Panel Data I

Francisco Villamil

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Today's goals

- Understand the structure and logic of panel data
- See why cross-sectional OLS can be biased due to unobserved heterogeneity
- Learn the fixed effects (within) estimator and its key intuition
- Add time fixed effects for two-way FE models
- Compare fixed effects and random effects; know when to use which
- Cluster standard errors correctly in panel settings

This lecture introduces panel data methods, which will occupy two sessions. The key motivating question is: why is it often not enough to run OLS on cross-sectional data, even with many controls? The answer — unobserved heterogeneity — leads naturally to fixed effects. Emphasize that the goal is not just to learn a new estimator, but to develop intuition about what variation is being used to identify the effect of interest.

Roadmap

Panel data: the basic structure

Unit i	Time t	y_{it}	x_{it}
1	2010	0.42	12.1
1	2011	0.51	13.0
1	2012	0.48	12.7
2	2010	0.61	9.4
2	2011	0.59	9.8
2	2012	0.64	10.2

- N units, each observed at T time points
- Data indexed (i, t): unit i at time t

Start concretely. Panel data simply means we observe the same units more than once. The key index is always (i, t): unit and time. Walk through the table: unit 1 appears in 2010, 2011, 2012; unit 2 also appears in all three years. This structure is very common in social science: countries surveyed annually, individuals re-interviewed across waves, firms reporting quarterly. The data structure itself is the motivation for the methods.

Panel data: examples

- **Cross-national**: GDP, democracy, conflict for 150+ countries × 50 years

- **Survey panels**: same individuals surveyed in 2010, 2014, 2018
  - → European Social Survey rotating panels
  - → British Household Panel Survey

- **Sub-national**: US states × years; municipalities × election cycles

- **Firms**: quarterly earnings reports for publicly traded companies

- **Running example**: US state-level presidential approval × years

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Give students a range of examples so they can connect this to their own research. The running example (US states over time) is intuitive and will anchor the technical material. Mention that panels can be balanced (all units observed in all periods) or unbalanced (some units missing in some periods). Most real datasets are unbalanced.

Why panel data? Three advantages

- **More observations**: $N \times T$ rows instead of N — more statistical power

- **Within-unit variation**: follow how y_{it} changes as x_{it} changes *for the same unit*
  - → Cleaner comparison than across different units

- **Control for unobserved heterogeneity**: the big one
  - → Units may differ in ways we cannot measure
  - → Panel structure lets us "absorb" those differences

The third advantage is the key motivation for everything that follows. Many unit-level characteristics are simply unobservable: political culture, institutional quality, historical legacy, personality. If these unobservables are correlated with the regressor, OLS is biased. Panel methods allow us to "difference out" these time-invariant unit characteristics without ever having to measure them. That is a very powerful idea.

Roadmap

Motivating example: presidential approval

Does unemployment drive down presidential approval?
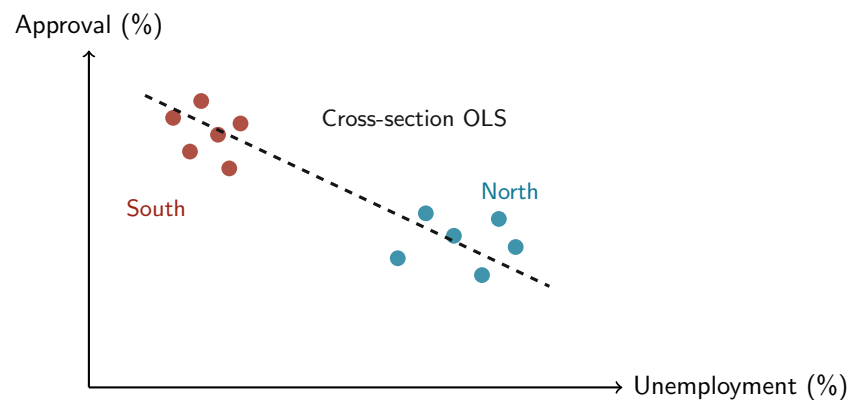
This is the classic illustration of omitted variable bias in a cross-section. Southern states in the US tend to have lower unemployment AND higher presidential approval (at least for Republican presidents, and historically for any incumbent). Northern states tend to have higher unemployment and lower approval. The cross-sectional slope picks up this between-group difference and confounds it with the actual effect of unemployment. The question is: how do we isolate the within-state effect of unemployment changes on approval changes?

