$\frac{1}{3}$

$N = 1$
$N = 1$

$[a, b]$

$$I = \int_a^b f\left(x\right) dx$$

$f(x) x x = a x = b$

$$I = \int_a^b \int_{u\left(x\right)}^{v\left(x\right)} f\left(x,\, y\right) dy\, dx$$

$f(x, y) x y x = a x = b y = u(x) y = v(x)$

$$I = \int_a^b f\left(x\right) dx \approx \int_a^b g\left(x\right) dx$$

$g(x)$

$$E = \int_a^b f\left(x\right) dx - \frac{b-a}{2}\left[f\left(a\right) + f\left(b\right)\right]$$

$f(x)$ $\overline{x} = \frac{a+b}{2}$

$$I = \int_a^b f(x)\, dx = \frac{b-a}{2}\,[f(a)+f(b)]+E$$

$$E =$$

$$f(x) = f(\overline{x}) + \frac{f'(\overline{x})}{1!}(x-\overline{x}) + \frac{f''(\overline{x})}{2!}(x-\overline{x})^2 + \dots$$

$$\begin{aligned}
\int_a^b f(x)\, dx &= f(\overline{x})\ x\,\big|_a^b + f'(\overline{x})\left\{\left.\frac{x^2}{2}\right|_a^b - \overline{x}\ x\,\big|_a^b\right\} + \frac{f''(\overline{x})}{2}\left\{\left.\frac{x^3}{3}\right|_a^b - \overline{x}\ x^2\,\big|_a^b\ \overline{x}^{\,2}\ x\,\big|_a^b\right\} \\
&= f(\overline{x})(b-a) + f'(\overline{x})\left\{\frac{(b-a)}{1}\ \underbrace{\frac{(b+a)}{2}}_{\overline{x}} - \overline{x}\,(b-a)\right\} + \\
&+ \frac{f''(\overline{x})}{2}\left\{\frac{b^3-a^3}{3} - \overline{x}\,(b^2-a^2) + \overline{x}^{\,2}\,(b-a)\right\} \\
&= f(\overline{x})(b-a) + \frac{1}{24}\,f''(\overline{x})(b-a)^3 + \dots
\end{aligned}$$

$$
\begin{aligned}
\frac{b-a}{2}\left[f(a)+f(b)\right] &= \frac{b-a}{2}\left\{ f(\overline{x}) + f'(\overline{x})\left[\underbrace{a-\frac{a+b}{2}}_{-\frac{1}{2}(b-a)}\right] + \frac{1}{2}\,f''(\overline{x})\left[\underbrace{a-\frac{a+b}{2}}_{-\frac{1}{2}(b-a)}\right]^2 + \dots\right.\\
&\qquad \left. + f(\overline{x}) + f'(\overline{x})\left[\underbrace{b-\frac{a+b}{2}}_{\frac{1}{2}(b-a)}\right] + \tfrac{1}{2}\,f''(\overline{x})\left[\underbrace{b-\frac{a+b}{2}}_{\frac{1}{2}(b-a)}\right]^2\right\}\\
&= \frac{b-a}{2}\left\{2\,f(\overline{x}) + f''(\overline{x})\,\frac{1}{4}\,(b-a)^2 + \dots\right\}\\
&= f(\overline{x})\,(b-a) + \frac{1}{8}\,f''(\overline{x})\,(b-a)^3 + \dots\\
& E \approx f''(\overline{x})\,(b-a)^3\left(\frac{1}{24}-\frac{1}{8}\right) \approx -\frac{1}{12}\,f''(\overline{x})\,(b-a)^3
\end{aligned}
$$

$$
\boxed{E \approx -\frac{1}{12}\,f''(\overline{x})\,h^3} \tag{1.5}
$$

$$
h = \frac{b-a}{N}
$$

$$
\boxed{E \approx -\frac{1}{12}\,\frac{(b-a)^3}{N^3}\sum_{i=1}^{N} f''(\overline{x}_i)} \tag{1.6}
$$

$$
\boxed{E \approx -\frac{1}{12}\,(b-a)\,h^2\,\overline{f}\,''}
$$

$$
\overline{f}\,'' = \sum_{i=1}^{N}\frac{f''(\overline{x}_i)}{N}
$$

- n
- $h\,\dfrac{b-a}{n}$

$$
\begin{aligned}
n=2 \Rightarrow I &= \frac{1}{2}\left[f(a)+f(a+h)\right]\,h + \frac{1}{2}\left[f(a+h)+f(b)\right]\,h\\
&= \frac{h}{2}\,\left[f(a)+2\,f(a+h)+f(b)\right]
\end{aligned}
$$

$$
\begin{aligned}
n=3 \Rightarrow I &= \frac{1}{2}\left[f(a)+f(a+h)\right]\,h + \frac{1}{2}\left[f(a+h)+f(a+2\,h)\right]\,h + \frac{1}{2}\,\left[f(a+2\,h)+f(b)\right]\,h\\
&= \frac{h}{2}\,\left\{f(a)+2\,\left[f(a+h)+f(a+2\,h)+f(a+3\,h)\right]+f(b)\right\}
\end{aligned}
$$

$$
n=4 \Rightarrow I = \frac{h}{2}\left\{f(a)+2\,\left[f(a+h)+f(a+2\,h)+f(a+3\,h)\right]+f(b)\right\}
$$

$$n = N \Rightarrow \boxed{I = \frac{h}{2} \left\{ f\,(a) + 2 \sum_{i=1}^{N-1} f\,(a + i\,h) + f\,(b) \right\}}$$

$$\overline{f}\,(x) = 1 + \left(\frac{x}{2}\right)^2,\ \ 0 \leq x \leq 2$$

$$\begin{aligned} V &= \int_0^2 \pi\, r^2\, dx = \int_0^2 \pi \left[1 + \frac{x^2}{4}\right]^2 dx = \int_0^2 \pi \left(1 + \frac{x^2}{2} + \frac{x^4}{16}\right) dx \\ &= \pi \left[x + \frac{x^3}{6} + \frac{x^5}{80}\right]\Bigg|_0^2 = \pi \left(2 + \frac{2^3}{6} + \frac{2^5}{80}\right) = 11.7286 \\ &? \end{aligned}$$

$$\frac{1}{3}$$

$$\begin{aligned} g\,(x_0 + s\,h) &= \binom{s}{0} \Delta^0 f_0 + \binom{s}{1} \Delta^1 f_1 + \binom{s}{2} \Delta^2 f_2 \\ g\,(x_0 + s\,h) &= 1\, f_0 + s\,(f_1 - f_0) + \frac{1}{2}\, s\,(s-1)\,(f_2 - 2\,f_1 + f_0) \end{aligned}$$

