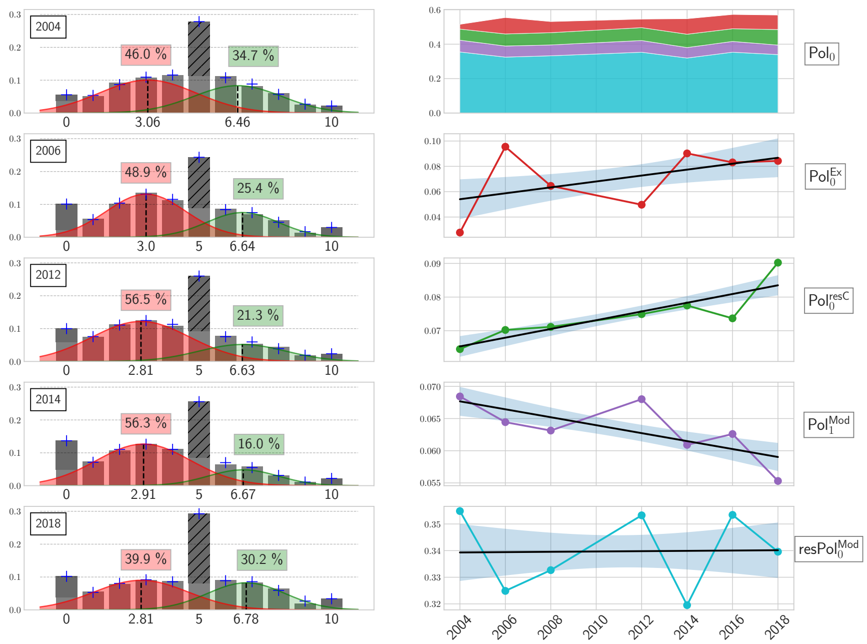

Figure 10 Analysis European unification (0=gone too far, 10=should go further) in the United Kingdom from 2004 until 2018. Left: Distributions and results of the Pentamodal Model including the proportion of moderate left and right and their according mean. Right: Pol_0 and the decomposed components of polarization

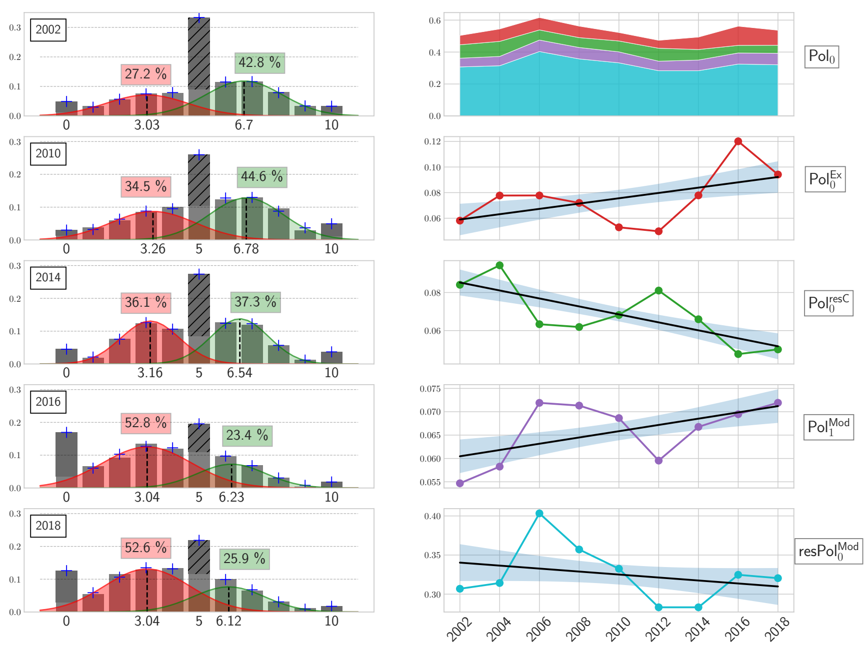

Figure 11 Analysis of Hungarian attitudes about the impact of immigration on the country's culture (0=underminded, 10=enriched) from 2002 to 2018. Left: Distributions and results of the Pentamodal Model including the proportion of moderate left and right and their according mean. Right: Pol_0 and the decomposed components of polarization

polarization increased from 2014 to 2016. Remarkably, Pol_0 remains mostly stable despite the shift of the mean attitude. What happened in Hungary shows more the characteristic of what is called group polarization which is the collective shift towards more extreme attitudes. This is not captured by bipolarization as measured by Pol_0 and its decomposition.

Appendix E shows further trends for Pol_0 and its four components for the three topics in Figure 22). The tables show the trends through the slope of a linear approximation. All Countries are included which participated in more than six rounds of the ESS. No Europe-wide trend of increasing or decreasing polarization could be detected for the three core topics, neither in total polarization nor in its components.

Conclusion

The measurement of societal polarization on one attitude dimension appears controversial in the literature. The question what is polarized and what not, seems trivial because there is agreement about the minimally and maximally polarized attitude landscape. However, in between there are many aspects which could be emphasized. We propose the measure of Pol_0 based on the measurement concept of Esteban and Ray (1994), because it can be decomposed into different components concerning different groups and identification-weighted components also introduced by Esteban and Ray (1994). Pol_0 is almost perfectly correlated with other "catch-all" polarization measures and focuses on the dispersion principle of DiMaggio et al (1996). Furthermore, it has an appealing probabilistic interpretation: It is the expected antagonism (distance in attitude) a pair of two randomly selected individuals from the population perceive.[9]

Motivated by empirical observations on 4,155 attitude landscapes on eleven-point scales in the ESS data set, we developed the Pentamodal Model which assumes groups of left and right moderates modeled by normal distribution, and left and right extremists, and centrists modeled as discrete groups focused on just one answer. The model allows the decomposition of attitude landscapes into these five groups. As a validation of our model, the introduced R^2 showed a mean of R^2=0.99 with only 0.6% of cases dropping below $R^2 < 0.95$. The model works across all countries and topics with few exceptions. Consequently, we are able to decompose Pol_0. We can measure the polarization perceived by extremists and centrists. The remaining polarization of the moderates can be further decomposed by specifying the identification weighted part Pol_1 following the framework of Esteban and Ray (1994).

