Art and Complexity
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# What Lies Between Order and Chaos?

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What is a pattern? How do we come to recognize patterns that we've never seen before? Formalizing and quantifying the notion of pattern and the process of pattern discovery go right to the heart of scientific practice. Over the last several decades science's view of nature's lack of structure—its unpredictability—underwent a major renovation with the discovery of deterministic chaos. Behind the veil of apparent randomness, many processes are highly ordered, following simple rules. As the new millennium begins, tools adapted from the theory of computation will bring empirical science to the brink of automatically discovering patterns and quantifying their structural complexity. For example, rather than interpreting a data stream according to a given model, we look at a model stream. The regularities found in the *way* models improve with learning is the basis for inferring universal laws on how complexity arises from the interaction of order and chaos.

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## 1 INTRODUCTION

During the Summer of 1927 Balthasar van der Pol, a Dutch engineer, listened to the tones produced by a neon glow lamp coupled to an oscillating electrical circuit.

Lacking modern electronic test equipment, he monitored the circuit's behavior by listening through a telephone ear piece. In what is probably one of the earlier experiments on electronic music, he discovered that, by tuning the circuit as if it were a musical instrument, fractions or subharmonics of a fundamental tone could be produced [27]. This is markedly unlike common musical instruments—such as the flute which is known for its purity of harmonics, or multiples of a fundamental tone. As van der Pol and a colleague reported in the September 10th issue of the British journal *Nature* that year "the turning of the condenser in the region of the third to the sixth subharmonic strongly reminds one of the tunes of a bag pipe."

There is a curious aside in the report, however. The experimenters noted that when tuning the circuit "often an irregular noise is heard in the telephone receivers before the frequency jumps to the next lower value." We now know that van der Pol had listened to deterministic chaos: the noise was produced in an entirely lawful, ordered way by the circuit itself. The *Nature* report stands as one of its first experimental discoveries. Other concerns were on the experimenters minds, for the report immediately continues "However, this is a subsidiary phenomenon...." With this remark their primary interest in the design of stable radio oscillators led them away from discovering the order in the chaos.

Much of our appreciation of nature depends on whether our minds or, more typically these days, our computers are prepared to discern its intricacies. When confronted by a phenomenon for which we are ill-prepared, we often simply fail to see it, though we may be looking directly at it.

Indeed, what is a "pattern" in nature? More to the point, how do we come to notice a "pattern" we've never seen before? How can we ever see past our own assumptions? Formalizing and quantifying the notion of pattern goes right to the heart of scientific practice. Over the last several decades our view of nature's lack of structure—its unpredictability—underwent a major renovation with the discovery of deterministic chaos. As the new millennium begins, ideas adapted from the theory of computation will bring empirical science to the brink of automatically discovering patterns and quantifying their structural complexity. One guide to this will be universal laws on how complexity arises from the interaction of order and chaos.

## 2 BACKGROUND

Van der Pol and his colleague J. van der Mark apparently were unaware that the deterministic mechanisms underlying the noises they'd heard had been rather keenly analyzed three decades earlier by the French mathematician Jules Henri Poincaré in his efforts to establish the orderliness of planetary motion. The motion of the planets about the sun is one of the hallmarks of regularity and predictability. But when mathematicians and physicists attempted to finally prove this, trouble arose. At the very close of the nineteenth century Poincaré in his treatise *Nouvelles Méthodes des Mécanique Céleste* focused on the collective motion of the sun, a planet, and a moon—the famous "three-body problem" [23]. After nearly 1500 pages of detailed successful analysis and simplification, he ran into deep complications in solving for their motions:

> If one seeks to visualize the pattern formed by these two [solution] curves...[their] intersections form a kind of lattice-work, a weave, a chain-link network of infinitely fine mesh; ... One will be struck by the complexity of this figure, which I am not even attempting to draw. Nothing can give us a better idea of the intricacy of the three-body problem, and of all the problems of dynamics in general....

