

Model Interpretation and Diagnostics

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Applied Quantitative Methods II
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- Understand simulation-based uncertainty
- Diagnose common regression problems: heteroskedasticity, non-linearity

Roadmap

Beyond Coefficient Tables

Predicted Values and Marginal Effects

Presenting Results

Diagnostics

Wrap-up

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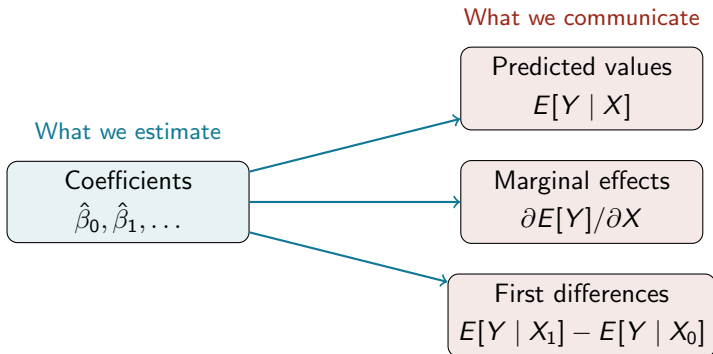
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 - Log transformations: coefficients are elasticities or semi-elasticities
 - Different scales: is $\hat{\beta} = 0.003$ big or small?

Coefficients vs. quantities of interest



You estimate a model with GDP (in dollars), population (in millions), and an interaction between them.

What does the coefficient on GDP tell you?

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- Example: “What is predicted income for a 35-year-old with a college degree?”
- In R:
 - `predictions(model)`
 - `predictions(model, newdata = datagrid(age = 35, college = 1))`

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 - `plot_predictions(model, condition = c("age", "female"))`
 - Separate lines/panels for each group

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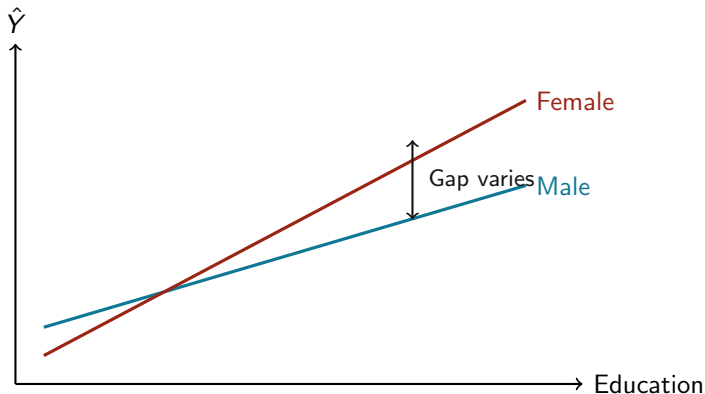
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→ `avg_slopes(model)`

Marginal effects with interactions

$$Y = \beta_0 + \beta_1 \text{Education} + \beta_2 \text{Female} + \beta_3 (\text{Education} \times \text{Female}) + \varepsilon$$



- ME of Education for males: β_1
- ME of Education for females: $\beta_1 + \beta_3$

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condition = "female")`
- This replaces the old manual approach of computing $\beta_1 + \beta_3 \cdot X_2$ by hand

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 - Comparing meaningful scenarios (e.g., min vs. max)

Choosing the right quantity

Quantity	Question	R function
Predicted value	What does the model predict here?	<code>predictions()</code>
Marginal effect	How much does Y change per unit of X ?	<code>slopes()</code>
Average ME	What is the average effect across the sample?	<code>avg_slopes()</code>
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- All compute standard errors and CIs automatically
- All work with `lm()`, `glm()`, and many other model types

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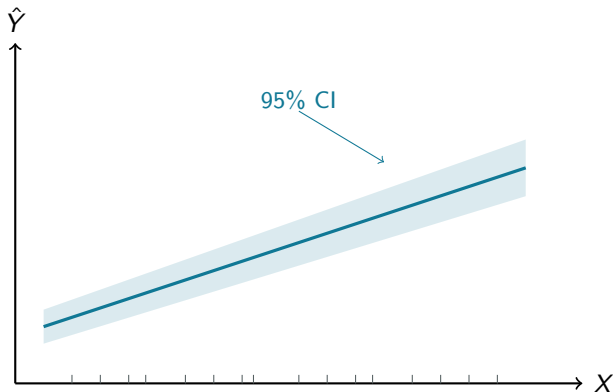
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- `modelplot(list("Model 1" = m1, "Model 2" = m2))`

Prediction plots: the gold standard



- Shows the **full relationship**, not just one number
- Any audience can read it
- `plot_predictions(model, condition = "x")`

Tables vs. plots: when to use which

	Table	Plot
Exact numbers needed	✓	
Many models side by side	✓	
Conveying one key relationship		✓
Non-specialist audience		✓
Interactions / non-linearities		✓
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- Best practice: prediction plots in the main text, tables in the appendix
- Both are easy to produce with `modelsummary` and `marginaleffects`