[9] With a more general definition of distance the measure also includes standard measures for diversity and concentration used for the measurement of ethnic or religious fragmentation, biological diversity (Simpson index), economic concentration (Herfindahl-Hirschman index), or the concentration of political party system (inverse of Taagepeera's effective number of parties). All these are essentially based on replacing the ordered attitude scale with a nominal scale of group labels (ethnic groups, religions, species, economic sectors, parties). Technically, the distance term $|i - j|$ is being replaced by one when $i \neq j$ and zero when $i = j$.

The Pentamodal Model and the decomposition of Pol_0 provide a reasonable way to assess how polarized a certain topic is in a certain European country in a particular year and what group drives it in comparison to a set of reference cases, e.g., other topics in the same country, other countries on the same topic or other years. A structured analysis can follow these questions:

1 How strong is the level of total of polarization Pol_0 compared to the reference set?
2 How much is the polarization driven by extremists $\mathrm{Pol}_0^{\mathrm{Ex}}$? (Compared to the other aspects and to other landscapes in the reference set.) A large value indicates that there are many excess extremists which can not be covered by the extremes in the moderate group.
3 How much is the polarization driven by the residual centrists $\mathrm{Pol}_0^{\mathrm{resC}}$? (Compared to the other aspects and to other landscapes in the reference set.) A large value may also indicate that many individuals in the population are not very interested in forming nuanced attitudes on this topic.
4 What is the level of the identification-weighted polarization of moderates $\mathrm{Pol}_1^{\mathrm{Mod}}$? This points towards a more peaked distribution of moderates instead of a uniform distribution.
5 Is there something else visible in the attitude landscapes and the fitted Pentamodal Model? This can be assessed visually in comparison with other attitude landscapes.

Using these guidelines for the three core political topics, we found that European unification and immigration are among the polarized ones, while left-right self-placement is among the topics with the lowest polarization mostly because of higher shares of centrists. Strong variation in topical polarization exists between different countries. For example, in Norway left-right self-placement is polarized (0.545, ranked sixth from 19 countries) but European unification not (0.458, ranked last of 19 countries); the other way round, in Estonia, European unification is polarized (0.621, ranked fifth) and left-right self-placement not (0.384, ranked last).

Overall, we find no indication of a general trend of increasing or decreasing polarization, neither in total nor in one of its components. The strongest general increase we found for the left-right self-placement in Denmark, which is to a large extend driven by a decrease of the amount of centrists. Given the political and media discourse, attitudes on Europe in the United Kingdom and on immigration in Hungary should show trends in polarization. We do not find this for the total polarization of the general population. However, we find that polarization on European unification in the United kingdom became more driven by extremists and centrists over the Brexit discourses and less by identification-weighted moderates. The polarization on immigration in Hungary became more driven by identification weighted moderates and extremists. At the same time there was a substantial shift of the average attitude towards anti-immigration attitudes which is not reflected in a substantial impact on total polarization. The polarization discussed in the media may reflect polarization on the European level.

With our measurement framework we provide a data-driven and more nuanced view on mass polarization such that it can be discussed with more specific definition and quantitative evidence. The polarization indices derived from the Pentamodal

Model can also serve future analyses of context conditions for polarization, such as the political system, income inequality, social cohesion, or other country-based indicators. It would also be interesting to explore relations to other concepts of polarization, for example, issue alignment.

Discussion

A topic of some debate is the contribution of social media to polarization (Barberá, 2014, 2015; Eady et al, 2019; Garimella and Weber, 2017). In many studies, researchers try to infer attitude values (typically liberal-conservative or left-right) from postings (e.g., tweets), the follower-followee network and context information. The population active on social media does not represent the general population well (Barberá and Rivero, 2015). Our work can help to compare polarization in social media with the general population. To that end, we provide the response rates extracted from the ESS documentations in Appendix A.

Our data exploration elicited that all attitude landscapes have multimodal structures and do not follow simple distributions. This raises the questions if these patterns emerge through processes of attitude dynamics in the population which are analyzed with agent-based models (Flache et al, 2017; Lorenz et al, 2021). The Pentamodal Model can provide a solid structure of empirical data as stylized facts to validate such models based on their macroscopic outcomes.

A remaining problem is that the polarization measure Pol_α produces similar values for skewed unimodal distributions and bimodal distributions of the moderates, visible in Figure 5 especially in the example B and F. So, the parameter α for weighting identification cannot not differentiate between all three different theoretical principles shown in Figure 1.

The Pentamodal Model has the assumption of endogenously given groups. Therefore, one may think of transferring the group-specific measurement concepts of Bramson et al (2016) more directly by using the parameters of the Gaussian functions of the two moderate groups, e.g., by defining group divergence as $|\mu_\mathrm{L} - \mu_\mathrm{L}|$, group consensus based on $-(\sigma_\mathrm{L} + \sigma_\mathrm{R})$, size parity based on the parity of w_ModL and w_ModL, and distinctness describing the ratio of the overlap of two groups. All these are feasible directions for future work and might help to further refine and improve the measurement of polarization. We refrained from it because in the current state the model parameters are very sensitive to small fluctuations within the overall distributions especially for answer 4 and 6. Due to the unknown fraction of moderates within answer 5 these fluctuations can have a big impact on group divergence, distinctness, and group consensus unlike $\mathrm{Pol}_1^\mathrm{Mod}$ and $\mathrm{resPol}_0^\mathrm{Mod}$ in our decomposition of Pol_0.

After rejecting the bi-polarization measure by Foster and Wolfson (2009) and Wang and Tsui (2000) for this study, one should consider these measures for future work. By satisfying Esteban and Ray (2012) axioms they qualify specifically for the bimodal distribution like the theoretical distribution of moderates. In combination with the Pentamodal Model, the different nature of the measure complicates the interpretation in relation to the other measures, but may enabling more comprehensive rankings. Nevertheless, all these measures follow a solid axiomatic foundation based on economic assumptions but otherwise enabling to enhance the research on political and mass polarization in the future.

Another direction of future research could be to simplify the Pentamodal Model through the identification of further regularities. Fitting eleven data points with a eight free parameters looks exaggerated. It could, for example, be that means of moderates were related, e.g., when one group was close to the center the other was not. We made some explorations to find such relations of parameters of the 4,155 attitude landscapes with the aim to construct a model with fewer parameters. We found some correlations between μ_L and σ_R and analogously between μ_R and σ_L. We did not start to simplify the model based on this finding, because correlations were small and theoretical plausibility was not very strong.

