

Applied Regression

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Applied Quantitative Methods II
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Today's goals

- Review regression as modeling conditional expectations

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- Understand multiple regression and control variables
- Learn how to model conditional relationships (interactions)
- Present results effectively with `modelsummary`

Roadmap

Regression Review

OLS Properties

Multiple Regression

Interaction Effects

Presenting Results

What question does regression answer?

- “What is the average value of Y for different values of X ? ”

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- This is the **conditional expectation function** (CEF)
- Written as: $E[Y|X]$
- Regression approximates this function

What does $E[\text{Income} | \text{Education}]$ look like?

Is it linear? Why or why not?

The regression model

The most common tool in social science:

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

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- X : explanatory variable(s)
- β : coefficients (what we estimate)
- ε : error term (what we can't explain)

The regression model in matrix form

With n observations and k variables:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

The OLS estimator:

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$$

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Don't memorize – just know this is what `lm()` computes for you

Linear regression as approximation

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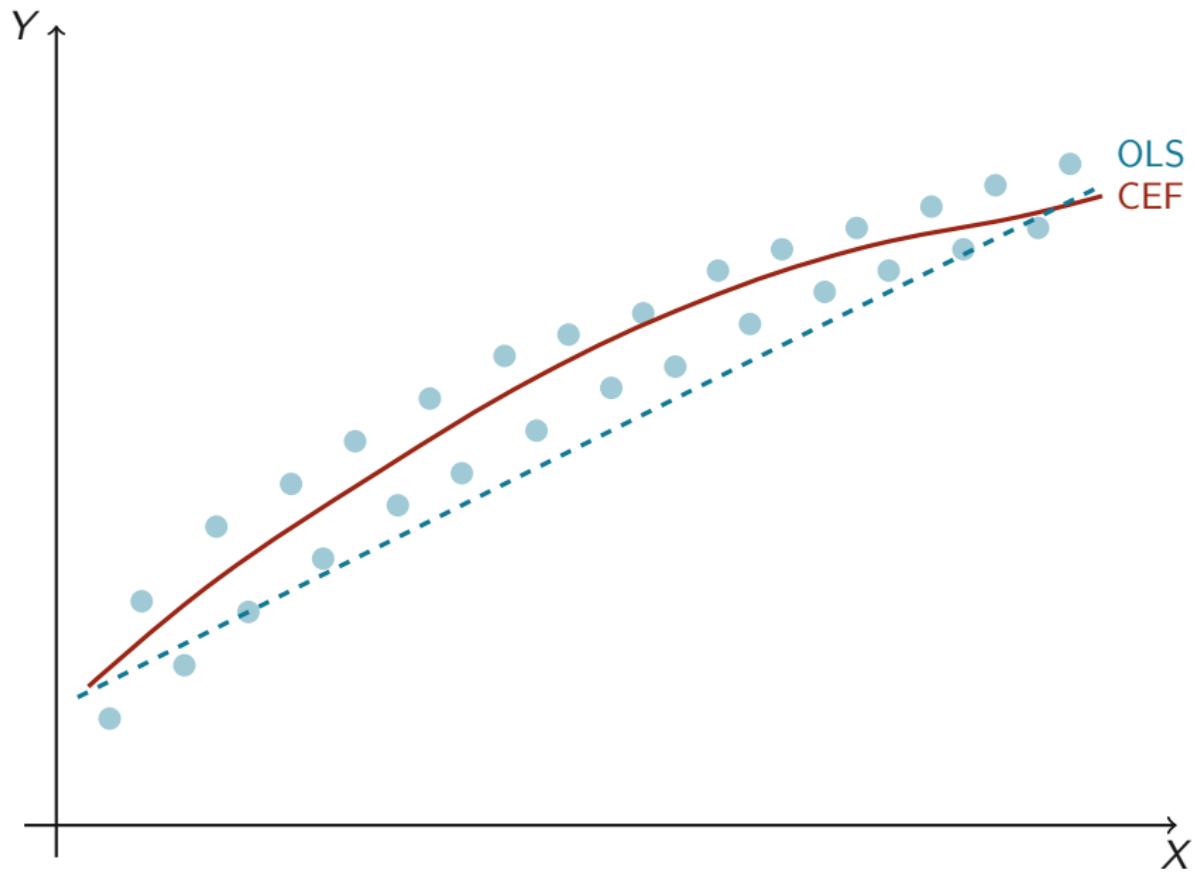
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- Why linear? Simple, interpretable, often good enough