$$\boxed{g\,(x_0 + s\,h) = f_0 + (f_1 - f_0)\, s + \frac{1}{2}\,(f_2 - 2\,f_1 + f_0)\,(s^2 - s)}$$

$$I=\int_{x_0}^{x_2} g\,(x)\,dx \quad \begin{array}{rcl} x & = & x_0+h\,s \left\{ \begin{array}{l} x=x_0\rightarrow s=0 \\ x=x_2\rightarrow s=2 \end{array}\right. \\ dx & = & h\,ds \end{array}$$

$$\begin{aligned} I &= \int_0^2 g\,(x_0+s\,h)\;h\;ds \\ &= h\left[f_0\,s+(f_1-f_0)\,\frac{s^2}{2}+\frac{1}{2}\,(f_2-2\,f_1+f_0)\,\left(\frac{s^3}{3}-\frac{s^2}{2}\right)\right]\Bigg|_0^2 \\ &= h\left[2\,f_0+(f_1-f_0)\,\frac{4}{2}+\frac{1}{2}\,(f_2-2\,f_1+f_0)\,\left(\underbrace{\frac{8}{3}-\frac{8}{4}}_{\frac{16-12}{6}=\frac{4}{6}=\frac{2}{3}}\right)\right] \\ &= h\left[2\,f_0+2\,f_1-2\,f_0+\frac{1}{3}\,f_2-\frac{2}{3}\,f_1+\frac{1}{3}\,f_0\right] \\ &= \frac{h}{3}\,\left[f_0+4\,f_1+f_2\right] \end{aligned}$$

- $\frac{1}{3}\,E\approx-\frac{h^5}{90}\,f^{(IV)}(\overline{x})$
  $f(x)\leq 3\,f^{(IV)}(\overline{x})=0$
- $\frac{1}{3}$

$$E\approx-(b-a)\,\frac{h^4}{180}\,\overline{f}^{(IV)}$$

$$\overline{f}^{(IV)}=\sum_{i=1}^{N/2}\frac{f^{(IV)}(\overline{x}_i)}{(N/2)}$$

- $\frac{3}{8}\,-\frac{3}{80}\,h^5\,f^{(IV)}(\overline{x})$
- $\frac{3}{8}\,E\approx-\frac{3}{240}\,(b-a)\,h^4\,\overline{f}^{(IV)}$

$I_n\;k=\frac{(b-a)}{N}$

$I_{2n}\;\overline{h}=\frac{(b-a)}{(N/2)}$

$$E_h\approx-\frac{1}{12}\,(b-a)\,\overline{f}''\,h^2\approx C\,h^2$$

$$E_{2\,h} \approx -\frac{1}{12}\,(b-a)\,\overline{f}''\,(2\,h)^2 \approx 4\,C\,h^2$$

$$I = I_h + E_h = I_{2\,h} + E_{2\,h}$$

$$E_h - E_{2\,h} = I_{2\,h} - I_h$$

$\rightarrow \Rightarrow$

$$\begin{aligned} & C\,h^2 - 4\,C\,h^2 = I_{2\,h} - I_h \\ \Rightarrow\quad & C = \frac{1}{3}\,h^{-2}\,(I_h - I_{2\,h}) \end{aligned}$$

$$E_h \approx \frac{1}{3}\,(I_h - I_{2\,h})$$

$$\boxed{I = I_h + E_h \approx I_h + \tfrac{1}{3}\,(I_h - I_{2\,h})}$$

$$\begin{array}{ll} I_{0.5} = 11.9895 & I_{0.25} = 11.7940 \\ \uparrow & \uparrow \\ N = 2 & N = 4 \end{array}$$

$$h = 0.25$$

$$I = 11.7940 + \frac{1}{3}\,(11.7940 - 11.9895) \quad \begin{array}{c} \approx 11.7288 \\ \uparrow \\ \approx I_{0.0156} \\ \uparrow \\ N = 128 \end{array}$$

$$\int_a^b f\,(x)\,dx = \alpha\,h\,\left[w_0\,f_0 + w_1\,f_1 + w_2\,f_2 + \ldots + w_n\,f_n\right] + E$$

αw

$$f_i = f\,(x_i)\,,\, x_i = a + i\,h\,,\, h = \frac{(b-a)}{N}$$

	α	$w_i\,,\, i = 0\,,\,\ldots\,,\,N$	
			$-\dfrac{1}{12}\,h^3\,f''$
			$-\dfrac{1}{90}\,h^5\,f^{(iv)}$
			$-\dfrac{3}{80}\,h^5\,f^{(iv)}$
			$-\dfrac{8}{945}\,h^7\,f^{(vi)}$

$hn = 1$

$$N = 1$$

$$\int_a^b f\,(x)\,dx = \alpha\,h\;\left[w_0\,f_0 + w_1\,f_1 + \ldots + w_{n+2}\,f_{n+2}\right] + E$$

	α	$w_i\,,\, i = 0\,,\,\ldots\,,\,N+2$	
			$-\dfrac{1}{4}\,h^3\,f''$
			$-\dfrac{28}{90}\,h^5\,f^{(iv)}$
			$-\dfrac{95}{144}\,h^5\,f^{(iv)}$
			$-\dfrac{41}{140}\,h^7\,f^{(vi)}$

$N = 1$

$$N = 1$$

$$I = \quad = \frac{1}{2}\,(f_0 + f_3)\,3\,h = \frac{1}{2}\,(f_1 + f_2)\,3\,h$$

$$= \frac{3}{2}\,h\,[0\,f_0 + 1\,f_1 + 1\,f_2 + 0\,f_3]$$

$$N = 2 \to g\left(x_1 + s\,h\right) = f_1 + \left(f_2 - f_1\right) s + \frac{1}{2}\left(f_3 - 2\,f_2 + f_1\right)\left(s^2 - s\right)$$

$$\begin{array}{ll} x = x_1 + h\,s & \displaystyle\int_{x_0}^{x_4} g\left(x\right) dx \quad = \int_{-1}^{3} g\left(x_1 + s\,h\right) h\,ds \\ x = x_0 \to s = -1 & \\ x = x_4 \to s = 3 & \qquad\qquad\qquad = \dfrac{4}{3}\,h\left\{0\,f_0 + 2\,f_1 - 1\,f_2 + 2\,f_3 + 0\,f_4\right\} \end{array}$$

$$I = \int_a^b f(x)dx = \int_a^b g(x)dx + E$$

$f_0, f_1, f_2 ... f_k$

$$g(x) = g(x_0 + sh) = \sum_{n=0}^{k} \binom{s}{n} \triangle^n f_0$$

$$x\left(s\right) = x_0 + sh$$

$$dx = h.ds$$

$$I = \int_a^b g(x)dx = \int_0^4 g(x(s))h.ds = h\int_0^4 g(x_0 + sh)ds$$

$$g(x) = g(x_0 + sh) = \sum_{n=0}^{k} \binom{s}{n} \triangle^n f_0$$

$$g(x_0 + sh) = g_0 + s(g_1 - g_0) + \frac{s(s-1)}{2}(g_2 - g_1 + g_0) + \frac{s^3 - 3s^2 + 2s}{6}(g_3 - 3g_2 + 3g_1 - g_0)$$

