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Type IIB superstring action in $AdS_5 \times S^5$ background

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Abstract

We construct the covariant κ-symmetric superstring action for a type IIB superstring on $AdS_5 \otimes S^5$ background. The action is defined as a 2d σ-model on the coset superspace $\frac{SU(2,2|4)}{SO(4,1)\times SO(5)}$ and is shown to be the unique one that has the correct bosonic and flat space limits. © 1998 Elsevier Science B.V.

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1. Introduction

After the construction of IIB supergravity (motivated by the development of superstring theory) [1–3] it was immediately realized that in addition to the flat ten-dimensional space $R^{1,9}$ this theory allows $AdS_5 \otimes S^5$ (+ self-dual 5-form background) as another maximally supersymmetric 'vacuum' [2]. Some aspects of this compactification on S^5 were studied in [4–6] (this led to the construction of $N = 8$ gauged supergravity in 5 dimensions which describes the 'massless' modes [7]). In particular, it was understood [4] that the Kaluza–Klein modes fall into unitary irreducible representations of $N = 8, D = 5$ anti de Sitter supergroup $SU(2,2|4)$ (which is the same

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as the $N = 4$ superconformal group in 4 dimensions [8]). The supergroup $SU(2,2|4)$ (with the even part $SU(2,2) \otimes SU(4) \simeq SO(4,2) \otimes SO(6)$ which is the isometry of the $AdS_5 \otimes S^5$ space) thus plays here the same central role as does the Poincaré supergroup in the flat vacuum.

Motivated by the recent duality conjecture between the type IIB string theory on $AdS_5 \otimes S^5$ background and $N = 4, D = 4$ Super Yang–Mills theory [9–13], one would like to study the corresponding string theory directly, using the world-sheet methods. This may allow to prove that $AdS_5 \otimes S^5$ space is an exact string solution, define the corresponding the 2d conformal theory, find the spectrum of string states, etc..

Since the $AdS_5 \otimes S^5$ space is supported by the Ramond–Ramond 5-form background, the NSR approach does not seem to apply in a straightforward way (while the non-local RR vertex operator is known in the flat space [14], it is most likely not sufficient to determine the complete form of the NSR string action when the space-time metric is curved).

The manifestly supersymmetric Green–Schwarz (GS) approach [15] seems a more adequate one when the RR fields are non-vanishing. While the formal superspace expression for the GS superstring action in a generic type IIB background (satisfying the supergravity equations of motion to guarantee the κ-invariance of the string action [16]) was presented in Ref. [17] (see also Ref. [18]), it is not very practical for finding the explicit form of the action in terms of the coordinate fields (x, θ): given a particular bosonic background, one is first to determine explicitly the corresponding $D = 10$ type IIB superfields which is a complicated problem not solved so far in any non-trivial case. [3]

The remarkable (super)symmetry of the $AdS_5 \otimes S^5$ background suggests that here one should apply an alternative approach which explicitly uses the special properties of this vacuum. Our aim below will be to find the counterpart of the covariant GS action in flat space for the type IIB string propagating in $AdS_5 \otimes S^5$ space-time by starting directly with the supergroup $SU(2,2|4)$ and constructing a κ-symmetric 2d σ-model on the coset superspace $\frac{SU(2,2|4)}{SO(4,1)\otimes SO(5)}$. The method is conceptually the same as used in [21] to reproduce the flat-space GS action as a WZW-type σ-model on the coset superspace ($D = 10$ super Poincaré group/Lorentz group $SO(9,1)$).

In Section 2 we shall describe the structure of the superalgebra $su(2,2|4)$ and define the invariant Cartan 1-forms L^A on the coset superspace (x, θ). [4]

In Section 3 we shall present the covariant $AdS_5 \otimes S^5$ superstring action in the coordinate-free form, i.e. in terms of the Cartan 1-forms L^A on the superspace. As in the flat space case [15,21] it is given by the sum the 'kinetic' or 'Nambu' term (2d integral of the quadratic term in L^A) plus a Wess–Zumino type term (3d integral of a closed 3-form $\mathcal{H}$ on the superspace), with the coefficient of the WZ term fixed uniquely

[3] In fact, the only known example of a covariant GS action in a curved RR background was recently constructed in [19] (in the case of a non-supersymmetric IIA magnetic 7-brane background) using an indirect method based on starting with the known supermembrane action [20] in flat $D = 11$ space.

[4] For some applications of the formalism of the Cartan forms on coset superspaces see Refs. [22–26].

by the requirement of κ-symmetry. In the zero-curvature (infinite radius) limit the action reduces to the standard flat space GS action [15,21].

In Section 4 we shall find the explicit 2d form of the action by choosing a specific parametrization of the Cartan 1-forms in terms of the fermionic coordinates θ. The resulting action may be viewed as the unique maximally supersymmetric and κ-symmetric extension of the bosonic string sigma model with $AdS_5 \otimes S^5$ as a target space. It is given by a 'covariantization' of the flat-space GS action plus terms containing higher powers of θ. Though we explicitly present only the θ^4 term, it is very likely that after an appropriate κ-symmetry gauge choice the full action will be determined by the θ^2 and θ^4 terms only. [5]

Some properties of the resulting string theory will be briefly discussed in Section 5. The string action depends on generalised 'supersymmetric' spinor covariant derivative and thus contains the expected coupling $(\partial x \partial x \bar{\theta} \gamma \ldots \gamma \theta \ e^{\phi} F_{\ldots})$ to the RR background. Since the action is uniquely determined by the $SU(2,2|4)$ symmetry, its classical 2d conformal invariance should survive at the quantum level: as in the WZW model case, the symmetries of the action prohibit any deformation of its structure (provided, of course, the regularisation scheme preserves these symmetries). The central role played by $SU(2,2|4)$ implies that not only the 'supergravity' (marginal) but also all 'massive' string vertex operators will belong to its representations.

In Appendix A we shall introduce explicit parametrisation of the coset superspace and define the corresponding Cartan superconnections. In Appendix B we shall explain the procedure to compute the expansion of the supervielbeins in powers of θ which is used to determine higher-order terms in the component 2d string action.

2. $su(2,2|4)$ superalgebra

Our staring point is the supergroup $SU(2,2|4)$. Since the string we are interested in propagates on the coset superspace $\frac{SU(2,2|4)}{SO(4,1)\otimes SO(5)}$ with the even part being

$$AdS_5 \otimes S^5 = \frac{SO(4,2)}{SO(4,1)} \otimes \frac{SO(6)}{SO(5)}$$

we shall present the corresponding superalgebra $su(2,2|4)$ in the $so(4,1) \oplus so(5)$ (or '5+5') basis. The even generators are then two pairs of translations and rotations – (P_a, J_{ab}) for AdS_5 and $(P_{a'}, J_{a'b'})$ for S^5 and the odd generators are the two $D = 10$ Majorana–Weyl spinors $Q_I^{\alpha\alpha'}$.

2.1. Notation

In what follows we use the following convention for indices:

[5] Such truncation of a complicated covariant GS action to $O(\theta^4)$ expression was found to happen after choice of the light-cone gauge $\gamma^+\theta = 0$ in the case of a curved RR background considered in [19].

$a,b,c = 0,1,\ldots,4$	$so(4,1)$ vector indices (AdS_5 tangent space)
$a',b',c' = 5,\ldots,9$	$so(5)$ vector indices (S^5 tangent space)
$\hat{a},\hat{b},\hat{c} = 0,1,\ldots,9$	combination of $(a,a'),(b,b'),(c,c')$ ($D=10$ vector indices)
$\alpha,\beta,\gamma,\delta = 1,\ldots,4$	$so(4,1)$ spinor indices (AdS_5)
$\alpha',\beta',\gamma',\delta' = 1,\ldots,4$	$so(5)$ spinor indices (S^5)
$\hat{\alpha},\hat{\beta},\hat{\gamma} = 1,\ldots,32$	$D=10$ MW spinor indices
$I,J,K,L = 1,2$	labels of the two sets of spinors

Similarly, $\hat{\mu} = (\mu,\mu')$ will denote the coordinate indices of $AdS_5 \otimes S^5$. The generators of the $so(4,1)$ and $so(5)$ Clifford algebras are 4×4 matrices γ_a and $\gamma_{a'}$

$$\gamma^{(a}\gamma^{b)} = \eta^{ab} = (-++++)\,, \qquad \gamma^{(a'}\gamma^{b')} = \eta^{a'b'} = (+++++)\,.$$

It will be useful to define also the ten 4×4 matrices $\hat{\gamma}^{\hat{a}}$

$$\hat{\gamma}^a \equiv \gamma^a\,, \qquad \hat{\gamma}^{a'} \equiv \mathrm{i}\gamma^{a'}\,. \tag{2.1}$$