Interpreting the slope coefficient

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 - Between groups that differ by 1 unit in X
- This is a **comparison**, not necessarily a causal effect

Two framings of β_1

- **Predictive framing:**
 - “Groups that differ by 1 in X differ by β_1 in Y , on average”
 - A comparison across units
- **Counterfactual framing:**
 - “If we changed X by 1, Y would change by β_1 ”
 - A statement about what would happen
- Same number, very different claims
- The counterfactual framing requires **causal assumptions**

Which framing — predictive or counterfactual —
does a randomized experiment give you?

Descriptive vs. Causal interpretation

- **Descriptive:** How do units with different X values compare?
 - “People with more education earn more, on average”
- **Causal:** What happens if we change X for a given unit?
 - “If we give someone more education, they will earn more”
- Same coefficient, very different claims!

Running a regression in R

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- Getting tidy output:
 - `broom::tidy(model)` — coefficients as a data frame
 - `broom::glance(model)` — model-level statistics (R^2 , etc.)
- These are much easier to work with than `summary()`

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A1–A4 are needed for unbiasedness; A5 for efficient SEs

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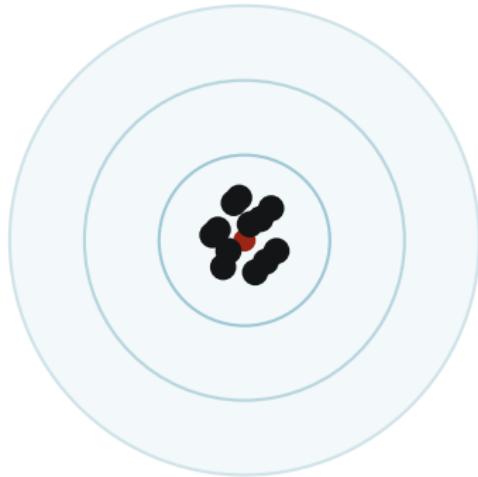
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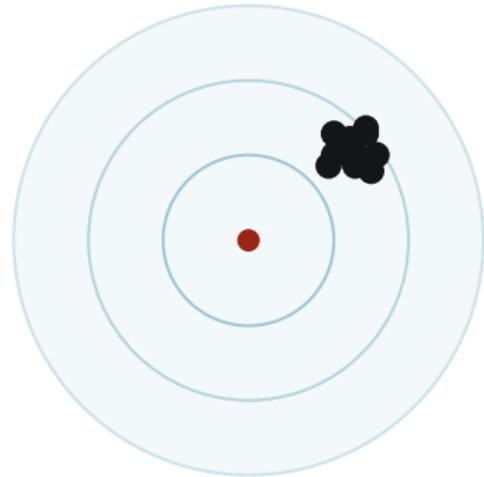
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- *Think of it like an unbiased dart thrower:
centered on the bullseye, but with some scatter*



Unbiased
centered on target



Biased
systematically off

Standard errors and uncertainty

OLS gives us $\hat{\beta}$, but how precise is it?

$$SE(\hat{\beta}_j) = \sqrt{\frac{\hat{\sigma}^2}{SST_j(1 - R_j^2)}}$$

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- The SE tells us how much $\hat{\beta}$ would vary across samples

