Applied Regression (I)

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Roadmap

Review: Key Concepts from AQMSS-I

The regression model

The most common tool in social science:

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

- Y: outcome we want to explain
- X: explanatory variable(s)
- β: coefficients (what we estimate)
- ε: error term (what we can't explain)

What regression tells us

- Regression estimates **conditional expectations**
- "What is the average Y for units with a given value of X?"

- The slope β_1 tells us:
  - → How much Y changes, on average
  - → When comparing units that differ by 1 in X

Descriptive vs. Causal interpretation

- **Descriptive**: How do units with different X values compare?
  - → "People with more education earn more, on average"

- **Causal**: What happens if we change X for a given unit?
  - → "If we give someone more education, they will earn more"

- Same coefficient, very different claims!

The challenge of causal inference

- Causal effects are about **counterfactuals**
- "What would have happened if things were different?"

- The problem: we can't observe counterfactuals
- We need strategies to infer them
- This will be a recurring theme throughout the course

Today's goals

- Understand regression as modeling conditional expectations
- Review the logic of OLS
- Discuss when regression can tell us about causation
- Learn how to think about control variables

Roadmap

Regression as Conditional Expectations

What question does regression answer?

- "What is the average value of Y for different values of X?"
- This is the **conditional expectation function** (CEF)
- Written as: $E[Y|X]$
- Regression approximates this function

Example: Income and support for redistribution

- Research question: How does income relate to support for redistribution?
- CEF: "What is the average support for redistribution among people earning $50k? Among those earning $100k?"
- We can estimate this with regression

Linear regression as approximation

- The true CEF might be complicated
- Linear regression fits the **best linear approximation**

- Even if the true relationship is non-linear
- The linear fit is still the best predictor among linear functions

- Why linear? Simple, interpretable, often good enough

The OLS formula

$$\hat{\beta} = \frac{Cov(X, Y)}{Var(X)}$$

- This gives us the slope that minimizes squared errors
- Intuition: how much does Y move when X moves?
- Scaled by how much X varies

Interpreting the slope coefficient

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

- β_1 represents:
  - → The difference in average Y
  - → Between groups that differ by 1 unit in X

- This is a **comparison**, not necessarily a causal effect

Roadmap

From Description to Causation

When can we interpret regression causally?

- Descriptive interpretation: always valid
  → "Higher income is associated with less support for redistribution"
- Causal interpretation: requires additional assumptions
  → "Increasing someone's income would decrease their support"
- The difference is crucial!

The potential outcomes framework

- Every unit has two potential outcomes:
  - → $Y(1)$: outcome if treated
  - → $Y(0)$: outcome if not treated
- Causal effect for unit i: $\tau_i = Y_i(1) - Y_i(0)$
- The fundamental problem: we only observe one of these

Why experiments work

- In an experiment, treatment is randomly assigned
- This means treated and control groups are comparable
- We can use the control group's outcomes as counterfactual
- The simple difference in means estimates the causal effect

The challenge with observational data

- Most social science data is observational
- Treatment is not randomly assigned

- Problem: treated and control groups may differ
- Not just in treatment, but in other ways too

- These differences can bias our estimates

Confounding

A **confounder** is a variable that:

- Affects both the treatment and the outcome
- Creates a spurious association between them

- Example: Education, income, and political preferences
- Education affects both income and political views
- Income-politics relationship may be partly spurious

The logic of controlling

- If we can identify the confounders...
- ...we can "control" for them in regression

- The idea: compare units with same confounder values
- This eliminates the spurious part of the association

- But: this requires knowing what the confounders are

Roadmap

Control Variables in Practice

Multiple regression

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$$

- β_1 now represents:
  - → The difference in average Y
  - → Between groups that differ by 1 in X_1
  - → **Holding X_2 constant**

- This is the "controlled" effect of X_1

How controlling works

- OLS with multiple variables "partials out" the controls
- Technically: we look at variation in X_1 that is unrelated to X_2
- This isolates the unique contribution of X_1

Omitted variable bias

- If we omit a confounder, our estimate will be biased
- The bias formula:

$$\text{Bias} = \beta_{\text{confounder}} \times \delta_{X,\text{confounder}}$$

- Depends on:
  - → How strongly the confounder affects Y
  - → How strongly the confounder relates to X

What makes a good control?

Good controls are variables that:

- Affect both the treatment and the outcome
- Are determined **before** the treatment
- Are not affected by the treatment

Pre-treatment confounders are the key!

Bad controls: Post-treatment variables

- Never control for variables caused by the treatment
- Example: Studying effect of job training on wages
  - → Don't control for job type (affected by training)
  - → Do control for education (determined before training)
- Controlling for post-treatment variables can *introduce* bias

Bad controls: Colliders

- A **collider** is caused by both X and Y
- Controlling for it creates a spurious association
- Example: NBA players
  - → Height and skill both affect being in NBA
  - → Among NBA players, height and skill are negatively correlated
  - → But not in the general population!

The limitations of controlling

- We can only control for what we observe and measure
- Unobserved confounders will still bias our estimates
- There's no purely statistical solution to this
- Need theory + research design, not just more controls

Summary: Key takeaways

- Regression estimates conditional expectations
- Causal interpretation requires additional assumptions
- Control variables help only if chosen correctly
- Controlling for the wrong variables can make things worse
- Always think about what you're comparing

For next week

- Read Angrist & Pischke (2008), chapters 1-3
- Read Urdinez & Cruz (2020), chapter 5
- Work on Problem Set 1

- Next session: More on regression in practice
  - → Interactions
  - → Non-linear relationships
  - → Standard errors and inference

Questions?