It is impossible to know his state of mind when, at the end of a Herculean effort to establish mathematically the observed fact of the solar system's stability, Poincaré realized the daunting complexity of the task. He dryly notes that it was "a point which gave me a great deal of trouble."

The puzzle of deterministic chaos is just one example from twentieth-century science that shows how the limitations of human understanding make nature appear "noisy," complicated, and unpredictable. One immediately thinks of quantum mechanics, another legacy from the early part of the twentieth century, as putting severe limits on purely objective measurements of nature [15]. But even in the rarefied world of the foundations of mathematics similar roadblocks appeared. Kurt Gödel demonstrated that logical consistency had to be traded-off against one's ability to prove the possible "truths" within a formal system—even a system as simple as arithmetic [21]. Alan Turing then showed, more concretely, that well-formulated questions in a formal system may have no constructive answers [26]. More recently, Gregory Chaitin has argued that there is an irreducible element of randomness in mathematics that limits its effectiveness [3].

Psychology and philosophy in the twentieth century were punctuated by a series of similar disappointments. Limitations and the complication they engender permeate much more than just mathematics and physics. Freud, to take one example, called into question the Western concept of a whole and knowable self controlling the mind. He viewed all apparently spontaneous arbitrary actions as being at the beck and call of the unconscious, of which one could have no knowledge [12, 13]. Derrida then deconstructed the remaining notion of a self, which was based, he thought, on erroneous notion of a metaphysical presence. Derrida, it seems, would have us believe that there is chaos in our own houses [11].

These limitations suggest that humans are strongly predisposed to make many unjustified, often unspoken, simplifying assumptions about nature and experience. At first, these assumptions are frustrated and the world appears complicated, structureless, and random. Once they are finally acknowledged and become an object of study, a "new" limitation on our knowledge is discovered.

This list of limitations, which could be easily extended, paints a rather pessimistic picture of the progress of human knowledge. But it also raises a constructive question: If things are so complicated, how do we ever discover patterns and regularity? Is a hurricane's path really unpredictable or is there some hidden order that we do not yet appreciate? How can the lawfulness producing deterministic chaos ever be extracted, if its outward appearance is random?

These questions highlight the very activity by which scientists penetrate the veil of complication and distill new laws from experiments. How do scientists balance the need for order against nature's seeming chaos? As certainly as we have come to appreciate our limitations, this century has fostered an unparalleled increase in our knowledge of nature. Somewhat ironically, the realization of each limit, rather than being only a disappointment, often showed nature to be much richer than before, when seen through the dark sunglasses of simplifying assumptions. The rapid increase in knowledge suggests there may be some driving force behind this progress. Can we be scientific about the practice of science? Is there a dynamic of discovery?

Certainly, whatever this dynamic is, it is not unfamiliar to us. The structural anthropologist Claude Levi-Strauss describes the process as he experienced it during his first treks in the 1930s into the Amazon [18]:

> Seen from the outside, the Amazonian forest seems like a mass of congealed bubbles, a vertical accumulation of green swellings; it is as if some pathological disorder had attacked the riverscape over its whole extent. But once you break through the surface-skin and go inside, everything changes: seen from within, the chaotic mass becomes a monumental universe. The forest ceases to be a terrestrial distemper; it could be taken for a new planetary world, as rich as our world, and replacing it.
>
> As soon as the eye becomes accustomed to recognizing the forest's various closely adjacent planes, and the mind has overcome its first impression of being overwhelmed, a complex system can be perceived.

Despite confronting what initially appears to be structurelessness, we seem to be able eventually to discover the hidden order.

One of the most fascinating spontaneous pattern discovery and learning processes is a child's acquisition of language. Imagine the trade-offs that an infant faces in balancing the initial apparent structurelessness of what it hears and its need to find order. Allison Gopnik, a child psychologist at UC Berkeley, has suggested that infants in their developmental succession of world views are like scientists, forming and testing hypotheses and rejecting those that are unhelpful, inconsistent, or too complicated [14].