We apply the Pentamodal Model only to questions on eleven-point scales from zero to ten. Scherpenzeel (2002) outline why the eleven-point scale has the most advantages in the context of the Swiss Household Panel. Nevertheless, many surveys use shorter scales like seven or five point scales. The groups in the Pentamodal Model should be reasonable also when using these scales, but these scales have lower numbers of answering option than the Pentamodal Model has parameters. So, for such scales it seems more appropriate to develop new models with similar heuristics.

We want to note that the European Social Survey would also allow to study the polarization aspects of issue alignment (constraint principle) issue partisanship (consolidation principle) of exogenously given groups, e.g., party supporters. However, this was beyond the scope of this study.

Potential limitations could apply from a psychometric viewpoint. This stance assumes that public opinion measures extracted from survey data contain measurement error. Their quality depends on item characteristics such as the bipolar eleven-point response scale used here. However, (Leung, 2011) advocate the use of 11-point scales as compared to other Likert scales because of increased sensitivity which did not seem to come at the cost of cognitive fatigue. The latter argument is frequently made against the use of long scale formats but could not be empirically supported by the findings: Varying the number of response categories did not affect (re-scaled) means and standard deviations, factor loadings, average item-to-item correlations or other psychometric scale properties. One can conclude that longer and more sensitive scale types do not distort the measurement of constructs. Our descriptive analysis shows that even more details may be observed thanks to sensitivity, for example, different types of pentamodal distributions. From a psychometric view, (Harzing, 2006) criticised cross-national comparisons. She found that countries exhibit different response styles regarding agreement bias and extreme answering. This was related to differences in cultural dimensions such as extraversion, uncertainty avoidance or collectivism (Harzing, 2006). This could be kept in mind as an alternative explanation for why we found no indication of a general, cross-national trend. Generally, the ESS survey program was designed to allow cross-national attitude comparisons. Explicit measures taken to improve comparability are outlined on the ESS website as well as in bi-annual data quality reports (Wuyts and Looseweldt, 2019).

These limitations only apply from a psychometric viewpoint, however we can also take survey answers by face value. Questions with ordered rating scales are not only asked as part of psychometric measurement instruments. They are used in psychotherapy and pain regulation (Berg and De Shazer, 1993; Farrar et al, 2001). E.g., a pain assessment of a patient is not to be judged by the therapist as potentially subject to measurement error, but as basis to judge the effectiveness of therapeutic interventions by the patient. In the form of stars, rating scales are used in online recommendation systems for movies, dining places, and all sorts of consumer products and services. That way, numerical attitudes towards products are communicated, negotiated, and judged in for interpersonal purposes and gain value in themselves.

In a similar way, ordered rating scales are also the basis of modern range voting systems, e.g., majority judgment, which Balinski and Laraki (2011) proposed using the example of French presidential election. In this voting system, each voter has to assess each candidate with a rating from "reject" to "excellent". The voting systems extracts the median rating for each candidate and declares the candidate with the highest median as the winner.[10] Range voting systems aim to reduce advantages for candidates which are politically polarizing. The Pentamodal Model may help to classify the empirical conditions when this can be empirically realized.

Competing interests
The authors declare that they have no competing interests.

Author's contributions
"All authors designed the study together. MG, JL, and NH analyzed the data, MG programmed the model, MG and JL carried out the analysis and wrote the manuscript following editions of NH and KB."

Acknowledgements
The authors benefited from a grant from the German Research Foundation (DFG 396901899) within the Project "ToRealSim - Towards realistic models of social influence". The article was prepared within the framework of the HSE University Basic Research Program.

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[10]Ties are broken by removing one-by-one votes with the common median's value from all tied candidates until median moves such that a winner can be declared.

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Appendix A: Data Details

The Table 4 shows the wording of the questions and the corresponding acronym. The Figure 12 gives an overview which of how often a topic occurred per country in the ESS data set.

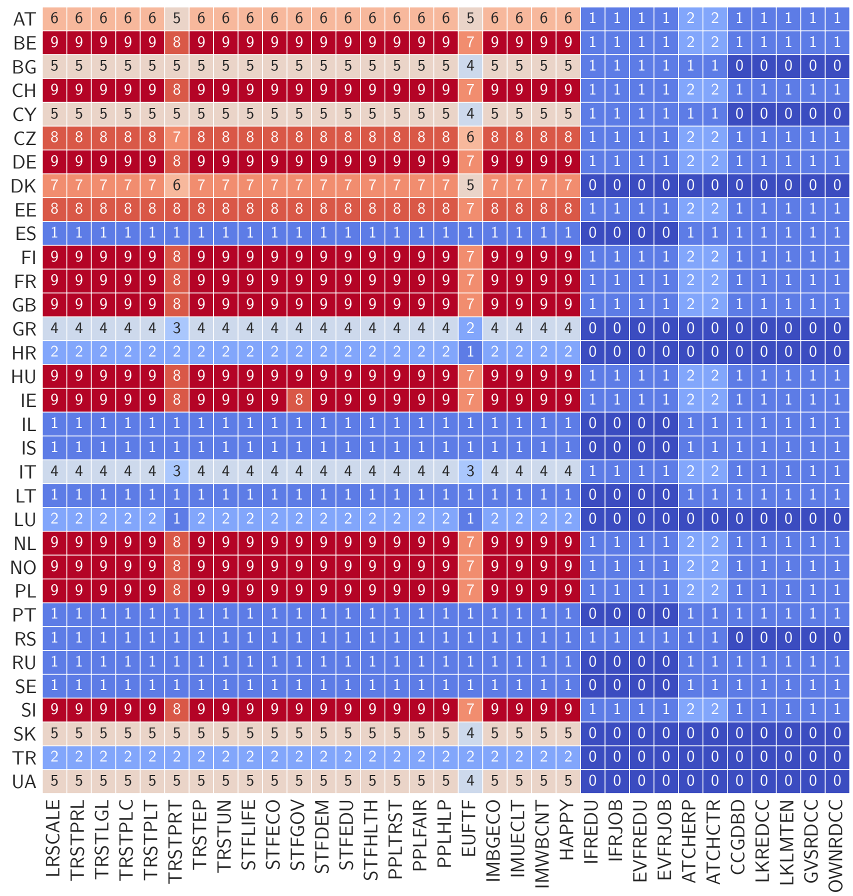

Figure 12 Amount of rounds a topic is represented in a country.

