Math Exercises
NMRC
2025

Note: The exercises denoted with $\ast$ are slightly more challenging.

1) Plot the following groups of functions in the same graph:

$$a) \quad y = x\,; \quad y = 2x\,; \quad y = 4x \tag{1}$$

$$b) \quad y = \sqrt{x}\,; \quad y = x\,; \quad y = x^2 \tag{2}$$

$$c) \quad y = e^{\frac{1}{2}x}\,; \quad y = e^{x}\,; \quad y = e^{2x} \tag{3}$$

2) Compute the following derivatives

$$\frac{d}{dx}3x^{x} \tag{4}$$

$$f'(z),\ \text{where } f(z) = 2e^{z} + \frac{1}{z^2} \tag{5}$$

$$\frac{d}{d\varphi}e^{4\varphi^2} \tag{6}$$

$$\ast \quad \frac{d}{dx}\left(\cos^2(x)\right) \tag{7}$$

$$\ast \quad \frac{d}{dx}\left(\frac{x}{1+e^{x^2}}\right) \tag{8}$$

3) $\ast$ Let $s(x) = \frac{1}{1+e^{-x}}$, i.e. the sigmoid function. Compute the derivative and show that $s'(x) = s(x)(1 - s(x))$ (note: this requires a bit of rewriting, consider if it might be useful to add $+1-1$). Finally, draw $s(x)$.

4) $\ast$ Consider a neuron modelled with a logistic function $f(x) = \frac{1}{1+e^{-kx}}$, with $k = 2$. Suppose you want this neuron to discriminate more strongly between negative and positive inputs: how would you change k to obtain this behavior?

5 $\ast$ For the hyperbolic tangent function $\tanh(x) = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$ compute the derivative and show that $\frac{d}{dx}\tanh(x) = 1 - \tanh^2(x)$

6) For the following functions, compute $\frac{\partial}{\partial x}f(x,y)$, $\frac{\partial}{\partial y}f(x,y)$, and (when applicable) $\frac{\partial}{\partial z}f(x,y)$.

$$f(x,y) = x^3 + 3xy + 4y^2 + x^2y \tag{9}$$
$$f(x,y,z) = x^2 + 3xyz \tag{10}$$
$$f(x,y) = x^y \tag{11}$$

7) Compute the gradient of the following functions:

$$f(x,y) = x^2y \tag{13}$$
$$f(x,y,z) = x^2y\cos z \tag{14}$$
$$f(x,y,z,t) = x^2yt^{-1}\cos z \tag{15}$$

8) Given the two following vectors $\mathbf{v}$ and $\mathbf{u}$, compute their dot product $\mathbf{v}\cdot\mathbf{u}$.

$$\mathbf{v} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \qquad \mathbf{u} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \tag{17}$$

9) Consider the two matrices A and B:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} \tag{18}$$

Compute the following:

$$A + B \tag{19}$$
$$A + 2B \tag{20}$$
$$A^T - 2B \tag{21}$$