We shall assume that $(\gamma^a)^\dagger = \gamma^0\gamma^a\gamma^0$, $(\gamma^{a'})^\dagger = \gamma^{a'}$ and that the Majorana condition is diagonal with respect to the two supercharges,

$$\bar{Q}_{\alpha\alpha' I} \equiv (Q_I^{\beta\beta'})^\dagger(\gamma^0)_\alpha^\beta\delta_{\alpha'}^{\beta'} = -Q_I^{\beta\beta'}C_{\beta\alpha}C_{\beta'\alpha'}\,. \tag{2.2}$$

Here $C = (C_{\alpha\beta})$ and $C' = (C_{\alpha'\beta'})$ are the charge conjugation matrices[6] of the $so(4,1)$ and $so(5)$ Clifford algebras which are used to raise and lower spinor indices, e.g., $Q_{\alpha\alpha' I} \equiv Q_I^{\beta\beta'}C_{\beta\alpha}C_{\beta'\alpha'}$. The bosonic generators will be assumed to be antihermitian: $P_a^\dagger = -P_a$, $P_{a'}^\dagger = -P_{a'}$, $J_{ab}^\dagger = -J_{ab}$, $J_{a'b'}^\dagger = -J_{a'b'}$. We shall use the 2×2 matrices $\epsilon^{IJ} = -\epsilon^{JI}$, $\epsilon^{12} = 1$, and $s^{IJ} \equiv \mathrm{diag}(1,-1)$ to contract the indices $I = 1,2$.[7] Unless stated otherwise, we shall always assume the summation rule over the repeated indices (irrespective of their position).

The 10-dimensional 32×32 Dirac matrices $\Gamma^{\hat{a}}$ of $SO(9,1)$ ($\Gamma^{(\hat{a}}\Gamma^{\hat{b})} = \eta^{\hat{a}\hat{b}}$) and the corresponding charge conjugation matrix $\mathcal{C}$ can be represented as

$$\Gamma^a = \gamma^a \otimes I \otimes \sigma_1\,, \qquad \Gamma^{a'} = I \otimes \gamma^{a'} \otimes \sigma_2\,, \qquad \mathcal{C} = C \otimes C' \otimes \mathrm{i}\sigma_2\,, \tag{2.3}$$

where I is the 4×4 unit matrix and σ_i are the Pauli matrices. Note that $C\gamma^{a_1\ldots a_n}$ are symmetric (antisymmetric) for $n = 2,3 \bmod 4$ ($n = 0,1 \bmod 4$). The same properties are valid for $C'\gamma^{a_1'\ldots a_n'}$.

A $D = 10$ positive chirality 32-component spinor Ψ is decomposed as follows: $\Psi = \psi\otimes\psi'\otimes\begin{pmatrix}1\\0\end{pmatrix}$ (16-component spinors θ^I and L^I below will correspond to 32-component

[6] To simplify the notation, we shall put primes on matrices and generators to distinguish between the objects corresponding to the two factors (AdS_5 and S^5) only in the cases when they do not carry explicit (primed) indices.

[7] As in the flat space case [15,21], s^{IJ} will appear in the WZ term of the GS action, indicating the breakdown of the formal $U(1)$ symmetry between the two Majorana-Weyl spinors of the same chirality, which is a symmetry of the type IIB superfield supergravity [3] but is broken in perturbative string theory, e.g., is absent in the action of the superstring in a type IIB supergravity background [17].

spinors of positive chirality). The Majorana condition $\bar{\Psi} \equiv \Psi^\dagger \Gamma^0 = \Psi^T C$ then takes the same form as in (2.2) but with sign plus (for a negative chirality spinor $\sim \binom{0}{1}$ the Majorana condition is the same as in (2.2)). A useful formula that explains the 10-dimensional origin of some of the expressions below is

$$K^{\hat{a}}\bar{\Psi}_1\Gamma^{\hat{a}}\Psi_2 = K^a\bar{\chi}_1\gamma^a\chi_2 + \mathrm{i}K^{a'}\bar{\chi}_1\gamma^{a'}\chi_2\,, \tag{2.4}$$

where Ψ_n $(n = 1,2)$ are the $D = 10$ Majorana-Weyl spinors of positive chirality, $\chi_n = \psi_n \otimes \psi'_n$, and $K^{\hat{a}}$ is a 10-vector. Here (and in similar expressions below) γ^a and $\gamma^{a'}$ stand for $\gamma^a \otimes I$ and $I \otimes \gamma^{a'}$.

2.2. Commutation relations

The commutation relations for the generators $T_A = (P_a, P_{a'}, J_{ab}, J_{a'b'}, Q_{\alpha\alpha' I})$ are [8]

$$\begin{aligned}
[P_a, P_b] &= J_{ab}\,, & [P_{a'}, P_{b'}] &= -J_{a'b'}\,,\\
[P_a, J_{bc}] &= \eta_{ab}P_c - \eta_{ac}P_b\,, & [P_{a'}, J_{b'c'}] &= \eta_{a'b'}P_{c'} - \eta_{a'c'}P_{b'}\,,\\
[J_{ab}, J_{cd}] &= \eta_{bc}J_{ad} + 3 \text{ terms}\,, & [J_{a'b'}, J_{c'd'}] &= \eta_{b'c'}J_{a'd'} + 3 \text{ terms}\,,\\
[Q_I, P_a] &= -\frac{\mathrm{i}}{2}\epsilon_{IJ}Q_J\gamma_a\,, & [Q_I, P_{a'}] &= \tfrac{1}{2}\epsilon_{IJ}Q_J\gamma_{a'}\,,\\
[Q_I, J_{ab}] &= -\tfrac{1}{2}Q_I\gamma_{ab}\,, & [Q_I, J_{a'b'}] &= -\tfrac{1}{2}Q_I\gamma_{a'b'}\,,
\end{aligned} \tag{2.5}$$

$$\begin{aligned}
\{Q_{\alpha\alpha' I}, Q_{\beta\beta' J}\} = \delta_{IJ}&\left[-2\mathrm{i}C_{\alpha'\beta'}(C\gamma^a)_{\alpha\beta}P_a + 2C_{\alpha\beta}(C'\gamma^{a'})_{\alpha'\beta'}P_{a'}\right]\\
&+\epsilon_{IJ}\left[C_{\alpha'\beta'}(C\gamma^{ab})_{\alpha\beta}J_{ab} - C_{\alpha\beta}(C'\gamma^{a'b'})_{\alpha'\beta'}J_{a'b'}\right].
\end{aligned}$$

The curvature (radius R) parameter of $AdS_5 \otimes S^5$ space can be introduced by rescaling the generators $P_a \to RP_a$, $P_{a'} \to RP_{a'}$, $J \to J$, $Q_I \to \sqrt{R}Q_I$. Then the limit $R \to \infty$ gives the subalgebra of $D = 10$ type IIB Poincaré superalgebra. [9]

2.3. Cartan 1-forms

To find the super-invariant and κ-invariant string action we will use the formalism of Cartan forms defined on the coset superspace. The left-invariant Cartan 1-forms

$$L^A = dX^M L^A_M\,, \qquad X^M = (x, \theta)$$

[8] Because of the presence of ϵ^{IJ}, some of the anticommutators are not diagonal with respect to the supercharges. This is related to the standard choice of diagonal Majorana condition (2). By obvious redefinitions we could make the commutation relations diagonal with respect two the supercharges but this would lead to a non-diagonal Majorana condition.

[9] The Poincaré superalgebra contains the generators $J_{aa'}$ which are absent in $su(2,2|4)$: the $SO(2,4)\otimes SO(6)$ isometry of $AdS_5 \otimes S^5$ leads in the limit $R \to \infty$ only to a subgroup of $SO(9,1)$ (times translations). While the $AdS_5 \otimes S^5$ background has a maximum number of 32 Killing spinors, it does not have a maximum number of 55 Killing vectors in 10 dimensions (the dimension of $SO(2,4) \otimes SO(6)$ is 30).

are given by

$$G^{-1}dG = L^A T_A \equiv L^a P_a + L^{a'} P_{a'} + \tfrac{1}{2} L^{ab} J_{ab} + \tfrac{1}{2} L^{a'b'} J_{a'b'} + L^{\alpha\alpha' I} Q_{\alpha\alpha' I}, \qquad (2.6)$$

where $G = G(x, \theta)$ is a coset representative in $SU(2,2|4)$. A specific choice of $G(x, \theta)$ which we shall use in Section 4 is described in Appendix A.

