

# Binary Outcomes

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## Today's goals

- Understand why OLS is problematic for binary outcomes
- Learn the linear probability model and its trade-offs
- Understand logistic regression and how to estimate it in R
- Interpret logit results using predicted probabilities and marginal effects

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## Roadmap

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## The Problem with Binary Outcomes

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## Binary outcomes are everywhere

- Many outcomes in social science are binary (yes/no):
  - Did someone vote?
  - Did a war break out?
  - Did a bill pass?
  - Did a country democratize?
- Our outcome  $Y \in \{0, 1\}$
- We want to model:  $P(Y = 1|X)$

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## What happens if we just use OLS?

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

- OLS gives us:  $E[Y|X] = \beta_0 + \beta_1 X$
- Since  $Y \in \{0, 1\}$ :  $E[Y|X] = P(Y = 1|X)$
- So OLS is modeling a probability as a linear function of  $X$
- This is the **linear probability model** (LPM)

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## The LPM: Simple and intuitive

- $\beta_1$  has a direct interpretation:
  - A one-unit increase in  $X$  changes the probability of  $Y = 1$  by  $\beta_1$
- Easy to estimate: just `lm(y ~ x, data = df)`
- Easy to interpret: same as OLS
- Many applied researchers use the LPM in practice

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## LPM limitations

- **Problem 1:** Predictions outside  $[0, 1]$ 
  - A linear function can produce  $\hat{P} < 0$  or  $\hat{P} > 1$
  - Probabilities must be between 0 and 1!
- **Problem 2:** Heteroskedasticity by construction
  - $\text{Var}(\epsilon|X) = P(1 - P)$ , which varies with  $X$
  - Standard errors from OLS are wrong (use robust SEs)
- **Problem 3:** Non-linearity at the extremes
  - True relationship between  $X$  and  $P(Y = 1)$  is S-shaped
  - LPM forces it to be linear

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## When is the LPM “good enough”?

- When probabilities are in the middle range (0.2–0.8)
  - The linear approximation is reasonable here
- When you care about **average marginal effects**
  - LPM and logit often give similar AMEs
- When simplicity of interpretation matters
- When is it **not** good enough?
  - Many observations near 0 or 1
  - You need predicted probabilities to be bounded
  - The relationship is clearly non-linear

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## Roadmap

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# Logistic Regression

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## The logistic function

$$P(Y = 1|X) = \frac{e^{X\beta}}{1 + e^{X\beta}} = \frac{1}{1 + e^{-X\beta}}$$

- This is an S-shaped (sigmoid) curve
- Output is always between 0 and 1
- Steep in the middle, flat at the extremes
- A natural model for probabilities

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## The logit model

We can rearrange the logistic function:

$$\log\left(\frac{P}{1-P}\right) = \beta_0 + \beta_1 X$$

- The left side is the **log-odds** (or “logit”)
- $\frac{P}{1-P}$  is the **odds** of the event
- The model is linear in the log-odds, not in the probability

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## Maximum likelihood estimation

- We can't use OLS for logistic regression
- Instead, we use **maximum likelihood estimation** (MLE)
- The intuition:
  - For each observation, the model predicts  $P(Y_i = 1)$
  - MLE finds the coefficients that make the observed data most likely
  - The “likelihood” is the product of these predicted probabilities
- No need to derive this—R does it for us

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## Estimating logit in R

- The function: `glm(y ~ x, family = binomial, data = df)`
- `glm`: generalized linear model
- `family = binomial`: tells R to use logistic regression
- The syntax is identical to `lm()`, just change to `glm()`
- Works with `broom::tidy()`, `modelsummary()`, etc.

