Applied Regression

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Applied Quantitative Methods II

MA in Social Sciences, Spring 2026

Today's goals

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- Understand multiple regression and control variables
- Learn how to model conditional relationships (interactions)
- Present results effectively with `modelsummary`

Roadmap

Regression Review

OLS Properties

Multiple Regression

Interaction Effects

Presenting Results

What question does regression answer?

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- This is the **conditional expectation function** (CEF)
- Written as: $E[Y|X]$
- Regression approximates this function

What does $E[\text{Income} \mid \text{Education}]$ look like?

Is it linear? Why or why not?

The regression model

The most common tool in social science:

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

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- Y: outcome we want to explain
- X: explanatory variable(s)
- β: coefficients (what we estimate)
- ε: error term (what we can't explain)

The regression model in matrix form

With n observations and k variables:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

The OLS estimator:

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$$

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Don't memorize – just know this is what `lm()` computes for you

Linear regression as approximation

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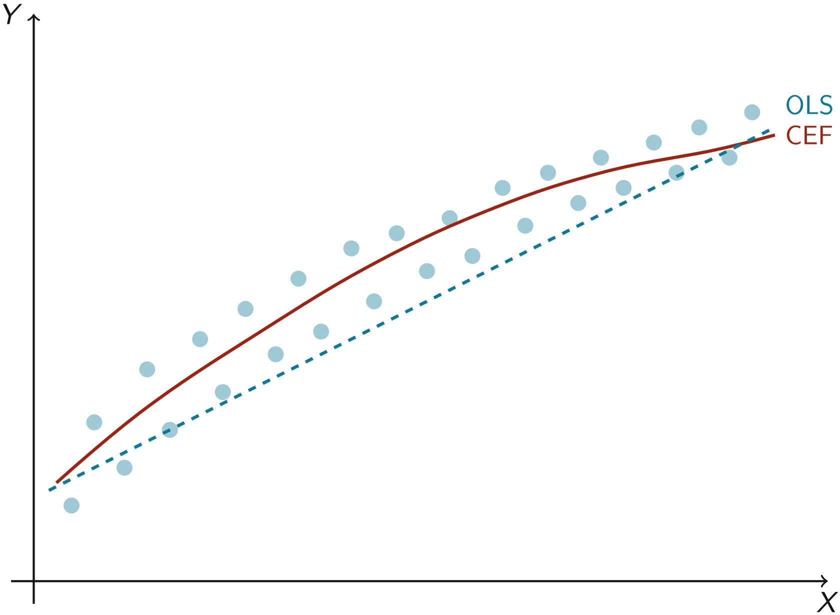
- Why linear? Simple, interpretable, often good enough


Y
OLS
CEF
X

Interpreting the slope coefficient

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- This is a **comparison**, not necessarily a causal effect

Two framings of β_1

- **Predictive framing**:
  - $\rightarrow$ "Groups that differ by 1 in X differ by β_1 in Y, on average"
  - $\rightarrow$ A comparison across units

- **Counterfactual framing**:
  - $\rightarrow$ "If we changed X by 1, Y would change by β_1"
  - $\rightarrow$ A statement about what would happen

- Same number, very different claims
- The counterfactual framing requires **causal assumptions**

Which framing — predictive or counterfactual —

does a randomized experiment give you?

Descriptive vs. Causal interpretation

- **Descriptive**: How do units with different X values compare?
  - $\rightarrow$ "People with more education earn more, on average"
- **Causal**: What happens if we change X for a given unit?
  - $\rightarrow$ "If we give someone more education, they will earn more"
- Same coefficient, very different claims!

Running a regression in R

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- Getting tidy output:
  - → `broom::tidy(model)` — coefficients as a data frame
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- These are much easier to work with than `summary()`

Roadmap

OLS assumptions

For OLS to work well, we need:

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A1–A4 are needed for unbiasedness; A5 for efficient SEs

OLS is unbiased (under A1–A4)

If assumptions A1–A4 hold:

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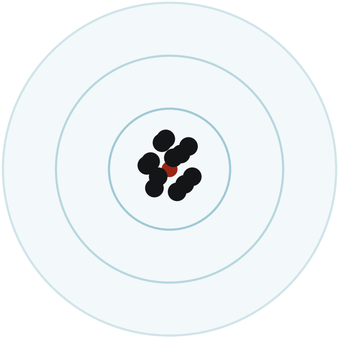
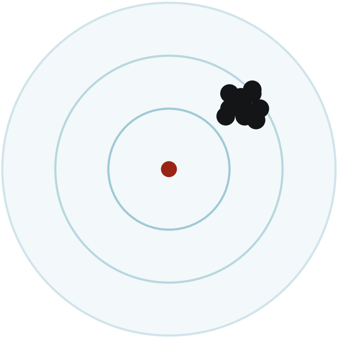
- *Think of it like an unbiased dart thrower:*
  centered on the bullseye, but with some scatter



Unbiased

centered on target



Biased

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Standard errors and uncertainty

OLS gives us $\hat{\beta}$, but how precise is it?

$$SE(\hat{\beta}_j) = \sqrt{\frac{\hat{\sigma}^2}{SST_j(1 - R_j^2)}}$$

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- The SE tells us how much $\hat{\beta}$ would vary across samples