L^a and $L^{a'}$ are the 5-beins, $L^{\alpha\alpha' I}$ are the two spinor 16-beins and L^{ab} and $L^{a'b'}$ are the Cartan connections. They satisfy the Maurer–Cartan equations implied by the structure of the $su(2,2|4)$ superalgebra

$$dL^a = -L^b \wedge L^{ba} - \mathrm{i}\bar{L}^I \gamma^a \wedge L^I, \qquad dL^{a'} = -L^{b'} \wedge L^{b'a'} + \bar{L}^I \gamma^{a'} \wedge L^I, \qquad (2.7)$$

$$dL^{ab} = -L^a \wedge L^b - L^{ac} \wedge L^{cb} + \epsilon^{IJ} \bar{L}^I \gamma^{ab} \wedge L^J, \qquad (2.8)$$

$$dL^{a'b'} = L^{a'} \wedge L^{b'} - L^{a'c'} \wedge L^{c'b'} - \epsilon^{IJ} \bar{L}^I \gamma^{a'b'} \wedge L^J, \qquad (2.9)$$

$$dL^I = -\frac{\mathrm{i}}{2} \gamma^a \epsilon^{IJ} L^J \wedge L^a + \tfrac{1}{2} \epsilon^{IJ} \gamma^{a'} L^J \wedge L^{a'} + \tfrac{1}{4} \gamma^{ab} L^I \wedge L^{ab} + \tfrac{1}{4} \gamma^{a'b'} L^I \wedge L^{a'b'}. \qquad (2.10)$$

The rescaling of the generators which restores the scale parameter R of $AdS_5 \otimes S^5$ corresponds to $L^a \to R^{-1} L^a$, $L^{a'} \to R^{-1} L^{a'}$, $L^{ab} \to L^{ab}$, $L^{a'b'} \to L^{a'b'}$, $L^I \to R^{-1/2} L^I$.

For comparison, let us note that in the flat superspace case

$$G(x, \theta) = \exp(x^{\hat{a}} P_{\hat{a}} + \theta^I Q_I), \quad [P_{\hat{a}}, P_{\hat{b}}] = 0, \quad \{Q_I, Q_J\} = -2\mathrm{i}\delta_{IJ} (C\Gamma^{\hat{a}}) P_{\hat{a}}, \qquad (2.11)$$

and thus the coset space vielbeins in $G^{-1}dG = L^A T_A$ are given by

$$L_0^{\hat{a}} = dx^{\hat{a}} - i\bar{\theta}^I \Gamma^{\hat{a}} d\theta^I, \qquad L_0^I = d\theta^I. \qquad (2.12)$$

3. Superstring action as $\frac{SU(2,2|4)}{SO(4,1) \otimes SO(5)}$ superspace sigma model

Our aim below will be to construct the superstring action that satisfies the following conditions (some of which are not completely independent):

(a) its bosonic part is the standard σ-model with the $AdS_5 \otimes S^5$ as a target space;

(b) it has global $SU(2,2|4)$ super-invariance;

(c) it is invariant under local κ-symmetry;

(d) it reduces to the standard Green–Schwarz type IIB superstring action in the flat-space ($R \to \infty$) limit.

We shall find that such action exists and is *unique*. Its leading θ^2 fermionic term will contain the required coupling to the RR 5-form field background. This is, of course, expected as the κ-symmetry implies the satisfaction of the IIB supergravity equations of motion [17] but the unique supergravity solution with the metric of $AdS_5 \otimes S^5$ has a non-trivial F_5 background ($F_5 \sim \epsilon_5$ in each of the two factors).

It is useful to recall that the flat-space GS superstring action may be written in the manifestly supersymmetric form in terms of the 1-forms (2.12) as the sum of the 'kinetic' term and a WZ term (integral of a closed 3-form) [21] [10]

$$I_0 = -\frac{1}{2}\int\limits_{\partial M_3} d^2\sigma \sqrt{g}\, g^{ij}\; L^{\hat{a}}_{0i} L^{\hat{a}}_{0j} + \mathrm{i}\int\limits_{M_3} s^{IJ} L^{\hat{a}}_0 \wedge \bar{L}^I_0 \Gamma^{\hat{a}} \wedge L^J_0, \tag{3.1}$$

where s^{IJ} is defined by $s^{11} = -s^{22} = 1$, $s^{12} = s^{21} = 0$ and the string tension is set to be $\frac{1}{2\pi\alpha'} = 1$. g_{ij} $(i,j = 0,1)$ is a world-sheet metric with signature $(-+)$ $(g = -\det g_{ij})$. The coefficient of the WZ term is fixed by the condition of local κ-invariance [15]. Using (2.12) one observes that the 3-form in the WZ term is exact and thus finds the explicit 2d form of the GS action [15]

$$I_0 = \int d^2\sigma\, \mathcal{L}_0 = \int d^2\sigma \left[-\tfrac{1}{2}\sqrt{g} g^{ij} (\partial_i x^{\hat{a}} - \mathrm{i}\bar{\theta}^I \Gamma^{\hat{a}} \partial_i \theta^I)(\partial_j x^{\hat{a}} - \mathrm{i}\bar{\theta}^J \Gamma^{\hat{a}} \partial_j \theta^J) \right.$$
$$\left. - \mathrm{i}\epsilon^{ij} s^{IJ} \bar{\theta}^I \Gamma^{\hat{a}} \partial_j \theta^J (\partial_i x^{\hat{a}} - \tfrac{1}{2}\mathrm{i}\bar{\theta}^K \Gamma^{\hat{a}} \partial_i \theta^K) \right], \tag{3.2}$$

in which the ϵ^{ij}-term is invariant under global supersymmetry only up to a total derivative. The action we shall find below is the generalisation of (3.2) to the case when the free bosonic $\partial^i x^{\hat{a}} \partial_i x^{\hat{a}}$ term is replaced by the σ-model on the $AdS_5 \otimes S^5$ space.

3.1. General structure of the action

As the flat-space action, the action for the type IIB superstring propagating in $AdS_5 \otimes S^5$ space-time will be given by the sum of the 'σ-model' term $I_{\rm kin}$ and a WZ term $I_{\rm WZ}$ which is the integral of a closed 3-form $\mathcal{H}$ over a 3-space M_3 which has the world-sheet as its boundary,

$$I = I_{\rm kin} + I_{\rm WZ}\,, \qquad I_{\rm WZ} = \mathrm{i}\int\limits_{M_3} \mathcal{H}\,, \qquad d\mathcal{H} = 0. \tag{3.3}$$

To satisfy the condition of $SU(2,2|4)$ invariance both $I_{\rm kin}$ and $\mathcal{H}$ should be constructed in terms of the Cartan 1-forms L^A. The basic observation is that under the action of an arbitrary element of the isometry group the vielbeins transform as tangent vectors of the stability subgroup. Thus any invariant of the stability subgroup ($SO(4,1) \otimes SO(5)$ in the present case) constructed in terms of L^A will be automatically invariant under the full isometry group ($SU(2,2)|4$), i.e. $SO(4,2) \otimes SO(6)$ and supersymmetry).

[10] This action may be compared with the standard bosonic WZW action on a group manifold: in a similar notation, if $G^{-1}dG = L^A T_A$, $\mathrm{tr}(T_A T_B) = c_{AB}$, $\mathrm{tr}(T_A[T_B, T_C]) = f_{ABC}$, then [27]

$$I_{\rm WZW} = k\Big[\int\limits_{\partial M_3} d^2\sigma \sqrt{g} g^{ij} c_{AB} L^A_i L^B_j + \frac{1}{6}\mathrm{i} \int\limits_{M_3} f_{ABC} L^A \wedge L^B \wedge L^C \Big].$$

The structure of $I_{\rm kin}$ is then fixed unambiguously by the conditions (a) and (b)

$$I_{\rm kin} = -\tfrac{1}{2} \int d^2\sigma \sqrt{g}\, g^{ij}\, L_i^{\hat a} L_j^{\hat a}. \tag{3.4}$$

Here the repeated indices are contracted with $\eta_{\hat a\hat b} = (\eta_{ab}, \eta_{a'b'})$ and $L_i^A = \partial_i X^M L_M^A$ are the induced components of the supervielbein.

