Model Interpretation and Diagnostics

Francisco Villamil

Applied Quantitative Methods II

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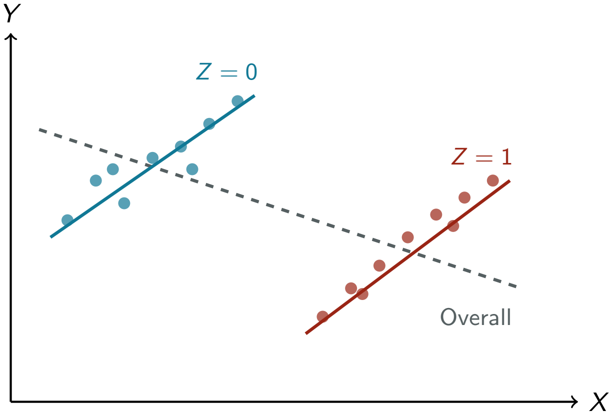
- Compute and plot predicted values, marginal effects, and first differences
- Present results with publication-quality tables and coefficient plots
- Understand simulation-based uncertainty
- Diagnose common regression problems: heteroskedasticity, non-linearity

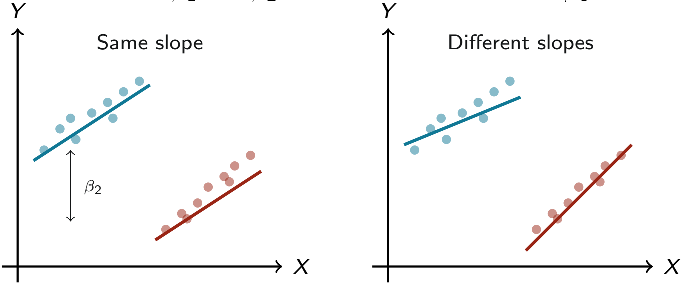
A motivating puzzle: Simpson's paradox



- Overall trend is **negative**, but within each group it's **positive**
- Ignoring Z gives the **wrong answer** — we need to account for it

Controlling vs. interacting

Control: $Y = \beta_1 X + \beta_2 Z$ **Interaction:** $+\beta_3 X \cdot Z$



- **Controlling**: adjusts the level (intercept) — the effect of X stays the same
- **Interacting**: allows the **slope** of X to differ across groups

Roadmap

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  - → Interactions: the coefficient on X_1 depends on X_2
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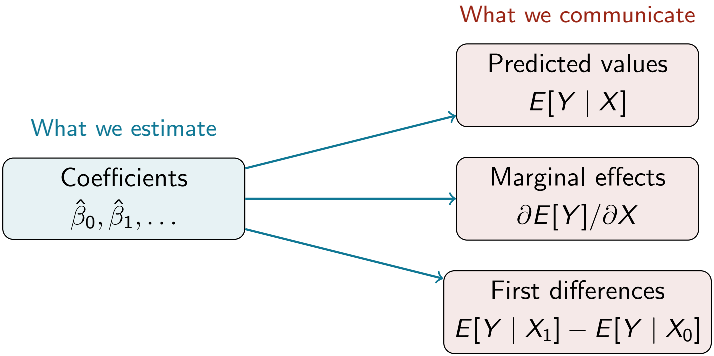
  - → Different scales: is $\hat{\beta} = 0.003$ big or small?

Coefficients vs. quantities of interest



You estimate a model with GDP (in dollars), population (in millions), and an interaction between them.

What does the coefficient on GDP tell you?

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- In R:
  - → `predictions(model)`
  - → `predictions(model,`
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  - → `plot_predictions(model,`
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  - → Separate lines/panels for each group

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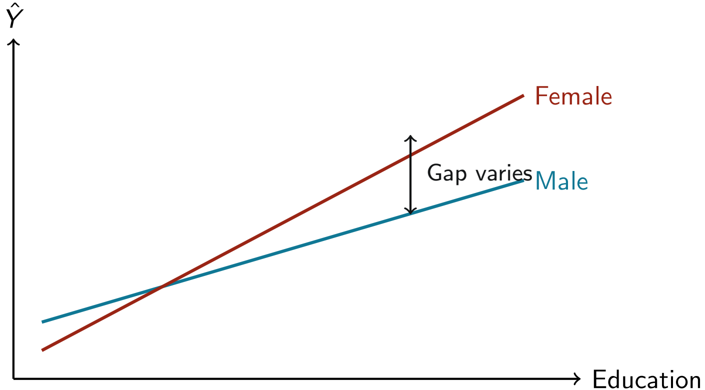
  - $\rightarrow$ `avg_slopes(model)`

0pt.50pt

Marginal effects with interactions

$$Y = \beta_0 + \beta_1 \text{Education} + \beta_2 \text{Female} + \beta_3(\text{Education} \times \text{Female}) + \varepsilon$$



0pt.50pt • ME of Education for males: β_1

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- This replaces the old manual approach of computing $\beta_1 + \beta_3 \cdot X_2$ by hand

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  - → Comparing meaningful scenarios (e.g., min vs. max)

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Quantity	Question	R function
Predicted value	What does the model predict here?	`predictions()`
Marginal effect	How much does Y change per unit of X?	`slopes()`
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- All work with `lm()`, `glm()`, and many other model types

