Moderovacie a renderovacie techniky

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https://github.com/frantisekdracek/Prezentacie/tree/main

Projection 3D to 2D

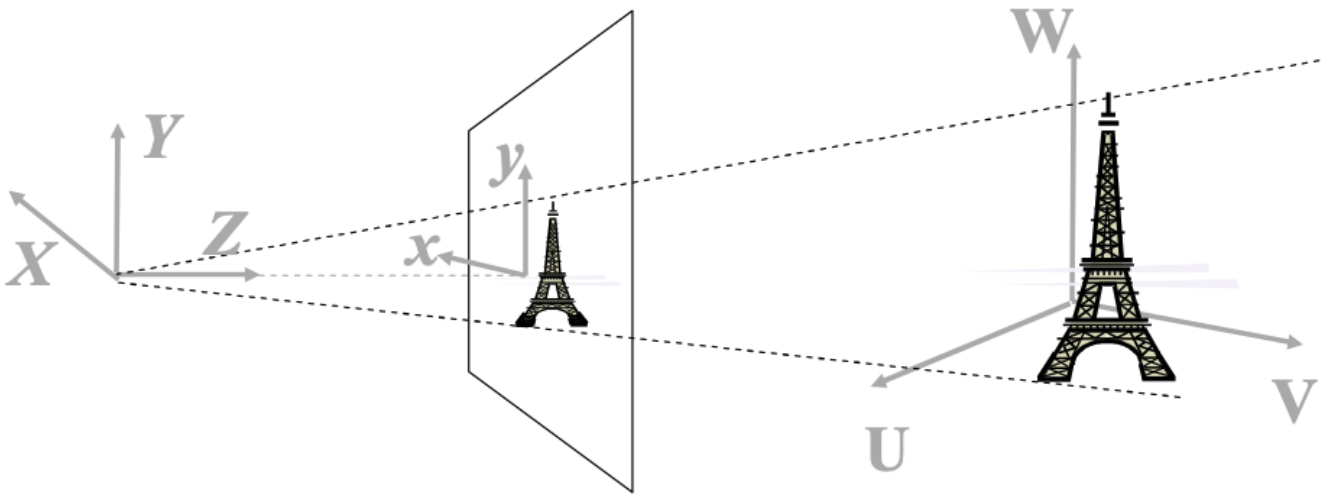
Basic projection

- need to projects 3D objects on 2D screens
- point a $\rightarrow x - y$ plane.
- simplest projection:

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{1}$$

Projection 3D to 2D

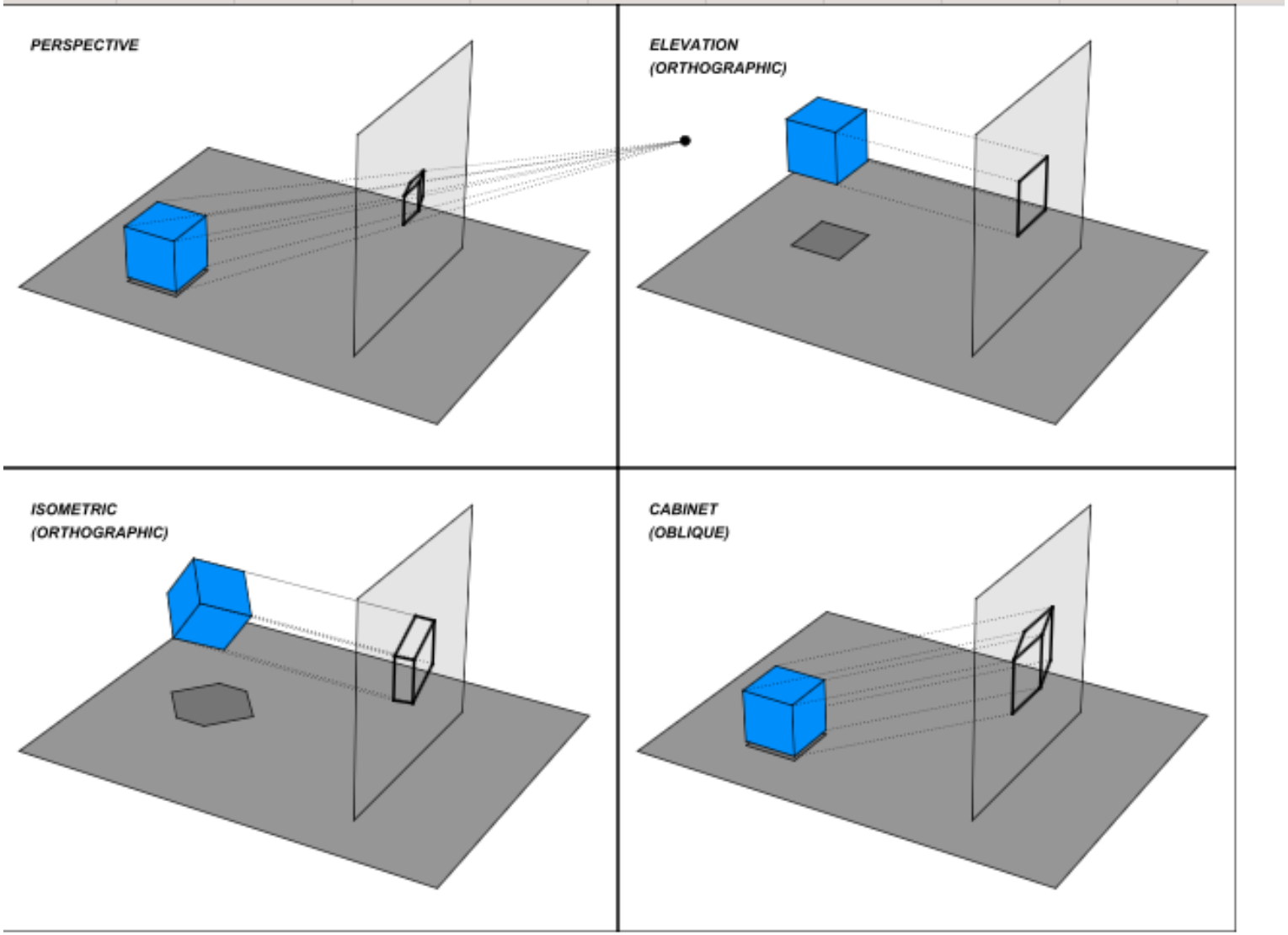
Perspective projection



Obr.: Example of perspective geometry projection

Projection 3D to 2D

Different types of projections



Obr.: Various projections of cube

Projection 3D to 2D

Scheme

world coordinates $\rightarrow$ camera coordinates $\rightarrow$ film coordinates $\rightarrow$ screen coordinates

Projection 3D to 2D

World to camera

- point in space a
- camera position c
- translate to origin: a − c
- homogeneous coordinates:

$$\begin{bmatrix} \tilde{a}_x \\ \tilde{a}_y \\ \tilde{a}_z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -c_x \\ 0 & 1 & 0 & -c_y \\ 0 & 0 & 1 & -c_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \\ 1 \end{bmatrix} \tag{2}$$

Projection 3D to 2D

World to camera

- camera orientation θ
- rotate camera coordinates: $R(\theta)$
- in homogenous:

$$R = \begin{bmatrix} R_{11} & R_{12} & R_{13} & 0 \\ R_{21} & R_{22} & R_{23} & 0 \\ R_{31} & R_{32} & R_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

- combined:

$$\begin{bmatrix} \tilde{\tilde{a}}_x \\ \tilde{\tilde{a}}_y \\ \tilde{\tilde{a}}_z \\ 1 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & 0 \\ R_{21} & R_{22} & R_{23} & 0 \\ R_{31} & R_{32} & R_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -c_x \\ 0 & 1 & 0 & -c_y \\ 0 & 0 & 1 & -c_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \\ 1 \end{bmatrix} = RT\mathrm{a} \tag{4}$$

Projection 3D to 2D

Camera to film - Perspective

- simple perspective:

$$\mathrm{P} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{5}$$

- general perspective matrix:

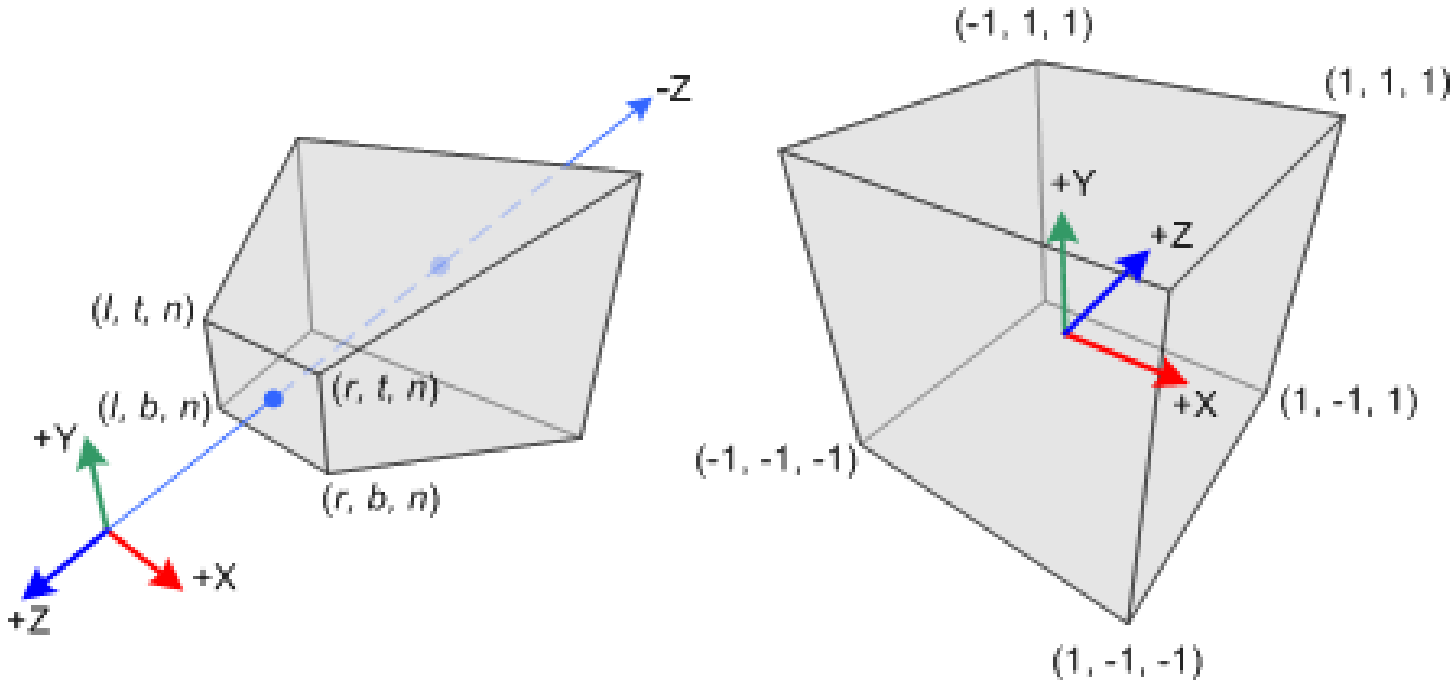
$$\mathrm{P} = \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix} \tag{6}$$

- homogenize resulting vector:

$$\begin{bmatrix} \tilde{\tilde{a}}_x \\ \tilde{\tilde{a}}_y \\ \tilde{\tilde{a}}_z \\ w \end{bmatrix} \rightarrow \begin{bmatrix} \tilde{\tilde{a}}_x / w \\ \tilde{\tilde{a}}_y / w \\ \tilde{\tilde{a}}_z / w \\ 1 \end{bmatrix} \tag{7}$$

Projection 3D to 2D

Camera to film - Perspective



Projection 3D to 2D

Camera to film - Orthographic

- simple perspective:

$$\mathrm{P} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

- general perspective matrix:

$$\mathrm{P} = \begin{bmatrix} 2/(r-l) & 0 & 0 & -(r+l)/(r-l) \\ 0 & 2/(t-b) & 0 & -(t+b)/(t-b) \\ 0 & 0 & -2/(f-n) & -(f+n)/(f-n) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