The only relevant 3-forms built out of L^A which are invariant under $SO(4,1)\otimes SO(5)$ are given by the following linear combination (k, k' are free parameters):

$$\mathcal{H}^I = kL^a \wedge \bar L^I \gamma^a \wedge L^I + k' L^{a'} \wedge \bar L^I \gamma^{a'} \wedge L^I, \quad I = 1,2. \tag{3.5}$$

Here we do *not* sum over the repeated indices I. Using the Maurer–Cartan equations, one finds (no summation over I)

$$\begin{aligned} d(L^a \wedge \bar L^I \gamma^a \wedge L^I) &= \epsilon^{IJ} L^a \wedge L^{a'} \wedge \bar L^I \gamma^a \gamma^{a'} \wedge L^J - \mathrm{i}\bar L^I \gamma^a \wedge L^I \wedge \bar L^J \gamma^a \wedge L^J \\ &\quad -\mathrm{i}\epsilon^{IJ} L^a \wedge L^b \wedge \bar L^I \gamma^{ab} \wedge L^J , \end{aligned} \tag{3.6}$$

$$\begin{aligned} d(L^{a'} \wedge \bar L^I \gamma^{a'} \wedge L^I) &= \mathrm{i}\epsilon^{IJ} L^a \wedge L^{a'} \wedge \bar L^I \gamma^a \gamma^{a'} \wedge L^J + \bar L^I \gamma^{a'} \wedge L^I \wedge \bar L^J \gamma^{a'} \wedge L^J \\ &\quad +\epsilon^{IJ} L^{a'} \wedge L^{b'} \wedge \bar L^I \gamma^{a'b'} \wedge L^J , \end{aligned} \tag{3.7}$$

so that to cancel the terms in $d\mathcal{H}^I$ which are given by the first terms in the r.h.s of (3.6), (3.7) we are to put $k' = \mathrm{i}k$. Then

$$\mathcal{H}^I = k(L^a \wedge \bar L^I \gamma^a \wedge L^I + \mathrm{i}L^{a'} \wedge \bar L^I \gamma^{a'} \wedge L^I), \quad I = 1,2, \tag{3.8}$$

$$\begin{aligned} d\mathcal{H}^I &= -\mathrm{i}k\ (\bar L^I \gamma^a \wedge L^I \wedge \bar L^J \gamma^a \wedge L^J - \bar L^I \gamma^{a'} \wedge L^I \wedge \bar L^J \gamma^{a'} \wedge L^J) \\ &\quad -\mathrm{i}k\ \epsilon^{IJ}(L^a \wedge L^b \wedge \bar L^I \gamma^{ab} L^J - L^{a'} \wedge L^{b'} \wedge \bar L^I \gamma^{a'b'} \wedge L^J). \end{aligned} \tag{3.9}$$

It is easily verified that the only possibility to obtain a closed 3-form is to consider

$$\mathcal{H} \equiv \mathcal{H}^1 - \mathcal{H}^2. \tag{3.10}$$

To prove this one uses the Fierz identity $(C\gamma^a)_{\alpha\delta}(C\gamma^a)_{\gamma\beta} = 2(C_{\alpha\beta}C_{\gamma\delta} - C_{\alpha\gamma}C_{\beta\delta})$ for $SO(4,1)$ and $\bar L^1 \gamma^{ab} \wedge L^2 = -\bar L^2 \gamma^{ab} \wedge L^1$ and similar relations for $SO(5)$ part. [11]

In the flat-space limit $\mathcal{H}$ reduces to the 3-form in the GS action (3.1). As in flat space, the value of the overall coefficient k in $\mathcal{H}$ is fixed to be 1 by the requirement of κ-symmetry of the whole action (which is proved in the next subsection). The final expression for the action written in the $SU(2,2|4)$ invariant form in terms of the vielbeins $L^a, L^{a'}$ and L^I thus has the same structure as the GS action (3.1),

$$I = -\tfrac{1}{2} \int_{\partial M_3} d^2\sigma \sqrt{g}\, g^{ij} L_i^{\hat a} L_j^{\hat a} + \mathrm{i} \int_{M_3} s^{IJ} L^{\hat a} \wedge \bar L^I \hat\gamma^{\hat a} \wedge L^J, \tag{3.11}$$

or, explicitly,

[11] Let us note that the first line in the expression (3.9) can be rewritten in terms of ten-dimensional spinors and Γ-matrices only, and, as in the flat-space case [15,21], the fact that $d\mathcal{H} = 0$ is a consequence of the famous identity for the $D = 10$ Dirac matrices $\Gamma^{\hat a}_{\hat\alpha(\hat\beta}\Gamma^{\hat a}_{\hat\gamma\hat\sigma)} = 0$ (first line in (3.9)) and the relation $\bar L^1 \gamma^{ab} \wedge L^2 = -\bar L^2 \gamma^{ab} \wedge L^1$ (second line in (3.9)).

$$I = -\tfrac{1}{2} \int_{\partial M_3} d^2\sigma \sqrt{g}\, g^{ij} (L_i^a L_j^a + L_i^{a'} L_j^{a'})$$
$$+ \mathrm{i} \int_{M_3} s^{IJ} (L^a \wedge \bar{L}^I \gamma^a \wedge L^J + \mathrm{i} L^{a'} \wedge \bar{L}^I \gamma^{a'} \wedge L^J), \qquad (3.12)$$

and, indeed, reduces to (3.1) in the flat-space limit.

Since the 3-form $\mathcal{H}$ is closed, in a local coordinate system it can be represented as $\mathcal{H} = d\mathcal{B}$; then the action takes the usual 2d σ-model form which will be considered in Section 4.

3.2. Invariance under κ-symmetry and equations of motion

The action (3.12) is invariant with respect to the local κ-transformations [28,15] which is useful to write down in terms of $\delta x^a \equiv \delta X^M L_M^a$, $\delta x^{a'} \equiv \delta X^M L_M^{a'}$, $\delta\theta^I \equiv \delta X^M L_M^I$

$$\delta_\kappa x^a = 0 , \qquad \delta_\kappa x^{a'} = 0 , \qquad \delta_\kappa \theta^I = 2 (L_i^a \gamma^a - \mathrm{i} L_i^{a'} \gamma^{a'}) \kappa^{iI} , \qquad (3.13)$$

$$\delta_\kappa (\sqrt{g} g^{ij}) = -16 \mathrm{i} \sqrt{g} (P_-^{jk} \bar{L}_k^1 \kappa^{i1} + P_+^{jk} \bar{L}_k^2 \kappa^{i2}). \qquad (3.14)$$

Here $P_\pm^{ij} \equiv \frac{1}{2} (g^{ij} \pm \frac{1}{\sqrt{g}} \epsilon^{ij})$, and the 16-component spinors κ^{iI} (the corresponding 32-component spinor has opposite chirality to that of θ) satisfy the (anti) self-duality constraints

$$P_-^{ij} \kappa_j^1 = \kappa^{i1} , \qquad P_+^{ij} \kappa_j^2 = \kappa^{i2} , \qquad (3.15)$$

which can be rewritten as

$$\frac{1}{\sqrt{g}} \epsilon^{ij} \kappa_j^1 = -\kappa^{i1}, \qquad \frac{1}{\sqrt{g}} \epsilon^{ij} \kappa_j^2 = \kappa^{i2}.$$

To demonstrate κ-invariance one uses the following expressions for the variations of the Cartan 1-forms:

$$\delta L^a = d\delta x^a + L^{ab} \delta x^b + L^b \delta x^{ba} + 2\mathrm{i} \bar{L}^I \gamma^a \delta\theta^I ,$$
$$\delta L^{a'} = d\delta x^{a'} + L^{a'b'} \delta x^{b'} + L^{b'} \delta x^{b'a'} - 2 \bar{L}^I \gamma^{a'} \delta\theta^I ,$$
$$\delta L^I = d\delta\theta^I + \frac{\mathrm{i}}{2} \epsilon^{IJ} (\delta x^a \gamma^a + \mathrm{i} \delta x^{a'} \gamma^{a'}) L^J - \frac{\mathrm{i}}{2} \epsilon^{IJ} (L^a \gamma^a + \mathrm{i} L^{a'} \gamma^{a'}) \delta\theta^J$$
$$- \tfrac{1}{4} (\delta x^{ab} \gamma^{ab} + \delta x^{a'b'} \gamma^{a'b'}) L^I + \tfrac{1}{4} (L^{ab} \gamma^{ab} + L^{a'b'} \gamma^{a'b'}) \delta\theta^I ,$$

where $\delta x^{ab} \equiv \delta X^M L_M^{ab}$, $\delta x^{a'b'} \equiv \delta X^M L_M^{a'b'}$. The crucial relation that allows one to check the κ-invariance of the action (and also to obtain the equations of motion) directly in terms of the coordinate-invariant Cartan forms is

$$\delta \mathcal{H} = d\Lambda, \qquad \Lambda \equiv \Lambda^1 - \Lambda^2, \qquad (3.16)$$

$$\Lambda^I \equiv \bar{L}^I \gamma^a \wedge L^I \delta x^a + 2 L^a \wedge \bar{L}^I \gamma^a \delta\theta^I + \mathrm{i} \bar{L}^I \gamma^{a'} \wedge L^I \delta x^{a'} + 2\mathrm{i} L^{a'} \wedge \bar{L}^I \gamma^{a'} \delta\theta^I . \qquad (3.17)$$

The equations of motion that follow from the action (3.12) are

$$\sqrt{g}g^{ij}(\nabla_i L^a_j + L^{ab}_i L^b_j) + \mathrm{i}\epsilon^{ij}s^{IJ}\bar{L}^I_i\gamma^a L^J_j = 0\,, \tag{3.18}$$

$$\sqrt{g}g^{ij}(\nabla_i L^{a'}_j + L^{a'b'}_i L^{b'}_j) - \epsilon^{ij}s^{IJ}\bar{L}^I_i\gamma^{a'} L^J_j = 0\,, \tag{3.19}$$

$$(\gamma^a L^a_i + \mathrm{i}\gamma^{a'}L^{a'}_i)(\sqrt{g}g^{ij}\delta^{IJ} - \epsilon^{ij}s^{IJ})L^J_j = 0\,, \tag{3.20}$$

where ∇_i is the g_{ij}-covariant derivative. These relations should be supplemented by the standard constraint

$$L^a_i L^a_j + L^{a'}_i L^{a'}_j = \tfrac{1}{2}g_{ij}g^{kl}(L^a_k L^a_l + L^{a'}_k L^{a'}_l)\,, \tag{3.21}$$

following from the variation of the action over g_{ij}.

