

Horizon Duality: Black Holes as Asymptotic Shells of a Fractal Cosmos (*Draft*)

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Abstract

We present a unified framework for black hole horizons and cosmic horizons based on a soliton–noise model of fundamental matter. In this picture, the black hole event horizon is not a mathematical coordinate singularity but a physical, dynamic phase transition: an “asymptotic shell” where matter condenses to the vacuum’s minimum scale length. We derive the thermodynamics of this shell, finding a temperature scaling $T \propto M^{-1/2}$ that implies a constant-power evaporation law ($P \approx \text{const}$), contrasting with the standard Hawking result. We further demonstrate a rigorous duality between the interior of such a shell and an expanding Friedmann–Robertson–Walker (FRW) universe. By matching clocks across the horizon, we show that the parent black hole’s finite evaporation time maps to a semi-infinite cosmological history for the interior, characterized by an effective phantom equation of state ($w < -1$) and an asymptotic approach to a Planck-scale heat death. This framework resolves the information paradox through scale-dependent bookkeeping and suggests an eternal, fractal cosmology of nested universes.

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1 Introduction

The similarity between the black hole event horizon and the cosmic particle horizon has long hinted at a deeper connection. Both are null trapping surfaces that limit causal access; both are associated with thermodynamic entropies and temperatures. However, standard General Relativity (GR) treats them as distinct kinematic entities. Furthermore, the standard black hole model suffers from the information loss paradox and the singularity problem, while standard cosmology faces challenges regarding the initial state (Big Bang) and the nature of dark energy.

This paper proposes that these are not separate problems. We introduce a *Soliton-Noise Framework* in which matter particles are stable wave-packets (solitons) interacting with a background noise field. In this view, gravity is an emergent thermodynamic force driving solitons toward regions of higher noise variance.

The central hypothesis of this work is **Horizon Duality**:

1. **Black Hole Horizons** are local regions where an intense spatial noise gradient drives matter to undergo a phase transition, collapsing to the minimum scale allowed by the vacuum (the Planck scale) and forming a physical shell.
2. **Cosmic Horizons** are the manifestation of the same phase transition viewed from the inside (or globally), where the universal evolution of the noise field creates an apparent recession of the vacuum structure.

We show that this model naturally regularizes the singularity (replacing it with a black-body cavity), predicts a falsifiable deviation in Hawking radiation (constant power), and provides a microphysical origin for the “phantom” dark energy ($w < -1$) required to reconcile the parent and child clocks.

1.1 Key Definitions & Notation

Symbol	Meaning
$\Lambda_{\text{obs}}, \Lambda_{\text{IR/UV}}$	Observer’s reference scale and IR/UV cutoffs for the <i>scale window</i> .
L_{min}	The window-adapted minimum cell; the effective Planck length.
$N_* \approx \alpha$	Critical occupancy for the quantum-to-gravitational phase transition.
c_s	Phase-wave speed (speed of light).
$\tau(x)$	Local noise proxy, related to noise density by $\tau^2 \propto \rho_N$.
K_{drag}	Radiative drag coefficient setting the speed-limit asymptote.

Consistency note (cross-document symbols). We standardize on c_s for the universal phase-wave/light-cone speed (any bare c should be read as c_s), and we relate the local proxy $\tau(x)$ to the cosmological noise density by $\tau^2 \propto \rho_N$ in weak-field regimes. We reserve $(\alpha_{\text{grad}}, \beta_{\text{pot}})$ for amplitude-sector parameters and $(\alpha_{\text{dim}}, \beta_{\text{dim}})$ for dimensional-bias parameters to avoid symbol overloading.

2 Foundations: The Soliton–Noise Mechanism

We summarize the core postulates derived in companion papers to establish the physical baseline.

2.1 Matter as Solitons

Fundamental particles are not point-like excitations but stable, localized wave packets (solitons) of a complex scalar order parameter $\Psi = w e^{i\phi}$. These solitons are sustained by a Mexican-hat self-interaction potential $V(w)$ that balances dispersive spreading. Crucially, the soliton’s stability depends on the local noise level: to minimize its free energy, it adjusts its characteristic size L_c and internal frequency ω_c to match the local vacuum texture.

2.2 The Noise Field and Emergent Gravity

Solitons source and interact with a background stochastic field ρ_N (the “noise”). This field acts as a refractive medium with index $\chi \approx 1 + \alpha\rho_N$.