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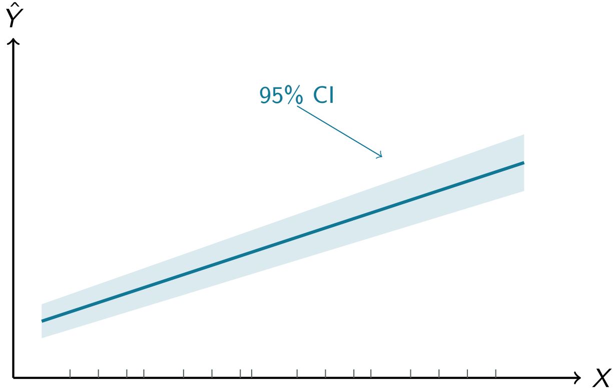
- `modelplot(list("Model 1" = m1, "Model 2" = m2))`

Prediction plots: the gold standard



- Shows the **full relationship**, not just one number
- Any audience can read it

Tables vs. plots: when to use which

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Conveying one key relationship		✓
Non-specialist audience		✓
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- Both are easy to produce with `modelsummary` and `marginaleffects`

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  - → Confidence intervals are automatic

Simulation: the logic

Estimate model $\hat{\beta}, \hat{V}$

```
m = lm(y ~ x, data = df)
```

↓

Draw $\tilde{\beta}^{(s)}$ from $N(\hat{\beta}, \hat{V})$, $s = 1, \ldots, S$

```
mvrnorm(1000, coef(m), vcov(m))
```

↓

For each draw, compute quantity of interest $Q^{(s)}$

```
sim_pred = sim_betas %*% x_scenario
```

↓

Use distribution of $Q^{(1)}, \ldots, Q^{(S)}$

```
quantile(sim_pred, c(0.025, 0.975))
```

Worked example: predicted values with uncertainty

```
m = lm(income ~ age + education + female, data = df)

# Using marginaleffects (delta method)
predictions(m,
  newdata = datagrid(age = 40, education = 16, female = 1))

# Using simulation
library(MASS)
sim_b = mvrnorm(1000, coef(m), vcov(m))
x = c(1, 40, 16, 1) # intercept, age, educ, female
sim_pred = sim_b %*% x
quantile(sim_pred, c(0.025, 0.5, 0.975))
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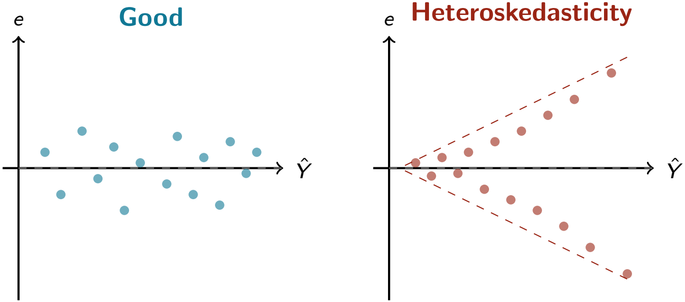
- How do we check? **Residual diagnostics**

Residuals vs. fitted values



- In R: `plot(model, which = 1)`
- Look for patterns: funnel shape, curves, clusters

You plot residuals vs. fitted values and see a clear funnel shape.

What does this tell you about your model?

What would you do about it?

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  - → Binary outcomes (always heteroskedastic, as in LPM)

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  - → If heteroskedastic: robust SEs are correct, classical are not

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- But interpretation changes!

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Log-level	$\log(Y) = \beta_0 + \beta_1 X$	$\Delta X = 1 \Rightarrow \%\Delta Y \approx 100 \cdot \beta_1$
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- Most common in practice: log-level and log-log

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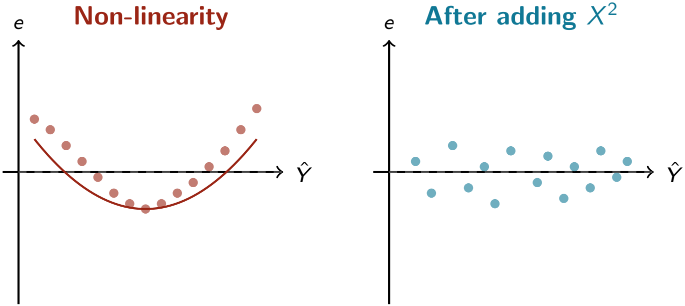
- Rule of thumb: use approximation when $\beta_1 \cdot \Delta X < 0.1$

Residual plots: checking linearity



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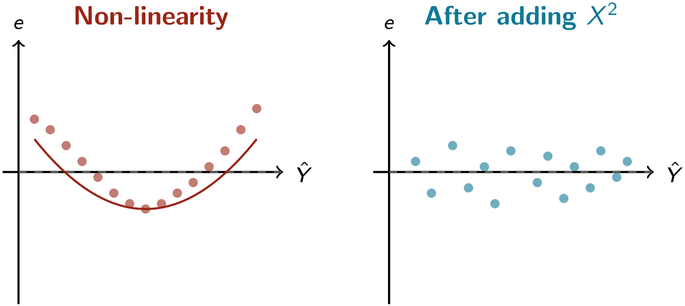

- A U-shape in residuals $\Rightarrow$ missing non-linearity
- Fixes: add X^2, use $\log(X)$, or use a more flexible model

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Roadmap

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- Predicted values, marginal effects, and first differences (all via `marginaleffects`)
- Present with plots (main text) and tables (appendix)
- Always use robust standard errors and check residual diagnostics
- Log-transform skewed variables (but interpret carefully)

For next week

- Complete Assignment 4
- Next session: Best Practices in Computing
  - → Reproducible workflows
  - → Project organization
  - → Writing clean R code

Questions?