MATH/CS 514 FINAL PROJECT

DUE: 11:59PM DEC. 9, 2025

For your MATH/CS 514 final project, you will write a 4-5 page paper about your choice of ODE system (see the suggestions at the end of this document). Your paper should include:

(1) An explanation of the physical phenomenon that the ODE system describes. What is this system used for?

(2) A numerical implementation of the ODE system using one of the methods discussed in Chapter 12 of the Süli and Mayers textbook (we will spend most of November on this chapter). Attach the code as a separate file (and do not include the code itself in your paper). Describe which method you've chosen to use and why. Describe how this method is implemented for your specific system (this can be pseudocode if you'd like).

(3) Analyze the performance of the ODE method that you've chosen for your system. Include evidence to demonstrate the convergence of your method.

(4) Depending on the initial conditions and parameters chosen for your system, interpret the output/dynamics of your ODE system in the context of item (1). It is helpful (and more fun) to include plots for this item.

Additional formatting guidelines:
Your paper should be single-spaced with 1 to 1.5 inch margins and 12pt font using Times, Times New Roman, Computer Modern (the default LaTeX font), or similar. You are strongly encouraged to include figures, so 4-5 pages should feel a lot shorter than it sounds!
Please be sure to cite any resources used (including ChatGPT or similar AI–feel free to use, but please cite!). You can use whichever bibliography format you are most familiar with (MLA, Chicago, BibTeX, etc.).

Some suggestions of ODE systems to consider are listed below. More information about each of them can be found in the book Nonlinear Dynamics and Chaos, by Steve Strogatz.

(1) The spruce budworm model:

$$\frac{dx}{dt} = rx(1 - x/k) - \frac{x^2}{1+x^2}\,.$$

Here $r, k > 0$ are parameters.

(2) An SIR model:

$$\begin{aligned}\frac{dS}{dt} &= -kSI\\ \frac{dI}{dt} &= kSI - \ell I\\ \frac{dR}{dt} &= \ell I\,.\end{aligned}$$

Here $k, \ell > 0$ are parameters.

(3) The Lotka Volterra model:

$$\begin{aligned}\frac{dx}{dt} &= x(1-y)\\ \frac{dy}{dt} &= \mu y(x-1)\,.\end{aligned}$$

Here $\mu > 0$ is a parameter.

(4) The Lorenz equations:

$$\begin{aligned}\frac{dx}{dt} &= \sigma(y-x)\\ \frac{dy}{dt} &= rx - y - xz\\ \frac{dz}{dt} &= xy - bz\,.\end{aligned}$$

Here $\sigma, r, b > 0$ are parameters.

(5) Motion of a particle in a double-well potential:

$$\begin{aligned}\frac{dx}{dt} &= y\\ \frac{dy}{dt} &= x - x^3\,.\end{aligned}$$

(6) The van der Pol oscillator:

$$\begin{aligned}\frac{dx}{dt} &= y\\ \frac{dy}{dt} &= -\mu(x^2-1)y - x\,.\end{aligned}$$

Here $\mu \geq 0$ is a parameter.