

Applied Regression

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Applied Quantitative Methods II

IC3JM, Spring 2026

Today's goals

- Review regression as modeling conditional expectations

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- Understand multiple regression and control variables

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- Learn how to model conditional relationships (interactions)

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- Understand multiple regression and control variables
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- Present results effectively with `modelsummary`

Roadmap

Regression Review

What question does regression answer?

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- Written as: $E[Y|X]$
- Regression approximates this function

The regression model

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$$Y = \beta_0 + \beta_1 X + \varepsilon$$

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- β : coefficients (what we estimate)
- ε : error term (what we can't explain)

Linear regression as approximation

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- Why linear? Simple, interpretable, often good enough

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- This is a **comparison**, not necessarily a causal effect

Descriptive vs. Causal interpretation

- **Descriptive:** How do units with different X values compare?
 - “People with more education earn more, on average”
- **Causal:** What happens if we change X for a given unit?
 - “If we give someone more education, they will earn more”
- Same coefficient, very different claims!

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- Getting tidy output:
 - `broom::tidy(model)` — coefficients as a data frame
 - `broom::glance(model)` — model-level statistics (R^2 , etc.)
- These are much easier to work with than `summary()`

Roadmap

Multiple Regression

Adding predictors

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 - **Holding X_2 constant**
- This is the “controlled” effect of X_1

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- This isolates the unique contribution of X_1

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 - How strongly the confounder relates to X

What makes a good control?

Good controls are variables that:

- Affect both the treatment and the outcome

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Pre-treatment confounders are the key!

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- Controlling for post-treatment variables can *introduce* bias

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 - Height and skill both affect being in NBA
 - Among NBA players, height and skill are negatively correlated
 - But not in the general population!

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- Example: `lm(income ~ factor(region), data = df)`
 - If reference is “North”, the “South” coefficient means: average income in South minus average income in North

Roadmap

Interaction Effects

When effects depend on context

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- We model this with **interaction terms**

The interaction model

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 (X \times Z) + \varepsilon$$

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- β_1 : effect of X when $Z = 0$
- β_2 : effect of Z when $X = 0$
- β_3 : how the effect of X changes as Z increases

The marginal effect of X

$$\frac{\partial Y}{\partial X} = \beta_1 + \beta_3 Z$$

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- Need to report effects at meaningful values of Z

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- But estimated jointly (shares the error variance)

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 - Marginal effect of X across values of Z

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- **Mistake 3:** Not showing how the effect varies
 - Plot the marginal effect across values of Z

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- In R: `marginaleffects::plot_predictions()`

Roadmap

Presenting Results

Why presentation matters

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 - Helps readers evaluate the **size** of effects

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- Highly customizable: statistics, labels, notes

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- Readers immediately see which effects are large vs. small

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Building sequential models

- Common strategy: show how results change as you add variables
- Step 1: Bivariate model (just X and Y)
- Step 2: Add control variables
- Step 3: Add interactions
- Present all three in one table:
→ `modelsummary(list(m1, m2, m3))`
- Shows robustness and what adding controls does to the estimate

Example workflow in R

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m1 <- lm(y ~ x, data = df)
m2 <- lm(y ~ x + z1 + z2, data = df)
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modelsummary(list(m1, m2, m3))
modelplot(list(m1, m2, m3))
plot_predictions(m3, condition = c("x", "z1"))
```

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- Multiple regression: “holding constant” interpretation
- Control variables help only if chosen correctly
- Interactions model conditional relationships
- Present results clearly: tables, coefficient plots, marginal effects

For next week

- Read Urdinez & Cruz (2020), chapter 8
- Read Gelman et al., chapters 13–14
- Complete Assignment 2
- Next session: Binary outcomes
 - Linear probability model vs. logistic regression
 - Interpreting logit results
 - Predicted probabilities and marginal effects

Questions?