

# Binary Outcomes

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- Learn the linear probability model and its trade-offs
- Understand logistic regression and how to estimate it in R
- Interpret logit results using predicted probabilities and marginal effects

# Roadmap

The Problem with Binary Outcomes

Logistic Regression

Interpreting Logit Results

Practice

# The Problem with Binary Outcomes

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  - Did a war break out?
  - Did a bill pass?
  - Did a country democratize?
- Our outcome  $Y \in \{0, 1\}$
- We want to model:  $P(Y = 1|X)$

# What happens if we just use OLS?

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- So OLS is modeling a probability as a linear function of  $X$
- This is the **linear probability model** (LPM)

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- Many applied researchers use the LPM in practice

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  - True relationship between  $X$  and  $P(Y = 1)$  is S-shaped
  - LPM forces it to be linear

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  - The relationship is clearly non-linear



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# Logistic Regression

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- Steep in the middle, flat at the extremes
- A natural model for probabilities

# The logit model

We can rearrange the logistic function:

$$\log \left( \frac{P}{1-P} \right) = \beta_0 + \beta_1 X$$

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- $\frac{P}{1-P}$  is the **odds** of the event
- The model is linear in the log-odds, not in the probability

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- No need to derive this—R does it for us

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- `glm`: generalized linear model
- `family = binomial`: tells R to use logistic regression
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- Works with `broom::tidy()`, `modelsummary()`, etc.

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- The change in probability depends on where you start

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- We need better tools to interpret logit models

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# Interpreting Logit Results

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# Predicted probabilities

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- “What is the predicted probability of  $Y = 1$  for a person with these characteristics?”
- In R:
  - `marginalEffects::predictions(model)`
  - Returns predicted probabilities for each observation
  - Or at specific values: `predictions(model, newdata = ...)`

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  - “On average, a one-unit increase in  $X$  changes the probability of  $Y = 1$  by  $\Delta P$ ”

# Average marginal effects (AME)

- The marginal effect varies across observations
- The AME averages across all observations
- In R: `marginalEffects::avg_slopes(model)`
- Interpretation (like OLS):
  - “On average, a one-unit increase in  $X$  changes the probability of  $Y = 1$  by  $\Delta P$ ”
- This is often comparable to the LPM coefficient

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- In R:
  - `avg_slopes(model, newdata = datagrid(age = 40, gender = "F"))`
- Useful when the marginal effect varies a lot across the sample

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- Much more informative than a table of log-odds

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  - Marginal effects at extreme values
- A good practice: estimate both and compare

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- None is perfect; use them as rough guides
- Reported automatically by `modelsummary()` and `performance::r2()`

# Roadmap

The Problem with Binary Outcomes

Logistic Regression

Interpreting Logit Results

Practice

# Practice

## Worked example: LPM vs. logit

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modelsummary(list("LPM" = lpm, "Logit" = logit))
avg_slopes(logit)                # compare to LPM coefficients
plot_predictions(logit, condition = "income")
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- In practice: estimate both, report the one most appropriate

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- Binary outcomes require special treatment
- LPM is simple but has known limitations
- Logit bounds probabilities between 0 and 1
- Log-odds and odds ratios are not intuitive—use marginal effects
- Predicted probabilities and AMEs are the best way to interpret logit
- Compare LPM and logit; they often agree on AMEs

# For next week

- Read Urdinez & Cruz (2020), chapter 5 (§5.6)
- Read Gelman et al., chapters 11–12
- Complete Assignment 3
  
- Next session: Model interpretation and diagnostics
  - Beyond coefficient tables
  - Visualizing model results
  - Publication-quality tables
  - Key diagnostics

Questions?