Natural language itself shows a balance between order and randomness [5]. On the one hand, there is a need for static structures, such as a vocabulary and a grammar, so that two people can communicate. Without a prior agreement on these there is no basis for understanding; each and every utterance would be unintelligible to the listener—a common experience for the world traveler. On the other hand, there would be no need to communicate if spoken utterances were completely predictable by the listener. In this case the language would be a rigidly fixed structure with all possible sentences uniquely identified and identifiable. But humans use language (typically) to communicate new information—facts, ideas, feelings, and other states of mind. And so, there must be an unknown or unexpected element in communication as far as the listener is concerned, if they are to stay engaged. Then again the "new" element cannot be so dominant that the result is a jumble of phonemes, words, and sentences. Natural language as a changeable and dynamic system must be a balance of new information unpredictable by the listener and of order so that communication is understandable.

Is there a general principle that guides the dynamic balance of order and chaos? And what is the result of this balance? In his *Process and Reality* [28], the

British philosopher Alfred North Whitehead comments on the interplay of order and chaos in art:

> The same principle is exhibited by the tedium arising from the unrelieved dominance of fashion in art. Europe, having covered itself with treasures of Gothic architecture, entered upon generations of satiation. These jaded epochs seem to have lost all sense of that particular form of loveliness. It seems as though the last delicacies of feeling require some element of novelty to relieve their massive inheritance from bygone system. Order is not sufficient. What is required, is something much more complex. It is order entering upon novelty; so that the massiveness of order does not degenerate into mere repetition; and so that the novelty is always reflected upon a background of system.

So is it complexity that is the result of the balance of order and chaos? But what is complexity? Are there any general principles that govern the interplay of order and chaos, that aid in detecting structure and pattern? How does genuinely new information arise from a structureless universe? Finally, why do humans presume that there is order to be found in a chaotic, uncharted nature? Recent work has begun to elucidate this drive toward finding regularity in nature and, in particular, the trade-offs between order and chaos that occur in the process of acquiring new knowledge.

## 3 COMPLEXITY

The weather is often considered a prime example of unpredictable behavior. The simple truth, though, is that it is quite predictable. Over the period of one minute (say), one can surely predict it. With a glance out the nearest window to note the sky's disposition, one can immediately report back a forecast. To predict over one hour, one would search to the horizon, noting much more of the sky's prevailing condition. Only then, and not without pause to consider how that might change during the hour, would one offer up a tentative prediction. If asked to forecast two weeks in advance one would probably not even attempt the task. Why even look out the window? The necessary amount of information and the time to assimilate it for a two-week forecast would be overwhelming. Despite the long-term unpredictability, a meteorologist can write down the equations of motion for the forces controlling the weather dynamics in each case. In this sense, the weather's behavior is symbolically specified in its entirety. How does unpredictability arise in such a situation?

One meteorologist, Edward Lorenz of MIT, did analyze the equations governing weather dynamics with this question in mind [19]. Focusing on particularly simple deterministic equations, in 1963 Lorenz proposed a mechanism—the butterfly effect—that actively amplifies even the most microscopic, and uncontrollable, events to macroscopic proportions; this is *the* mechanism underlying *deterministic chaos*. Imagine that a meteorologist is allowed to use as much historical weather data and as much computer time as needed for a moderately accurate four-day forecast. What Lorenz found was that if the meteorologist tries to extend the forecast for one additional day, while maintaining the same degree of accuracy, *twice* as much historical data and computer time are required. The result is that, if deterministic chaos is present, there is an irreducible error in long-term predictions, since the required resources grow so quickly: If the resources required for predicting double with each additional day, extending the forecast by only ten days requires a thousand-fold increase, rapidly overwhelming any effort to forecast.