The cross-sectional slope is negative.

Does that mean unemployment *causes* lower approval?

What else might explain this pattern?

Give students 30–60 seconds to think or turn to a neighbor. Expected answers: Southern states have historically different political cultures, economic structures, partisan composition. What we see is the difference *between* groups of states, not the effect of unemployment changes *within* any particular state. This is the omitted variable problem in action.

The problem: unit-level confounders

- Units differ in many **unobserved** ways:
  - → Political culture, history, institutional quality
  - → Personality (in individual panels)
  - → Industrial structure, geography

- If these unobservables correlate with x_{it} **and** y_{it}: OLS is biased

- The model:

$$y_{it} = \alpha_i + \beta x_{it} + \varepsilon_{it}$$

- α_i = unit-specific intercept (the unobserved heterogeneity)
- Cross-section OLS ignores $\alpha_i \Rightarrow$ omitted variable bias

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The key insight: if we run OLS ignoring the α_i terms, we get biased estimates of β whenever α_i is correlated with x_{it}. In the presidential approval example, the "South" variable captures part of α_i: southern states have high α_i (higher approval baseline) and low x_{it} (lower unemployment). Omitting α_i from the regression means part of its effect gets attributed to x_{it}. This is a classic omitted variable bias story, but the variable being omitted is the whole set of unit-level characteristics, not just one.

Simpson's paradox: the intuition
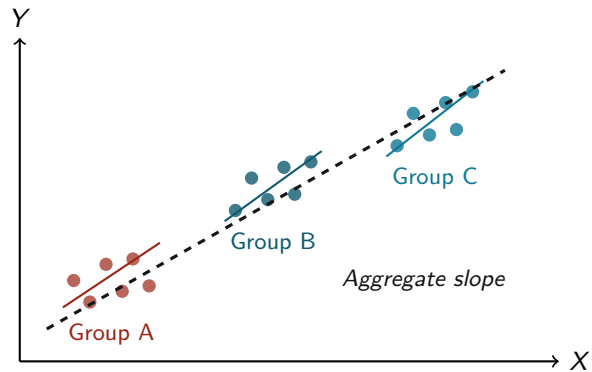
- Within each group: positive slope
- Cross-section OLS: also positive, but for the **wrong reason**
- The group-level differences dominate the estimate

This diagram shows that even when the aggregate relationship looks right, it can be driven entirely by between-group differences rather than the true within-group relationship. Here all three groups show a positive within-group slope, and the overall slope is also positive — but in other cases the signs can differ (the classic Simpson's paradox). The point is that we want to estimate the within-group (within-unit) relationship. Fixed effects achieve exactly this.

Roadmap

Fixed effects: the key idea

```
┌─────────────────────────┐
│   Each unit has its own  │
│   baseline level ($\alpha_i$)  │
└─────────────────────────┘
            │
            ▼
┌─────────────────────────┐
│  Include a **dummy variable** │
│      for each unit       │
└─────────────────────────┘
            │
            ▼
┌─────────────────────────┐
│   $\alpha_i$ is **absorbed**   │
│     into the intercepts  │
└─────────────────────────┘
            │
            ▼
┌─────────────────────────┐
│       $\hat{\beta}$ uses only    │
│    **within-unit** variation  │
└─────────────────────────┘
```
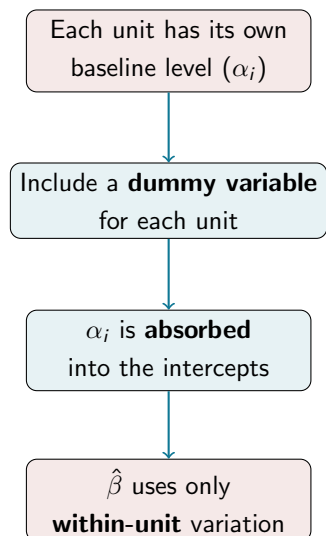
This is the conceptual core of the lecture. Fixed effects is just a fancy way of saying: include a dummy variable for each unit. If Alabama always has higher approval than Oregon (regardless of unemployment), that difference gets captured by the Alabama and Oregon dummies. The coefficient on unemployment is then identified purely from within-state changes over time. No more cross-sectional confounding from unobserved state characteristics.