First, the response rates within ESS are assessed. If response rates decreases the error in the distributions could increase and the validation of our model would be deficient. But like Koen Beullens (2018) already investigated and in our observations the ESS show no systematical decrease in response rates. In the following table 5 the response rates from the documentation report is shown which confirm the result with a slight overall decrease in response rates.

Appendix B: Exploration of peaks in attitude landscapes

Investigating the peaks of all attitude landscapes ("total") and the ones of the core political topics are assessed in the Figure 13. First row showing the frequency of the total amount of peaks in the explicit topics. Revealing that within the three topics mostly 4 peaks occur whereby over all attitude landscape 3 peaks are more likely. Furthermore in all case of LRSCALE distribution answer 5 conforms with the largest share. In the entire data set over 80 % of distributions have a peak at 5, being also the highest one in over 50 %. Concluding the theory that answer 5 is evaluated as neutral answer. Certain answers are unpopular over the whole survey. Answer 1,2,4,6, and 9, as shown in the last distribution in 13, are significantly less common.

Appendix C: Further motivation and evidence for the Pentamodal Model of attitude landscapes

This part is about perceiving the theory from a different angle. As motivation the ESS data set also provides the possibilities of analysing endogenous groups. Using political groups like voters for a political party might enabling further support for normal distributed moderates. Political parties can also be assigned on a left-right scale, like done in the manifesto project (Merz et al, 2016) or PARLGOV (Döring and Manow, 2012). Assuming that voters not share the opinion of the party categorically, they will distribute around a certain mean. Considered rationally, different opinions within a group should also occur with the same probability to each side of the maximum. Bauer et al (2016) showed that people deviate around a mean which we approximate as a normal distribution. As an example, Figure 14 displays CDU (German conservative party) voters on the left-right scale including a fit for a normal distribution. The fit works, except for irregularities at answers 5 and 9. As a result of the normal

ID	Description
ATCHCTR	How emotionally attached to [country] (0=Not at all, 10=Very)
ATCHERP	How emotionally attached to Europe (0=Not at all, 10=Very)
CCGDBD	Climate change good or bad impact across world (0=Extremely bad, 10=Extremely good)
EUFTF	European unification go further or gone too far (0=Already gone too far, 10=Go further)
EVFREDU	Everyone in country fair chance achieve level of education they seek (0=Not at all, 10=Completely)
EVFRJOB	Everyone in country fair chance get job they seek (0=Not at all, 10=Completely)
GVSRDCC	How likely, governments in enough countries take action to reduce climate change (0=Not at all, 10=Extremely)
HAPPY	How happy are you (0=Extremely unhappy, 10=Extremely happy)
IFREDU	Compared other people in country, fair chance achieve level of education I seek (0=Not at all, 10=Completely)
IFRJOB	Compared other people in country, fair chance get job I seek (0=Not at all, 10=Completely)
IMBGECO	Immigration bad or good for country's economy (0=Bad, 10=Good)
IMUECLT	Country's cultural life undermined or enriched by immigrants (0=Undermined, 10=Enriched)
IMWBCNT	Immigrants make country worse or better place to live (0=Worse, 10=Better)
LKLMTEN	How likely, large numbers of people limit energy use (0=Not at all, 10=Extremely)
LKREDCC	Imagine large numbers of people limit energy use, how likely reduce climate change (0=Not at all, 10=Extremely)
LRSCALE	Placement on left right scale (0=Left, 10=Right)
OWNRDCC	How likely, limiting own energy use reduce climate change (0=Not at all, 10=Extremely)
PPLFAIR	Most people try to take advantage of you, or try to be fair (0=Take advantage, 10=Try to be fair)
PPLHLP	Most of the time people helpful or mostly looking out for themselves (0=Look out for themselves, 10=Try to be helpful)
PPLTRST	Most people can be trusted or you can't be too careful (0=Can't be too careful, 10=Can be trusted)
STFDEM	How satisfied with the way democracy works in country (0=Extremely dissatisfied, 10=Extremely satisfied)
STFECO	How satisfied with present state of economy in country (0=Extremely dissatisfied, 10=Extremely satisfied)
STFEDU	State of education in country nowadays (0=Extremely bad, 10=Extremely good)
STFGOV	How satisfied with the national government (0=Extremely dissatisfied, 10=Extremely satisfied)
STFHLTH	State of health services in country nowadays (0=Extremely bad, 10=Extremely good)
STFLIFE	How satisfied with life as a whole (0=Extremely dissatisfied, 10=Extremely satisfied)
TRSTEP	Trust in the European Parliament (0=No trust at all, 10=Complete trust)
TRSTLGL	Trust in the legal system (0=No trust at all, 10=Complete trust)
TRSTPLC	Trust in the police (0=No trust at all, 10=Complete trust)
TRSTPLT	Trust in politicians (0=No trust at all, 10=Complete trust)
TRSTPRL	Trust in country's parliament (0=No trust at all, 10=Complete trust)
TRSTPRT	Trust in political parties (0=No trust at all, 10=Complete trust)
TRSTUN	Trust in the United Nations (0=No trust at all, 10=Complete trust)

Table 4 ID-string, abbreviated wording, and answering scale of all questions used from the European Social Survey (ESS).

