Introduction to Splines

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B-splines

Definition 1. Let $\tau_1 \leq \cdots \leq \tau_n$ be an arbitrary sequence of nodes. Then the B-splines $N_{i,k}(t)$ of order $k = 1, \ldots, n$ and $i = 1, \ldots, n - k$ are recursively defined by

$$N_{i,1}(t) := \begin{cases} 1 & \text{if } \tau_i \leq t < \tau_{i+1} \\ 0 & \text{else} \end{cases} \tag{1a}$$

$$N_{i,k}(t) := \frac{t - \tau_i}{\tau_{i+k-1} - \tau_i} N_{i,k-1}(t) + \frac{\tau_{i+k} - t}{\tau_{i+k} - \tau_{i+1}} N_{i+1,k-1}(t) \tag{1b}$$
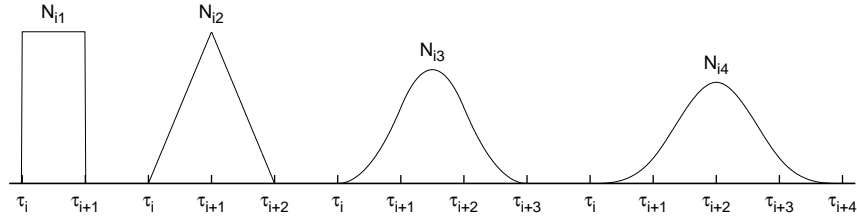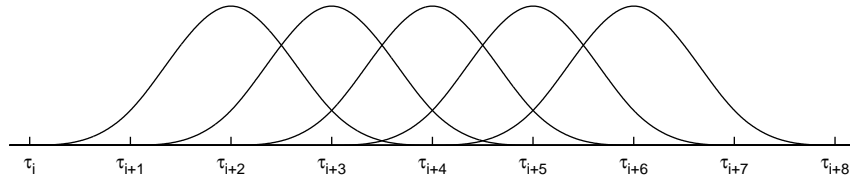

Figure 1: B-splines of order $k = 1, 2, 3, 4$
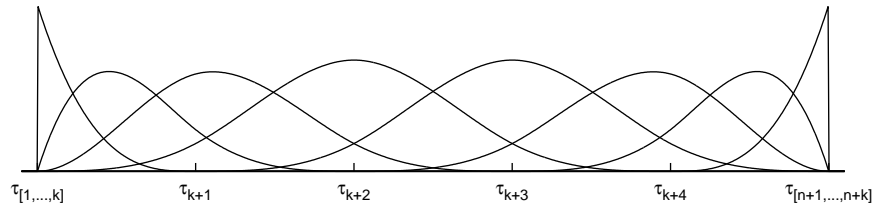

Figure 2: Uniform qubic $(k = 4)$ B-splines

Figure 3: Qubic B-splines with boundaries

2