The within (demeaning) estimator

Starting from $y_{it} = \alpha_i + \beta x_{it} + \varepsilon_{it}$, subtract unit means:

$$\underbrace{y_{it} - \bar{y}_i}_{\tilde{y}_{it}} = \beta \underbrace{(x_{it} - \bar{x}_i)}_{\tilde{x}_{it}} + \underbrace{(\varepsilon_{it} - \bar{\varepsilon}_i)}_{\tilde{\varepsilon}_{it}}$$

- α_i **cancels out** — the unit effect is gone
- Regressing $\tilde{y}_{it}$ on $\tilde{x}_{it}$ gives the FE estimator
- Uses only variation *within* each unit over time
- Works because α_i is constant: $\bar{\alpha}_i = \alpha_i$

The demeaning derivation is the cleanest way to see why FE works. By subtracting the unit mean from both sides, α_i disappears: it appears in both y_{it} and $\bar{y}_i$ and thus cancels. What remains is demeaned Y and demeaned X. Regressing one on the other gives the within-unit estimator. Walk through this slowly. The key punchline: since α_i cancels, it doesn't matter whether it's correlated with x_{it} or not. That's the magic of FE.

FE = dummies for each unit

- Mathematically equivalent to including unit dummies:

$$y_{it} = \sum_{i=1}^{N} \alpha_i D_i + \beta x_{it} + \varepsilon_{it}$$

  where $D_i = 1$ if observation belongs to unit i

- "Least squares dummy variable" (LSDV) estimator
- Same $\hat{\beta}$, different computational approach
- **Key implication**: cannot estimate effect of **time-invariant** variables
    - → If z_i does not vary over time, it is collinear with D_i
    - → Example: "South" dummy, gender, country of birth

The equivalence between FE and dummy variables is important for understanding both what FE does and what it cannot do. Because we include a separate intercept for every unit, any variable that is constant within a unit is perfectly collinear with those unit dummies. So we literally cannot estimate the effect of gender, race, or country in a unit FE model. This is not a bug but a feature of the design: FE is absorbing all time-invariant variation, observed or not.

Presidential approval: what FE does
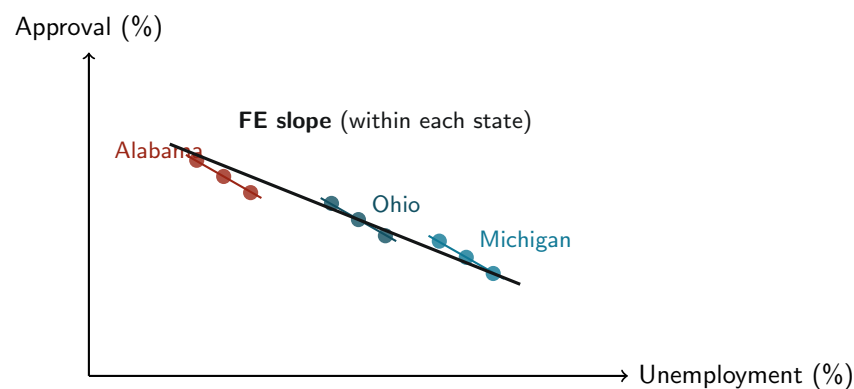
- Each state has its own intercept; the slope is shared
- FE estimates: as *this state's* unemployment rises, its approval falls

This visualization shows how FE works with the running example. Instead of comparing Alabama to Michigan (which would confound structural differences), FE asks: when Alabama's unemployment goes up by 1 point, how much does Alabama's approval change? When Michigan's unemployment goes up, how much does Michigan's approval change? The separate intercepts absorb the between-state differences; the common slope captures the within-state relationship. Emphasize that the FE slope is identified from the within-state time variation only.

