SYMMETRIES AND TRANSFORMATIONS OF CHIRAL $N = 2, D = 10$ SUPERGRAVITY ☆

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An SU(1,1) symmetry is identified in chiral $N = 2$, $D = 10$ supergravity. Complete supersymmetry transformation formulas are obtained for each of the fields in the theory in a covariant formalism with manifest global SU(1,1) and local U(1) symmetries.

Chiral $N = 2, D = 10$ supergravity [1] has been formulated to first order in κ (the gravitational coupling constant) in terms of an unconstrained scalar light-cone superfield [2]. The methods that were used are difficult to extend to higher orders, and therefore it would be helpful to develop new ones for completing the construction of the theory. The usual procedure [3] of constructing supergravity theories by starting with kinetic terms and building up interactions and supersymmetry transformation formulas by the Noether method is not applicable, because a manifestly-covariant action does not exist for this theory. As explained elsewhere [4], this is due to the presence of an antisymmetric tensor gauge field with a self-dual field strength, something that can only exist in Minkowski space when the spacetime dimension is 2 mod 4. There are covariant equations of motion, however, which it ought to be possible to discover without reference to an action principle. When they are known one can try to re-express them in terms of the light-cone superfield to find the light-cone superspace action for the complete field theory. This could then be generalized to give the complete superfield theory description of type II superstrings [1,2]. In this Letter we present the various symmetry transformation formulas of each of the fields for the full interacting theory. This information is sufficient to deduce all the equations of motion [5].

From ref. [2], we know that there is a conserved charge U associated with the U(1) symmetry that rotates the two supersymmetry generators. The free-theory spectrum consists of a complex scalar A with $U = 2$, a complex Weyl spinor λ with $U = 3/2$, a complex antisymmetric tensor $A_{\mu\nu}$ with $U = 1$, a complex Weyl gravitino ψ_μ with $U = 1/2$, a real graviton $h_{\mu\nu}$ with $U = 0$, and a real fourth rank antisymmetric tensor $A_{\mu\nu\rho\lambda}$ (that has a self-dual field strength) with $U = 0$.

A first step in constructing the nonlinear theory is to find the covariant transformations and field equations of the linearized (free) theory, which is invariant under the rigid $N = 2$ super-Poincaré and local abelian transformations. The linearized transformations are obtained by considering field variations with all possible terms compatible with U conservation and dimensional requirements. Their coefficients are determined by requiring that the transformation algebra closes, using the free field equations, to give rigid $N = 2$ super-Poincaré and local abelian transformations. The result of this calculation is

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$$\delta h_{\mu\nu} = -2\,\mathrm{Im}(\bar\epsilon\gamma_{(\mu}\psi_{\nu)}) + \partial_{(\mu}\xi_{\nu)}, \tag{1a}$$

$$\delta A = \bar\epsilon^*\lambda, \tag{1b}$$

$$\delta A_{\mu\nu} = \bar\epsilon\gamma_{\mu\nu}\lambda + 4\mathrm{i}\bar\epsilon^*\gamma_{[\mu}\psi_{\nu]} + 2\partial_{[\mu}\Lambda_{\nu]}, \tag{1c}$$

$$\delta A_{\mu\nu\rho\lambda} = 2\mathrm{Re}(\bar\epsilon\gamma_{[\mu\nu\rho}\psi_{\lambda]}) + 4\partial_{[\mu}\Lambda_{\nu\rho\lambda]}, \tag{1d}$$

$$\delta\lambda = \gamma^\mu\epsilon^*\partial_\mu A - \tfrac{1}{24}\mathrm{i}\,\gamma^{\mu\nu\rho}\epsilon f_{\mu\nu\rho}, \tag{1e}$$

$$\delta\psi_\mu = \tfrac{1}{2}\gamma^{\nu\rho}\partial_\nu h_{\mu\rho} + \tfrac{1}{480}\mathrm{i}\,\gamma^{\rho_1\ldots\rho_5}\gamma_\mu\epsilon f_{\rho_1\ldots\rho_5}$$
$$+ \tfrac{1}{96}(\gamma_\mu^{\nu\rho\lambda}f_{\nu\rho\lambda} - 9\gamma^{\rho\lambda}f_{\mu\rho\lambda})\epsilon^* + \partial_\mu\eta, \tag{1f}$$

where square brackets denote antisymmetrization and round brackets symmetrization with unit weight. Also, ϵ is the parameter of rigid supersymmetry and ξ_μ, η, Λ_μ, and $\Lambda_{\mu\nu\rho}$ are parameters of the local abelian gauge transformations. The field strengths are

$$f_{\mu\nu\rho} = 3\partial_{[\mu}A_{\nu\rho]}, \quad f_{\mu\nu\rho\lambda\sigma} = 5\partial_{[\mu}A_{\nu\rho\lambda\sigma]}. \tag{2a,b}$$

The free field equations required for closure of the algebra are

$$\gamma^\mu\partial_\mu\lambda = \gamma^{\mu\nu\rho}\partial_\nu\psi_\rho = 0, \tag{3}$$

and self duality of $f_{\mu\nu\rho\lambda\sigma}$.