$$+ \frac{s(s-1)(s-2)(s-3)}{24}(g_4 - 4g_3 + 6g_2 - 4g_1 + g_0)$$

$$h\int_0^4 g_0 + s(g_1 - g_0) + \frac{s(s-1)}{2}(g_2 - g_1 + g_0) + \frac{s^3 - 3s^2 + 2s}{6}(g_3 - 3g_2 + 3g_1 - g_0) + \frac{s(s-1)(s-2)(s-3)}{24}(g_4 - 4g_3 + 6g_2 - 4g_1 + g_0)$$

g_k

$$I \equiv h\left(\frac{14}{45}g_0 + \frac{64}{45}g_1 + \frac{24}{45}g_2 + \frac{64}{45}g_3 + \frac{14}{45}g_4\right)$$

$$I \equiv \frac{2h}{45}(7g_0 + 32g_1 + 12g_2 + 32g_3 + 7g_4)$$

$f''f(x)$

f^{IV}

$nnnn+1$

$n=2n=3$

$$I=\int_{-1}^{1} f\,(x)\,dx = w_1\,f\,(x_1)+w_2\,f\,(x_2)+E$$

$E=0f(x)=1f(x)=xf(x)=x^2f(x)=x^3$

$$\begin{array}{lll} \int_{-1}^{1} 1\,dx & =2=w_1+w_2 & (a)\\ \int_{-1}^{1} x\,dx & =0=w_1\,x_1+w_2\,x_2 & (b)\\ \int_{-1}^{1} x^2\,dx & =\dfrac{2}{3}=w_1\,x_1^2+w_2\,x_2^2 & (c)\\ \int_{-1}^{1} x^3\,dx & =0=w_1\,x_1^3+w_2\,x_2^3 & (d) \end{array}$$

$x=?$

$x_1x_2x=0$

$$x_2=-x_1$$

$$\begin{array}{l}(1.15.b)\Rightarrow w_1\,x_1-w_2\,x_1=x_1\,(w_1-w_2)=0\\ (1.15.a)\Rightarrow w_1+w_2=2\neq 0\Rightarrow w_1=w_2=1\end{array}$$

$$0=1\,x_1^3+1\,(-x_1)^3=x_1^3-x_1^3$$

$$(1.15.c)\Rightarrow \frac{2}{3}=x_1^2+(-x_1)^2=2\,x_1^2\Rightarrow x_1^2=\frac{1}{3}$$

$$x_1=\frac{1}{\sqrt{3}}$$

$$x_2=-\frac{1}{\sqrt{3}}$$

$n x_1 x_2 x_n n$

$$\boxed{\int_{-1}^{1} f\left(x\right) dx \approx \sum_{k=1}^{N} w_k\, f\left(x_k\right)}$$

n

$$P_n\left(x\right) = \frac{1}{2^N\, N!}\, \frac{d^N\left(x^2-1\right)^N}{dx^N}$$

$$\begin{aligned}
P_0\left(x\right) &= \frac{1}{2^0\, 0!}\left(x^2-1\right)^0 = 1\\
P_1\left(x\right) &= \frac{1}{2^1\, 1!}\,\frac{d}{dx}\left(x^2-1\right)^1 = x\\
P_2\left(x\right) &= \frac{1}{2^2\, 2!}\,\frac{d^2}{dx^2}\left(x^2-1\right)^2 = \frac{1}{2}\left(3\,x^2-1\right)
\end{aligned}$$

$\pm\,x_i$	w_i
$\pm$	
0	$0.888888889 = 8/9$
± 0.774596669	$0.555555556 = 5/9$
± 0.339981043	0.652145155
± 0.861136312	0.347854845

$$\begin{aligned}
I &= \int_a^b f\left(x\right) dx =\\
&= \frac{b-a}{2}\int_{-1}^{1} \bar{f}\left(\xi\right) d\xi\\
&= \frac{b-a}{2}\sum_{k=1}^{N} w_k\, \bar{f}\left(\xi_k\right)
\end{aligned}$$

$$\begin{aligned}
x &= \frac{a+b}{2} + \xi\left(\frac{b-a}{2}\right)\\
dx &= \frac{b-a}{2}\, d\xi
\end{aligned}$$

$$I=\int_0^2 \pi\left[1+(x/2)^2\right]^2 dx$$

$$\left.\begin{array}{l} b=2 \\ a=0 \end{array}\right\} \begin{array}{ll} x & =\dfrac{0+2}{2}+\dfrac{2-0}{2}\,\xi=1+\xi \Rightarrow x=1+\xi \\ dx & =d\xi \end{array}$$

$n=22n-1=4-1=3$

$n=3\leq$

$n=3$

$$\xi_1=-0.774596669 \rightarrow f_1=\pi\left\{1+\left[(1+\xi_1)/2\right]^2\right\}^2=3.2219064 w_1=5/9$$
?

$$\begin{aligned} I & =\frac{5}{9}3.2219064+\frac{8}{9}4.9087385+\frac{5}{9}10.0356146 \\ & =11.7286 \end{aligned}$$

$f(x)2n-1n$

$f(x)\int_{-1}^{1} f(x)\,dx\,2n-1$

$$f(x)=c(x)\,P_N(x)+r(x)$$

$P_n(x)n-1$

$$\int_{-1}^{1} f(x)\,dx=\underbrace{\int_{-1}^{1} c(x)\,P_N(x)\,dx}_{=0P_N}+\int_{-1}^{1} r(x)\,dx$$

$$<N-1$$

$$\underbrace{\boxed{\int_{-1}^{1} P_m(x)\,dx=\begin{cases} 0 & n\neq m \\ \dfrac{2}{2n+1} & m=n \end{cases}}}$$

$$\int_{-1}^{1} f(x)\,dx=\int_{-1}^{1} r(x)\,dx$$

$x_i P_n$

$$f\left(x_i\right) = \underbrace{c\left(x_i\right) P_n\left(x_i\right)}_{0} + r\left(x_i\right) \Rightarrow f\left(x_i\right) = r\left(x_i\right)$$

