

# Introduction to Splines

Franz Webersberger

November 28, 2014

## B-splines

**Definition 1.** Let  $\tau_1 \leq \dots \leq \tau_n$  be an arbitrary sequence of nodes. Then the B-splines  $N_{i,k}(t)$  of order  $k = 1, \dots, n$  and  $i = 1, \dots, n - k$  are recursively defined by

$$N_{i,1}(t) := \begin{cases} 1 & \text{if } \tau_i \leq t < \tau_{i+1} \\ 0 & \text{else} \end{cases} \quad (1a)$$

$$N_{i,k}(t) := \frac{t - \tau_i}{\tau_{i+k-1} - \tau_i} N_{i,k-1}(t) + \frac{\tau_{i+k} - t}{\tau_{i+k} - \tau_{i+1}} N_{i+1,k-1}(t) \quad (1b)$$

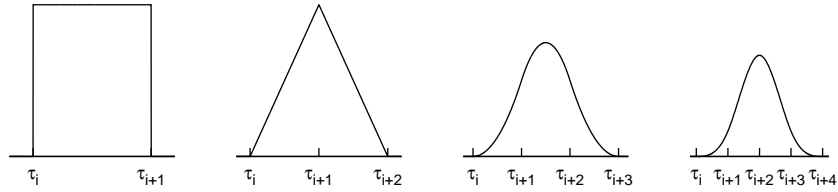


Figure 1: B-splines of order  $k = 1, 2, 3, 4$

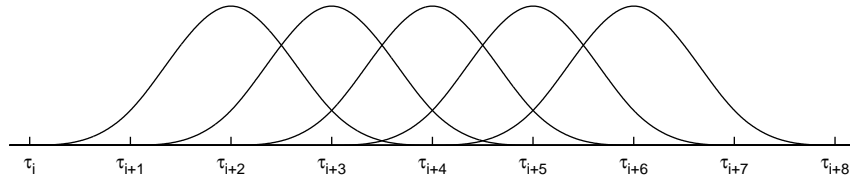


Figure 2: Uniform qubic ( $k = 4$ ) B-splines

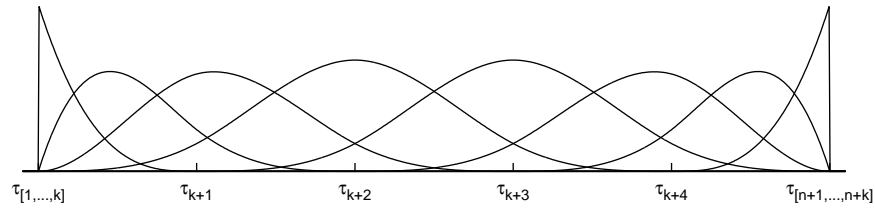


Figure 3: Qubic B-splines with boundaries