The transformations of the interacting theory can be deduced by making the rigid supersymmetry parameter ϵ a function of x^μ and demanding that the algebra closes order by order in κ. This requires adding terms to the transformation formulas as well as to the closure relations of the algebra. To first order in κ, this involves replacing all derivatives by supercovariantized ones and identifying the local abelian gauge parameter $\kappa\eta(x)$ with the (now) local supersymmetry parameter $\epsilon(x)$. For a discussion of this method see ref. [6].

The extension of this procedure to include all orders in κ is helped enormously by recognizing that the U(1) symmetry can be extended to an SU(1,1) symmetry, quite analogous to that of $N = 4, D = 4$ supergravity [7]. The physical SU(1,1) acts nonlinearly on the scalars, but it can be described in a linear form by adding an auxiliary real scalar field and compensating for it with an additional local U(1) gauge symmetry. Then the physical scalars can be identified with the coset SU(1,1)/U(1), just as the 70 scalars in $N = 8, D = 4$ supergravity are identified with $E_{7,7}$/SU(8) [8]. In this description λ and ψ_μ transform only under the local U(1), while $A^\alpha_{\mu\nu}$ (with $\alpha = 1,2$) transforms as a doublet of the global SU(1,1) and is inert under the local U(1). The scalars are described by the SU(1,1) matrix

$$\begin{pmatrix} V^1_- & V^1_+ \\ V^2_- & V^2_+ \end{pmatrix} = \exp\left[\kappa\begin{pmatrix} \mathrm{i}\varphi & A \\ A^* & -\mathrm{i}\varphi \end{pmatrix}\right], \tag{4}$$

where A is the physical complex scalar and φ is the real auxiliary scalar. The subscripts on $V^\alpha_\pm$ label local U(1) representations ($U = \pm 1$), while the superscript is an SU(1,1) doublet index. It is unnecessary to explicitly introduce the inverse matrix, because

$$\epsilon_{\alpha\beta}V^\alpha_- V^\beta_+ = 1. \tag{5}$$

It is convenient to define

$$P_\mu = -\epsilon_{\alpha\beta}V^\alpha_+\partial_\mu V^\beta_+, \quad Q_\mu = -\mathrm{i}\epsilon_{\alpha\beta}V^\alpha_-\partial_\mu V^\beta_+, \tag{6a,b}$$

since under local U(1) transformations

$$\delta V^\alpha_\pm = \pm\mathrm{i}\Sigma V^\alpha_\pm, \tag{7a}$$

$$\delta Q_\mu = \partial_\mu\Sigma, \quad \delta P_\mu = 2\mathrm{i}\Sigma P_\mu, \tag{7b}$$

$$\delta\psi_\mu = \tfrac{1}{2}\mathrm{i}\Sigma\psi_\mu, \quad \delta\lambda = \tfrac{3}{2}\mathrm{i}\Sigma\lambda, \tag{7c}$$

$$\delta A^\alpha_{\mu\nu} = \delta A_{\mu\nu\rho\lambda} = \delta e^r_\mu = 0. \tag{7d}$$

The gauge potentials $A^\alpha_{\mu\nu}$ and $A_{\mu\nu\rho\lambda}$ enter into field strenghts as follows:

$$F^\alpha_{\mu\nu\rho} = 3\partial_{[\mu}A^\alpha_{\nu\rho]}, \tag{8a}$$

$$F_{\mu\nu\rho\lambda\sigma} = 5\partial_{[\mu}A_{\nu\rho\lambda\sigma]} + \tfrac{5}{8}\mathrm{i}\kappa\,\epsilon_{\alpha\beta}A^\alpha_{[\mu\nu}F^\beta_{\rho\lambda\sigma]}. \tag{8b}$$

These expressions are invariant under the local gauge transformations

$$\delta A^\alpha_{\mu\nu} = 2\partial_{[\mu}\Lambda^\alpha_{\nu]}, \tag{9a}$$

$$\delta A_{\mu\nu\rho\lambda} = 4\partial_{[\mu}\Lambda_{\nu\rho\lambda]} - \tfrac{1}{4}\mathrm{i}\kappa\,\epsilon_{\alpha\beta}\Lambda^\alpha_{[\mu}F^\beta_{\nu\rho\lambda]}. \tag{9b}$$

We also define

$$G_{\mu\nu\rho} = -\epsilon_{\alpha\beta}V^\alpha_+F^\beta_{\mu\nu\rho}, \tag{10}$$

which is SU(1,1) invariant and has charge $U = 1$ with respect to local U(1) transformations.

