Introduction to Splines

Franz Webersberger November 28, 2014

B-splines

Definition 1. Let $\tau_1 \leq \cdots \leq \tau_n$ be an arbitrary sequence of nodes. Then the B-splines $N_{i,k}(t)$ of order $k=1,\ldots,n$ and $i=1,\ldots,n-k$ are recursively defined by

$$N_{i,1}(t) := \begin{cases} 1 & \text{if } \tau_i \le t < \tau_{i+1} \\ 0 & \text{else} \end{cases}$$
 (1a)

$$N_{i,k}(t) := \frac{t - \tau_i}{\tau_{i+k-1} - \tau_i} N_{i,k-1}(t) + \frac{\tau_{i+k} - t}{\tau_{i+k} - \tau_{i+1}} N_{i+1,k-1}(t)$$
 (1b)

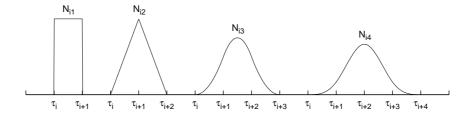


Figure 1: B-splines of order k = 1, 2, 3, 4

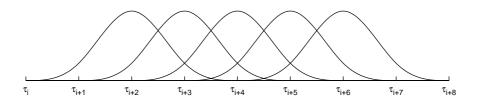


Figure 2: Uniform qubic (k = 4) B-splines

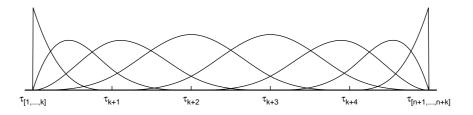


Figure 3: Qubic B-splines with boundaries