Applied Regression (I)

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Applied Quantitative Methods II

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Review: Key Concepts from AQMSS-I

The regression model

The most common tool in social science:

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- Y: outcome we want to explain
- X: explanatory variable(s)
- β: coefficients (what we estimate)
- ε: error term (what we can't explain)

What regression tells us

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  - $\rightarrow$ How much Y changes, on average
  - $\rightarrow$ When comparing units that differ by 1 in X

Descriptive vs. Causal interpretation

- **Descriptive**: How do units with different X values compare?
  - $\rightarrow$ "People with more education earn more, on average"
- **Causal**: What happens if we change X for a given unit?
  - $\rightarrow$ "If we give someone more education, they will earn more"
- Same coefficient, very different claims!

The challenge of causal inference

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- We need strategies to infer them
- This will be a recurring theme throughout the course

Today's goals

- Understand regression as modeling conditional expectations

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- Learn how to think about control variables

Regression as Conditional Expectations

What question does regression answer?

- "What is the average value of Y for different values of X?"

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- Regression approximates this function

Example: Income and support for redistribution

- Research question: How does income relate to support for redistribution?

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- We can estimate this with regression

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- Why linear? Simple, interpretable, often good enough

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- Scaled by how much X varies

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- This is a **comparison**, not necessarily a causal effect

From Description to Causation

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  - $\rightarrow$ "Increasing someone's income would decrease their support"

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- The difference is crucial!

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- The fundamental problem: we only observe one of these

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- The simple difference in means estimates the causal effect

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- These differences can bias our estimates

Confounding

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- Creates a spurious association between them

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- Example: Education, income, and political preferences
- Education affects both income and political views
- Income-politics relationship may be partly spurious

The logic of controlling

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- But: this requires knowing what the confounders are

Control Variables in Practice

Multiple regression

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- This is the "controlled" effect of X_1

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- This isolates the unique contribution of X_1

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- Depends on:
  - $\rightarrow$ How strongly the confounder affects Y
  - $\rightarrow$ How strongly the confounder relates to X

What makes a good control?

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Pre-treatment confounders are the key!

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- Example: Studying effect of job training on wages
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- Controlling for post-treatment variables can *introduce* bias

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- Example: NBA players
  - $\rightarrow$ Height and skill both affect being in NBA
  - $\rightarrow$ Among NBA players, height and skill are negatively correlated
  - $\rightarrow$ But not in the general population!

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- There's no purely statistical solution to this
- Need theory + research design, not just more controls

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- Control variables help only if chosen correctly
- Controlling for the wrong variables can make things worse
- Always think about what you're comparing

For next week

- Read Angrist & Pischke (2008), chapters 1-3
- Read Urdinez & Cruz (2020), chapter 5
- Work on Problem Set 1

- Next session: More on regression in practice
  - → Interactions
  - → Non-linear relationships
  - → Standard errors and inference

Questions?