There are a rather limited number of possible terms compatible with the SU(1,1) × U(1) structure that can appear in the local supersymmetry transformation formulas. Thus there are just some constant coefficients to determine. Aside from the freedom due to arbitrariness in field normalizations, the following transformation formulas are determined uniquely by requiring closure of the algebra on the bosonic fields [‡1]:

$$\delta e^r_\mu = -2\kappa\ \mathrm{Im}(\bar{\epsilon}\gamma^r\psi_\mu), \tag{11a}$$

$$\delta V^\alpha_+ = \kappa V^\alpha_- \bar{\epsilon}^* \lambda, \quad \delta V^\alpha_- = \kappa V^\alpha_+ \bar{\epsilon}\lambda^*, \tag{11b}$$

$$\delta A^\alpha_{\mu\nu} = V^\alpha_+ \bar{\epsilon}^*\gamma_{\mu\nu}\lambda^* + V^\alpha_-\bar{\epsilon}\gamma_{\mu\nu}\lambda + 4\mathrm{i}V^\alpha_+\bar{\epsilon}\gamma_{[\mu}\psi^*_{\nu]} + 4\mathrm{i}V^\alpha_-\bar{\epsilon}^*\gamma_{[\mu}\psi_{\nu]}, \tag{11c}$$

$$\delta A_{\mu\nu\rho\lambda} = 2\mathrm{Re}(\bar{\epsilon}\gamma_{[\mu\nu\rho}\psi_{\lambda]}) - \tfrac{3}{8}\mathrm{i}\kappa\ \epsilon_{\alpha\beta}A^\alpha_{[\mu\nu}\delta A^\beta_{\rho\lambda]}, \tag{11d}$$

$$\delta\lambda = (\mathrm{i}/\kappa)\gamma^\mu\epsilon^*\hat{P}_\mu - \tfrac{1}{24}\mathrm{i}\,\gamma^{\mu\nu\rho}\ \epsilon\hat{G}_{\mu\nu\rho}, \tag{11e}$$

$$\begin{aligned}\delta\psi_\mu = {} & \kappa^{-1}\mathrm{D}_\mu\epsilon + \tfrac{1}{480}\mathrm{i}\,\gamma^{\rho_1\ldots\rho_5}\gamma_\mu\epsilon\hat{F}_{\rho_1\ldots\rho_5} \\ & + \tfrac{1}{96}(\gamma_\mu{}^{\nu\rho\lambda}\hat{G}_{\nu\rho\lambda} - 9\gamma^{\rho\lambda}\hat{G}_{\mu\rho\lambda})\epsilon^* \\ & - \tfrac{7}{16}\kappa(\gamma_\rho\lambda\bar{\psi}_\mu\gamma^\rho\epsilon^* - \tfrac{1}{1680}\gamma_{\rho_1\ldots\rho_5}\lambda\bar{\psi}_\mu\gamma^{\rho_1\ldots\rho_5}\epsilon^*) \\ & + \tfrac{1}{32}\mathrm{i}\kappa\,[(\tfrac{9}{4}\gamma_\mu\gamma^\rho + 3\gamma^\rho\gamma_\mu)\epsilon\bar{\lambda}\gamma_\rho\lambda \\ & - (\tfrac{1}{24}\gamma_\mu\gamma^{\rho_1\rho_2\rho_3} + \tfrac{1}{6}\gamma^{\rho_1\rho_2\rho_3}\gamma_\mu)\epsilon\bar{\lambda}\gamma_{\rho_1\rho_2\rho_3}\lambda \\ & + \tfrac{1}{960}\gamma_\mu\gamma^{\rho_1\ldots\rho_5}\epsilon\bar{\lambda}\gamma_{\rho_1\ldots\rho_5}\lambda].\end{aligned} \tag{11f}$$