Country	2002	2004	2006	2008	2010	2012	2014	2016	2018	Mean
Austria		62.41	63.96	62.26	59.60		51.58	52.54	50.87	57.60
Belgium	59.21	61.37	61.01	58.86	53.43	58.74	57.03	56.77	56.18	58.07
Bulgaria			64.75	74.98	81.43	74.74			72.59	73.70
Croatia				45.70	54.49				43.24	47.81
Cyprus			67.32	78.74	69.74	76.75			53.38	69.19
Czech Republic	43.33	55.29		69.49	70.16	68.40	67.93	68.45	67.36	63.80
Denmark	67.56	65.11	50.78	53.88	55.40	49.43	51.85		48.82	55.35
Estonia		79.27	64.97	57.37	56.21	67.83	59.94	68.44	64.06	64.76
Finland	73.21	70.77	64.40	68.44	59.45	67.27	62.67	57.67	52.39	64.03
France	43.09	43.57	45.97	49.38	47.05	52.05	50.94	52.38	48.14	48.06
Germany	55.68	52.60	54.47	47.99	30.52	33.76	31.41	30.61	27.57	40.51
Greece	79.99	78.78		74.27	65.60					74.66
Hungary	69.86	66.52	66.06	61.29	49.15	64.53	52.70	42.71	40.78	57.07
Iceland		51.28				54.65		45.81	40.50	48.06
Ireland	64.46	62.51	56.76	51.55	65.17	67.94	60.74	64.46	62.02	61.73
Israel	70.99			77.69	72.85	78.06	74.35	74.37		74.72
Italy	43.72	60.84				36.04		49.74	53.00	48.67
Latvia			71.20	57.88					38.93	56.00
Lithuania				52.41	39.41	49.61	68.87	64.03	59.21	55.59
Luxembourg	43.90	50.02								46.96
Netherlands	67.85	64.33	59.80	49.83	60.03	55.09	58.61	52.99	51.21	57.75
Norway	65.01	66.24	65.52	60.44	58.04	54.92	53.94	52.82	44.24	57.91
Poland	73.24	74.35	70.19	71.20	70.26	74.87	65.84	69.63	60.80	70.40
Portugal	68.81	70.52	72.76	75.74	67.08	77.12	43.00	45.00	34.92	61.66
Romania			71.80	67.95						69.88
Russia			69.45	67.93	66.64	67.01		63.41		66.89
Slovakia		63.29	73.19	72.55	74.66	74.09			39.55	66.22
Slovenia	70.52	70.24	65.05	59.10	64.39	57.74	52.31	55.93	64.14	62.16
Spain	53.22	54.83	65.94	66.79	68.52	70.28	67.85	67.66	53.81	63.21
Sweden	69.46	65.77	65.88	62.16	50.99	52.44	50.10	43.01	39.00	55.42
Switzerland	33.46	46.85	51.54	49.88	53.31	51.73	52.70	52.21	51.87	49.28
Turkey		50.70		65.24						57.97
Ukraine		66.59	66.42	61.50	64.41	59.10				63.60
United Kingdom	55.52	50.64	54.57	55.77	56.30	53.06	43.56	42.82	40.96	50.36
Mean	60.57	61.72	63.35	62.20	60.15	61.01	56.09	55.37	53.42	60.13

Table 5 Response rates in % taken from the documentation report

Figure 13 Top: Total amount of peaks within the distributions. Middle: Distribution of which answer is chosen to most for the core topics and the complete data set. Bottom: Distribution of peaks over the attitude landscape.

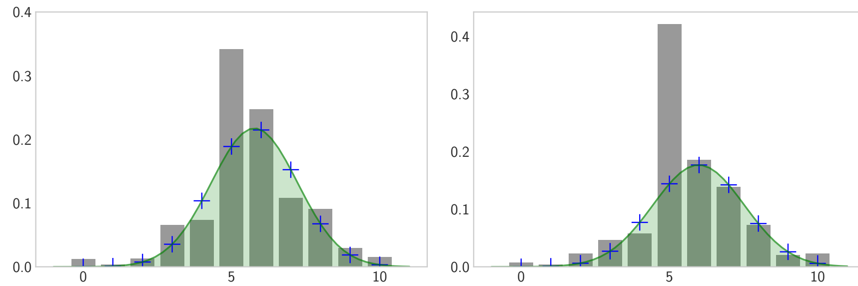

Figure 14 Distribution of CDU-Voters (2nd Vote) on the Left-Right Scale in 2004 (left) and 2014 (right)

distributions, some moderate lefts must answer larger than 5 and some moderate right smaller than 5. As an example, these people may have a more left mindset but still vote for a conservative party or the other way around. Proving this with Figure 14, in which partisans of a conservative party (CDU, left-right scale > 5 (Döring and Manow, 2018)) evaluate themselves up to 2 on the left-right scale. Looking at the results from various years this theory is far from perfect and endogenous groups are not an adequate solution. In our model w_C has a big influence of politically undecided or people answer 5 instead of "don't know" or refusal (Downey and Huffman, 2001). To prove this the question of political interest in combination with the LRSCALE, distributions of groups with different amount of political interest is shown in Figure 15.

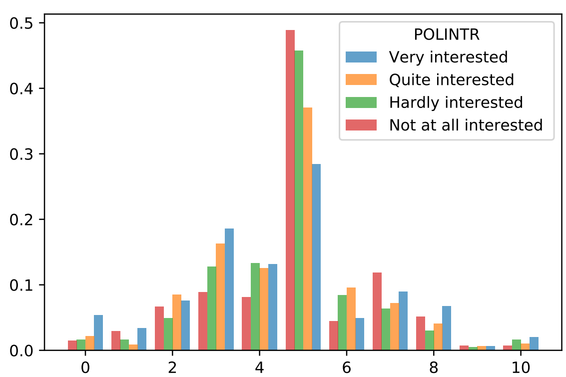

Figure 15 Distribution of the left-right scale in Germany in the year 2002 with different level of political interest.

The peak at 5 in Figure 14 also justifies this stated theory.

Appendix D: Details about the parameter estimation and the goodness-of-fit

Figure 16 shows the distributions of all 9 parameters. Here, some similarities and differences can be observed. Whereas the weights for the moderates w_{ModR} and w_{ModL} share similar shape, the standard deviation σ_{R} and σ_{L} are differentiating. σ_{L} is mostly over 1 whereby σ_{R} rarely exceeds 2 and has a more distinct shape. Due to the lack of peaks on the left hand side of the scale (see Fig. 13), the moderate left deviate more.