We now measure this degree of long-term unpredictability using the *entropy rate*, a quantity introduced by Claude Shannon in his theory of communication that measures the degree of surprise when receiving messages produced by some source [25]. The Russian mathematician Andrei Kolmogorov adapted this to view a deterministic chaotic system—such as the three-body problem—as an information source [16]: As observers, we are surprised when our predictions of its behavior fail. If we measure the state of a system to an accuracy of one part in a thousand and if the system doubles that measurement uncertainty every second, then after one second we know the system state to only one part in five hundred. In terms of Shannon's entropy rate, the system has produced one bit of information, since we can resolve only half as many distinct states. At that rate of information production, the system is completely unpredictable after only ten seconds.

Lorenz's work suggested that unpredictability was inherent in very large systems, such as the weather, not only in systems with a few components, such as the three-body problem analyzed by Poincaré. Their work left open the question of how a chaotic system is structured to support a given degree of unpredictability. In 1982 Norman Packard and I proposed that the structural complexity of a process, such as the weather, could be measured by the decay in one's ability to predict its behavior as one accumulates additional information [8]. We called this complexity the *excess entropy*, since it captured the initial apparent disorder above the long-term unpredictability.

To see how this works we envisioned a meteorologist making a succession of observations. Initially, before any measurements are made, the weather could be anything; the meteorologist is ignorant of the prevailing conditions and forecasts have nothing to do with the actual weather. It is highly unpredictable; the entropy rate apparent to the meteorologist is very large. After a few observations, though, the meteorologist knows the current condition and has the possibility of noticing regularities: Are the conditions changing? By how much and in what way? The additional information allows much better forecasts, certainly, than before observations were begun. As more information is accumulated through succeeding observations, the accuracy of forecasts continues to improve until the ceiling imposed by the weather's inherent unpredictability is reached.

The excess entropy was invented to monitor just how this increase to optimal forecasting comes about. To see how it differs from the entropy rate, which sets the ceiling on long-term unpredictability, consider three different types of weather. The first is a sunny day, with clear and calm skies. This weather behavior is very easy to predict: once we know the current wind velocity, temperature, and humidity, we forecast that they will continue. If we make further observations, there are no surprises; the entropy rate is zero, the system is not chaotic. We also come to notice the regularity very quickly. Just one observation of the temperature, wind velocity, and so on, is all that is required to set up the forecast. In this case, the excess entropy is low, since only a few observations are required to know the prevailing (exactly predictable) conditions.

The second example comes from the other extreme. Imagine we are in the crush of a horrendous storm, wild winds and sudden downpours pelt the land, with no chance of letting up. This weather behavior—the change in wind direction, the variation in local temperature and humidity—is very difficult to predict. We are maximally uncertain about the weather: we keep looking out the window for an update and are constantly surprised; the entropy rate is high. We come to appreciate this high unpredictability very quickly, after only a few observations. We also immediately realize that it's not really worth the effort to accumulate detailed observations and have our computers (say) develop a forecast, since the conditions are so changeable. In this case, as for the calm weather, the excess entropy is low. Independent of the weather's predictability, only a few observations are required to learn its condition (highly unpredictable). In other words, highly predictable *and* highly unpredictable behaviors are *simple*, since the method of forecasting is so straightforward. For the calm weather we simply report that our first observations will continue. For the stormy whether, we make our forecasts by flipping a coin. In both cases, after a while we don't even bother to look out the window.

The genuinely interesting cases fall between these two extremes. Instead of our forecasts being either exactly right or almost always wrong, imagine weather that regularly alternates between clear skies and cloud bursts. When it is clear, we certainly want to know this, since for that period our forecasts will be correct. It is also useful to know when the weather switches to being stormy. Since our forecasts, then, will be wrong on average, we can reduce our effort to predict and go back to simply guessing. To make optimal forecasts in this situation, we must monitor the weather closely: Is it clear or stormy? Since half our forecasts are wrong, the entropy rate is somewhere between zero and the maximal value: there are some elements that are predictable. But it takes a long time to appreciate just what those elements are and the amount of effort used to take advantage of them for optimal forecasts is quite high. The result is that the excess entropy is large, unlike that found at the extremes of predictability. This intermediate behavior is more "complex" than either extreme. One needs more observations to know the prevailing conditions, our models need to be more sophisticated, and the effort to forecast is larger. In short, more information is required for optimal prediction in this intermediate case.