- **Scale Adaptation:** A soliton adapts its characteristic size to the local noise intensity: $L_c \propto 1/\chi$.
- **Emergent Gravity:** Gradients in the noise field create gradients in the refractive index. Solitons drift toward regions of higher ρ_N (higher χ , smaller L_c) to lower their internal energy. This thermodynamic drift is identified as the gravitational acceleration $\mathbf{a}_{\text{grav}} \propto -\nabla\rho_N$.
- **Scale Invariance:** The physics is conformally invariant. A local observer’s rulers and clocks, being made of the same solitonic substance, co-scale with the noise field. All locally measured constants (e.g. c_s) remain invariant.

2.3 Thermodynamic Drag and the Planck Floor

An accelerating soliton experiences a Doppler-shifted noise spectrum (an Unruh-like effect). This interaction dissipates kinetic energy into the vacuum, imposing a resistance to acceleration:

$$P_{\text{diss}} \propto \gamma^4 a^2. \quad (1)$$

Balancing mechanical work against this dissipation yields ultra-relativistic suppression $a(v) \propto \gamma^{-1}$ under sustained forcing. Under extreme acceleration—such as infall into a black hole—the drag dominates. The soliton sheds internal energy by shrinking. However, it cannot shrink indefinitely: it hits the **Window Floor** (L_{min}), the effective Planck length defined by the observer’s coarse-graining scale. This floor is not an absolute limit of the substrate but of any particular observer’s descriptive capacity.

3 The Black Hole as an Asymptotic Shell

In this framework, a black hole is not a vacuum solution with a central singularity. It is a macroscopic object defined by a phase transition at the horizon.

3.1 Formation: The “Snowplow” Mechanism

The formation of the shell is a runaway phase transition driven by the noise gradient.

1. **Infall and Acceleration:** Matter falls down the noise gradient $-\nabla\rho_N$, accelerating toward c_s .
2. **Drag and Scale Collapse:** The extreme acceleration invokes thermodynamic drag (1). The soliton sheds energy by shrinking ($L_c \rightarrow L_{\text{min}}$).
3. **Decoupling:** As the soliton’s scale collapses to the Planck floor, its cross-section for interacting with the macroscopic gradient vanishes. It becomes “blind” to the force that created it.
4. **Transition to Constituent:** The soliton is now a maximally downshifted particle moving at $v \approx c_s$. From the external perspective, the effective radial velocity asymptotically vanishes at the horizon while internal time continues.

The Snowplow Effect. Since constituents decouple from the macroscopic gradient, they pile up at the transition surface. The horizon acts as a “snowplow,” sweeping infalling matter into a finite-thickness shell Σ rather than letting it fall to $r = 0$.

What about matter already inside R_S during collapse? During dynamical collapse there is, of course, a period when ordinary solitonic matter occupies radii that later lie inside the null trapping surface. The shell model does not claim that no worldline ever crosses $r < R_S$; it claims that, *after trapping forms and coarse-graining is applied*, the only long-lived, externally accountable mass-energy resides in a finite-thickness interface layer. Two mechanisms enforce this:

- **Phase separation:** A supercritical interior ($\tau_{\text{eff}} \gtrsim \tau_{\text{max}}$) is not a hospitable phase for amplitude solitons. The region inside the evolving surface $\Sigma(t)$ rapidly relaxes to an optically thick, radiation-dominated bath. Any remaining amplitude-soliton content is either thermalised or swept into the interface—exactly as quasiparticles accumulate at

a moving phase boundary in a first-order transition. A minimal coarse model treats the noise proxy as an order parameter:

$$F[\tau] \sim \int d^3x \left[\frac{\kappa_\tau}{2} |\nabla \tau|^2 + V(\tau; \rho_m) \right], \quad (2)$$

with relaxational dynamics $\partial_t \tau \propto -\delta F / \delta \tau$. Increasing ρ_m during collapse tilts V so that the interior prefers a higher- τ state; $\Sigma(t)$ is the associated moving front.

- **Trapping bottleneck:** Once the refractive/metric trapping condition is reached ($\theta_{\text{out}} \rightarrow 0$), outward transport becomes exponentially inefficient in external time. This pins the accumulating interface near R_S , yielding a stable shell/photosphere.

Geometric horizon vs. thermodynamic onset. These are adjacent but distinct:

- **Geometric:** The event horizon is the null trapping surface ($\theta_{\text{out}} = 0$). It governs photon escape and is purely kinematic.
- **Thermodynamic:** The Unruh-like heating raises the effective noise across the shell, driving irreversible downshifting over a finite microphysical band. For massless excitations, only the geometric trapping applies; for massive solitons, both effects operate.