Note that like the '2d+3d' form (3.12) of the action (but not its 2d form discussed in the next section, cf. Eq. (3.2)), the equations of motion admit a manifestly covariant representation in terms of the Cartan 1-forms. There is a certain similarity with the equations of motion of the WZW model. In the conformal gauge $\sqrt{g}g^{ij} = \eta^{ij}$ these equations (like in the case of the σ-model on G/H space) imply the existence of $\dim G$ conserved currents. The 'chirality' (presence of ϵ^{ij}-terms) of the above equations has a purely fermionic nature. A more direct analogy with the WZW model may be possible after a certain bosonisation of the fermionic degrees of freedom.

4. Explicit 2-dimensional form of the action

To find the explicit form of the action in terms of the coordinate 2d field θ which generalises (3.2) we are to choose a particular parametrization of the coset representative G in (2.6):

$$G(x,\theta) = g(x)g(\theta)\,, \qquad g(\theta) = \exp(\theta^I Q_I)\,. \tag{4.1}$$

Here $g(x)$ is a coset representative of $[SO(4,2)\otimes SO(6)]/[SO(4,1)\otimes SO(5)]$, i.e. $x = (x^\mu, x^{\mu'})$ provides a certain parametrization of $\mathrm{AdS}_5 \otimes S^5$ which we will not need to specify in what follows.

To represent the WZ term in (3.3), (3.11) as an integral over the 2-dimensional space we use the standard trick of rescaling $\theta \to \theta_t \equiv t\theta$,

$$I_{\mathrm{WZ}} = I_{\mathrm{WZ}}(t=1)\,, \qquad I_{\mathrm{WZ}}(t) = \mathrm{i}\int_{M_3} \mathcal{H}_t\,, \quad \mathcal{H}_t = \mathcal{H}(\theta_t)\,. \tag{4.2}$$

Then (3.16) implies

$$\partial_t I_{\mathrm{WZ}}(t) = \mathrm{i}\int_{\partial M_3} \partial_t \Lambda\,, \qquad \partial_t\Lambda = -2s^{IJ}L^{\hat{a}}_t \bar{\theta}^I \hat{\gamma}^{\hat{a}} L^J_t\,, \tag{4.3}$$

where $L^A_t \equiv L^A(\theta_t)$. We have used (3.17) and that $\partial_t\theta_t = \theta$ and $\partial_t x^{\hat{a}} = 0$. Thus

$$I_{\rm WZ} = -2{\rm i}\int_0^1 dt \int d^2\sigma\, \epsilon^{ij} s^{IJ} L^{\hat a}_{it}\bar\theta^I \hat\gamma^{\hat a} L^J_{jt}. \tag{4.4}$$

Eq. (4.4) together with equations from Appendices A and B provide a setup for a systematic calculation of the action I (3.12) as an expansion in powers of θ.

The expansions of the Cartan 1-forms are given by (see Appendix B, cf. Eq. (2.12))

$$\begin{aligned} L^a &= e^a - {\rm i}\bar\theta^I\gamma^a D\theta^I + \tfrac{1}{12}{\rm i}\epsilon^{IJ}(\bar\theta^I\theta^J\bar\theta^K\gamma^a D\theta^K - \bar\theta^I\gamma^a\gamma^{a'}\theta^J\bar\theta^K\gamma^{a'}D\theta^K) \\ &\quad + \tfrac{1}{24}{\rm i}\epsilon^{KL}(-\bar\theta^I\gamma^{abc}\theta^I\bar\theta^K\gamma^{bc}D\theta^L + \bar\theta^I\gamma^a\gamma^{b'c'}\theta^I\bar\theta^K\gamma^{b'c'}D\theta^L) + \dots, \end{aligned} \tag{4.5}$$

$$\begin{aligned} L^{a'} &= e^{a'} + \bar\theta^I\gamma^{a'} D\theta^I + \tfrac{1}{12}\epsilon^{IJ}(\bar\theta^I\theta^J\bar\theta^K\gamma^{a'} D\theta^K - \bar\theta^I\gamma^a\gamma^{a'}\theta^J\bar\theta^K\gamma^{a}D\theta^K) \\ &\quad + \tfrac{1}{24}\epsilon^{KL}(-\bar\theta^I\gamma^{a'b'c'}\theta^I\bar\theta^K\gamma^{b'c'}D\theta^L + \bar\theta^I\gamma^{a'}\gamma^{bc}\theta^I\bar\theta^K\gamma^{bc}D\theta^L) + \dots, \end{aligned} \tag{4.6}$$

$$\begin{aligned} L^I &= D\theta^I + \tfrac{1}{6}\epsilon^{IJ}(-\gamma^a\theta^J\bar\theta^K\gamma^a D\theta^K + \gamma^{a'}\theta^J\bar\theta^K\gamma^{a'}D\theta^K) \\ &\quad + \tfrac{1}{12}\epsilon^{KL}(\gamma^{ab}\theta^I\bar\theta^K\gamma^{ab}D\theta^L - \gamma^{a'b'}\theta^I\bar\theta^K\gamma^{a'b'}D\theta^L) + \dots, \end{aligned} \tag{4.7}$$

where e^a, $e^{a'}$, ω^{ab}, $\omega^{a'b'}$ are 5-beins and Lorentz connections of AdS_5 and S^5, and the generalised spinor covariant differential $D\theta^I$ is defined by [12]

$$D\theta^I \equiv D^{IJ}\theta^J, \qquad D^{IJ}D^{JK} = 0, \tag{4.8}$$

$$\begin{aligned} D^{IJ} &\equiv \delta^{IJ}\mathcal{D} - \tfrac{1}{2}{\rm i}\epsilon^{IJ}e^{\hat a}\hat\gamma^{\hat a} \\ &= \delta^{IJ}\left[d + \tfrac{1}{4}(\omega^{ab}\gamma^{ab} + \omega^{a'b'}\gamma^{a'b'})\right] - \tfrac{1}{2}{\rm i}\epsilon^{IJ}(e^a\gamma^a + {\rm i}e^{a'}\gamma^{a'}). \end{aligned} \tag{4.9}$$

The θ^2 and θ^4 terms in $I_{\rm WZ}$ are determined by

$$\partial_t^2 A|_{t=0} = -2e^{\hat a}\wedge s^{IJ}\bar\theta^I\hat\gamma^{\hat a}D\theta^J, \tag{4.10}$$

$$\begin{aligned} \partial_t^4 A|_{t=0} &= -24{\rm i}\bar\theta^1\hat\gamma^{\hat a}D\theta^1\wedge\bar\theta^2\hat\gamma^{\hat a}D\theta^2 - 2e^{\hat a}\wedge s^{IJ}\bar\theta^I\hat\gamma^{\hat a}\partial_t^3 L^J_t|_{t=0} \\ &= -24{\rm i}\bar\theta^1\hat\gamma^{\hat a}D\theta^1\wedge\bar\theta^2\hat\gamma^{\hat a}D\theta^2 + 4e^a\wedge\bar\theta^1\gamma^{ab}\theta^2\bar\theta^I\gamma^b D\theta^I \\ &\quad -4{\rm i}e^{a'}\wedge\bar\theta^1\gamma^{a'b'}\theta^2\bar\theta^I\gamma^{b'}D\theta^I \\ &\quad + s^{IJ}\epsilon^{KL}e^a\wedge(-\bar\theta^I\gamma^{abc}\theta^J\bar\theta^K\gamma^{bc}D\theta^L + \bar\theta^I\gamma^a\gamma^{b'c'}\theta^J\bar\theta^K\gamma^{b'c'}D\theta^L) \\ &\quad + {\rm i}s^{IJ}\epsilon^{KL}e^{a'}\wedge(\bar\theta^I\gamma^{a'b'c'}\theta^J\bar\theta^K\gamma^{b'c'}D\theta^L - \bar\theta^I\gamma^{a'}\gamma^{bc}\theta^J\bar\theta^K\gamma^{bc}D\theta^L). \end{aligned}$$