These examples serve to illustrate a general principle that as one moves across the spectrum of predictability—from ordered to random behavior—the "complexity" is maximized in the middle. The excess entropy is one measure of how processes are structured and it is a necessary tool for our understanding how nature comes to appear more or less predictable to an observer. Since it was introduced, a number of similar proposals to measure "physical complexity" have appeared [29]. Like the excess entropy, each alternative attempts to capture the amount of information processing that a system employs to produce its unpredictability. Their main failing, however, is that they do not tell us *how* that information is processed.

As a first step to address this, in 1987 Bruce McNamara and I showed how one could extract from experimental data the underlying equations of motion, a compact representation of the governing forces [7]. Although our approach addressed certain issues of automated modeling, its main problem was that there appeared to be no way estimate from the symbolic equations of motion how much information processing was being performed by the system.

To remedy this, in 1989 Karl Young and I introduced a method to reconstruct from observations the hidden computational mechanisms underlying unpredictable behavior [9]. We adapted several ideas from the earliest days of computers, in particular those introduced by Noam Chomsky, the MIT linguist [4]. To Chomsky, the activity of building a grammar for a language was analogous to the construction of a scientific theory from experimental data. He proposed a range of distinct grammar types in order to capture different classes of linguistic capability. Though the essential aspects of human language still elude this approach, Chomsky's classification scheme was a boon for the study of computer languages and various types of computational device. What Young and I did was turn Chomsky's analogy between linguistic- and scientific-theory building inside out. We viewed the goal of a scientist as extracting from experimental data the linguistic structure of natural processes. This differs from *pattern recognition* in which data is compared against a pre-existing palette of patterns. Moreover, ours is not a qualitative approach but a quantitative one.

We developed a procedure—$\epsilon$-machine reconstruction—to automate the discovery of grammatical rules hidden in experimental data. The rules were the "significant" patterns or regularities that govern the process which produced the data and that could be used to develop optimal predictions. The collection of the rules so discovered forms a "theory" of the process, in the sense that they model its mechanisms and allow us to make predictions about behavior that has yet to be observed.

In several ways, $\epsilon$-machine reconstruction is analogous to a procedure, introduced by Norman Packard, Doyne Farmer, Rob Shaw, and myself, in 1980 for transforming experimental data into a geometric view of the "strange attractors" underlying deterministic chaos. In this light, the work with McNamara showed how this geometric approach could be extended to produce compact symbolic equations that governed the behavior on the attractors.

One fallout of $\epsilon$-machine reconstruction was a much more refined notion—the *statistical complexity*—of information processing structures found in nature. Just as the excess entropy is complementary to the entropy rate, the statistical complexity as a measure of computation is complementary to the algorithmic notions of randomness introduced by Andrei Kolmogorov and Gregory Chaitin [2, 17]. Roughly speaking, the statistical complexity measures the amount of memory in a process; while Kolmogorov and Chaitin's *algorithmic entropy rate* measures how random a process is, when viewed as a computer. Thus, there can be a range of (structurally distinct) processes that each appears to be equally unpredictable, but that use different amounts of memory to produce that apparent randomness.

Young and I also introduced a useful graphical device—the complexity-entropy diagram—that reveals the range of information processing that natural systems can exhibit [9]. The complexity-entropy diagram is analogous to the thermodynamic *phase diagrams* introduced in the nineteenth century to map out the states of matter—solid, liquid, gas—at different conditions of temperature, pressure, and volume. It's different, though, in that it is based not on varying physical parameters, but on information processing coordinates: the rate at which information is produced (entropy rate) and how much memory is used to produce it (statistical complexity).