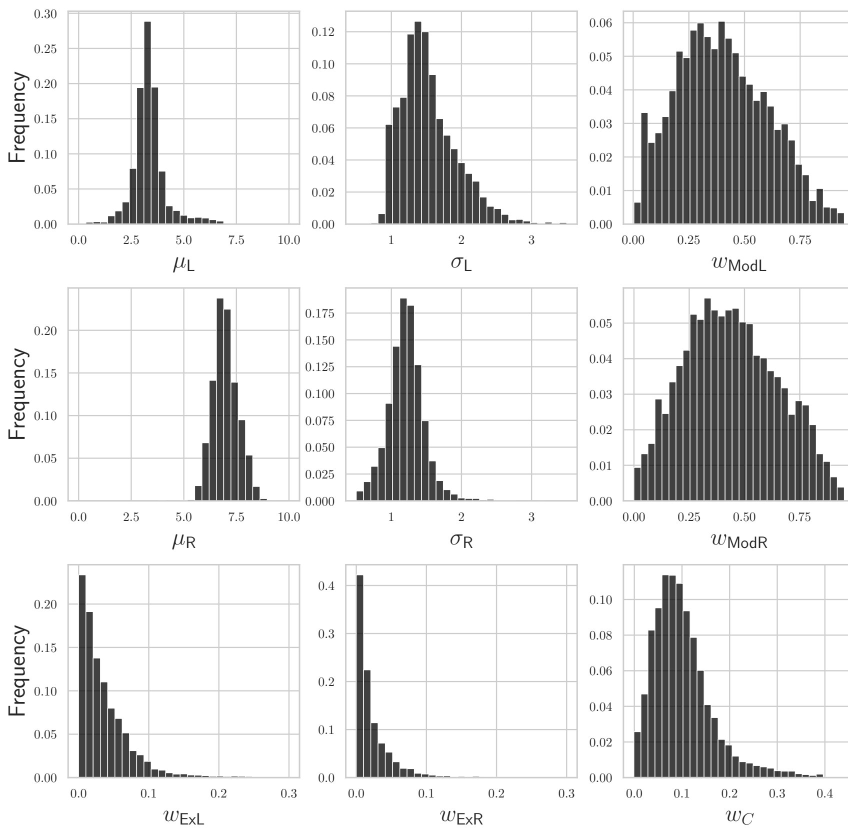

Figure 16 Distribution of the parameters using the Pentamodal Model on the ESS data set.

As further investigation of the good-of-fit the distribution of residuum is analyzed. Shown in Figure 17 is the complete data set. Due to explicit weights for right, left extremists, and centrist the box plot illustrates the least variation between empirical and model data for these groups. For the scale that is only influenced by the moderates distribution (1 to 4 and 6 to 9), the difference becomes considerably larger. Nevertheless, the box plot show small variation from the zero-line. For answer 7 and 9 the model is mostly producing too small results whereas for answer 1 and 8 results are slightly too big. These can be a result of under- or overfitting and the overall worse result for right hand side of the scale (answers > 5) can be a result of a greater amount of peak and difficulties to fit them accordingly (Fig. 13).

Figure 18 shows how the fitting parameter for assessing the three penalty terms influences the median R^2 value. Furthermore, the distribution of the three weights w_{ExL}, w_{ExR}, and w_C only for values below zero is shown on the right hand side. All three weights are close to zero and rarely exceed values of < -0.0005. Confirming the benefits of the penalty weight β.

Figure 19 shows the four attitude landscapes and Pentamodal Models with the worst coefficients of determination ($R^2 < 0.9$) in our data set. Their analysis indicates the limitation of our model.

These examples have shapes that were not considered when creating the model. Most importantly, all four landscapes show an unusually high p_1, which even appears as a peak in three of them. Also the case of EUFTF in Austria 2018 which the second most polarized EUFTF in Figure 8 has a peak at attitude one. Overall, this is extremely rare (see also Figure 13 in Appendix B). Such landscapes prevent a good fit of confined normal distributions within the definition of the Pentamodal Model. If the p_1 becomes particularly large, either the corresponding μ must shift close to extreme resulting in way too high extreme bins in the model due to overlapping tails, or, if μ remains moderate between extreme and neutral, p_1 is estimated way too low.

We speculate that some unusually large p_1 values result from errors in the survey interview on the phone. A possible explanation is that some interviewers asked people for values between 1 and 10 instead of 0 and 10. This would need further investigation which we omitted because it is out of the scope of this study and the number of cases is low.

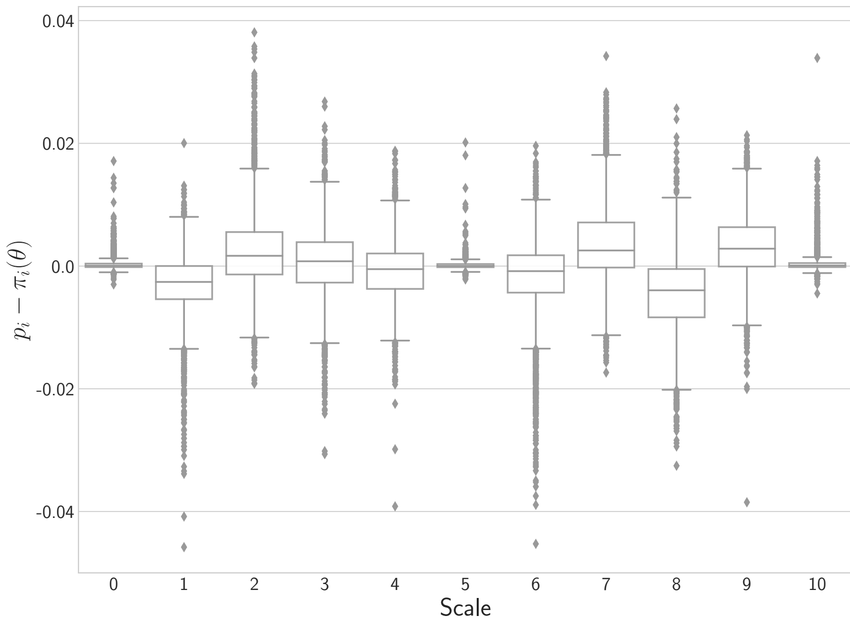

Figure 17 Distribution of the residuum between the empirical and model data.

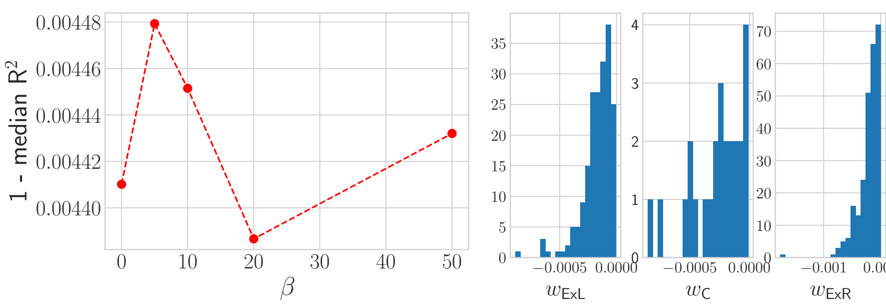

Figure 18 Left: 1 - Median R^2 for different penalty parameter β shows best result for $\beta = 20$ Right: Absolute distributions of negative residues for the weights w_{ExL}, w_C, and w_{ExR} with $\beta = 20$.

