

2.a) Find the Jacobian of the coordinate transformation

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syms theta r x y
J=[-r*sin(theta), r*cos(theta)
    cos(theta), sin(theta)]
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J =

$$\begin{pmatrix} -r \sin(\theta) & r \cos(\theta) \\ \cos(\theta) & \sin(\theta) \end{pmatrix}$$

2.a) Express the inverse transformation of partial derivatives

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J_theta_r=inv(J)
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J_theta_r =

$$\begin{pmatrix} -\frac{\sin(\theta)}{r \cos(\theta)^2 + r \sin(\theta)^2} & \frac{\cos(\theta)}{\cos(\theta)^2 + \sin(\theta)^2} \\ \frac{\cos(\theta)}{r \cos(\theta)^2 + r \sin(\theta)^2} & \frac{\sin(\theta)}{\cos(\theta)^2 + \sin(\theta)^2} \end{pmatrix}$$

2.c) Check that the inverse transformation (7) satisfies the chain rule

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d_dx_1=diff(J_theta_r(1,1),theta)+diff(J_theta_r(1,2),r)
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d_dx_1 =

$$-\frac{\cos(\theta)}{r \cos(\theta)^2 + r \sin(\theta)^2}$$

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d_dy_1=diff(J_theta_r(2,1),theta)+diff(J_theta_r(2,2),r)
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d_dy_1 =

$$-\frac{\sin(\theta)}{r \cos(\theta)^2 + r \sin(\theta)^2}$$

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theta_1=atan(y/x);
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d_dx_2=diff(theta_1,x)
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d_dx_2 =

$$-\frac{y}{x^2 \left(\frac{y^2}{x^2} + 1 \right)}$$