When we analyzed the boundaries between chaotic and predictable systems, we realized that the analogy with nineteenth-century thermodynamics was deeper than we had first thought. Just as water changes state in going from ice to liquid with increasing temperature, certain classes of information processing systems show phase transitions between order and chaos. The ordered regime is analogous to a crystalline solid; it literally corresponds to fixed crystalline patterns in time (periodic behaviors). The chaotic regime is analogous to a gas, in which the molecular motion is much more disordered. We demonstrated that at a order-chaos phase transition a new and qualitatively more powerful type of computation appears [10].

While different classes of natural process have their own computational-phase diagrams, our work suggested there are universal laws governing the interplay of the entropy rate and statistical complexity. It also indicated that there is organization at a higher level of understanding than the accounting of energy flows typically done in physics: the level of how natural systems store and process information and perform computations. Curiously, this view of the increase of complexity at the onset of chaos says, in a self-reflexive way, something more about the process of building scientific theories [5, 6].

## 4 THEORY

A key modeling dichotomy that runs throughout all of science is that between order and randomness. Imagine a scientist in the laboratory confronted after days of hard work with the results of a recent experiment—summarized prosaically as a simple numerical recording of instrument responses. The question arises, What fraction of the particular numerical value of each datum confirms or denies the hypothesis being tested and how much is essentially irrelevant information, merely "noise" or "error"?

This dichotomy is probably clearest within science, but it is not restricted to it, being a constant presence in the creation of artworks or in the engineering of artificial systems: What part of what we see or design is meaningful or functional? In many ways, this caricature of scientific investigation—"artificial science"?—gives a framework for understanding the necessary balance between order and randomness that appears whenever there is an "observer" trying to detect structure or pattern in its environment. The general puzzle of discovery then is: Which part of a measurement series does an observer ascribe to "randomness" and which part to "order" and "predictability?" Aren't we all in our daily activities to one extent or another "scientists" trying to ferret out the usable from the unusable information in our lives?

Given this basic dichotomy one can then ask: How does an observer actually make the distinction? The answer requires understanding how an observer models data—that is, the method by which elements in a representation, a "model," are justified in terms of given data.

A fundamental point is that *any* act of modeling makes a distinction between data that is accounted for—the ordered part—and data that is not described—the apparently random part. However, where to draw the line between theory and error is not so clear. The problem of building too complicated a model to fit all those things you want to explain is a familiar one in science. Jorge Luis Borges, the Argentine writer, illustrates the pitfall of "overfitting" in a faux critique of a nonexistent *Celestial Emporium of Benevolent Knowledge* [1], thusly:

> On those remote pages it is written that animals are divided into (a) those that belong to the Emperor, (b) embalmed ones, (c) those that are trained, (d) suckling pigs, (e) mermaids, (f) fabulous ones, (g) stray dogs, (h) those that are included in this classification, (i) those that tremble as if they were mad, (j) innumerable ones, (k) those drawn with a very fine camel's brush hair, (l) others, (m) those that have just broken a flower vase, (n) those that resemble flies from a distance.

As a general theory of "animal" the *Celestial Emporium* strikes us as being, at some points, too general and, at others, far too specialized, including too much "noise." Even without being a trained zoologist, one suspects that when presented with a candidate "animal" previously unknown to us, the *Celestial Emporium* may very well not help us in deciding whether or not it is an animal. As a scheme it does not generalize very well.