In satisfaction with health, Estonia has $p_0 = 0$ and peaks at p_1, p_4, p_6, and p_8. With two peaks at each side of the centrists, it is impossible to fit two normal distributions properly to it. This also looks like an erroneous a shift of answer categories. This is again speculation.

Finally, Ukraine's distribution is shaped like a sloped uniform distribution with a distinct greater amount of answers for bins p_0, p_4, and p_5. Distinguishing five groups within this distribution within the Pentamodal Model is impossible.

The development of the model was focused on core political topics where a group of moderate left and right is a natural assumption. On a trust or a satisfaction scale this is not so much the case. Empirically, satisfaction and trust scales and are responsible for 22 (65 %) of the cases with $R^2 < 0.95$.

Appendix E: Further Results for Cross-Country and Time Trend Analysis

In this section the further results from ranking polarization cross-country in our core political topics will be presented. Additionally, an overview of time trends in these 3 topics is included. First, the ranking on left-right polarization 20 shows less deviation between the most and least polarized country. Comparing high polarized countries like Hungary or Cyprus with less polarized ones like the Netherlands or Estonia, the histograms share an analogous structure. Furthermore all measurements of the decomposition do not correlate with overall ranking. To evaluate the polarization of a single country, one has to assess case by case. On the topic of immigration (IMUECLT) the behaviour on the higher ranks is similar to the topic of European unification (compare Fig. 8). At the lower end Pol_0, moderate individual-weighted polarization becomes larger ($\mathrm{Pol}_1^{\mathrm{Mod}}$) instead of smaller. This phenomenon

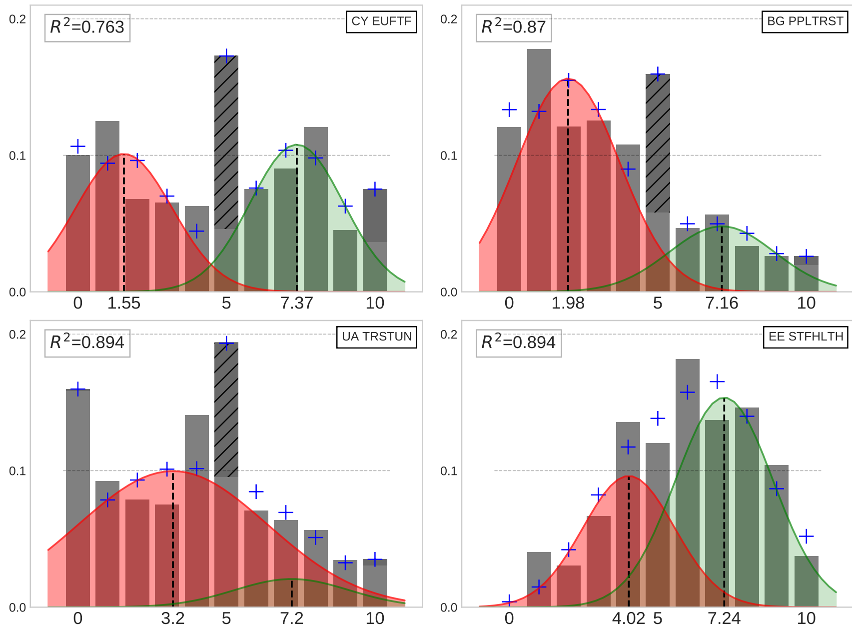

Figure 19 The four attitude landscapes where the Pentamodal Model achieves the worst goodness-of-fit ($R^2 < 0.9$).

originates from higher agreement for immigration in these countries. Therefore more people answer this question around the mean (μ_R). This concludes one of the reasons $\mathrm{Pol}_1^{\mathrm{Mod}}$ correlates negatively with Pol_0. and Figure 21 Figure 22 represent the overall polarization increase and decrease in Europe within our core political topics. For all topics we lack a general trend. Only for the LRSCALE more countries show an increase but most of them have a little to none increase which are not validated with the area of confidence.

Country	Pol_0	Pol_0^{Ex}	Pol_0^{resC}	Pol_1^{Mod}	$resPol_0^{Mod}$
HU	0.584	0.082	0.084	0.059	0.36
CY	0.569	0.132	0.124	0.035	0.278
RS	0.566	0.164	0.141	0.029	0.233
BG	0.559	0.086	0.102	0.053	0.318
PL	0.553	0.099	0.101	0.051	0.302
NO	0.545	0.059	0.055	0.086	0.344
SI	0.524	0.118	0.122	0.034	0.249
IT	0.524	0.051	0.048	0.08	0.345
FR	0.482	0.062	0.083	0.054	0.284
FI	0.471	0.036	0.073	0.072	0.29
CZ	0.454	0.033	0.065	0.067	0.288
CH	0.442	0.018	0.08	0.061	0.283
NL	0.437	0.026	0.062	0.072	0.277
BE	0.432	0.044	0.079	0.055	0.254
GB	0.416	0.05	0.088	0.046	0.233
DE	0.412	0.044	0.085	0.051	0.231
AT	0.41	0.036	0.089	0.044	0.24
IE	0.385	0.029	0.075	0.049	0.231
EE	0.384	0.042	0.099	0.034	0.209

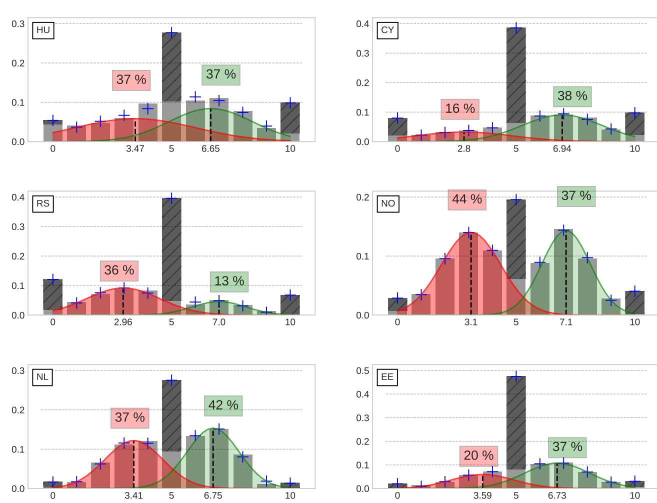

Figure 20 Left-right scale (LRSCALE) round 9 ranking. Table and histograms are sorted by Pol_0. Histograms from top left to bottom right