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## Interpreting logit output: Log-odds

- The direct output gives coefficients in **log-odds**
- $\beta_1 = 0.5$  means:
  - A one-unit increase in  $X$  increases the log-odds by 0.5
- This is hard to interpret!
- Nobody thinks in log-odds

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## Interpreting logit output: Odds ratios

- Exponentiate the coefficient:  $e^{\beta_1}$  = odds ratio
- In R: `exp(coef(model))`
- $e^{0.5} \approx 1.65$  means:
  - A one-unit increase in  $X$  **multiplies** the odds by 1.65
  - Or: the odds increase by 65%
- Slightly more intuitive, but still not probabilities
- The change in probability depends on where you start

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## Why coefficients alone are not enough

- In OLS:  $\beta_1$  = change in  $Y$  for one-unit change in  $X$  (always)
- In logit: the change in **probability** depends on:
  - The current value of  $X$
  - The values of all other variables
- A coefficient of  $\beta_1 = 0.5$  could mean:
  - Going from  $P = 0.01$  to  $P = 0.016$  (tiny change)
  - Going from  $P = 0.50$  to  $P = 0.62$  (large change)
- We need better tools to interpret logit models

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## Roadmap

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## Interpreting Logit Results

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## Predicted probabilities

- The most intuitive way to interpret logit models
- “What is the predicted probability of  $Y = 1$  for a person with these characteristics?”
- In R:
  - `marginaleffects::predictions(model)`
  - Returns predicted probabilities for each observation
  - Or at specific values: `predictions(model, newdata = ...)`

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## Average marginal effects (AME)

- The marginal effect varies across observations
- The AME averages across all observations
- In R: `marginaleffects::avg_slopes(model)`
- Interpretation (like OLS):
  - “On average, a one-unit increase in  $X$  changes the probability of  $Y = 1$  by  $\Delta P$ ”
- This is often comparable to the LPM coefficient

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## Marginal effects at representative values

- Instead of averaging, evaluate at specific values
- Example: “What is the effect of education on voting for a 40-year-old woman?”
- In R:  
→ `avg_slopes(model, newdata = datagrid(age = 40, gender = "F"))`
- Useful when the marginal effect varies a lot across the sample

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## Plotting predicted probabilities

- The best way to communicate logit results
- Show how  $P(Y = 1)$  changes across values of  $X$
- Include confidence bands
- In R:  
→ `plot_predictions(model, condition = "x")`  
→ Plots the S-curve with uncertainty
- Much more informative than a table of log-odds

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## Comparing LPM and logit

- In many cases, LPM and logit give similar results
  - Especially for average marginal effects
  - Especially when probabilities are in the 0.2–0.8 range
- Where they differ:
  - Predicted probabilities near 0 or 1
  - LPM can go outside [0, 1]; logit cannot
  - Marginal effects at extreme values
- A good practice: estimate both and compare

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## Model fit for logit

- No  $R^2$  in the usual sense (MLE, not OLS)
- Alternative measures:
  - **Pseudo- $R^2$** : compares model to null model (McFadden)
  - **AIC**: penalized likelihood (lower = better)
  - **Classification**: what percent does the model correctly predict?
  - **ROC curve**: trade-off between true and false positives
- None is perfect; use them as rough guides
- Reported automatically by `modelsummary()` and `performance::r2()`

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## Roadmap

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Practice

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## Worked example: LPM vs. logit

```
lpm <- lm(vote ~ age + income + educ, data = df)
logit <- glm(vote ~ age + income + educ,
              family = binomial, data = df)
modelsummary(list("LPM" = lpm, "Logit" = logit))
avg_slopes(logit)          # compare to LPM coefficients
plot_predictions(logit, condition = "income")
```

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## Decision tree: When to use which?

- **Use LPM when:**
  - You want simple, quick interpretation
  - Probabilities are in the middle range
  - You mainly care about average effects
- **Use logit when:**
  - You need bounded predicted probabilities
  - Many observations have extreme probabilities (near 0 or 1)
  - You want to properly account for the binary nature of  $Y$
- In practice: estimate both, report the one most appropriate

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## Summary: Key takeaways

- Binary outcomes require special treatment
- LPM is simple but has known limitations
- Logit bounds probabilities between 0 and 1
- Log-odds and odds ratios are not intuitive—use marginal effects
- Predicted probabilities and AMEs are the best way to interpret logit
- Compare LPM and logit; they often agree on AMEs

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## For next week

- Read Urdinez & Cruz (2020), chapter 5 (§5.6)
- Read Gelman et al., chapters 11–12
- Complete Assignment 3
- Next session: Model interpretation and diagnostics
  - Beyond coefficient tables
  - Visualizing model results
  - Publication-quality tables
  - Key diagnostics

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Questions?

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