In principle, a balance between order and randomness can be reached and used to define a "best" model for a given data set. A balance can be found by minimizing the model's size while simultaneously minimizing the amount of apparent randomness or error. The first part is a version of Ockham's dictum [22]: causes should not be multiplied beyond necessity. The second part is a basic tenet of science: obtain the best prediction of nature. Neither component of this balance can be minimized alone, otherwise absurd "best" models would be selected. Minimizing the model size alone leads to huge error, since the smallest (null) model captures no regularities—all of the data appears to be noise; minimizing the error alone produces a huge model, which is simply the data itself and manifestly not a useful encapsulation of what happened in the laboratory. So both model size and the induced error must be minimized together in selecting a "best" model. Typically, the sum of the model size and the total error are minimized [24].

From the viewpoint of scientific methodology the key element missing in this view of modeling is how to measure structure or regularity. Just how structure is measured determines where the order-randomness border is set. This particular problem can be solved in principle: we take the size of the candidate model as the measure of structure. Then the size of the "best" model is a measure of the data's intrinsic structure. If we believe the data is a faithful representation of the raw behavior of the underlying process, this then translates into a measure of structure in the natural phenomenon originally studied.

After a little reflection one realizes, though, that this does not really solve the problem of quantifying structure. In fact, it simply elevates it to a higher level of abstraction. Measuring structure as the length of the description of the "best" model assumes one has chosen a language in which to describe models. The catch is that this representation choice builds in its own biases. In a given language some regularities can be compactly described, in other languages the same regularities can be quite baroquely expressed. For example, on the one hand, it is well known that, sentence for sentence, the German language expression of a thought is longer than the English equivalent. On the other, the sentiment captured in the single German word "freudenschade" has no equivalent in English and is translated to the longer phrase "happiness at other's distress". Change the language and the same regularities can require more or less description. And so, given that there is no prior God-given knowledge of the appropriate language for nature, a measure of structure in terms of the description length is, at root, arbitrary.

And so we are left with a deep puzzle, one that precedes measuring structure: How is structure discovered in the first place? If the scientist knows beforehand the appropriate representation for an experiment's possible behaviors, then the amount of that kind of structure can be extracted from the data as outlined above. In this case, the prior knowledge about the structure is verified by the data if a compact, predictive model results. But what if it is not verified? What if the hypothesized structure is simply not appropriate? Perhaps we've started out our data analysis with the wrong assumptions, the wrong representation. The "best" model could be huge or, worse, appear upon closer and closer analysis to diverge in size. The standard example of this is the Fourier—or frequency or sinusoidal or periodic—representation of the on-off "square wave". The Fourier representation describes the square wave as consisting of an infinite number of active frequencies; when, in fact, the square wave is described quite compactly (and exactly) as a "half on, half off" signal. The situation of an infinitely large model is clearly not tolerable. For one thing, it is impractical to manipulate. These situations indicate that the behavior is so new as to not fit (finitely) into current understanding. Then what do we do?

This is the problem of *innovation*. How can an observer ever break out of inadequate model classes and discover appropriate ones? How can incorrect assumptions be changed? How is anything new ever discovered, if it must always be expressed in the current language? If the problem of innovation can be solved, then, as all of the preceding development indicated, there is a framework which specifies how to be quantitative in detecting and measuring structure. One approach to this problem is hierarchical $\epsilon$-machine reconstruction [5]. In this, one starts with the simplest assumptions about the world and then builds a succession of more sophisticated languages as the assumptions prove inadequate. $\epsilon$-Machine reconstruction plays a central role in this because we use it to discover regularities, not in the raw data, but in a series of increasingly accurate models. Thus, we replace the data stream with a "model stream" and the regularities discovered form the basis of a new language that describes how less-accurate models are transformed into more-accurate ones.

# 5 CONCLUSION: THE MIDDLE GROUND

Copernicus said that the earth is not the center of the universe; Freud believed that our conscious self is the tip of an unknowable psychological iceberg. Gödel proved that there are limits to logical analysis; Turing, that answers can be beyond our reach; Poincaré that determinism leads to unpredictability; and Heisenberg that physical determinism fails on short temporal and small spatial scales.

