1.c) Confirm that the type of (3) is the same as the type of (1)

$$\nabla^2 \phi = 0$$
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

Comparing with a general linear PDE:

$$\left(A\partial_x^2 + B\partial_{\mathrm{xy}} + C\partial_y^2\right)u(x, y) = 0$$

and the factor

$$s = -\frac{B}{2A} \pm \frac{1}{2A}\sqrt{B^2 - 4\mathrm{AC}}$$

$$A = 1,\ \ B = 0,\ \ C = 1$$

and

$s = 0 \pm \frac{1}{2}\sqrt{-4} = \pm\sqrt{-1}$, a complex value.

The factors of the discriminate:

$$B^2 - 4\mathrm{AC} = -4 \quad < 0$$

According to (1.24) of the script the type of (3) is **elliptic**, which is the same as of (1).

1.d) Confirm that the boundary conditions are equivalent to (1c,1d)

$\frac{\partial \phi}{\partial r} = 0$ at Γ_C:

$$x : \frac{\partial \phi}{\partial r} = \frac{\partial \phi}{\partial x}\frac{\partial x}{\partial r} = u\frac{\partial}{\partial r}r\cos(\Theta) = u\cos(\Theta)$$

$$y : \frac{\partial \phi}{\partial r} = \frac{\partial \phi}{\partial y}\frac{\partial y}{\partial r} = v\frac{\partial}{\partial r}r\sin(\Theta) = v\ \sin(\Theta)$$

This is equivalent to $\vec{u} \cdot \vec{n}$

$\phi = x \ \text{ at } \Gamma_0 :$

means $\frac{\partial \phi}{\partial x} = \frac{\partial x}{\partial x} = 1$

with $\vec{u} = \nabla\phi$

leads to $\vec{u} = u(1, 0)$

$\frac{\partial \phi}{\partial \Theta} = 0$ at Γ_F:

$$x : \frac{\partial \phi}{\partial \Theta} = \frac{\partial \phi}{\partial x}\frac{\partial x}{\partial \Theta} = u\frac{\partial}{\partial \Theta} r\cos(\Theta) = u\, r\,(-\sin(\Theta))$$

$$y : \frac{\partial \phi}{\partial \Theta} = \frac{\partial \phi}{\partial y}\frac{\partial y}{\partial \Theta} = v\frac{\partial}{\partial \Theta} r\sin(\Theta) = v\;\, r\cos(\Theta)$$

This is equivalent to $\vec{u} \cdot \vec{n}$