Fixed effects in R

- Preferred: `fixest` package (fast, flexible, clustered SEs built in)

  ```
  library(fixest)
  feols(approval ~ unemp | state, data = df)
  ```

- Alternative: `plm` package

  ```
  library(plm)
  pdata.frame(df, index = c("state","year"))
  plm(approval ~ unemp, data = pdf, model = "within")
  ```

- The | in `feols()` separates regressors from fixed effects
- Tables with `modelsummary()` work seamlessly with `feols` objects

Walk through the syntax carefully. In `feols()`, everything to the left of | is a standard regressor; everything to the right is a set of fixed effects. So `feols(y ~ x1 + x2 | unit, data = df)` regresses y on x_1 and x_2 with unit fixed effects. The `plm` approach is older and slower but worth knowing since students will encounter it in papers. The key argument in `plm` is `model = "within"` (other options: `"pooling"` for OLS, `"random"` for RE). Remind students that `modelsummary()` works with `feols` objects and will display the fixed effect label in the table footer.

Roadmap

FE controls for everything time-invariant.

But what if something happens in 2008 that affects *all* states simultaneously?

The 2008 financial crisis is the perfect motivator here. Unemployment spiked everywhere at once. Presidential approval also shifted. Unit FE cannot separate the unemployment effect from the common macro shock: every state's unemployment went up, every state's approval went down. The unit demeaning removes the 2008 mean *for each state*, but the common shock is still in the within-unit variation. This motivates adding time fixed effects.

A new threat: time trends and common shocks

- Unit FE removes time-invariant unit characteristics

- But what about events that affect **all** units at the same time?
  - → A global recession hits every state simultaneously
  - → A presidential scandal lowers approval everywhere
  - → A pandemic affects all countries in 2020

- If these shocks also correlate with x_{it}: new bias

- Solution: add **time fixed effects** γ_t

After selling unit FE, immediately introduce the new problem it does not solve. Unit FE eliminates time-invariant confounders; time FE eliminates unit-invariant confounders (i.e., events that hit all units equally in a given period). If unemployment rises everywhere in a recession, and presidential approval also falls everywhere in that recession, a model without time FE would partly attribute the approval drop to unemployment even if the two are unrelated. Time FE fix this by absorbing the "common trend" in approval that affects all states each year.

Two-way fixed effects model

$$y_{it} = \alpha_i + \gamma_t + \beta x_{it} + \varepsilon_{it}$$

- α_i: unit FE — absorbs all time-invariant unit characteristics
- γ_t: time FE — absorbs all unit-invariant time shocks
- $\hat{\beta}$: identified from variation **within units**, **across time**, **net of common trends**

- In R:

      feols(approval ~ unemp | state + year, data = df)

The two-way FE model is the workhorse of modern panel data analysis in political science and economics. Adding time FE is as simple as adding a second variable after the | in `feols()`. Emphasize the interpretation: $\hat{\beta}$ now captures how much approval deviates from a state's own average when unemployment deviates from that state's average, *in the same year as all other states*. The year fixed effects soak up everything that is common to all states in a given year. This is a very demanding specification but also a very credible one.

What TWFE absorbs

Type of variation	Unit FE	Two-way FE
Time-invariant unit differences	absorbed	absorbed
Unit-invariant time shocks	not absorbed	absorbed
Within-unit, across-time variation	used for $\hat{\beta}$	used for $\hat{\beta}$

- TWFE is conservative: only uses within-unit, net-of-time-trends variation
- Leaves less variation to identify $\hat{\beta} \Rightarrow$ larger standard errors
- But more credible: fewer threats to identification

This table summarizes what gets absorbed by each specification. TWFE is more demanding than unit FE alone. Students often worry that adding time FE will wipe out the effect. Reassure them: if the true effect exists, TWFE will still find it, because there is still within-unit variation in x_{it} even after removing year means. What TWFE removes is the spurious correlation driven by common shocks. A coefficient that survives TWFE is particularly credible. The price is higher standard errors (less variation left), but that's appropriate uncertainty.