$r\left(x\right)$ $N-1$ $N-1$ N

$$r\left(x\right) = \sum_{i=1}^{N} \left[\prod_{j=1\ j\neq i}^{N} \frac{x-x_j}{x_i-x_j}\right] r\left(x_i\right)$$

$$r_1 \frac{x-x_2}{x_1-x_2} \frac{x-x_3}{x_1-x_3} \frac{x-x_4}{x_1-x_4}$$

$$r_2 \frac{x-x_1}{x_2-x_1} \frac{x-x_3}{x_2-x_3} \frac{x-x_4}{x_2-x_4}$$

$$r_3 \frac{x-x_1}{x_3-x_1} \frac{x-x_2}{x_3-x_2} \frac{x-x_4}{x_3-x_4}$$

N $P_N\left(x\right)$

$$r\left(x\right) = \sum_{i=1}^{N} \left[\prod_{j=1\ j\neq i}^{N} \frac{x-x_j}{x_i-x_j}\right] f\left(x_i\right)$$

$\rightarrow\Rightarrow$

$$\int_{-1}^{1} f\left(x\right) dx = \int_{-1}^{1} r\left(x\right) dx = \sum_{i=1}^{N} f\left(x_i\right) \underbrace{\int_{-1}^{1} \left[\prod_{j=1\ j\neq i}^{N} \frac{x-x_j}{x_i-x_j}\right] dx}_{w_i}$$

$$r_4 \frac{x-x_1}{x_4-x_1}\frac{x-x_2}{x_4-x_2}\frac{x-x_3}{x_4-x_3}$$

$$\int_{-1}^{1} f(x)dx \sum_{k=1}^{N} w_k f(x_k)$$

$x_1, x_2, x_3 ... x_n P_N(x) = 0 P_N(x)$

$$P_4(x) = \frac{1}{2^4 4!}\frac{d^4(x^2-1)^4}{dx^4}$$

$$\frac{1}{2^4 4!} = \frac{1}{384}$$

$$\frac{1}{384}\frac{d^4(x^2-1)^4}{dx^4} = \frac{1}{384}\frac{d^4}{dx^4}(x^8-4x^6+6x^4-4x^2+1) = \frac{1}{8}(35x^4-30x^2+3)$$

$$P_4(x) = 0$$

$$(35x^4-30x^2+3) = 0$$

$x^2 = a 35a^2 - 30a + 3 = 0 a_1 = \frac{15+2\sqrt{30}}{35} a_2 = \frac{15-2\sqrt{30}}{35}$

$x^2 = a x_1 = \sqrt{\frac{15+2\sqrt{30}}{35}}; x_2 = -\sqrt{\frac{15+2\sqrt{30}}{35}}; x_3 = \sqrt{\frac{15-2\sqrt{30}}{35}}; x_4 = -\sqrt{\frac{15-2\sqrt{30}}{35}};$

$x_1 = 0,861136312; x_2 = -0,861136312; x_3 = 0,339981043; x_4 = -0,339981043;$

$w_1 = w_2 w_3 = w_4.$

$$w_i = \int_{-1}^{1} \left[\prod_{j=1;j\neq i}^{N} \frac{x-x_j}{x_i-x_j} \right] dx$$

w_1

$$w_1 = \int_{-1}^{1} \left[\prod_{j=1;j\neq i}^{N} \frac{x-x_j}{x_i-x_j} \right] dx = \int_{-1}^{1} \left[\frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} \right] dx$$

$$w_1 = w_2 = 0,347854845$$

w_3

$$w_3 = \int_{-1}^{1} \left[\prod_{j=1;j\neq i}^{N} \frac{x-x_j}{x_i-x_j} \right] dx = \int_{-1}^{1} \left[\frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} \right] dx$$

$$w_3 = w_4 = 0,652145155$$

$$\int_{-1}^{1} f(x)dx 0,347854845 f(0,861136312) + 0,347854845 f(-0,861136312)$$

$$+0,652145155 f(0,339981043) + 0,652145155 f(-0,339981043)$$

$$I = \int_{-\infty}^{\infty} e^{-x^2} f(x)\, dx \approx \sum_{k=1}^{N} w_k\, f(x_k)$$

	$\pm x_i$	w_i
	$\pm$0.70710678	0.88622692
	0.00000000	1.18163590
	$\pm$1.22474487	0.29540897
	$\pm$0.52464762	0.80491409
	$\pm$1.65068012	0.08131283
	0.00000000	0.94530872
	$\pm$0.95857246	0.39361932
	$\pm$2.02018287	0.01995324

$$I = \int_0^{\infty} e^{-x}\, f\,(x)\, dx \approx \sum_{k=1}^{N} w_k\, f\,(x_k)$$

	x_i	w_i
	0.58578643	0.85355339
	3.41421356	0.14644660
	0.41577455	0.71109300
	2.24428036	0.27851973
	6.28994508	0.01038926
	0.32254768	0.60315410
	1.74576110	0.35741869
	4.53662029	0.03888791
	9.39507091	0.00053929

$$I = \int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}\, f\,(x)\, dx \approx \sum_{k=1}^{N} w_k\, f\,(x_k)$$

$x_k = \cos \frac{k-1/2}{N}\, \pi \;\; k = 1, 2, \ldots, N$

$$w_k = \frac{\pi}{N} \forall k$$

$$I = \int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}\, f\,(x)\, dx \approx \frac{\pi}{N} \sum_{k=1}^{N} f\,(x_k)$$

$f(x) \leq 2n - 1$

$$\vec{A} \cdot \vec{B} = |\vec{A}|\, |\vec{B}|\, \cos\theta$$

$$\vec{A} \cdot \vec{B} = \sum_{i=1}^{N} A_i\, B_i$$

$$\begin{aligned} \vec{A} \cdot \vec{B} = 0 \quad & \Rightarrow \cos\theta = 0 \\ & \Rightarrow \theta = \frac{\pi}{2} + n\,\pi\, ; n = 0, 1, \ldots \end{aligned}$$

$\vec{A}\vec{B}$

$$\sum_{i} f\left(x_{i}\right) g\left(x_{i}\right) \equiv \sum_{i} G\left(x\right) \rightarrow \int f\left(x\right) g\left(x\right) dx = 0$$

$\int f\left(x\right) g\left(x\right) dx = 0 \, f(x)g(x)$

P_x

$$\vec{V} = C_1 \, \vec{B}_1 + C_2 \, \vec{B}_2$$

$$\left.\begin{array}{l} \vec{B}_3 \cdot \vec{B}_1 = 0 \\ \vec{B}_3 \cdot \vec{B}_2 = 0 \end{array}\right\} \Rightarrow \vec{B}_3 \cdot \vec{V} = 0$$

$$\int_{-1}^{1} P_m(x)\,P_n(x)\,dx = \begin{cases} 0 & n \neq m \\ \dfrac{2}{2\,n+1} & m = n \end{cases}$$

$n-1 \Rightarrow n$

$\dfrac{\sin x}{x}$ $x=0$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1\,;\,\frac{\sin(0)}{0}$$