Using these expressions we find the following result for the action (3.12)

$$I = \int d^2\sigma\, \mathcal{L}, \qquad \mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + O(\theta^6), \tag{4.11}$$

where $\mathcal{L}_1$ is essentially a 'covariantisation' of the flat-space GS Lagrangian, i.e. it has the same structure as $\mathcal{L}_0$ in (3.2) but with $\partial_i\theta \to D_i\theta$ and 5-beins contracting target-space indices

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[12] The remarkable $D^2 = 0$ property (see Appendix A) is the condition of integrability of the Killing spinor equation $D_\mu\epsilon^I = 0$.

$$\mathcal{L}_1 = -\tfrac{1}{2}\sqrt{g}g^{ij}(e_i^{\hat a} - \mathrm{i}\bar\theta^I\hat\gamma^{\hat a}D_i\theta^I)(e_j^{\hat a} - \mathrm{i}\bar\theta^I\hat\gamma^{\hat a}D_j\theta^I)$$
$$-\mathrm{i}\epsilon^{ij}e_i^{\hat a}(\bar\theta^1\hat\gamma^{\hat a}D_j\theta^1 - \bar\theta^2\hat\gamma^{\hat a}D_j\theta^2) + \epsilon^{ij}\bar\theta^1\hat\gamma^{\hat a}D_i\theta^1\bar\theta^2\hat\gamma^{\hat a}D_j\theta^2\,, \tag{4.12}$$

where

$$e_i^{\hat a} = (e_i^a, e_i^{a'})\,, \qquad e_i^a \equiv e_\mu^a(x)\partial_i x^\mu\,, \qquad e_i^{a'} \equiv e_{\mu'}^{a'}(x')\partial_i x^{\mu'}\,. \tag{4.13}$$

The additional θ^4 term $\mathcal{L}_2$ is given by

$$\mathcal{L}_2 = \mathcal{L}_{2\mathrm{kin}} + \mathcal{L}_{2\mathrm{WZ}}\,,$$
$$\mathcal{L}_{2\mathrm{kin}} = \tfrac{1}{24}\sqrt{g}g^{ij}\Big(\mathrm{i}e_i^a\Big[2\epsilon^{IJ}(-\bar\theta^I\theta^J\bar\theta^K\gamma^a D_j\theta^K + \bar\theta^I\gamma^a\gamma^{a'}\theta^J\bar\theta^K\gamma^{a'}D_j\theta^K)$$
$$+\epsilon^{KL}(\bar\theta^I\gamma^{abc}\theta^I\bar\theta^K\gamma^{bc}D_j\theta^L - \bar\theta^I\gamma^a\gamma^{b'c'}\theta^I\bar\theta^K\gamma^{b'c'}D_j\theta^L)\Big]$$
$$+e_i^{a'}\Big[2\epsilon^{IJ}(-\bar\theta^I\theta^J\bar\theta^K\gamma^{a'}D_j\theta^K + \bar\theta^I\gamma^a\gamma^{a'}\theta^J\bar\theta^K\gamma^a D_j\theta^K)$$
$$+\epsilon^{KL}(\bar\theta^I\gamma^{a'b'c'}\theta^I\bar\theta^K\gamma^{b'c'}D_j\theta^L - \bar\theta^I\gamma^{a'}\gamma^{bc}\theta^I\bar\theta^K\gamma^{bc}D_j\theta^L)\Big]\Big)\,,$$
$$\mathcal{L}_{2\mathrm{WZ}} = \tfrac{1}{24}\epsilon^{ij}\Big[4(\mathrm{i}e_i^a\bar\theta^1\gamma^{ab}\theta^2\bar\theta^I\gamma^b D_j\theta^I + e_i^{a'}\bar\theta^1\gamma^{a'b'}\theta^2\bar\theta^I\gamma^{b'}D_j\theta^I)$$
$$+\mathrm{i}s^{IJ}\epsilon^{KL}e_i^a(-\bar\theta^I\gamma^{abc}\theta^J\bar\theta^K\gamma^{bc}D_j\theta^L + \bar\theta^I\gamma^a\gamma^{b'c'}\theta^J\bar\theta^K\gamma^{b'c'}D_j\theta^L)$$
$$+s^{IJ}\epsilon^{KL}e_i^{a'}(-\bar\theta^I\gamma^{a'b'c'}\theta^J\bar\theta^K\gamma^{b'c'}D_j\theta^L + \bar\theta^I\gamma^{a'}\gamma^{bc}\theta^J\bar\theta^K\gamma^{bc}D_j\theta^L)\Big]\,.$$

We have not determined explicitly the θ^6 terms in I (this is, in principle, straightforward using the method explained above), but it is likely that after an appropriate κ-symmetry gauge choice the expression for the action will contain only θ^4 terms at most, i.e. the *full* action will be given by $\mathcal{L}_1 + \mathcal{L}_2$ presented above.

5. Some properties of the action

The presence of the 'mass' term linear in $\gamma^{\hat a}$ in the covariant derivative D (4.9) may be interpreted as being due to a non-trivial self-dual 5-form background and is directly connected with the global supersymmetry of the action. Essentially the same derivative $D_{\hat\mu} = \partial_{\hat\mu} + \frac{1}{4}\omega_{\hat\mu}^{\hat a\hat b}\Gamma_{\hat a\hat b} + c\Gamma^{\hat\mu_1\ldots\hat\mu_5}\Gamma_{\hat\mu}\ e^\phi F_{\hat\mu_1\ldots\hat\mu_5}$ appears in the Killing spinor equation of type IIB supergravity (see, e.g., Ref. [29]).

The leading order coupling to the RR background field

$$\partial x^{\hat\lambda}\partial x^{\hat\nu}\bar\theta\Gamma_{\hat\lambda}\Gamma^{\hat\mu_1\ldots\hat\mu_5}\Gamma_{\hat\nu}\theta\ e^\phi F_{\hat\mu_1\ldots\hat\mu_5}$$

is, indeed, contained in the $e^{\hat a}\bar\theta\gamma^{\hat a}D\theta$ term in (4.12) [13]

[13] Note that $\gamma^{a_1\ldots a_5} = \mathrm{i}\epsilon^{a_1\ldots a_5}$, $\gamma^{a'_1\ldots a'_5} = \epsilon^{a'_1\ldots a'_5}$, and $(F_5)_{\mathrm{AdS}_5} = Q\epsilon_5$, $(F_5)_{S^5} = Q\epsilon_5$, where Q is related to the RR charge. Also, $e^\phi = g_s$ =const and the radius $R \sim (g_sQ)^{1/4}$ is set equal to 1.

$$\mathcal{L}_1(\theta^2) = \mathrm{i}(\sqrt{g}g^{ij}\delta^{IJ} - \epsilon^{ij}s^{IJ})e_i^{\hat{a}}\bar{\theta}^I\gamma^{\hat{a}}D_i\theta^J$$
$$\rightarrow \ldots + \tfrac{1}{2}(\sqrt{g}g^{ij}\delta^{IK} - \epsilon^{ij}s^{IK})\epsilon^{KJ}e_{\hat{\mu}}^{\hat{a}}e_{\hat{\nu}}^{\hat{b}}\partial_i x^{\hat{\mu}}\partial_j x^{\hat{\nu}}\bar{\theta}^I\gamma^{\hat{a}}\gamma^{\hat{b}}\theta^J. \qquad (5.1)$$

The presence of terms of higher orders in θ reflect the curved nature of the background metric.

The uniqueness of the action, which is completely fixed by the requirement of $SU(2,2|4)$ supersymmetry, suggests, by analogy with the WZW model, that it defines an exact 2d conformal field theory in which non-conformal-invariance of the bosonic coset space $\mathrm{AdS}_5 \otimes S^5$ σ-model is compensated by contributions from the fermionic terms. The fact that the action contains no free parameters (except for the radius of the background) that can be renormalised implies that the corresponding $\mathrm{AdS}_5 \otimes S^5$ + 5-form background should be an *exact* string solution. In general, the global symmetry implies that only the coefficients in front of the 'kinetic' and WZ terms in (3.11) may be renormalised. However, the analogy with WZW model suggests that global (super)symmetry should prohibit renormalisation of the coefficient in front of the *WZ term* (which is manifestly supersymmetric when written in the global 3d form).[14] But then the local κ-symmetry relating the two terms in (3.11) should imply that the coefficient of the 'kinetic' term is also not renormalised,[15] i.e. that (3.11) is a conformal model. The remaining question is why its central charge is not shifted from its flat-space value. Possible corrections to the dilaton β-function are proportional to invariants constructed from curvature and F_5 and are constant in the given background. The crucial point is that the leading-order correction vanishes since $R = (F_5)^2 = 0$ because of the cancellation between the AdS_5 and S^5 contributions. Higher-order corrections should vanish in a special 'κ-symmetric' scheme.[16]

The absence of both perturbative and non-perturbative [32] string corrections to this vacuum (see also Ref. [33]) may be related to the 32 fermionic zero-modes associated with the global supersymmetry of the action (implying, as in flat space, the existence of certain non-renormalisation theorems).