In these expressions we have introduced the supercovariant quantities

$$\hat{P}_\mu = P_\mu - \kappa^2\bar{\psi}^*_\mu\lambda, \tag{12a}$$

$$\hat{G}_{\mu\nu\rho} = G_{\mu\nu\rho} - 3\kappa\bar{\psi}_{[\mu}\gamma_{\nu\rho]}\lambda - 6\mathrm{i}\kappa\bar{\psi}^*_{[\mu}\gamma_\nu\psi_{\rho]}, \tag{12b}$$

$$\hat{F}_{\mu\nu\rho\lambda\sigma} = F_{\mu\nu\rho\lambda\sigma} - 5\kappa\bar{\psi}_{[\mu}\gamma_{\nu\rho\lambda}\psi_{\sigma]} - \tfrac{1}{16}\kappa\ \bar{\lambda}\gamma_{\mu\nu\rho\lambda\sigma}\lambda. \tag{12c}$$

Also,

$$\mathrm{D}_\mu\epsilon = (\partial_\mu + \tfrac{1}{4}\omega_\mu{}^{rs}\gamma_{rs} - \tfrac{1}{2}\mathrm{i}Q_\mu)\epsilon, \tag{13}$$

where the spin connection is the supercovariant expression given by

$$\omega_{\mu\nu\rho} = \Omega_{\nu\rho\mu} - \Omega_{\mu\nu\rho} + \Omega_{\mu\rho\nu}, \tag{14a}$$

$$\Omega_{\rho\mu\nu} = e^r_\nu\partial_{[\rho}e_{\mu]r} - \kappa^2\ \mathrm{Im}(\bar{\psi}_\mu\gamma_\nu\psi_\rho). \tag{14b}$$

The symmetry algebra has several interesting features. We note first that

$$[\delta(\Lambda.), \delta(\epsilon)] = \delta(\Lambda\ldots), \tag{15}$$

with

$$\Lambda_{\mu\nu\rho} = -\tfrac{3}{16}\mathrm{i}\kappa\ \epsilon_{\alpha\beta}\Lambda^\alpha_{[\mu}\delta(\epsilon)A^\beta_{\nu\rho]}. \tag{16}$$

The commutator of two local supersymmetry transformations gives all six types of local symmetry transformations:

$$[\delta(\epsilon_1), \delta(\epsilon_2)] = \delta(\xi) + \delta(l) + \delta(\epsilon) + \delta(\Lambda.) + \delta(\Lambda\ldots) + \delta(\Sigma). \tag{17}$$

The general-coordinate-transformation parameter is

$$\xi^\mu = 2\ \mathrm{Im}(\bar{\epsilon}_1\gamma^\mu\epsilon_2). \tag{18a}$$

The local-Lorentz-transformation parameter is

$$\begin{aligned} l^{rs} = {} & \omega_\rho{}^{rs}\xi^\rho - \tfrac{1}{3}\kappa\ \hat{F}^{rs\mu\nu\rho}\mathrm{Re}(\bar{\epsilon}_1\gamma_{\mu\nu\rho}\epsilon_2) \\ & + \tfrac{3}{4}\kappa\ \mathrm{Im}(\hat{G}^{rs\rho}\bar{\epsilon}_1\gamma_\rho\epsilon^*_2 + \tfrac{1}{18}\hat{G}_{\mu\nu\rho}\bar{\epsilon}_1\gamma^{rs\,\mu\nu\rho}\ \epsilon^*_2) \\ & + \mathrm{Re}(3\bar{\epsilon}_1\gamma^{rs\rho}\epsilon_2\bar{\lambda}\gamma_\rho\lambda - \tfrac{1}{4}\bar{\epsilon}_1\gamma^{rs\rho_1\rho_2\rho_3}\epsilon_1\bar{\lambda}\gamma_{\rho_1\rho_2\rho_3}\lambda \\ & - \tfrac{5}{2}\bar{\epsilon}_1\gamma_\rho\epsilon_2\bar{\lambda}\gamma^{rs\rho}\lambda \\ & - \tfrac{1}{960}\bar{\epsilon}_1\{\gamma^{rs}, \gamma^{\rho_1\ldots\rho_5}\}\epsilon_2\bar{\lambda}\gamma_{\rho_1\ldots\rho_5}\lambda). \end{aligned} \tag{18b}$$

The local-supersymmetry parameter is

$$\epsilon = -\kappa\psi_\rho\xi^\rho - \tfrac{7}{16}\kappa(\gamma_\rho\lambda\bar{\epsilon}_1\gamma^\rho\epsilon^*_2 - \tfrac{1}{1680}\gamma_{\rho_1\ldots\rho_5}\lambda\bar{\epsilon}_1\gamma^{\rho_1\ldots\rho_5}\epsilon^*_2). \tag{18c}$$