$+\infty-\infty$

$x^{1/2}$ $x=0$

$$\begin{aligned} \int_0^1 x^{-1/2}\,dx &= \lim_{h\to 0} \int_h^1 x^{-1/2}\,dx = \lim_{h\to 0} \left.\frac{x^{1/2}}{1/2}\right|_h^1 = \lim_{h\to 0}(2\sqrt{1} - 2\sqrt{h}) \\ &= 2\sqrt{1} = 2 \end{aligned}$$

$$I = \int_{-\infty}^{\infty} exp\left(-x^2\right) dx$$

$$I = \int_0^1 \frac{1}{\sqrt{x}\left(e^x + 1\right)} dx$$

- $x = 0 f(x) \to \infty x \to 0$

$$I = \int_0^1 x^{0.7}\ \cos\left(x\right) dx$$

- $x = 0$

$$I = \int_{-\infty}^{\infty} f\left(x\right) dx I = \int_{-\infty}^{b} f\left(x\right) dx I = \int_{a}^{\infty} f\left(x\right) dx$$

$$I = \int_{-\infty}^{\infty} e^{-x^2}\, dx$$

$f(x)[-\infty, \infty]$

$$I = h \sum_{i=-M}^{M} f\left(x_i\right)$$

$x_i = i * h M \infty M * h$

$$I = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-x^2}$$

$$I \approx \frac{1}{\sqrt{\pi}} \int_{-10}^{10} e^{-x^2}$$

$$N = 20 \rightarrow I = 1.000104$$

$$N = 40 \rightarrow I = 1.000001$$

$$N = 80 \rightarrow I = 1.000000$$

$I = 1.000000$

$$I = \int_{a}^{b} f\left(x\right) dx$$

$abf(x)ab$

$[a,b][+\infty,-\infty]$

$$\xi\xi\left(?\right) \begin{cases} \xi\left(a\right) = -\infty \\ \xi\left(b\right) = +\infty \end{cases}$$

$$x = x\left(\xi\right)$$

$$I = \int_{a}^{b} f\left(x\right) dx = \int_{-\infty}^{\infty} f\left(x\left(\xi\right)\right) \left(\frac{dx}{d\xi}\right) dx$$

$$x\left(\xi\right) = \frac{1}{2}\left[a + b + \left(b - a\right) tanh\left(\xi\right)\right]$$

$$tanh\left(\xi\right) = \frac{e^{\xi} - e^{-\xi}}{e^{\xi} + e^{-\xi}}$$

$$\xi = +\infty \Rightarrow tanh\left(+\infty\right) = 1 \Rightarrow x\left(+\infty\right) = b$$
$$\xi = -\infty \Rightarrow tanh\left(-\infty\right) = -1 \Rightarrow x\left(-\infty\right) = a$$

$$\xi\left(x\right)=tanh^{-1}\left(\frac{2\,x-a-b}{b-a}\right)\;,x\in\left[a,\,b\right]$$

$\xi=2.64665\Rightarrow tanh=0.99\Rightarrow x\approx b$

?

$$\xi\left(x\right)=\frac{1}{2}\,\left[a+b+\left(b-a\right)tanh\,\left(\frac{\pi}{2}\,sinh\left(\xi\right)\right)\right]$$

$$sinh\left(\xi\right)=\frac{e^{\xi}-e^{-\xi}}{2}$$

$$cosh\left(\xi\right)=\frac{e^{\xi}+e^{-\xi}}{2}$$

$$\frac{dx}{d\xi}=\frac{\left(b-a\right)\frac{\pi}{4}\,cosh(\xi)}{cosh^{2}\,\left[\frac{\pi}{2}\,sinh\left(\xi\right)\right]}$$

$\rightarrow$

$$I=h\sum_{k=-N}^{N}f\left(x_{k}\right)\,\left(\frac{dx}{d\xi}\right)_{k}$$

$$\xi_{k}=k*h$$

N

$\xi_{k}\Rightarrow cosh^{2}\left[\frac{\pi}{2}\,sinh\left(\xi_{k}\right)\right]\rightarrow\frac{1}{4}\,exp\,\left[\frac{\pi}{2}\,exp\left(\xi\right)\right]\frac{dx}{d\xi}$

$$\begin{aligned}&\frac{1}{4}\,exp\,\left[\frac{\pi}{2}\,exp\left(\xi_{k}\right)\right]\approx\underbrace{2\,\cdot\,10^{38}}\\ \Rightarrow\quad&\xi_{k}\approx4\Rightarrow N\,h<4\end{aligned}$$

$$\xi\approx6.1\Rightarrow f\left(\xi\right)\approx3.6\,\cdot\,10^{303}$$

$$I=\int_{0}^{2}\sqrt{1+\frac{1}{x}}\,dx\;\;\begin{array}{l}a=0\\ b=2\end{array}$$

$$x = 0 \; \sqrt{1+\frac{1}{x}}$$

$$\begin{aligned} x_k &= \frac{1}{2}\left[0+2+(2-0)\, tanh\, \left(\frac{\pi}{2}\, sinh\,(\xi)\right)\right] \\ x_k &= \frac{1}{2}\left[2+2\, tanh\, \left(\frac{\pi}{2}\, sinh\,(\xi_k)\right)\right] \\ &= 1+tanh\, \left(\frac{\pi}{2} sinh\,(\xi_k)\right) \end{aligned}$$

ξ

$$I = \int_a^b \left[\int_{c\,(x)}^{d\,(x)} \; f\,(x,\; y)\, dy\right]\; dx$$

$$G\,(x) = \int_{c\,(x)}^{d\,(x)} \; f\,(x,\; y)\, dy$$

$$I = \int_a^b \; G\,(x)\, dx$$

$$I = \sum_{i=0}^{N} \; w_i\, G\,(x_i)$$

$$G\,(x_i) = \sum_{j=0}^{M} \; w_j\, f\,(x_i,\; y_j)$$

$$I = \int_a^b \left[\int_{c\,(x)}^{d\,(x)} \; sin\,(x+y)\,dy \right] \, dx$$

$$\frac{1}{3}$$

$$\begin{aligned} & a = 1 \\ & b = 3 \\ & c\,(x) = ln\,(x) \\ & d\,(x) = 3 + e^{x/5} \end{aligned}$$

$$\begin{aligned} I \; & = \frac{H_x}{3} \; [G\,(x_0) + 4\,G\,(x_1) + G\,(x_2)] \\ & \approx \frac{H_x}{3} \left[\int_{ln\,(1)}^{3+e^{(1/5)}} sin\,(1+y)\,dy \; + \; 4 \int_{ln\,(2)}^{3+e^{(2/5)}} sin\,(2+y)\,dy \; + \int_{ln\,(3)}^{3+e^{(3/5)}} sin\,(3+y)\,dy \right] \\ & \approx \frac{1}{3} \left[\int_0^{4.2214} sin\,(1+y)\,dy \; + \; 4 \int_{0.6931}^{4.4918} sin\,(2+y)\,dy \; + \int_{1.0986}^{4.8221} sin\,(3+y)\,dy \right] \end{aligned}$$

