Binary Outcomes

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Today's goals

- Understand why OLS is problematic for binary outcomes
- Learn the linear probability model and its trade-offs
- Understand logistic regression and how to estimate it in R
- Interpret logit results using predicted probabilities and marginal effects

Roadmap

The Problem with Binary Outcomes

Logistic Regression

Interpreting Logit Results

Practice

The Problem with Binary Outcomes

Binary outcomes are everywhere

- Many outcomes in social science are binary (yes/no):
  - → Did someone vote?
  - → Did a war break out?
  - → Did a bill pass?
  - → Did a country democratize?
- Our outcome $Y \in \{0, 1\}$
- We want to model: $P(Y = 1|X)$

What happens if we just use OLS?

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

- OLS gives us: $E[Y|X] = \beta_0 + \beta_1 X$
- Since $Y \in \{0, 1\}$: $E[Y|X] = P(Y = 1|X)$
- So OLS is modeling a probability as a linear function of X
- This is the **linear probability model** (LPM)

The LPM: Simple and intuitive

- β_1 has a direct interpretation:
  - → A one-unit increase in X changes the probability of $Y = 1$ by β_1
- Easy to estimate: just `lm(y ~ x, data = df)`
- Easy to interpret: same as OLS
- Many applied researchers use the LPM in practice

LPM limitations

- **Problem 1**: Predictions outside $[0, 1]$
  - → A linear function can produce $\hat{P} < 0$ or $\hat{P} > 1$
  - → Probabilities must be between 0 and 1!
- **Problem 2**: Heteroskedasticity by construction
  - → $\text{Var}(\varepsilon|X) = P(1 - P)$, which varies with X
  - → Standard errors from OLS are wrong (use robust SEs)
- **Problem 3**: Non-linearity at the extremes
  - → True relationship between X and $P(Y = 1)$ is S-shaped
  - → LPM forces it to be linear

When is the LPM “good enough”?

- When probabilities are in the middle range (0.2–0.8)
  - → The linear approximation is reasonable here
- When you care about **average marginal effects**
  - → LPM and logit often give similar AMEs
- When simplicity of interpretation matters

- When is it **not** good enough?
  - → Many observations near 0 or 1
  - → You need predicted probabilities to be bounded
  - → The relationship is clearly non-linear

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Logistic Regression

The logistic function

$$P(Y=1|X)=\frac{e^{X\beta}}{1+e^{X\beta}}=\frac{1}{1+e^{-X\beta}}$$

- This is an S-shaped (sigmoid) curve
- Output is always between 0 and 1
- Steep in the middle, flat at the extremes
- A natural model for probabilities

The logit model

We can rearrange the logistic function:

$$\log\left(\frac{P}{1-P}\right) = \beta_0 + \beta_1 X$$

- The left side is the **log-odds** (or "logit")
- $\frac{P}{1-P}$ is the **odds** of the event
- The model is linear in the log-odds, not in the probability

Maximum likelihood estimation

- We can't use OLS for logistic regression
- Instead, we use **maximum likelihood estimation** (MLE)
- The intuition:
  - → For each observation, the model predicts $P(Y_i = 1)$
  - → MLE finds the coefficients that make the observed data most likely
  - → The "likelihood" is the product of these predicted probabilities
- No need to derive this—R does it for us

Estimating logit in R

- The function: `glm(y ~ x, family = binomial, data = df)`

- `glm`: generalized linear model
- `family = binomial`: tells R to use logistic regression

- The syntax is identical to `lm()`, just change to `glm()`
- Works with `broom::tidy()`, `modelsummary()`, etc.

