Moderovacie a renderovacie techniky

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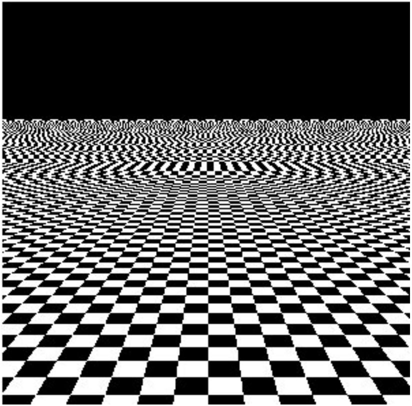
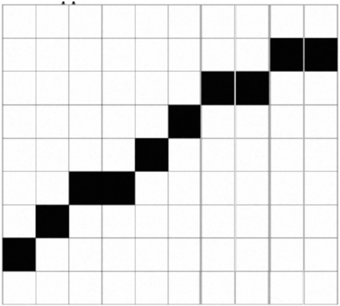
https://github.com/frantisekdracek/Prezentacie/tree/main

Aliasing

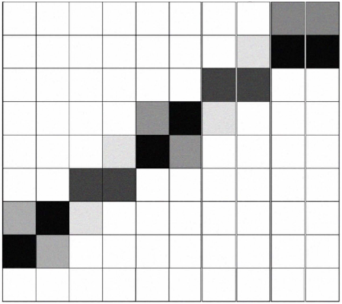


Obr.: An example of anti-aliasing

Aliasing



Obr.: Line with aliasing



Obr.: After antialiasing

Fourier series

- $$f(t) = \sum_{k=-\infty}^{k=\infty} c_k \mathrm{e}^{\mathrm{i}\frac{2\pi k}{T}t} \tag{1}$$

- $$c_k = \frac{1}{T}\int_{\frac{T}{2}}^{\frac{T}{2}} f(t)\mathrm{e}^{-\mathrm{i}\frac{2\pi k}{T}t} dt \tag{2}$$

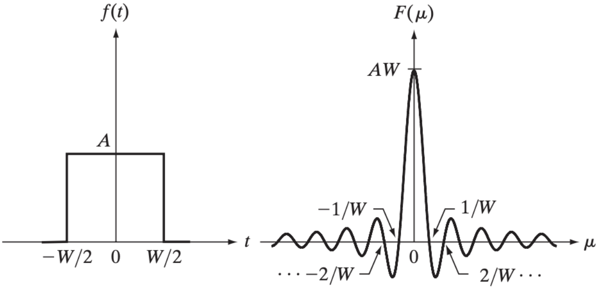
Fourier transform

- $$f(t) = \int_{k=-\infty}^{k=\infty} F(k)\mathrm{e}^{\mathrm{i}2\pi kt} dk \tag{3}$$

- $$F(k) = \int_{\infty}^{\infty} f(t)\mathrm{e}^{-\mathrm{i}2\pi kt} dt \tag{4}$$

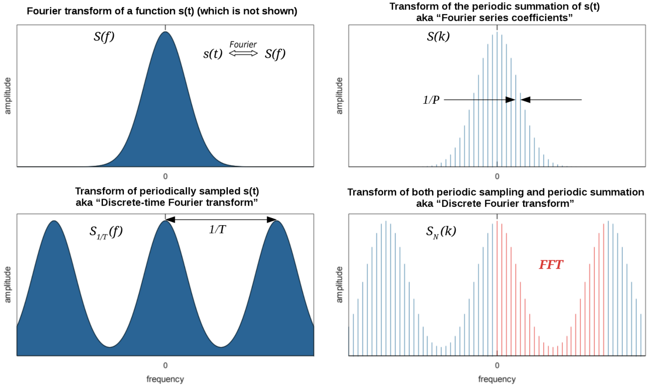
- $$\int_{\infty}^{\infty} \mathrm{e}^{-\mathrm{i}2\pi kt} dt = \delta(t) \tag{5}$$

Fourier transform example



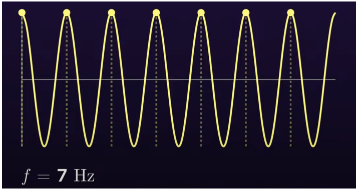
Obr.: FT of rect function.

Fourier transform of sampled function

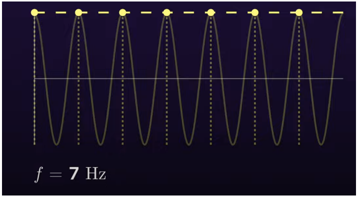


Obr.: FT of sampled function

Sampling

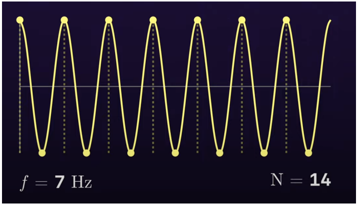


Obr.: Insufficient sampling

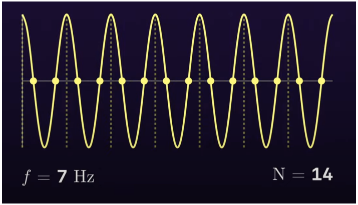


Obr.: Interpolation

Sampling

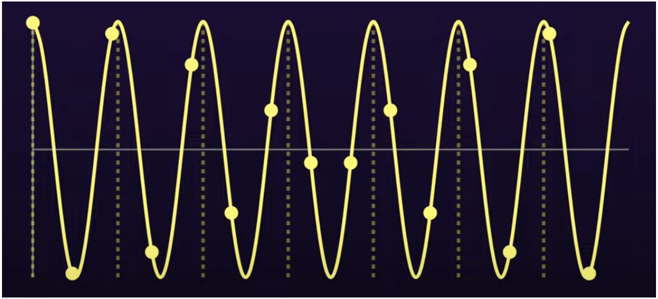


Obr.: Insufficient sampling



Obr.: Interpolation

Sampling



Obr.: Sampling theorem: $f_{sampling} < 2f_{max}$

Fourier transform of sampled function

4.7.3 Summary of Steps for Filtering in the Frequency Domain

The material in the previous two sections can be summarized as follows:

1. Given an input image $f(x, y)$ of size $M \times N$, obtain the padding parameters P and Q from Eqs. (4.6-31) and (4.6-32). Typically, we select $P = 2M$ and $Q = 2N$.
2. Form a padded image, $f_p(x, y)$, of size $P \times Q$ by appending the necessary number of zeros to $f(x, y)$.
3. Multiply $f_p(x, y)$ by $(-1)^{x+y}$ to center its transform.
4. Compute the DFT, $F(u, v)$, of the image from step 3.
5. Generate a real, symmetric filter function, $H(u, v)$, of size $P \times Q$ with center at coordinates $(P/2, Q/2)$.† Form the product $G(u, v) = H(u, v)F(u, v)$ using array multiplication; that is, $G(i, k) = H(i, k)F(i, k)$.
6. Obtain the processed image:

$$g_p(x, y) = \left\{\text{real}\left[\Im^{-1}[G(u, v)]\right]\right\}(-1)^{x+y}$$

   where the real part is selected in order to ignore parasitic complex components resulting from computational inaccuracies, and the subscript p indicates that we are dealing with padded arrays.

7. Obtain the final processed result, $g(x, y)$, by extracting the $M \times N$ region from the top, left quadrant of $g_p(x, y)$.

Obr.: FT of sampled function

Thank you!