The beautiful irony is that the result of each one of these concessions is an appreciation that the natural world is richer; that it is more structurally complex than we had previously thought. As individuals and as a culture we seem to be continually in a self-generated illusory state: saddled with implicit and naive assumptions about our ability to understand and control nature. These assumptions are only effective by dint of coincidence—in the sense that they are **not** nature, only feeble reflections of it. One might be tempted to view intellectual history as unkind, a continuing stripping away of these illusions. On retrospect, though, with each new fall, new knowledge and new understanding emerges.

Stepping back a bit, we now know that complexity arises in the middle ground, at the onset of chaos—the order-disorder border. Natural systems that evolve with and learn from interaction with their immediate environment exhibit both structural order and dynamical chaos. Order is the foundation of communication between elements at any level of organization, whether that refers to a population of neurons, bees, or humans. For an organism order is the distillation of regularities abstracted from observations. An organism's very form is a functional manifestation of its ancestor's evolutionary and its own developmental memory.

A completely ordered universe, however, would be dead. Chaos is necessary for life. Behavioral diversity, to take an example, is fundamental to an organism's survival. No organism can model the environment in its entirety. Approximation becomes essential to any system with finite resources. Chaos, as we now understand it, is the dynamical mechanism by which nature develops constrained and useful randomness. And from it follow diversity and the ability to anticipate the uncertain future.

There is a tendency, whose laws we dimly comprehend, for natural systems to balance order and chaos, to move to the interface between predictability and uncertainty. The result is increased complexity. This often appears as a change in a system's computational capability. The present state of evolutionary progress suggests that one need go even further and postulate a force that drives in time toward successively more sophisticated and qualitatively different computation. We can look back to times in which there were no systems that attempted to model themselves, as we do now. This is certainly one of the outstanding puzzles: How can lifeless and disorganized matter exhibit such a drive? And the question goes to the heart of many disciplines, ranging from philosophy and cognitive science to evolutionary and developmental biology and particle astrophysics. The dynamics of chaos, the appearance of pattern and organization, and the complexity quantified by computation will be inseparable components in its resolution.

Are these considerations too abstract to apply to contemporary social issues? I think not. At the very minimum, in a mathematical setting, understanding the interaction of order and chaos and the resulting complexity gives us a powerful set of metaphors for understanding more complicated (possibly complex) systems, such as human culture. In his *Process and Reality* [28], Whitehead saw a rather similar dynamic at work:

> The social history of mankind exhibits great organizations in their alternating functions of conditions for progress, and of contrivances for stunting humanity. The history of the Mediterranean lands, and of western Europe, is the history of the blessing and the curse of political organizations, of religious organizations, of schemes of thought, of social agencies for large purposes. The moment of dominance, prayed for, worked for, sacrificed for, by generations of the noblest spirits, marks the turning point where the blessing passes into the curse. Some new principle of refreshment is required. The art of progress is to preserve order amid change, and to preserve change amid order. Life refuses to be embalmed alive. The more prolonged the halt in some unrelieved system of order, the greater the crash of the dead society.

Can we as individuals come to appreciate the dynamic balance of order and chaos? Will our societies self-organize into a dynamic that moves beyond the least common denominator results characteristic of human groupings, toward an organization that is appreciative of diversity, understands the role of regularity, and that is truly and constructively complex? Economies, the scientific community, international relations, and other societal groupings are extremely large, complicated systems. Nonetheless, in the more limited and abstract realm of mathematics and physics we are beginning to see some glimmers of order amid the chaos, to appreciate the constructive role of randomness, and to understand the dynamic interplay of order and chaos. What lies between order and chaos? The answer now seems remarkably simple: Human innovation. The novelist and lepidopterist Vladimir Nabokov appreciated more deeply, than many, the origins of creativity in this middle, human ground [20]:

> There is, it would seem, in the dimensional scale of the world a kind of delicate meeting place between imagination and knowledge, a point, arrived at by diminishing large things and enlarging small ones, that is intrinsically artistic.

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