Interpreting logit output: Log-odds

- The direct output gives coefficients in **log-odds**

- $\beta_1 = 0.5$ means:

  → A one-unit increase in X increases the log-odds by 0.5

- This is hard to interpret!
- Nobody thinks in log-odds

Interpreting logit output: Odds ratios

- Exponentiate the coefficient: e^{β_1} = odds ratio
- In R: `exp(coef(model))`
- $e^{0.5} \approx 1.65$ means:
  - → A one-unit increase in X **multiplies** the odds by 1.65
  - → Or: the odds increase by 65%
- Slightly more intuitive, but still not probabilities
- The change in probability depends on where you start

Why coefficients alone are not enough

- In OLS: β_1 = change in Y for one-unit change in X (always)
- In logit: the change in **probability** depends on:
  - → The current value of X
  - → The values of all other variables
- A coefficient of $\beta_1 = 0.5$ could mean:
  - → Going from $P = 0.01$ to $P = 0.016$ (tiny change)
  - → Going from $P = 0.50$ to $P = 0.62$ (large change)
- We need better tools to interpret logit models

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Interpreting Logit Results

Predicted probabilities

- The most intuitive way to interpret logit models
- "What is the predicted probability of $Y = 1$ for a person with these characteristics?"
- In R:
  - → `marginaleffects::predictions(model)`
  - → Returns predicted probabilities for each observation
  - → Or at specific values: `predictions(model, newdata = ...)`

Average marginal effects (AME)

- The marginal effect varies across observations
- The AME averages across all observations
- In R: `marginaleffects::avg_slopes(model)`
- Interpretation (like OLS):
  - → "On average, a one-unit increase in X changes the probability of $Y = 1$ by ΔP"
- This is often comparable to the LPM coefficient

Marginal effects at representative values

- Instead of averaging, evaluate at specific values
- Example: "What is the effect of education on voting for a 40-year-old woman?"
- In R:
  - → `avg_slopes(model, newdata = datagrid(age = 40, gender = "F"))`
- Useful when the marginal effect varies a lot across the sample

Plotting predicted probabilities

- The best way to communicate logit results
- Show how $P(Y = 1)$ changes across values of X
- Include confidence bands
- In R:
  - → `plot_predictions(model, condition = "x")`
  - → Plots the S-curve with uncertainty
- Much more informative than a table of log-odds

Comparing LPM and logit

- In many cases, LPM and logit give similar results
  - → Especially for average marginal effects
  - → Especially when probabilities are in the 0.2–0.8 range
- Where they differ:
  - → Predicted probabilities near 0 or 1
  - → LPM can go outside $[0, 1]$; logit cannot
  - → Marginal effects at extreme values
- A good practice: estimate both and compare

Model fit for logit

- No R^2 in the usual sense (MLE, not OLS)
- Alternative measures:
  - → **Pseudo-R^2**: compares model to null model (McFadden)
  - → **AIC**: penalized likelihood (lower = better)
  - → **Classification**: what percent does the model correctly predict?
  - → **ROC curve**: trade-off between true and false positives
- None is perfect; use them as rough guides
- Reported automatically by `modelsummary()` and `performance::r2()`

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Worked example: LPM vs. logit

```
lpm <- lm(vote ~ age + income + educ, data = df)
logit <- glm(vote ~ age + income + educ,
             family = binomial, data = df)
modelsummary(list("LPM" = lpm, "Logit" = logit))
avg_slopes(logit)            # compare to LPM coefficients
plot_predictions(logit, condition = "income")
```

Decision tree: When to use which?

- **Use LPM when:**
  - → You want simple, quick interpretation
  - → Probabilities are in the middle range
  - → You mainly care about average effects
- **Use logit when:**
  - → You need bounded predicted probabilities
  - → Many observations have extreme probabilities (near 0 or 1)
  - → You want to properly account for the binary nature of Y
- In practice: estimate both, report the one most appropriate

Summary: Key takeaways

- Binary outcomes require special treatment
- LPM is simple but has known limitations
- Logit bounds probabilities between 0 and 1
- Log-odds and odds ratios are not intuitive—use marginal effects
- Predicted probabilities and AMEs are the best way to interpret logit
- Compare LPM and logit; they often agree on AMEs

For next week

- Read Urdinez & Cruz (2020), chapter 5 (§5.6)
- Read Gelman et al., chapters 11–12
- Complete Assignment 3

- Next session: Model interpretation and diagnostics
  - → Beyond coefficient tables
  - → Visualizing model results
  - → Publication-quality tables
  - → Key diagnostics

Questions?