Assignment Block B

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Theory

Markov random fields

Let Y denote a GMRF with mean vector μ and precision matrix Q.

i) To show: $\mathbb{E}[Y_i|Y_{-i}] = \mu_i - \frac{1}{q_{ii}} \sum_{j \in \mathrm{ne}(i)} q_{ij}(y_j - \mu_j)$

Proof:

ii) To show: $\mathbb{V}[Y_i|Y_{-i}] = \frac{1}{q_{ii}}$

Proof:

iii) To show: $\mathbb{C}\mathrm{or}[Y_i, Y_j|Y_{-ij}] = -\frac{q_{ij}}{\sqrt{q_{ii}q_{jj}}}$

Proof:

Local charcteristics

Suppose S is a finite set equipped with a symmetry relation $\sim$. For count outcomes, a specification commonly used in disease mapping is the *auto-Poisson* model where

$$\pi(y_i|y_{S\setminus i}) = \exp(-\mu_i)\frac{\mu_i^{y_i}}{y_i}$$
$$\log(\mu_i) = -\sum_{j \in \mathrm{ne}(i)} y_j \; i \neq j$$

for $y \in \mathbb{N}_0$, $i \in S$. Determine the canonical potential with respect to $a = 0$.

Computation

Guerry's data on social morals