Country	Pol_0	$\mathrm{Pol}_0^{\mathrm{Ex}}$	$\mathrm{Pol}_0^{\mathrm{resC}}$	$\mathrm{Pol}_1^{\mathrm{Mod}}$	$\mathrm{resPol}_0^{\mathrm{Mod}}$
RS	0.708	0.252	0.111	0.036	0.309
IT	0.629	0.127	0.04	0.081	0.38
FR	0.6	0.088	0.062	0.078	0.373
AT	0.597	0.095	0.053	0.077	0.371
GB	0.596	0.103	0.053	0.081	0.359
BG	0.585	0.02	0.072	0.084	0.41
SI	0.579	0.082	0.085	0.063	0.35
DE	0.568	0.068	0.052	0.085	0.363
EE	0.545	0.071	0.072	0.068	0.334
PL	0.542	0.089	0.077	0.064	0.312
HU	0.537	0.094	0.05	0.072	0.32
CY	0.535	0.076	0.052	0.079	0.328
IE	0.529	0.055	0.049	0.092	0.334
NO	0.508	0.054	0.074	0.077	0.303
CZ	0.504	0.052	0.041	0.084	0.327
CH	0.492	0.041	0.039	0.094	0.318
BE	0.489	0.044	0.055	0.09	0.3
FI	0.442	0.032	0.048	0.1	0.261
NL	0.435	0.025	0.046	0.097	0.266

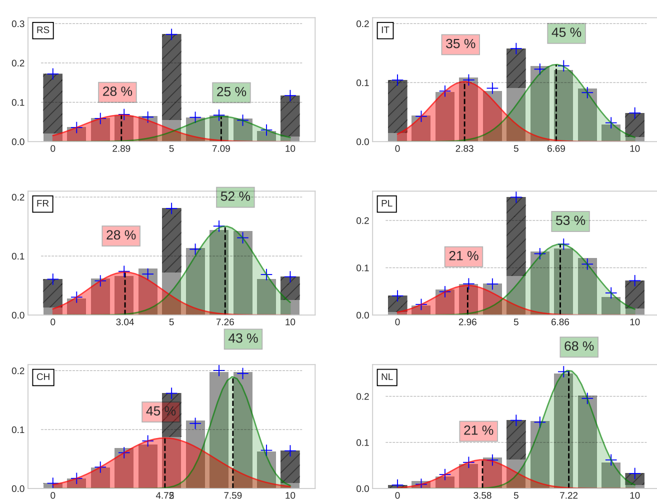

Figure 21 Immigration undermining or enriching culture (IMUECLT) 2018 ranking. Table and histograms are sorted by Pol_0. Histograms from top left to bottom right

Country	Pol_0	Pol_0^{Ex}	Pol_0^{resC}	Pol_1^{Mod}	$resPol_0^{Mod}$
SI	0.0054	0.0032	-0.0006	0.0005	0.0022
FI	0.0034	-0.0005	-0.0007	0.0003	0.0042
PL	0.003	0.0004	-0.0005	0.0002	0.003
GB	0.0023	0.0034	0.0011	-0.0005	-0.0017
DE	0.0017	-0.0014	-0.0023	0.0013	0.0041
NO	0.0012	0.0001	-0.0002	0.0	0.0012
DK	0.001	0.001	-0.0012	0.0008	0.0004
FR	0.0005	-0.001	-0.0013	0.0009	0.0019
CH	-0.0005	0.0001	-0.0011	0.0001	0.0004
IE	-0.0006	0.0026	-0.0008	0.0003	-0.0027
NL	-0.0006	-0.0005	-0.0003	0.0002	-0.0001
BE	-0.0008	-0.0001	-0.0008	0.0006	-0.0005
EE	-0.0008	0.0007	0.0006	-0.0007	-0.0014
CZ	-0.0012	0.0018	-0.002	0.0009	-0.0019
HU	-0.0013	0.0021	-0.0021	0.0007	-0.0019

Country	Pol_0	Pol_0^{Ex}	Pol_0^{resC}	Pol_1^{Mod}	$resPol_0^{Mod}$
SI	0.0039	0.002	-0.0013	0.0007	0.0024
PL	0.0037	-0.0004	0.0	-0.0001	0.0042
GB	0.0031	0.0023	0.0013	-0.0006	0.0001
IE	0.0018	0.0029	0.0003	-0.0002	-0.0011
EE	0.0001	0.0022	0.0014	-0.001	-0.0024
NL	-0.0005	0.0016	0.0007	-0.0006	-0.0024
FR	-0.0006	-0.0006	0.0002	0.0001	-0.0002
DE	-0.0012	-0.0016	-0.001	0.0007	0.0007
FI	-0.0016	-0.0009	-0.0001	0.0	-0.0007
HU	-0.0016	-0.0026	-0.0011	0.001	0.001
CH	-0.0017	0.0001	0.001	-0.0011	-0.0016
NO	-0.0023	-0.001	0.0009	-0.0009	-0.0014
BE	-0.0042	-0.0022	0.0003	-0.0001	-0.0022

Country	Pol_0	Pol_0^{Ex}	Pol_0^{resC}	Pol_1^{Mod}	$resPol_0^{Mod}$
DK	0.0086	0.0006	-0.0015	0.0016	0.0079
NO	0.0044	0.0013	-0.0005	0.0004	0.0032
GB	0.0037	0.0007	0.0001	0.0005	0.0024
PL	0.003	0.0011	0.0001	0.0004	0.0015
CH	0.0025	0.0003	-0.0002	0.0003	0.0021
HU	0.0023	0.001	-0.0015	0.0006	0.0022
IE	0.0015	0.0012	-0.0005	0.0005	0.0003
SI	0.0015	0.0014	0.0003	0.0	-0.0002
DE	0.0012	0.0007	0.0001	-0.0001	0.0005
FI	0.0012	-0.0	-0.0008	0.0005	0.0015
BE	0.0002	-0.0003	-0.0005	0.0005	0.0006
FR	-0.001	-0.0005	0.0004	-0.0003	-0.0006
NL	-0.0014	-0.0003	0.0004	-0.0003	-0.0013
EE	-0.0015	0.0008	0.0006	-0.0009	-0.002
CZ	-0.0055	-0.0007	-0.001	0.0001	-0.0039

Figure 22 Slopes of a linear fit applied to polarization measurements for three topics from top to bottom (IMUECLT, EUFTF, LRSCALE). Slopes represent the increase or decrease per year. Tables are sorted by slope of Pol_0.