One may check the conformal invariance directly by a one-loop calculation (using a 'quantum' κ-symmetry 'light-cone' gauge $\Gamma^+\theta = 0$ as in Ref. [34]). It is fairly clear that the RR coupling (5.1) will produce an extra F_5^2 contribution to the renormalisation of the $(\partial x)^2$ term in the action, leading to the required conformal invariance condition (supergravity equation of motion) $R_{\hat{\mu}\hat{\nu}} \sim e^{\phi}(F_5^2)_{\hat{\mu}\hat{\nu}}$.

[14] This is related to the fact that the components of the 3-form $\mathcal{H}$ in (3.8), (3.10) are covariantly constant (recall that the contributions to the β-function of the B_{mn} coupling in the standard bosonic sigma model always involve derivatives of $H = dB$).

[15] In the WZW model case similar fact is effectively related (via the Polyakov–Wiegmann identity) to the existence of on-shell conserved chiral currents and associated affine symmetry [27,30].

[16] The situation should be similar to that in the case of the NS–NS $\mathrm{AdS}_3 \otimes S^3$ background described by supersymmetric $SL(2,R) \times SU(2)$ WZW model (with both factors having the same level k). In this case [31] the non-trivial part of the central charge is $c = 3k_+/(k_+ - 2) + 3k_-/(k_- + 2) = (3 + 2/k) + (3 - 2/k) = 6$, where the leading-order correction cancels between the two factors and the shifts of the levels $k_+ = k + 2$, $k_- = k - 2$ (implying the absence of all higher-order corrections) take place in a special supersymmetric scheme.

Since the action we constructed has manifest $SU(2,2|4)$ symmetry, the string spectrum should be classified by representations of this supergroup. In particular, the marginal vertex operators should be in one-to-one correspondence with the unitary irreducible representations of $SU(2,2|4)$ associated [4] with the modes of type IIB supergravity on $AdS_5 \otimes S^5$.

Some basic features of this relation can be seen directly from the main $\mathcal{L}_1$ (4.12) part of the action. Here we make only qualitative comments to indicate some implications of the presence of the RR background on the conditions on vertex operators. Let us formally ignore the curvature of the background and fix the l.c. gauge: $x^+ \sim \tau$, $\Gamma^+\theta = 0$. One can then show that the presence of the above RR vertex (5.1) induces the mass term in the equation of conformal invariance for the graviton perturbation $(\Box + 2)h_{\mu\nu} = 0$ (this is directly related to the computation of the 1-loop β-function mentioned above). The coupling term for the RR 2-form perturbation $\tilde{H}$ together with the 5-form background term (5.1) contained in the action are represented by $\partial x^\mu \partial x^\lambda \bar{\theta}\gamma^\mu\gamma^{\rho\sigma}\theta\tilde{H}_{\rho\sigma\lambda} + \partial x^\mu \partial x^\nu \bar{\theta}\gamma^\mu\gamma^\nu\theta$, or, in the l.c. gauge, by $\bar{\theta}\gamma^-\gamma^{ij}\theta\tilde{H}_{jk+} + \partial x^i\bar{\theta}\gamma^i\gamma^-\theta$ $(i,j,k = 1,2,3)$. The resulting additional contribution to the NS-NS 2-form β-function is proportional to $\tilde{H}_{jk+}\partial x^i\langle(\bar{\theta}\gamma^-\gamma^i\theta)(\bar{\theta}\gamma^-\gamma^{jk}\theta)\rangle$. Since $\mathrm{tr}(\gamma^i\gamma^j\gamma^k) = \epsilon^{ijk}$ and $\langle(\theta\theta)(\theta\theta)\rangle \sim \ln\varepsilon$ we get $\partial_\mu H_{\mu i+} = \epsilon_{ijk}\tilde{H}_{jk+}$, which is the l.c. gauge ($H_{\mu\nu-} = 0$, $\tilde{H}_{\mu\nu-} = 0$) version of the well-known equation [2] $\partial_\mu H_{\mu\nu\rho} \propto \epsilon_{\nu\rho\lambda_1\lambda_2\lambda_3}\tilde{H}_{\lambda_1\lambda_2\lambda_3}$ 'mixing' the two 2-form tensors in the presence of the 5-form background.

This consequence of the non-vanishing RR background is also directly related to the presence of only one set of $SO(6)$ gauge vectors among the marginal perturbations. This is, of course, consistent with the supergravity spectrum of KK compactification on S^5 but is different from what one would naively expect on the basis of analogy with the WZW model. [17] The reason lies in the chirality of the string action (3.12), (4.12) in the fermionic sector. The latter implies the existence of only one marginal $SO(6)$ vertex operator associated with the super-extension of conserved currents of S^5 σ-model: in contrast to the group space case, the coset space S^5 has only one copy of $SO(6)$ as an isometry, with the marginality of the corresponding vertex depending on the non-zero RR 5-form background. [18]

Further progress in unraveling the properties of this string theory obviously depends on understanding how to fix a suitable κ-symmetry gauge which will lead to a substantial simplification of the action. To be able to define the corresponding 2d conformal field theory one may need also to identify the proper variables in which the world-sheet description may become more transparent. These are expected to be related to the conserved (super)currents. In the case of a bosonic σ-model defined on a coset space (e.g., AdS_5) the conserved currents $\mathcal{J}_i^p = k_a^p e_i^a$, $\mathcal{J}_i^{pq} = k_a^{pq} e_i^a$ are constructed in terms of the vielbeins (projected to the world-sheet) e_i^a and the Killing vectors k_μ^p, $k_\mu^{pq} = -k_\mu^{qp}$ $(\nabla_{(\mu}k_{\nu)}^p = 0$, $\nabla_{(\mu}k_{\nu)}^{pq} = 0)$ which generate translations and rotations (see in this

[17] A.T. is grateful to M. Douglas for raising this issue.

[18] Put differently, the 2d parity inversion does not lead to a new marginal operator as in the presence of the RR background it cannot be accompanied by a field transformation like $G \to G^{-1}$ in the WZW model case.

connection Refs. [35,36]). The currents are conserved but not chiral in general. We expect that the corresponding supercurrents constructed using also Killing spinors should play an important role in the 2d theory defined by (3.12).

In conclusion, let us mention that an approach similar to the one described in this paper can be used to construct actions of the IIB superstring on $AdS_3 \times S^3 \times T^4$ space or of the $D = 11$ membrane on $AdS_4 \times S^7$. In the former case there are two possible choices for a WZ term leading to the (1/2 supersymmetric) actions corresponding to the cases of non-trivial NS–NS or RR 2-form field backgrounds.

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Appendix A. Coset superspace parametrisation and some general relations

In the parametrisation $G(x,\theta) = g(x)g(\theta)$ (4.1) of an element of $\frac{SU(2,2|4)}{SO(4,1)\otimes SO(5)}$, $g(x)$ represents the bosonic coset $\frac{SO(4,2)\otimes SO(6)}{SO(4,1)\otimes SO(5)}$ and defines

$$g^{-1}dg = e^{\hat{a}}P_{\hat{a}} + \tfrac{1}{2}\omega^{\hat{a}\hat{b}}J_{\hat{a}\hat{b}} \tag{A.1}$$

the 5-beins $e^{\hat{a}} = (e^a, e^{a'})$ and Lorentz connections $\omega^{\hat{a}\hat{b}} = (\omega^{ab}, \omega^{a'b'})$ of $AdS_5 \otimes S^5$ (here $J_{\hat{a}\hat{b}} \equiv (J_{ab}, J_{a'b'})$, i.e. $\omega^{\hat{a}\hat{b}}$ and $\Omega^{\hat{a}\hat{b}}$ below do not contain 'cross terms'). They satisfy the standard (zero torsion, constant curvature) relations

$$de^a + \omega^{ab}\wedge e^b = 0\,, \qquad de^{a'} + \omega^{a'b'}\wedge e^{b'} = 0\,, \tag{A.2}$$

$$d\omega^{ab} + \omega^{ac}\wedge\omega^{cb} = -e^a\wedge e^b\,, \qquad d\omega^{a'b'} + \omega^{a'c'}\wedge\omega^{c'b'} = e^{a'}\wedge e^{b'}\,. \tag{A.3}$$

On the odd part of the superspace it is useful to introduce the following Cartan forms:

$$\mathrm{g}^{-1}d\mathrm{g} = \Omega^{\hat{a}}P_{\hat{a}} + \tfrac{1}{2}\Omega^{\hat{a}\hat{b}}J_{\hat{a}\hat{b}} + \Omega^I Q_I\,, \qquad \mathrm{g} = \mathrm{g}(\theta)\,, \tag{A.4}$$

where, by definition,

$$\Omega^{\hat{a}} = d\theta^I\Omega_I^{\hat{a}}\,, \qquad \Omega^{\hat{a}\hat{b}} = d\theta^I\Omega_I^{\hat{a}\hat{b}}\,, \qquad \Omega^I = d\theta^J\Omega_J^I. \tag{A.5}$$

