Moderovacie a renderovacie techniky

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26. novembra 2025

https://github.com/frantisekdracek/Prezentacie/tree/main

Delaunay Triangulation

- **Input:** A set of points P.
- **The Delaunay Condition:** A triangulation is Delaunay if **no point** from P is inside the circumcircle of any triangle in the mesh.
- **Properties:**
  - It maximizes the minimum angle of all the triangles (avoids long, skinny "sliver"triangles).
  - It is the dual graph of the Voronoi diagram.

Obr.: Every triangle's circumcircle is empty of other points.

Algorithm (Bowyer-Watson)

1. **Initialization:** Create a "super-triangle"that is large enough to enclose all points in P. Add it to the triangle list.
2. **Iterate:** For every point p in the list of points:
   - Identify **Bad Triangles**: specific triangles whose *circumcircles* contain the new point p.
   - Find the **Boundary Polygon**:
     - Iterate through the edges of all "Bad Triangles".
     - If an edge is **not shared** by two bad triangles, it is part of the boundary of the polygonal hole.
   - **Remove:** Delete all "Bad Triangles"from the mesh.
   - **Re-triangulate:** Create new triangles by connecting the new point p to the vertices of the Boundary Polygon.
3. **Cleanup:** Remove any triangles that share a vertex with the original super-triangle.

Mathematical Check: In-Circle Test

To check if a point D lies inside the circumcircle of triangle A, B, C, we calculate the determinant.

If points A, B, C are in counter-clockwise order, point D is **inside** the circumcircle if:

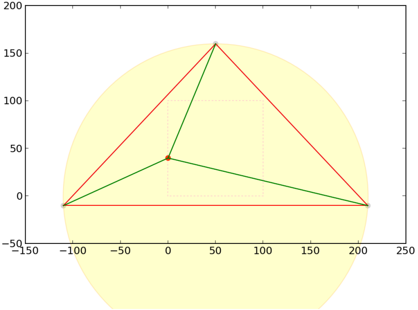
$$\det \begin{bmatrix} A_x & A_y & A_x^2 + A_y^2 & 1 \\ B_x & B_y & B_x^2 + B_y^2 & 1 \\ C_x & C_y & C_x^2 + C_y^2 & 1 \\ D_x & D_y & D_x^2 + D_y^2 & 1 \end{bmatrix} > 0$$

$$\begin{vmatrix} A_x & A_y & A_x^2 + A_y^2 & 1 \\ B_x & B_y & B_x^2 + B_y^2 & 1 \\ C_x & C_y & C_x^2 + C_y^2 & 1 \\ D_x & D_y & D_x^2 + D_y^2 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} A_x - D_x & A_y - D_y & (A_x - D_x)^2 + (A_y - D_y)^2 \\ B_x - D_x & B_y - D_y & (B_x - D_x)^2 + (B_y - D_y)^2 \\ C_x - D_x & C_y - D_y & (C_x - D_x)^2 + (C_y - D_y)^2 \end{vmatrix} > 0$$

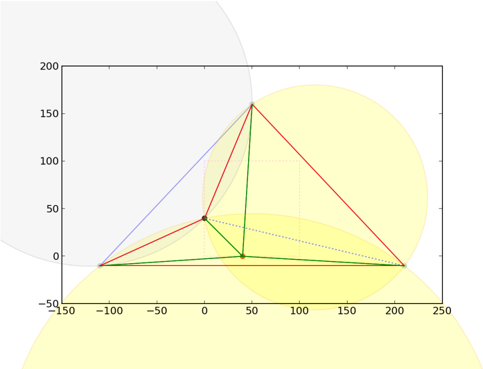
Obr.: The determinant lifts points to a paraboloid ($z = x^2 + y^2$).

Example: Step 1



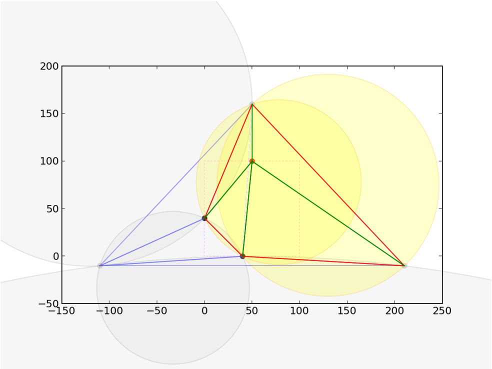
Obr.: Start with the Super-Triangle and insert the 1st point.

Example: Step 2



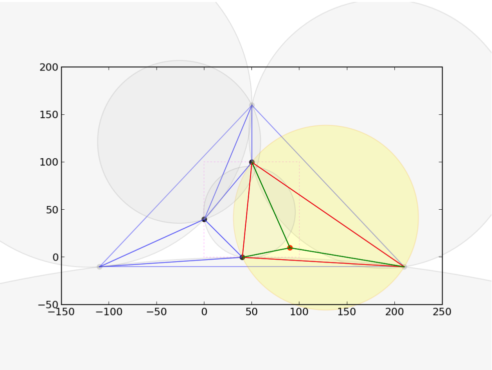
Obr.: Insert 2nd point. New edges connect to existing valid vertices.

Example: Step 3



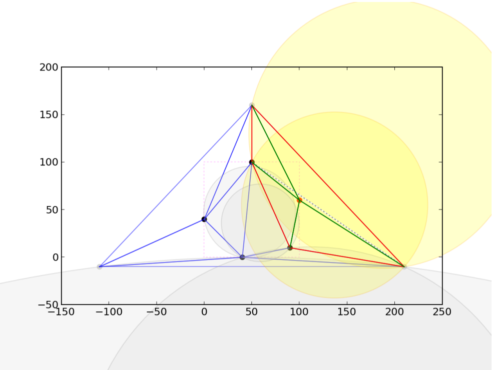
Obr.: Insert 3rd point. Check for circumcircle violations.

Example: Step 4



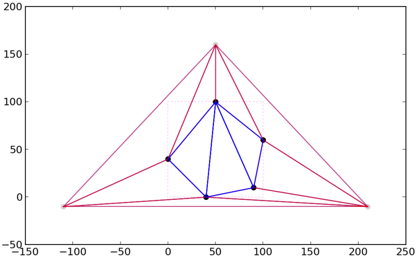
Obr.: Insert 4th point. Invalid triangles (circumcircle hit) are removed, creating a cavity.

Example: Step 5



Obr.: The cavity is re-triangulated with the new point.

Final Result



Obr.: Remove super-triangle vertices. The final Delaunay mesh remains.

Thank you!