Projection 3D to 2D

Film to screen

Scaling and positioning:

$$S = \begin{bmatrix} s_x & 0 & 0 & t_x \\ 0 & s_y & 0 & t_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

Projection 3D to 2D

Clipping

Points that are located behind camera will be projected on screen too Those points need to be removed – clipping

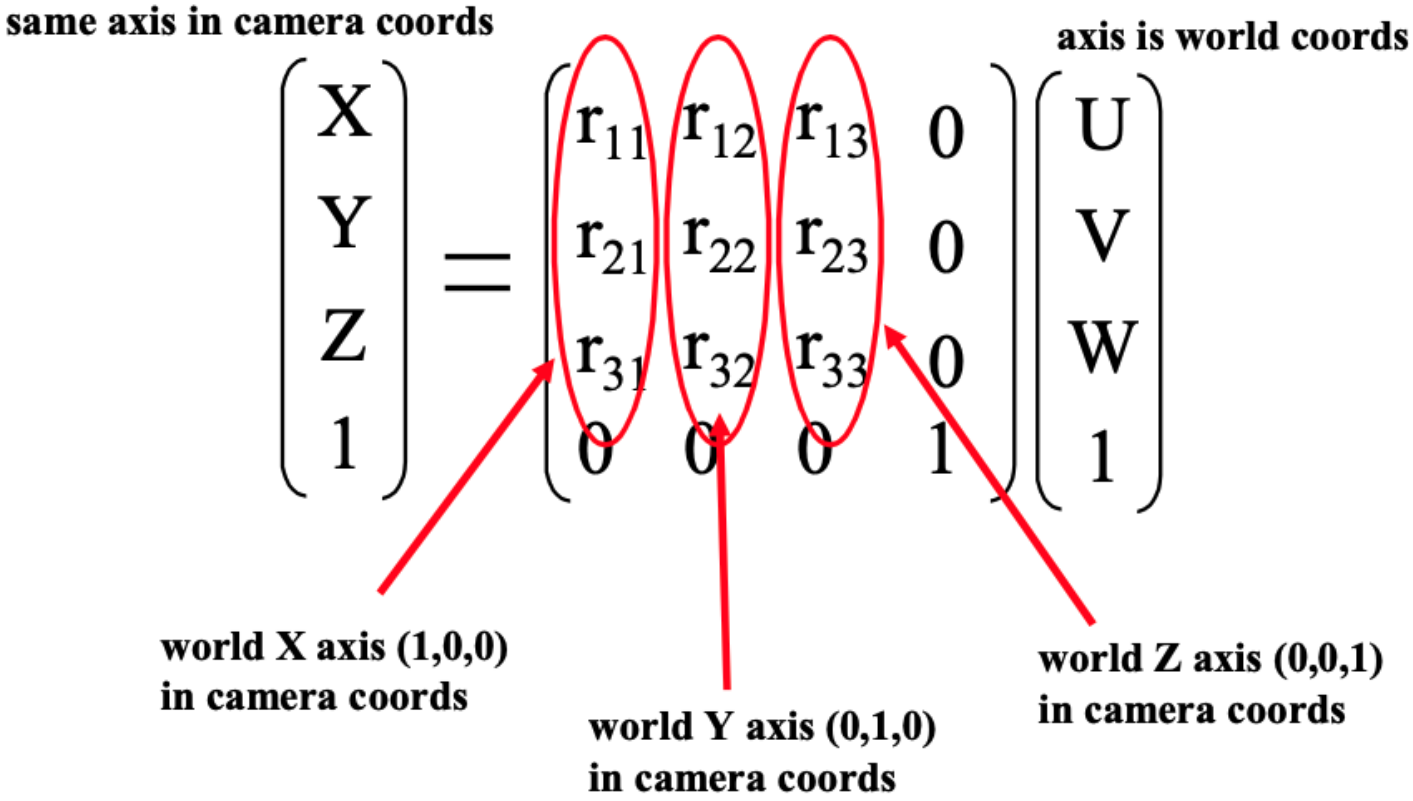
Rotations

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix} \tag{11}$$

$$R_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \tag{12}$$

$$R_z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{13}$$

rotation from axes



rotation from axes

Thank you!