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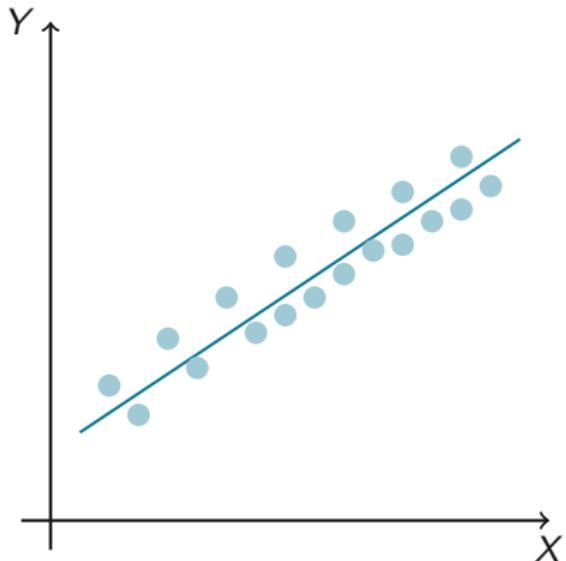
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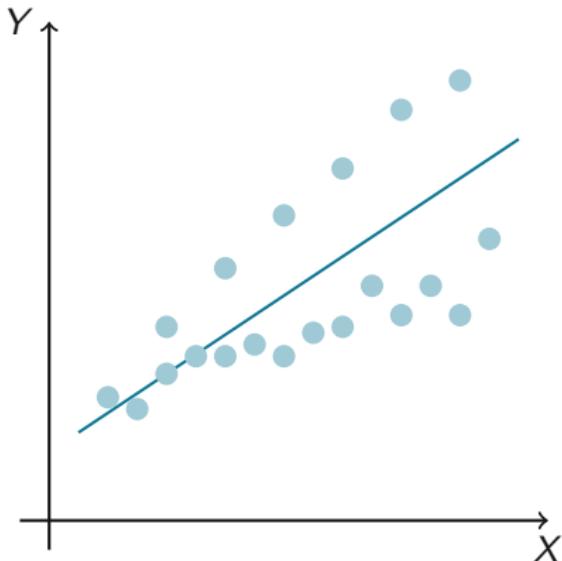
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- The estimates $\hat{\beta}$ are still unbiased!
- But the standard errors are wrong



Homoskedastic
constant spread



Heteroskedastic
spread increases with X

Solution: robust standard errors

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- There is no real cost when errors are homoskedastic

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- This is the “controlled” effect of X_1

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- This isolates the unique contribution of X_1

Omitted variable bias

If we omit a relevant variable X_2 , the short regression gives:

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- Zero only if $\hat{\beta}_2 = 0$ or $\tilde{\delta} = 0$

OVB in practice: education and income

	Short regression (omits ability)	Long regression (includes ability)
Education (β_1)	\$5,000	\$3,000
Ability (β_2)	—	\$5,000

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- Because ability ↑ education *and* ability ↑ income

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Pre-treatment confounders are the key!

You study the effect of job training on wages.

Is *current job type* a good or bad control?

Why?

Bad controls: Post-treatment variables

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- Example: Studying effect of job training on wages
 - Don't control for job type (affected by training)
 - Do control for education (determined before training)

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- Example: NBA players
 - Height and skill both affect being in NBA
 - Among NBA players, height and skill are negatively correlated
 - But not in the general population!

Categorical predictors

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- Example: `lm(income ~ factor(region), data = df)`
 - If reference is “North”, the “South” coefficient means: average income in South minus average income in North

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OLS Properties

Multiple Regression

Interaction Effects

Presenting Results

When effects depend on context

- Sometimes, the effect of X on Y depends on another variable Z

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- We model this with **interaction terms**

The interaction model

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 (X \times Z) + \varepsilon$$

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- β_1 : effect of X when $Z = 0$
- β_2 : effect of Z when $X = 0$
- β_3 : how the effect of X changes as Z increases

The marginal effect of X

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- Need to report effects at meaningful values of Z

