

Short-Vector Cooley-Tukey DCT Rules

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$$\begin{aligned}
 \text{DCT-}4_{2mn} &\rightarrow \underbrace{(\text{I}_n \oplus \text{J}_n \oplus \dots)}_{m \text{ times}} \\
 &\quad ((M_m^{2m}(\text{I}_m \oplus (-\text{I}_m)) \text{PRDFT-}3_{2m,-1}^\top) \otimes \text{I}_n) \\
 &\quad L_{2m}^{2mn} \overline{D_{m,n}^{2mn}} L_n^{2mn} \\
 &\quad (\text{I}_m \otimes (L_2^{2n} \text{PRDFT-}3_{2n}(\text{I}_n \oplus (-\text{I}_n)) K_2^{2n})) L_{2n}^{2mn} \\
 &\quad \underbrace{(\text{I}_m \oplus \text{J}_m \oplus \dots)}_{n \text{ times}} \\
 \text{DCT-}2_{2mn} &\rightarrow (2\text{I}_1 \oplus \text{I}_{2mn-1}) S_{2mn}^\top \text{DCT-}4_{2mn} E_{2mn} \\
 \text{DCT-}3_{2mn} &\rightarrow \text{DCT-}2_{2mn}^\top \\
 D_{m,n}^{2mn} &= \text{diag}_{0 \leq i < 2mn} \left(\omega_{16mn}^{(2\lfloor \frac{i}{2m} \rfloor + 1)(2(i \bmod m) + 1)} \right) \\
 E_n &= \text{diag}_{0 \leq i < n} \left(\frac{1}{2 \cos \frac{(2i+1)\pi}{4n}} \right)
 \end{aligned}$$