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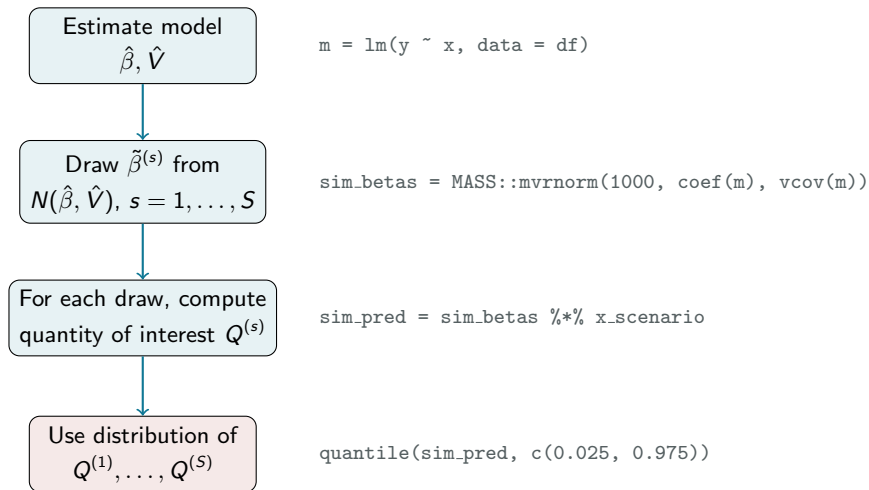
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 - Easy to combine multiple quantities
 - Confidence intervals are automatic

Simulation: the logic



Worked example: predicted values with uncertainty

```
m = lm(income ~ age + education + female, data = df)

# Using marginalesffects (delta method)
predictions(m, newdata = datagrid(age = 40, education = 16,
female = 1))

# Using simulation
library(MASS)
sim_b = mvrnorm(1000, coef(m), vcov(m))
x = c(1, 40, 16, 1) # intercept, age, educ, female
sim_pred = sim_b %*% x
quantile(sim_pred, c(0.025, 0.5, 0.975))
```

- Both approaches give (nearly) identical results

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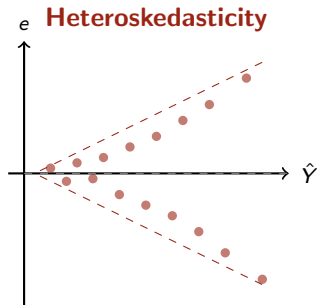
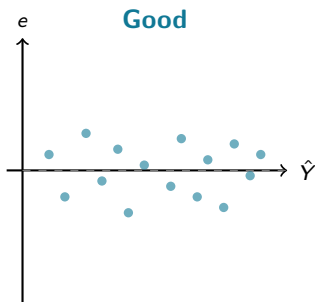
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 - Results may be driven by a few observations (outliers)
- How do we check? **Residual diagnostics**

Residuals vs. fitted values



- In R: `plot(model, which = 1)`
- Look for patterns: funnel shape, curves, clusters

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 - Binary outcomes (always heteroskedastic, as in LPM)

Robust standard errors

- The fix: **heteroskedasticity-consistent (HC) standard errors**

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 - If heteroskedastic: robust SEs are correct, classical are not

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- But interpretation changes!

Interpreting logs

Model	Equation	Interpretation of β_1
Level-level	$Y = \beta_0 + \beta_1 X$	$\Delta X = 1 \Rightarrow \Delta Y = \beta_1$
Log-level	$\log(Y) = \beta_0 + \beta_1 X$	$\Delta X = 1 \Rightarrow \% \Delta Y \approx 100 \cdot \beta_1$
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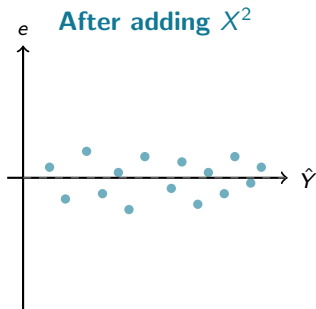
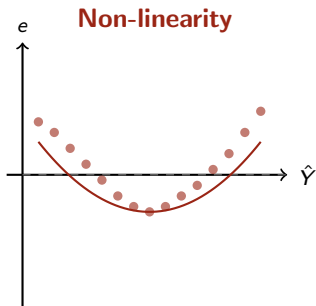
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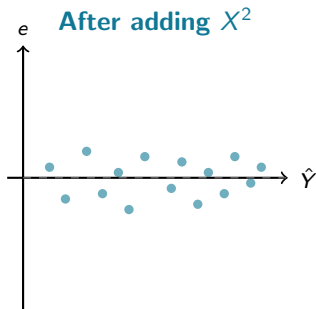
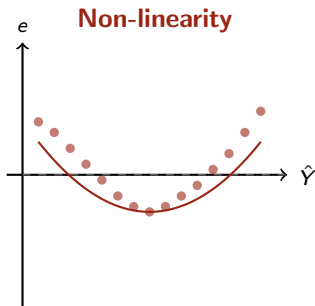
- Log-log: β_1 is the **elasticity**
- Log-level: β_1 is a semi-elasticity
- Most common in practice: log-level and log-log

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- Fixes: add X^2 , use $\log(X)$, or use a more flexible model

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 - Report both results

Roadmap

Beyond Coefficient Tables

Predicted Values and Marginal Effects

Presenting Results

Diagnostics

Wrap-up

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- Present results with plots (prediction plots, coefficient plots)
- Tables go in the appendix, plots in the main text
- Always check diagnostics: residual plots, heteroskedasticity
- Always use robust standard errors
- Use log transformations for skewed variables (but interpret carefully)

For next week

- Complete Assignment 4
- Next session: Best Practices in Computing
 - Reproducible workflows
 - Project organization
 - Writing clean R code

Questions?