Teaching evaluations: worked example

- Dataset: instructors evaluated across multiple courses
  - → Outcome: `Eval` (student evaluation score)
  - → Predictors: `Apct` (attractive), `Enrollment`, `Required`
- OLS ignores that instructors differ systematically:
  - → Friendliness, teaching experience, subject difficulty
- Unit FE (instructor):

  ```
  feols(Eval ~ Apct + Enrollment + Required
     | instructor, data = evals)
  ```

- $\hat{\beta}_{\text{Apct}}$: compares same instructor's courses, not different instructors

The teaching evaluations dataset is a nice pedagogical example because students find it intuitive. Does attractiveness improve evaluations? A cross-sectional comparison is biased: attractive instructors might also be more outgoing, teach easier subjects, etc. With instructor FE, we compare the same instructor's evaluations across different courses, controlling for the time-varying variables. The FE estimate of attractiveness is then: "compared to this instructor's other courses, do courses where the instructor is rated more attractive get higher evaluations?" That is a much sharper question. The answer in the literature: the effect is smaller and sometimes disappears with FE.

Roadmap

Random effects: a different assumption

$$y_{it} = \alpha + \beta x_{it} + \underbrace{\eta_i}_{\text{unit random effect}} + \varepsilon_{it}$$

- $\eta_i \sim N(0, \sigma_\eta^2)$: unit-specific deviation, treated as **random**

- **Key assumption**: $\eta_i \perp x_{it}$ (unit effects uncorrelated with regressors)

- If this holds: RE is more **efficient** than FE

- If this fails: RE is **biased**; FE is consistent

- Unlike FE: can estimate effects of **time-invariant** variables

Random effects treats the unit-specific term as part of the error, not as a parameter to estimate. This is a weaker treatment — it uses both within and between variation — which makes it more efficient if the assumption holds. But the assumption is strong: that unit effects are uncorrelated with regressors. In most political science applications (countries, regions, individuals) this assumption is questionable. Unit effects often reflect exactly the unobserved confounders we are worried about. That is why FE is usually preferred. RE is worth knowing mainly to understand the Hausman test and when comparing specifications.

FE vs. RE: the tradeoff

	Fixed Effects	Random Effects
Assumption	α_i correlated with X?	$\eta_i \perp X$
Consistency	Always (if $T \to \infty$)	Only if $\eta_i \perp X$
Efficiency	Less efficient	More efficient
Time-invariant vars	Cannot estimate	Can estimate

- If you are unsure: **use FE**

- More conservative, more credible

- RE requires an untestable assumption; FE does not

This table is the key reference. The critical point: FE is consistent regardless of whether unit effects are correlated with the regressors. RE is only consistent if they are not. In practice, we nearly always have reason to worry about correlation (that is the whole point of using panel data), so FE is the default. The efficiency advantage of RE is real but usually modest, and it comes at the cost of a questionable assumption. The practical advice "when in doubt, use FE" is widely accepted in the literature.

Hausman test: FE vs. RE

- **Hausman test** (1978): formally test whether $\eta_i \perp x_{it}$

- H_0: no correlation between unit effects and regressors (RE is consistent)

- H_1: correlation exists (only FE is consistent)

- If $p < 0.05$: reject $H_0 \Rightarrow$ use FE

- If $p > 0.05$: fail to reject $\Rightarrow$ RE or FE both OK

- In R (plm package):
  ```
  fe_mod = plm(y ~ x, data = pdf, model = "within")
  re_mod = plm(y ~ x, data = pdf, model = "random")
  phtest(fe_mod, re_mod)
  ```

The Hausman test compares the FE and RE estimates. Under H_0 (RE is correct), both FE and RE are consistent and they should produce similar estimates. Under H_1 (unit effects correlated with regressors), FE is consistent but RE is not; the estimates will diverge. The test statistic captures this divergence. In practice the Hausman test often rejects, confirming the need for FE. Even when it doesn't reject, many researchers still prefer FE for its more credible assumptions. Note that the Hausman test requires using `plm` rather than `fixest`, since `phtest()` is a `plm` function.

Roadmap

Why panel data violates iid

- OLS standard errors assume errors are **independent** across observations

- In panel data: observations from the *same unit* are correlated over time
  - → Alabama in 2010 and Alabama in 2011 are not independent
  - → Same unit experiences same shocks, trends, institutions

- Ignoring this: standard errors are **too small**

- Too-small SEs $\Rightarrow$ inflated t-statistics $\Rightarrow$ false positives

- Solution: **cluster standard errors by unit**

Serial correlation within units is one of the most common problems in panel data analysis and one of the most commonly ignored. The intuition: if we have 50 states over 20 years = 1000 observations, we do not really have 1000 independent pieces of information. Alabama's 20 observations all share the same political culture, economic structure, and history. Clustering by state tells the standard error estimator to treat the 20 Alabama observations as a single cluster, reducing the effective sample size. The result: standard errors are larger (correct) and test statistics are smaller (more conservative).