Continuous \times categorical interactions

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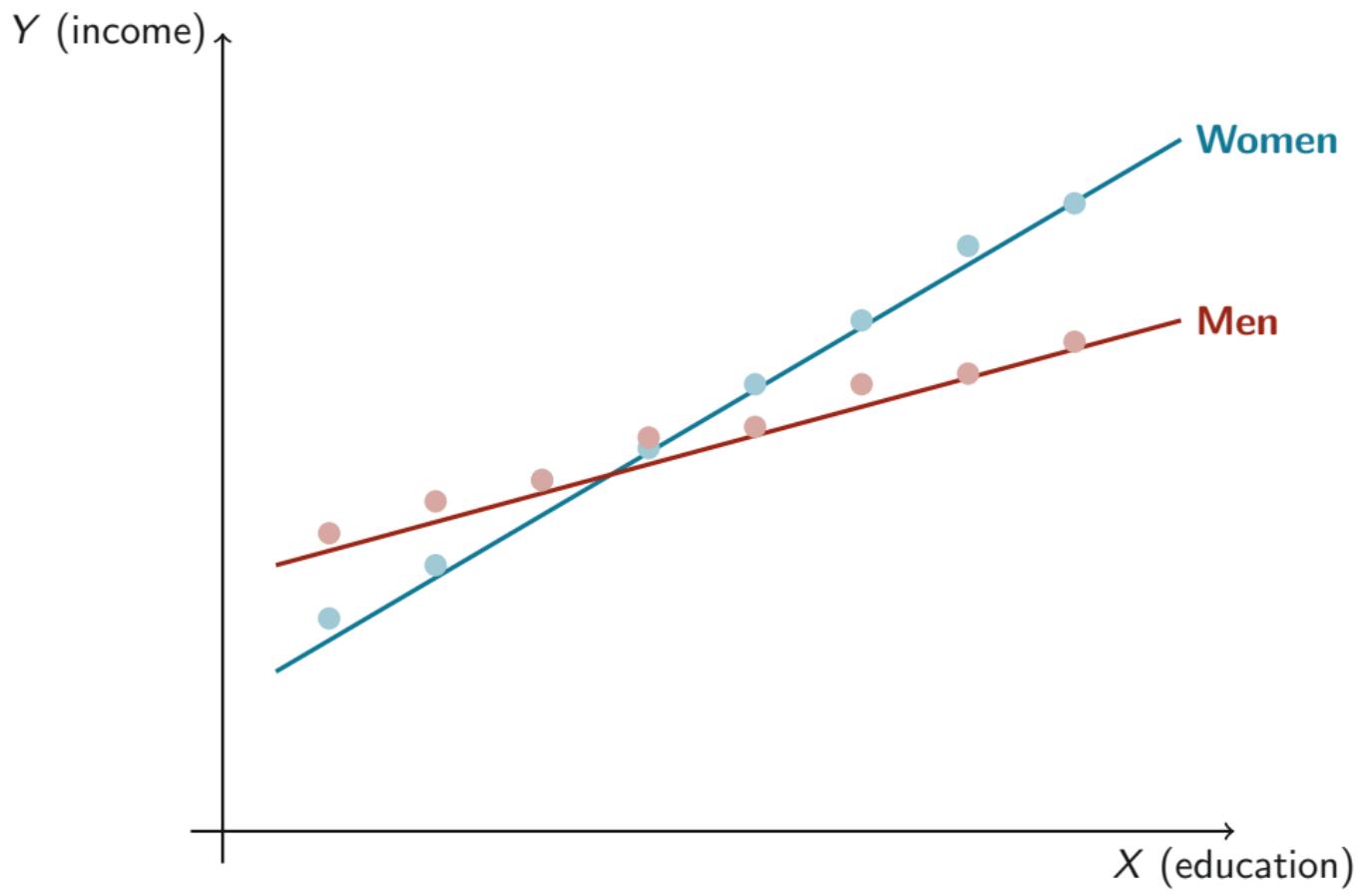
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- But estimated jointly (shares the error variance)



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 - Plot the marginal effect across values of Z

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- In R: `marginaleffects::plot_predictions()`

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 - Helps readers evaluate the **size** of effects

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- Highly customizable: statistics, labels, notes

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 - Easy to compare multiple models
- Often more effective than tables for communicating results
- Readers immediately see which effects are large vs. small

Building sequential models

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→ `modelsummary(list(m1, m2, m3))`
- Shows robustness and what adding controls does to the estimate

Example workflow in R

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m1 <- lm(y ~ x, data = df)
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modelplot(list(m1, m2, m3))
plot_predictions(m3, condition = c("x", "z1"))
```

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- Present results clearly: tables, coefficient plots, marginal effects

For next week

- Read Urdinez & Cruz (2020), chapter 8
- Read Gelman et al., chapters 13–14
- Complete Assignment 2
- Next session: Binary outcomes
 - Linear probability model vs. logistic regression
 - Interpreting logit results
 - Predicted probabilities and marginal effects

Questions?