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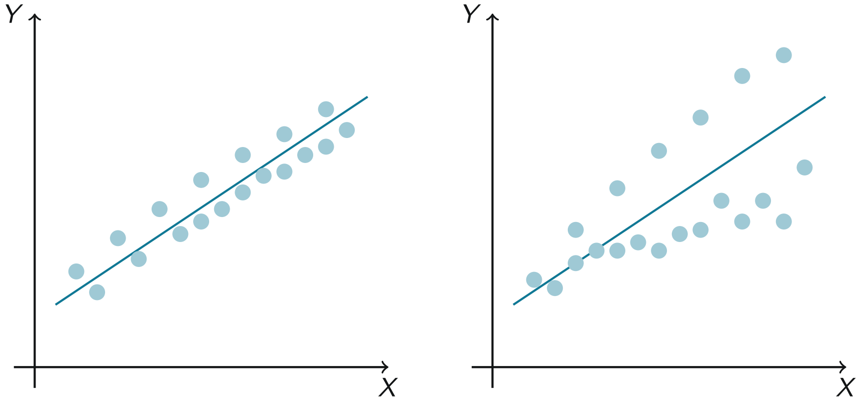
- The estimates $\hat{\beta}$ are still unbiased!
- But the standard errors are wrong



Homoskedastic
constant spread

Heteroskedastic
spread increases with X

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- There is no real cost when errors are homoskedastic

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- This means too many “significant” results

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- With few clusters ($<$ 30–50), even clustered SEs can be unreliable

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Adding predictors

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  - $\rightarrow$ **Holding X_2 constant**

- This is the "controlled" effect of X_1

How controlling works

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- Technically: we look at variation in X_1 that is unrelated to X_2
- This isolates the unique contribution of X_1

Omitted variable bias

If we omit a relevant variable X_2, the short regression gives:

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- Zero only if $\hat{\beta}_2 = 0$ *or* $\tilde{\delta} = 0$

OVB in practice: education and income

	Short regression (omits ability)	Long regression (includes ability)
Education (β_1)	\$5,000	\$3,000
Ability (β_2)	—	\$5,000

- Auxiliary regression: $\tilde{\delta} = 0.4$ (ability on education)

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- Bias $=$ \$2,000 — we overestimate by 67%!
- Because ability $\uparrow$ education *and* ability $\uparrow$ income

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Pre-treatment confounders are the key!

You study the effect of job training on wages.

Is *current job type* a good or bad control?

Why?

Bad controls: Post-treatment variables

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  - → Do control for education (determined before training)

Bad controls: Post-treatment variables

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- Example: Studying effect of job training on wages
  - → Don't control for job type (affected by training)
  - → Do control for education (determined before training)
- Controlling for post-treatment variables can *introduce* bias

Bad controls: Colliders

- A **collider** is caused by both X and Y

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  - $\rightarrow$ If reference is "North", the "South" coefficient means: average income in South minus average income in North

Roadmap

Regression Review

OLS Properties

Multiple Regression

Interaction Effects

Presenting Results

When effects depend on context

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  - $\rightarrow$ Effect of democracy on growth may depend on economic development
- We model this with **interaction terms**

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$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 (X \times Z) + \varepsilon$$

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- β_2: effect of Z when $X = 0$
- β_3: how the effect of X changes as Z increases

The marginal effect of X

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- Need to report effects at meaningful values of Z

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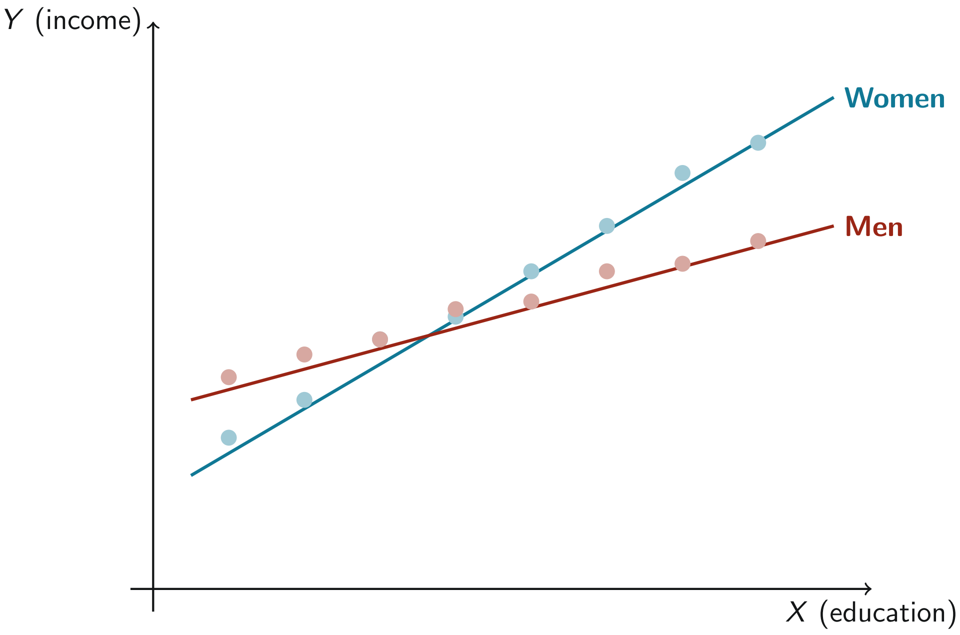
  → One slope for men, a different slope for women
- Equivalent to fitting separate regressions by group
- But estimated jointly (shares the error variance)


Y (income)
Women
Men
X (education)

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- In R: `marginaleffects::plot_predictions()`

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- Highly customizable: statistics, labels, notes

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- Often more effective than tables for communicating results
- Readers immediately see which effects are large vs. small

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- Shows robustness and what adding controls does to the estimate

Example workflow in R

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m1 <- lm(y ~ x, data = df)
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plot_predictions(m3, condition = c("x", "z1"))
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- Present results clearly: tables, coefficient plots, marginal effects

For next week

- Read Urdinez & Cruz (2020), chapter 8
- Read Gelman et al., chapters 13–14
- Complete Assignment 2

- Next session: Binary outcomes
  - → Linear probability model vs. logistic regression
  - → Interpreting logit results
  - → Predicted probabilities and marginal effects

Questions?