For the exponential parametrization g $=\exp(\theta^I Q_I)$ the Cartan superconnections satisfy the following constraints:

$$\theta^I\Omega_I^{\hat{a}} = 0\,, \qquad \theta^I\Omega_I^{\hat{a}\hat{b}} = 0\,, \qquad \theta^J\Omega_J^I = \theta^I. \tag{A.6}$$

It is easy to prove that

$$G^{-1}dG = e^{-\theta Q}De^{\theta Q}, \qquad G(x,\theta) = g(x)g(\theta) = g(x)e^{\theta Q}, \tag{A.7}$$

where the covariant differential is given by

$$D = d + e^{\hat a}P_{\hat a} + \tfrac{1}{2}\omega^{\hat a\hat b}J_{\hat a\hat b}, \quad D^2 = 0. \tag{A.8}$$

Eq. (A.7) can be re-written as follows:

$$G^{-1}dG = (e^{\hat a} + \Omega^{\hat a}_{\rm cov})P_{\hat a} + \tfrac{1}{2}(\omega^{\hat a\hat b} + \Omega^{\hat a\hat b}_{\rm cov})J_{\hat a\hat b} + \Omega^I_{\rm cov}Q_I\,, \tag{A.9}$$

where $\Omega^A_{\rm cov}$ are obtained from Ω^A by the replacement $d \to D$:

$$\Omega^{\hat a}_{\rm cov} = D\theta^I\Omega^{\hat a}_I\,, \qquad \Omega^{\hat a\hat b}_{\rm cov} = D\theta^I\Omega^{\hat a\hat b}_I\,, \qquad \Omega^I_{\rm cov} = D\theta^J\Omega^I_J. \tag{A.10}$$

The explicit form of the covariant differential $D\theta^I \equiv D^{IJ}\theta^J$,

$$D^{IJ} \equiv \delta^{IJ}(d + \tfrac{1}{4}\omega^{\hat a\hat b}\gamma^{\hat a\hat b}) - \tfrac{1}{2}{\rm i}\epsilon^{IJ}e^{\hat a}\hat\gamma^{\hat a}, \quad D^{IJ}D^{JK} = 0, \tag{A.11}$$

was already given in (4.9). Comparing (A.9) with the defining relation (2.6), we get the following representation for the Cartan 1-forms in terms of the 5-beins, Lorentz connections and superconnections

$$L^{\hat a} = e^{\hat a} + \Omega^{\hat a}_{\rm cov}\,, \qquad L^{\hat a\hat b} = \omega^{\hat a\hat b} + \Omega^{\hat a\hat b}_{\rm cov}\,, \qquad L^I = \Omega^I_{\rm cov}. \tag{A.12}$$

To compute the Cartan 1-forms we are thus to calculate the Cartan superconnections (A.5), make the covariantization (A.10) and then use the expressions (A.12).

Appendix B. θ-expansion of superconnections and 1-forms

Making the rescaling $\theta \to t\theta$ we get the defining equation

$$e^{-t\theta Q}de^{t\theta Q} = \Omega^{\hat a}_tP_{\hat a} + \tfrac{1}{2}\Omega^{\hat a\hat b}_tJ_{\hat a\hat b} + \Omega^I_tQ_I. \tag{B.1}$$

The forms Ω^A we are interested in are given by $\Omega^A_t|_{t=1}$. Eq. (B.1) implies the following differential equations:

$$\partial_t\Omega^a_t = -2{\rm i}\bar\theta^I\gamma^a\Omega^I_t\,, \qquad \partial_t\Omega^{a'}_t = 2\bar\theta^I\gamma^{a'}\Omega^I_t\,, \tag{B.2}$$

$$\partial_t\Omega^{ab}_t = 2\epsilon^{IJ}\bar\theta^I\gamma^{ab}\Omega^J_t\,, \qquad \partial_t\Omega^{a'b'}_t = -2\epsilon^{IJ}\bar\theta^I\gamma^{a'b'}\Omega^J_t\,, \tag{B.3}$$

$$\partial_t\Omega^I_t = d\theta^I - \frac{\rm i}{2}\epsilon^{IJ}\gamma^a\theta^J\Omega^a_t + \tfrac{1}{2}\epsilon^{IJ}\gamma^{a'}\theta^J\Omega^{a'}_t + \tfrac{1}{4}\gamma^{ab}\theta^I\Omega^{ab}_t + \tfrac{1}{4}\gamma^{a'b'}\theta^I\Omega^{a'b'}_t\,, \tag{B.4}$$

with the initial conditions

$$\Omega^{\hat a}_t|_{t=0} = \Omega^{\hat a\hat b}_t|_{t=0} = \Omega^I_t|_{t=0} = 0. \tag{B.5}$$

According to (B.1), the power of θ in the expansion of Ω^A coincides with that of t in the expansion of Ω^A_t. Making use of (B.2), (B.4) and (B.5) we find

$$\partial_t^2 \Omega_t^a|_{t=0} = -2\mathrm{i}\bar\theta^I\gamma^a d\theta^I\,, \qquad \partial_t^2 \Omega_t^{a'}|_{t=0} = 2\bar\theta^I\gamma^{a'} d\theta^I\,, \tag{B.6}$$

$$\partial_t^2 \Omega_t^{ab}|_{t=0} = 2\epsilon^{IJ}\bar\theta^I\gamma^{ab} d\theta^J\,, \qquad \partial_t^2 \Omega_t^{a'b'}|_{t=0} = -2\epsilon^{IJ}\bar\theta^I\gamma^{a'b'} d\theta^J\,, \tag{B.7}$$

$$\begin{aligned}
\partial_t^3 \Omega_t^I|_{t=0} &= \epsilon^{IJ}(-\gamma^a\theta^J\bar\theta^K\gamma^a d\theta^K + \gamma^{a'}\theta^J\bar\theta^K\gamma^{a'} d\theta^K) \\
&\quad + \tfrac{1}{2}\epsilon^{KL}(\gamma^{ab}\theta^I\bar\theta^K\gamma^{ab} d\theta^L - \gamma^{a'b'}\theta^I\bar\theta^K\gamma^{a'b'} d\theta^L)\,, \\
\partial_t^4 \Omega_t^a|_{t=0} &= 2\mathrm{i}\epsilon^{IJ}(\bar\theta^I\theta^J\bar\theta^K\gamma^a d\theta^K - \bar\theta^I\gamma^a\gamma^{a'}\theta^J\bar\theta^K\gamma^{a'} d\theta^K) \\
&\quad + \mathrm{i}\epsilon^{KL}(-\bar\theta^I\gamma^{abc}\theta^I\bar\theta^K\gamma^{bc} d\theta^L + \bar\theta^I\gamma^a\gamma^{b'c'}\theta^I\bar\theta^K\gamma^{b'c'} d\theta^L)\,, \\
\partial_t^4 \Omega_t^{a'}|_{t=0} &= 2\epsilon^{IJ}(\bar\theta^I\theta^J\bar\theta^K\gamma^{a'} d\theta^K - \bar\theta^I\gamma^a\gamma^{a'}\theta^J\bar\theta^K\gamma^a d\theta^K) \\
&\quad + \epsilon^{KL}(-\bar\theta^I\gamma^{a'b'c'}\theta^I\bar\theta^K\gamma^{b'c'} d\theta^L + \bar\theta^I\gamma^{a'}\gamma^{bc}\theta^I\bar\theta^K\gamma^{bc} d\theta^L)\,.
\end{aligned}$$

From (A.12) and (B.6) we find the leading terms in the θ-expansion of the Cartan 1-forms,

$$L^a = e^a - \mathrm{i}\bar\theta^I\gamma^a D\theta^I + \dots\,, \qquad L^{a'} = e^{a'} + \bar\theta^I\gamma^{a'} D\theta^I + \dots\,, \tag{B.8}$$

$$L^{ab} = \omega^{ab} + \epsilon^{IJ}\bar\theta^I\gamma^{ab} D\theta^J + \dots\,, \qquad L^{a'b'} = \omega^{a'b'} - \epsilon^{IJ}\bar\theta^I\gamma^{a'b'} D\theta^J + \dots\,, \tag{B.9}$$

$$L^I = D\theta^I + \dots \tag{B.10}$$

Eq. (A.12) and the above expressions for $\partial_t^n \Omega_t^A|_{t=0}$ allow one to find the higher order corrections to the Cartan 1-forms given in (4.5)–(4.7). These are used to calculate higher-order terms in the string action.

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