$$\begin{aligned} G\,(x_0) \; & \approx \frac{2.11070}{3} \; [sin\,(1+0) + 4\,sin\,(1+2.11070) + sin\,(1+4.2214)] \\ & = 0.064581 \\ G\,(x_1) \; & \approx \frac{1.89935}{3} \; [sin\,(2+0.6931) + 4\,sin\,(2+2.59245) + sin\,(2+4.4918)] \\ & = -2.1086 \\ G\,(x_2) \; & \approx \frac{1.86175}{3} \; [sin\,(3+1.0986) + 4\,sin\,(3+2.96035) + sin\,(3+4.8221)] \\ & = -0.67454 \end{aligned}$$

$$I \approx \frac{1}{3} \; [0.064581 + (4)\,(-2.1086) - 0.67454] = -3.0148$$

$f(x)F(x)$

$$\underbrace{F'(x) = f(x)}_{\square}$$

$f(x) \geq 0 y = f(x)$

$f(x)F(x)F'(x) = f(x)Ff$

$f(x) \lim_{h \to 0} f(x+h) = f(x) \forall x$

$$\frac{f(x+h) - f(x)}{h}$$

$$\begin{aligned}
& \lim_{h \to 0} (f(x+h) - f(x)) \\
= \; & \lim_{h \to 0} h \, \frac{f(x+h) - f(x)}{h} \\
= \; & \lim_{h \to 0} h \, \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 0
\end{aligned}$$

$$\boxed{\lim_{h \to 0} f(x+h) = f(x)}$$

$$f\left(x_{i+1}\right)=f\left(x_i+h\right)=f\left(x_i\right)+f'(x_i)\,h+\frac{1}{2}\,f''(x_i)\,h^2+\frac{1}{6}\,f'''(x_i)\,h^3+\frac{1}{24}\,f^{(iv)}\,h^4\ \ldots$$

$$f\left(x_{i-1}\right)=f\left(x_i-h\right)=f\left(x_i\right)-f'(x_i)\,h+\frac{1}{2}\,f''(x_i)\,h^2-\frac{1}{6}\,f'''(x_i)\,h^3+\frac{1}{24}\,f^{(iv)}\,h^4+\ldots$$

$$f\left(x_{i+2}\right)=f\left(x_i+2\,h\right)=f\left(x_i\right)+f'(x_i)\,2\,h+\frac{1}{2}\,f''(x_i)\,4\,h^2+\frac{1}{6}\,f'''(x_i)\,8\,h^3+\frac{1}{24}\,f^{(iv)}\,16\,h^4+\ldots$$

$$f\left(x_{i-2}\right)=f\left(x_i-2\,h\right)=f\left(x_i\right)-f'(x_i)\,2\,h+\frac{1}{2}\,f''(x_i)\,4\,h^2-\frac{1}{6}\,f'''(x_i)\,8\,h^3+\frac{1}{24}\,f^{(iv)}\,16\,h^4+\ldots$$

$$f\left(x_{i+3}\right)=f\left(x_i+3\,h\right)=f\left(x_i\right)+f'(x_i)\,3\,h+\frac{1}{2}\,f''(x_i)\,9\,h^2+\frac{1}{6}\,f'''(x_i)\,27\,h^3+\frac{1}{24}\,f^{(iv)}\,81\,h^4+\ldots$$

$$f\left(x_{i-3}\right)=f\left(x_i-3\,h\right)=f\left(x_i\right)-f'(x_i)\,3\,h+\frac{1}{2}\,f''(x_i)\,9\,h^2+\frac{1}{4}\,f'''(x_i)\,27\,h^3+\frac{1}{24}\,f^{(iv)}\,81\,h^4+\ldots$$

$x_i x_{i+1}$

$$f'(x_i) = \frac{f\left(x_{i+1}\right) - f\left(x_i\right)}{h} - \frac{1}{2}\, f''(x_i)\, h - \frac{1}{6}\, f'''(x_i)\, h^2 - \ldots$$

$$f'(x_i) = \frac{f\left(x_{i+1}\right) - f\left(x_i\right)}{h} + O\left(h\right)$$

$$O\left(h\right) = -\frac{1}{2}\, f''(x_i)\, h$$

$x_i x_{i+1} x_{i+2}$

$$4\, f\left(x_{i+1}\right) - f\left(x_{i+2}\right) = 3\, f\left(x_i\right) + 2\, h\, f'(x_i) - \frac{2}{3}\, h^3\, f'''(x_i)$$

$$f'(x_i) = \frac{-f\left(x_{i+2}\right) + 4\, f\left(x_{i+1}\right) - 3\, f\left(x_i\right)}{2\, h} + O\left(h^2\right)$$

$$O\left(h^2\right) = \frac{1}{3}\, h^2\, f'''(x_i)$$

$x_i x_{i+1} x_{i+2} x_{i+3}$
$\frac{1}{2}\frac{1}{9}$

$$\begin{array}{rcrrrrrr}
f_{i+1} & = & f_i & +h\, f_i' & +\dfrac{h^2}{2}\, f_i'' & +\dfrac{h^3}{6}\, f_i''' & +\dfrac{h^4}{24}\, f_i'''' & +\ldots \\
-\dfrac{f_{i+2}}{2} & = & -\dfrac{f_i}{2} & -h\, f_i' & -\dfrac{2\, h^2}{2}\, f_i'' & -\dfrac{4\, h^3}{6}\, f_i''' & -\dfrac{8\, h^4}{24}\, f'''' & -\ldots \\
+\dfrac{f_{i+3}}{9} & = & \dfrac{f_i}{9} & +\dfrac{h}{3}\, f_i' & +\dfrac{h^2}{2}\, f_i'' & +\dfrac{3\, h^3}{6}\, f_i''' & +\dfrac{9\, h^4}{24}\, f'''' & +\ldots \\
\hline
\multicolumn{8}{c}{\dfrac{f_{i+3}}{9} - \dfrac{f_{i+2}}{2} + f_{i+1} = \dfrac{f_i}{9} - \dfrac{f_i}{2} + f_i + \dfrac{h}{3}\, f_i' + \dfrac{h^4}{12}\, f'''' + \ldots}
\end{array}$$

$$f_i' = \frac{2\, f_{i+3} - 9\, f_{i+2} + 18\, f_{i+1} - 11\, f_i}{6\, h} + O\left(h^3\right)$$

