Moderovacie a renderovacie techniky

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https://github.com/frantisekdracek/Prezentacie/tree/main

Tone mapping

- HDR merging: LDR -> HDR e.g: combine several pictures with different exposure into HDR image
- tone mapping: HDR -> LDR , HDR image to display's dynamic range

Tone mapping

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Debevec algorithm

paper

$$Z_{ij} = f(E_i \delta t_j), \tag{1}$$

E_i radiance, δt_j exposure time, i spatial index and j time index.

$$g(Z_{ij}) = \ln(f^{-1}(Z_{ij}) = \ln(E_i) \ln(\delta t_j), \tag{2}$$

Minimalization in least squares sense:

$$\mathcal{O} = \sum_{i=1}^{N} \sum_{j=1}^{P} (g(Z_{ij}) - \ln(E_i) - \ln(\delta t_j))^2, \tag{3}$$

N number of pixels, P number of images.

Debevec algorithm

Regularization term to prevent excessive curliness:

$$\mathcal{O} = \sum_{i=1}^{N} \sum_{j=1}^{P} (g(Z_{ij}) - \ln(E_i) - \ln(\delta t_j))^2 + \lambda \sum_{Z_{min}+1}^{Z_{max}-1} g''(x)^2 \tag{4}$$

Weight term ensuring better fit:

$$\mathcal{O} = \sum_{i=1}^{N} \sum_{j=1}^{P} w(Z_{ij})(g(Z_{ij}) - \ln(E_i) - \ln(\delta t_j))^2 + \lambda \sum_{Z_{min}+1}^{Z_{max}-1} w(Z_{ij}) g''(x)^2 \tag{5}$$

$$w(z) = \begin{cases} z - Z_{min} & \text{if } z \leq \frac{1}{2}(Z_{min} + Z_{max}) \\ Z_{max} - z & \text{if } z \geq \frac{1}{2}(Z_{min} + Z_{max}) \end{cases} \tag{6}$$

Sampling

We dont have to use all pixels:

$$NP \geq Z_{max} - Z_{min} + N \tag{7}$$

Sampling matrix:

$$S = 256 \left\{ \underbrace{\begin{bmatrix} \dots & \dots & \dots \\ \dots & S_{zj} & \dots \\ \dots & \dots & \dots \end{bmatrix}}_{P} \right. \tag{8}$$

- Select middle exposure image .
- for selected z localize position of all pixels with intensity z. $[(k_1, l_1,), (k_2, l_2), \dots]$
- select one random pair (k, l).
- find intenisty values on other images correspoding to (k, l)
- $S_{z,j} = Z_{i(k,l),j}$

Least squares
Standard linear regression

Linear regression problem:

$$Ax = b, \tag{9}$$

Least squares solution:

$$\mathcal{O} = \min ||Ax - b||^2 \tag{10}$$

By taking gradient of condition and setting it to zero $\nabla_x \mathcal{O} \overset{!}{=} 0$:

$$x = (A^T A)^{-1} A^T b \tag{11}$$

Linear regression problem:

$$Ax = b, \tag{12}$$

Least squares solution with Ridge regularization:

$$\mathcal{O} = \min ||Ax - b||^2 + \lambda ||x||^2 \tag{13}$$

Solution:

$$x = (A^T A + \lambda \mathbb{I})^{-1} A^T b \tag{14}$$

Equivalent to standard linear regression with modified matrix $\tilde{A}$.

$$\tilde{A} = \begin{bmatrix} A \\ \sqrt{\lambda}\mathbb{I} \end{bmatrix}. \tag{15}$$

Least squares

Second derivative regularization

Linear regression problem:

$$\mathrm{Ax} = \mathrm{b}, \tag{16}$$

Least squares solution with second derivative regularization:

$$\mathcal{O} = \min ||\mathrm{Ag}(x) - \mathrm{b}||^2 + \lambda ||\mathrm{g}''(x)||^2 \tag{17}$$

Equivalent to:

$$\tilde{\mathrm{A}} = \begin{bmatrix} A \\ \sqrt{\lambda}\mathrm{D} \end{bmatrix}, \tag{18}$$

where:

$$\mathrm{D} = \begin{bmatrix} 1 & -2 & 1 & 0 & \cdots & \cdots \\ 0 & 1 & -2 & 1 & \cdots & \cdots \\ 0 & 0 & 1 & -2 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \end{bmatrix} \tag{19}$$

Debevec to simple least squares

transform problem to matrix

$$\begin{bmatrix} \mathrm{A} & \mathrm{E} \\ \mathrm{D}_w & 0 \end{bmatrix} \begin{bmatrix} g(0) \\ g(1) \\ \vdots \\ g(255) \\ \ln(E_1) \\ \vdots \end{bmatrix} - \left[\ln(\nabla t_1))\right], \tag{20}$$

where

$$\mathrm{D}_w = \begin{bmatrix} w(1) & -2w(1) & w(1) & 0 & \cdots & \cdots \\ 0 & w(2) & -2w(2) & w(2) & \cdots & \cdots \\ 0 & 0 & w(3) & -2w(3) & w(1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \end{bmatrix} \tag{21}$$

Debevec to simple least squares

$$b = \begin{bmatrix} w(Z_{1,1}) \ln(\nabla t_1) \\ w(Z_{1,2}) \ln(\nabla t_2) \\ \vdots \\ w(Z_{1,P}) \ln(\nabla t_1) \\ w(Z_{2,1}) \ln(\nabla t_1) \\ \vdots \\ w(Z_{2,P}) \ln(\nabla t_P) \\ \vdots \\ w(Z_{N,1}) \ln(\nabla t_1) \\ \vdots \\ w(Z_{N,P}) \ln(\nabla t_P) \end{bmatrix} \tag{22}$$

Debevec to simple least squares

Matrix Ais zero matrix except following terms:

$$A_{P*(i-1)+j,Z_{i,j}} = w(Z_{i,j}) \tag{23}$$

Matrix Eis zero matrix except following terms:

$$E_{P*(i-1)+j,i} = w(Z_{i,j}) \tag{24}$$

Algorithm

```
# 1. Add data-fitting constraints:
k = 0
for i in range(num_samples):
    for j in range(num_images):
        z_ij = intensity_samples[i, j]
        w_ij = weighting_function(z_ij)
        mat_A[k, z_ij] = w_ij
        mat_A[k, (intensity_range + 1) + i] = -w_ij
        mat_b[k, 0] = w_ij * log_exposures[j]
        k += 1

# 2. Add smoothing constraints:
for z_k in range(z_min + 1, z_max):
    w_k = weighting_function(z_k)
    mat_A[k, z_k - 1] = w_k * smoothing_lambda
    mat_A[k, z_k     ] = -2 * w_k * smoothing_lambda
    mat_A[k, z_k + 1] = w_k * smoothing_lambda
    k += 1

# 3. Add color curve centering constraint:
mat_A[k, (z_max - z_min) // 2] = 1
```

Reconstructing radiance map

$$\ln(E_i) = \frac{1}{P}\sum_{j=1}^{P}(g(Z_{ij}) - \ln(\nabla t_j)) \tag{25}$$

With weights:

$$\ln(E_i) = \frac{\sum\limits_{j=1}^{P} w(Z_{ij})(g(Z_{ij}) - \ln(\nabla t_j))}{\sum\limits_{j=1}^{P} w(Z_{ij})} \tag{26}$$

Reconstructing radiance map

- Last step involves computing radiance map for every channel and combining them to a resulting HDR image.
- Follow with tone mapping techniques to map image to LDR for displaying purpose.

Thank you!