The local-gauge transformation parameters are

$$\Lambda^\alpha_\mu = A^\alpha_{\mu\rho}\xi^\rho + (2\mathrm{i}/\kappa)(V^\alpha_+\bar{\epsilon}_1\gamma_\mu\epsilon^*_2 + V^\alpha_-\bar{\epsilon}^*_1\gamma_\mu\epsilon_2), \tag{18d}$$

$$\Lambda_{\mu\nu\rho} = A_{\mu\nu\rho\lambda}\xi^\lambda - (1/2\kappa)\mathrm{Re}(\bar{\epsilon}_1\gamma_{\mu\nu\rho}\epsilon_2) + \tfrac{3}{8}\epsilon_{\alpha\beta}A^\alpha_{[\mu\nu}(V^\beta_+\bar{\epsilon}_1\gamma_{\rho]}\epsilon^*_2 + V^\beta_-\bar{\epsilon}^*_1\gamma_{\rho]}\epsilon_2). \tag{18e}$$

The local U(1) parameter is

$$\Sigma = -Q_\rho\xi^\rho + 2\kappa^2\ \mathrm{Im}(\bar{\epsilon}_1\lambda^*\bar{\epsilon}^*_2\lambda). \tag{18f}$$

Eq. (17) has been verified for each of the bosonic

[‡1] We use the conventions $\{\gamma^r, \gamma^s\} = 2\eta^{rs}$, where η = diag $(+ - \ldots -)$. Also $\gamma^{r_1\ldots r_N} = \gamma^{[r_1}\gamma^{r_2}\ldots\gamma^{r_N]}$, $\gamma_{11} = \gamma^0\gamma^1\ldots\gamma^9$, $\gamma_{11}\psi_\mu = -\psi_\mu$, $\gamma_{11}\lambda = \lambda$ and $\gamma_{11}\epsilon = -\epsilon$.

fields. This provides numerous consistency checks on eqs. (11)–(18). In the case of e^r_μ, $V^\alpha_\pm$, and $A^\alpha_{\mu\nu}$, closure is achieved without invoking equations of motion. However, in the case of $A_{\mu\nu\rho\lambda}$ it is necessary to use its field equation – namely the self-duality of the field strength $\hat{F}$ in eq. (12c) – to achieve closure. Requiring closure for the Fermi fields determines their equations of motion. These equations can then be subjected to supersymmetry transformations to determine the remaining bosonic field equations [5]. Alternatively, one could deduce the superspace torsions and chiral U(1) field strengths from the x-space transformation laws. Analysis of the Bianchi identities of the theory would then confirm closure on the Fermi fields and yield the equations of motion [9].

It should be interesting to investigate the field equations of this theory for solutions that describe spontaneous compactification. There may be solutions that differ in important ways from those that can be obtained starting from $D = 11$ supergravity. In any case, chiral $N = 2, D = 10$ supergravity seems more promising than $D = 11$ supergravity for the following reasons: (1) It has a chiral structure. (2) It has more symmetry [the SU(1,1)] than the nonchiral $N = 2, D = 10$ supergravity theory obtained by dimensional reduction of the $D = 11$ theory. (3) It corresponds to the massless sector of type II superstring theory, which may be a finite quantum theory. (4) It has an elegant description in terms of a light-cone superfield.

References

[1] M.B. Green and J.H. Schwarz, Phys. Lett. 109B (1982) 444;
J.H. Schwarz, Phys. Rep. 89 (1982) 223.

[2] M.B. Green and J.H. Schwarz, Phys. Lett. 122B (1983) 143;
M.B. Green, J.H. Schwarz and L. Brink, Caltech preprint 68-972, to be published in Nucl. Phys. B.

[3] For a review, see P. Van Nieuwenhuizen, Phys. Rep. 68 (1981) 189.

[4] N. Marcus and J.H. Schwarz, Phys. Lett. 115B (1982) 111.

[5] J.H. Schwarz, in preparation.

[6] P. West, Representations of supersymmetry, in: Supergravity '81, eds. S. Ferrara and J.G. Taylor;
J.H. Schwarz, Nucl. Phys. B173 (1980) 311.

[7] E. Cremmer, J. Scherk and S. Ferrara, Phys. Lett. 74B (1978) 61.

[8] E. Cremmer and B. Julia, Nucl. Phys. B159 (1979) 141.

[9] P. Howe and P. West, in preparation.