$$O\left(h^3\right) = -\frac{1}{4}\, h^3\, f_i''''$$

$x_i x_{i-1}$

$$f'(x_i) = \frac{f\,(x_i) - f\,(x_{i-1})}{h} + O\,(h)$$

$$O\,(h) = \frac{1}{2}\,f''(x_i)\,h$$

$x_i x_{i-1} x_{i-2}$

$$f'(x_i) = \frac{3\,f\,(x_i) - 4\,f\,(x_{i-1}) + f\,(x_{i-2})}{2\,h} + O\,(h^2)$$

$$O\,(h^2) = \frac{1}{3}\,h^2\,f'''(x_i)$$

$x_i x_{i-1} x_{i-2} x_{i-3}$

$$f'(x_i) = \frac{11\,f_i - 18\,f_{i-1} + 9\,f_{i-2} - 2\,f_{i-3}}{6\,h} + O\,(h^3)$$

$$O\,(h^3) = \frac{1}{4}\,h^3\,f_i''''$$

$x_{i+1} x_{i-1}$

$$f'(x_i) = \frac{f\,(x_{i+1}) - f\,(x_{i-1})}{2\,h} + O\,(h^2)$$

$$O\,(h^2) = -\frac{1}{6}\,h^2\,f'''(x_i)$$

$x_{i+2} x_{i+1} x_{i-1} x_{i-2} f'' f'''$

$$f_i' = \frac{-f_{i+2} + 8\,f_{i+1} - 8\,f_{i-1} + f_{i-2}}{12\,h} + O\,(h^2)$$

$$O\,(h^2) = \frac{1}{30}\,h^4\,f^{\,(iv)}(x_i)$$

$x_i x_{i+1} x_{i+2} f'(x_i)$

$$f_i'' = \frac{f_{i+2} - 2\,f_{i+1} + f_i}{h^2} + O\,(h)$$

$$O\,(h) = -h\,f_i'''$$

$$f_i'' = \frac{f_i - 2\,f_{i+1} + f_{i-2}}{h^2} + O\,(h)$$

$$O\,(h) = h\,f_i'''$$

$$f_i'' = \frac{f_{i+1} - 2\,f_i + f_{i-1}}{h^2} + O\,(h^2)$$

$$O\,(h^2) = -\frac{1}{2}\,h^2\,f_i''''$$

- $f^{(p)} p + 1$
- p
-

$$\Delta f_i = f_{i+1} - f_i$$
$$\nabla f_i = f_i - f_{i-1}$$

$$\delta f_i = f_{i+\frac{1}{2}} - f_{i-\frac{1}{2}}$$
$$\delta f_{i+\frac{1}{2}} = f_{i+1} - f_i$$

$$f_{i+\frac{1}{2}} = f\,\left(x_i + \frac{h}{2}\right)$$

$$\begin{aligned}\Delta^2 f_i &= \Delta\left(\Delta f_i\right) = \Delta\left(f_{i+1} - f_i\right) = \Delta f_{i+1} - \Delta f_i \\ &= f_{i+2} - f_{i+1} - \left(f_{i+1} - f_i\right) = f_{i+2} - 2\, f_{i+1} + f_i\end{aligned}$$

$$\boxed{\Delta^2 f_i = f_{i+2} - 2\, f_{i+1} + f_i}$$

$$\begin{aligned}\nabla^2 f_i &= \nabla\left(\nabla f_i\right) = \nabla\left(f_i - f_{i-1}\right) = \nabla f_i - \nabla f_{i-1} \\ &= f_i - f_{i-1} - \left(f_{i-1} - f_{i-2}\right) = f_i - 2\, f_{i-1} + f_{i-2}\end{aligned}$$

$$\boxed{\nabla^2 f_i = f_i - 2\, f_{i-1} + f_{i-2}}$$

$$\begin{aligned}\delta^2 f_i &= \delta\left(\delta f_i\right) = \delta\left(f_{i+\frac{1}{2}} - f_{i-\frac{1}{2}}\right) = \delta f_{i+\frac{1}{2}} - \delta f_{i-\frac{1}{2}} \\ &= f_{i+1} - f_i - \left(f_i - f_{i-1}\right) = f_{i+1} - 2\, f_i + f_{i-1}\end{aligned}$$

$$\boxed{\delta^2 f_i = f_{i+1} - 2\, f_i + f_{i-1}}$$

$$\begin{aligned}\Delta\nabla f_i &= \Delta\left(\nabla f_i\right) = \Delta\left(f_i - f_{i-1}\right) = \Delta f_i - \Delta f_{i-1} \\ &= f_{i+1} - f_i - f_i + f_{i-1} = f_{i+1} - 2\, f_i + f_{i-1}\end{aligned}$$

$$\boxed{\Delta\nabla f_i = \delta^2\, f_i = f_{i+1} - 2\, f_i + f_{i-1}}$$

$$\begin{aligned}\nabla\Delta f_i &= \nabla\left(\Delta f_i\right) = \nabla\left(f_{i+1} - f_i\right) = \nabla f_{i+1} - \nabla f_i \\ &= f_{i+1} - f_i - f_i + f_{i+1} = f_{i+1} - 2\, f_i + f_{i-1}\end{aligned}$$

$$\boxed{\nabla\Delta f_i = \delta^2\, f_i = f_{i+1} - 2\, f_i + f_{i-1}}$$

$\delta^2 = \Delta\nabla = \nabla\Delta$

$nm = \dfrac{n}{2}$

$$\nabla^m \Delta^m = \delta^n$$

$$\begin{array}{l|l} & \dfrac{d^2}{dx^2} \approx \dfrac{\Delta^2}{\Delta x^2} \\ \dfrac{d}{dx} \approx \dfrac{\Delta}{\Delta x} & \dfrac{d^2}{dx^2} \approx \dfrac{\nabla^2}{\nabla x^2} \\ \dfrac{d}{dx} \approx \dfrac{\nabla}{\nabla x} & \dfrac{d^2}{dx^2} \approx \dfrac{\nabla}{\nabla x}\left(\dfrac{\Delta}{\Delta x}\right) = \dfrac{\Delta}{\Delta x}\left(\dfrac{\nabla}{\nabla x}\right) \\ \dfrac{d}{dx} \approx \dfrac{\delta}{\delta x} & \dfrac{d^2}{dx^2} \approx \dfrac{\delta^2}{\delta x^2} \end{array}$$

$$g\,(x) = g\,(x_k + s\,h) = \sum_{n=0}^{N} \binom{s}{n}\;\Delta^n\,f_k$$

$$s = \frac{x - x_k}{h}$$

$$\binom{s}{n} = \frac{s!}{n!\,(s-n)!}$$

$$\begin{aligned} g\,(x) &= g\,(x_k + s\,h) \\ &= f_k + s\,\Delta\,f_k + \frac{1}{2}\,s\,(s-1)\,\Delta^2\,f_k + \frac{1}{6}\,s\,(s-1)\,(s-2)\,\Delta^3\,f_k \\ &+ \frac{1}{24}\,s\,(s-1)\,(s-2)\,(s-3)\,\Delta^4\,f_k + \ldots + \binom{s}{n}\;\Delta^N\,f_k \end{aligned}$$