Clustering in practice

- `feols()` clusters by the FE variable automatically:
  ```
  feols(y ~ x | state, data = df)
  # SEs already clustered by state
  ```

- Explicit clustering:
  ```
  feols(y ~ x | state, data = df, cluster = ~state)
  ```

- In `modelsummary()` tables:
  ```
  modelsummary(m, vcov = ~state)
  ```

- Rule of thumb: cluster at the level of treatment assignment
  - → If state gets the treatment, cluster by state
  - → If individual gets the treatment, cluster by individual

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The good news: `feols()` clusters by default by the first fixed effect, so students using `feols()` are automatically protected. The bad news: students using `lm()` with manual dummies or `plm()` need to be explicit. Emphasize the rule of thumb: cluster at the level at which the treatment is assigned (or at which errors are likely to be correlated). In most political science panel data with state or country units, clustering by unit is correct. Two-way clustering (by both unit and time) is sometimes used in economics but less common in political science.

Comparing specifications: a template

	(1) OLS	(2) Unit FE	(3) TWFE
Unemployment	−0.82***	−0.51**	−0.43**
	(0.15)	(0.18)	(0.16)
State FE	No	Yes	Yes
Year FE	No	No	Yes
Clustered SE	No	Yes	Yes
N	1000	1000	1000

- Coefficient shrinks as we add FEs: the OLS estimate was partly confounded
- Report all three specifications for transparency
- Preferred specification: (3)

This template table is something students should memorize as a presentation format. Reporting multiple specifications side by side is standard practice: it shows how the estimate changes as we add controls, unit FE, and time FE. If the coefficient is stable across columns, we have evidence of robustness. If it shrinks dramatically (as here), we learn that the naive OLS estimate was partially driven by confounding. The shrinkage from −0.82 to −0.43 suggests that about half of the original cross-sectional estimate was confounded by unit and time effects. Standard practice in panel data papers is to show at minimum columns (2) and (3), often with column (1) for reference.

Roadmap

Summary: key takeaways

- Panel data: N units $\times$ T time periods; indexed (i, t)

- Unobserved heterogeneity α_i biases cross-sectional OLS
  - $\rightarrow$ If α_i correlates with x_{it}: omitted variable bias

- **Fixed effects** (within estimator): demeans data by unit, eliminates α_i
  - $\rightarrow$ Cannot estimate time-invariant variables

- **Two-way FE**: add time dummies to remove common shocks

- **FE vs. RE**: use FE when unit effects may correlate with X; use Hausman test

- Always **cluster standard errors** by unit

Recap the five main ideas. (1) Data structure: the (i, t) index. (2) The problem: unobserved unit heterogeneity biases OLS when correlated with regressors. (3) FE solution: demeaning removes the unit effect. (4) TWFE: also remove common time shocks. (5) Clustering: account for serial correlation within units. These are the building blocks for more advanced panel methods next session. Stress that these are now standard in applied work — almost any paper using panel data will have TWFE with clustered SEs as its main specification.

For next session

- Complete Assignment 5

- Read the assigned paper using panel FE

- Next session: Panel Data II
  - $\rightarrow$ Difference-in-Differences (DiD)
  - $\rightarrow$ Event studies
  - $\rightarrow$ Staggered treatment timing
  - $\rightarrow$ Recent advances in DiD (Callaway–Sant'Anna, etc.)

Preview the next session. DiD is a natural extension of two-way FE: it adds the notion of a treatment group vs. a control group, and uses the pre/post variation within each group. The identification assumption (parallel trends) is a strengthening of the FE assumptions. Recent work has shown that the standard staggered DiD estimator can be biased when treatment effects are heterogeneous across cohorts — this is an active area of research that students will see if they read recent applied papers.

Questions?