$g\,(x) = g\,(x_k + s\,h) = \sum_{n=0}^{N} \binom{s}{n}\;\Delta^n\,f_k\,nn+1$

$n = 2 \Rightarrow$

$$g\,(x) = f_k + s\,\Delta\,f_k + \frac{1}{2}\,s\,(s-1)\,\Delta^2\,f_k$$

$$g'\,(x) = \frac{1}{h}\,\left[\Delta\,f_k + \frac{1}{2}\,(2\,s-1)\,\Delta^2\,f_k\right]$$

$s = 012$

$$g'\,(x_k) = \frac{1}{2\,h}\,\left[2\,\Delta\,f_k - \Delta^2\,f_k\right] = \frac{1}{2\,h}\,\left[-f_{k+2} + 4\,f_{k+1} - 3\,f_k\right]$$

$$g'\,(x_{k+1}) = \frac{1}{2\,h}\,\left[2\,\Delta\,f_k + \Delta^2\,f_k\right] = \frac{1}{2\,h}\,\left[f_{k+2} - f_k\right]$$

$$g'\left(x_{k+2}\right)=\frac{1}{2\,h}\,\left[2\,\Delta\,f_k+3\,\Delta^2\,f_k\right]=\frac{1}{2\,h}\,\left[3\,f_{k+2}-4\,f_{k+1}+f_k\right]$$

$k=i$

$$g'\left(x_i\right)=\frac{1}{2\,h}\,\left[2\,\Delta\,f_i-\Delta^2\,f_i\right]=\frac{1}{2\,h}\,\left[-f_{i+2}+4\,f_{i+1}-3\,f_i\right]$$

$k+1=i$

$$g'\left(x_i\right)=\frac{1}{2\,h}\,\left[2\,\Delta\,f_{i-1}+\Delta^2\,f_{i-1}\right]=\frac{1}{2\,h}\,\left[f_{i+1}-f_{i-1}\right]$$

$k+2=i$

$$g'\left(x_i\right)=\frac{1}{2\,h}\,\left[2\,\Delta\,f_{i-2}+3\,\Delta^2\,f_{i-2}\right]=\frac{1}{2\,h}\,\left[3\,f_i-4\,f_{i-1}+f_{i-2}\right]$$

$n=2n=3$

$$\frac{1}{6}\,s\,(s-1)\,(s-2)\,\Delta^3 f_k$$

$$\frac{1}{6\,h}\,\left[3\,s^2-6\,s+2\right]\,\Delta^3\,f_k$$

$$s=0\rightarrow\frac{1}{3\,h}\,\Delta^3 f_k Forward$$

$$s=1\rightarrow\frac{1}{6\,h}\,\Delta^3 f_k Centrada$$

$$s=2\rightarrow\frac{1}{3\,h}\,\Delta^3 f_k Backward$$

$g(x)n$

$$\frac{d^n}{dx^n}\,g\,(x)=\frac{1}{h^n}\,\Delta^n\,f_i$$

$$\Delta^n f_i \approx h^n f^{(n)}(x)$$

$$\frac{1}{3} h^2 f_k''' , s = 0$$

$$-\frac{1}{6} h^2 f_k''' , s = 1$$

$$\frac{1}{3} h^2 f_k''' , s = 2$$

$$f_{,x} = \frac{\partial f\,(x,\, y)}{\partial x}(x, y) = (x_0, y_0)$$

$y = y_0$ $f(x, y_0)$ $f_{,x}$

$$f_{,x} \approx \frac{f\,(x_0 + \Delta x,\, y_0) - f\,(x_0,\, y_0)}{\Delta x} Forward$$

$$f_{,x} \approx \frac{f\,(x_0 + \Delta x,\, y_0) - f\,(x_0 - \Delta x,\, y_0)}{2\,\Delta x} Central$$

$$f_{,x} \approx \frac{f\,(x_0,\, y_0) - f\,(x_0 - \Delta x,\, y_0)}{\Delta x} Backward$$

$f_{,xx} f_{,yy} f_{,xy}$

$$f_{,xx} = \frac{\partial^2}{\partial x^2} f \approx \frac{f\,(x_0 + \Delta x,\, y_0) - 2\,f\,(x_0,\, y_0) + f\,(x_0 - \Delta x,\, y_0)}{\Delta x^2}$$

$$f_{,yy} = \frac{\partial^2}{\partial y^2} f \approx \frac{f\,(x_0,\, y_0 + \Delta y) - 2\,f\,(x_0,\, y_0) + f\,(x_0,\, y_0 - \Delta y)}{\Delta y^2}$$

$$\begin{aligned} f_{,xy} = \frac{\partial^2}{\partial x\,\partial y} f \quad &\approx \frac{f\,(x_0 + \Delta x,\, y_0 + \Delta y) - f\,(x_0 + \Delta x,\, y_0 - \Delta y)}{4\,\Delta x\,\Delta y} \\ &+ \frac{-f\,(x_0 - \Delta x,\, y_0 + \Delta y) + f\,(x_0 - \Delta x,\, y_0 - \Delta y)}{?} \end{aligned}$$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0\ x\in(0,\,1)\ y\in(0,\,1)$$

$$u=0\begin{cases} y=1\\ x=0\\ x=1\end{cases}$$

$$u\,(x,\,0)=sin\,(\pi\,x)x\in[0,\,1]$$

$(i,\,j)$

$$\frac{\partial^2 u}{\partial x^2}=\frac{1}{h_x^2}\,(u_{i,j+1}-2\,u_{i,j}+u_{i,j-1})$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{1}{h_y^2}\,(u_{i+1,j}-2\,u_{i,j}+u_{i-1,j})$$

$h_x=h_y=h$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\approx\frac{1}{h^2}\,(u_{i+1,j}+u_{i-1,j}+u_{i,j+1}+u_{i,j-1}-4\,u_{i,j})=0$$

$$\begin{array}{ccc} & \boxed{+1} & \\ \boxed{+1} & \boxed{-4} & \boxed{+1} \\ & \boxed{+1} & \end{array}$$

$$\begin{array}{r} P1: \\ P2: \\ P3: \\ P4: \\ P5: \\ P6: \\ P7: \\ P8: \\ P9: \end{array} \begin{bmatrix} -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -4 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{Bmatrix} + \begin{Bmatrix} sin\left(\frac{\pi}{4}\right) \\ sin\left(\frac{\pi}{2}\right) \\ sin\left(\frac{3\pi}{4}\right) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \{0\}$$