3.2 Mean-Field Nuance: Screened Links vs. the Exterior Monopole

An immediate worry is whether strictly local, screened interactions can reproduce a clean exterior $1/r$ gravitational tail for distant observers. The answer is yes: each Planck-scale constituent emits only a short-range Yukawa halo $\sim e^{-r/R_i}/r$, but because $R_i \ll R_{\text{shell}}$, the exponential factor is effectively unity in the far field ($r \gg R_{\text{shell}}$). Summing over N constituents therefore reproduces a monopolar $1/r$ tail with total mass $M = \sum_i E_i/c_s^2$, while inside the shell the far-side contributions are exponentially suppressed and gradients cancel.

Same mass does not mean same noise profile. The above statement is about *integrated source strength* (the monopole fixing the $1/r$ tail), not about the detailed spectrum/correlation structure of the emitted noise. A maximally downshifted shell is “window-limited”: most microscopic fluctuations lie at or below the external UV edge and therefore renormalise into effective couplings plus a monopole. Ordinary dense matter retains resolvable microstructure and can carry multipoles and longer correlations. See SI Sec. ?? for a more systematic discussion of what observables probe beyond the scalar intensity $\tau^2(x)$.

3.3 Equilibrium: The “Breathing” Shell

The shell is a self-contained, dynamic system in a remarkable equilibrium.

The Energy Balance. The two key thermodynamic processes are:

- **Outward Cooling (P_{out}):** The shell constituents form a near-2D, strongly coupled network. Even when geometrically dilute ($d \gg L_{\text{min}}$), its collective vibration modes radiate energy both inward (creating the blackbody core) and outward (as Hawking-like radiation). The outward component escapes because the radiating photosphere sits (on average) just outside the geometric trapping surface; as the shell “breathes,” its optical depth fluctuates, modulating the escape fraction without violating null trapping.
- **Inward Heating (P_{in}):** The non-geodesic proper acceleration of constituents (due to repeated shell collisions) allows them to absorb energy from vacuum fluctuations via the Unruh effect.

At equilibrium, $P_{\text{in}} = P_{\text{out}}$. This balance maintains the shell’s temperature and stability.

Confinement. The shell is prevented from dispersing by the prohibitive energy cost of constituents “re-inflating” in the lower-density noise field outside. It is prevented from collapsing by the immense outward radiation pressure from the blackbody core it creates.

The Blackbody Core. The core of the black hole is not empty. The hot inner wall of the shell fills the interior with a thermal bath of phase solitons (photons), creating a perfect blackbody cavity. For an external observer, the cavity fills in a light-crossing time $\sim R_S/c_s$. A Planck-scale observer co-moving with a constituent, however, measures vastly dilated clocks and shrunken rulers; the same photon flight spans billions of local years.

The Temperature Paradox. An external observer calculates the core temperature to be near the Planck temperature ($\sim 10^{32}$ K), with radiation wavelengths near the Planck length. However, for a constituent-scale observer on the shell:

- Their rulers are $\sim 10^{20} \times$ smaller than ours.
- Planck-length photons appear as extremely long-wavelength radiation.
- The temperature they measure is $\sim 10^{20} \times$ *colder* than what we perceive. “Hot” and “cold” are relative to the observer’s scale.

Fate of Inward-Moving Matter. Any amplitude soliton that receives a thermal kick into the core is immediately stopped by the extraordinarily dense radiation field. Radiative drag (photon friction from Doppler shifting and aberration) dissipates its kinetic energy back into the bath. Matter is effectively confined to the shell.

Covariant Power Balance. We can summarise the shell thermodynamics with a covariant, frame-consistent closure at the level of power per proper area. Let $\mathcal{P}_{\text{in}}(R)$ be the areal Unruh heating rate and $\mathcal{P}_{\text{out}}(R) = \sigma_{\text{SB}} T(R)^4$ the Stefan–Boltzmann leakage. At steady equilibrium:

$$\mathcal{P}_{\text{in}}(R) = \mathcal{P}_{\text{out}}(R) = P_0 \quad (\text{constant per proper area}). \quad (3)$$

Deviations (anisotropy, binaries) enter as small, trackable modulations of \mathcal{P}_{in} .

3.4 Shell Thermodynamics and the Temperature Law

The shell’s temperature is determined by the characteristic acceleration of its constituents. We derive the key scaling from two complementary viewpoints.

Shell Geometry. The shell contains $N \propto M$ constituents over area $A \propto R_S^2 \propto M^2$. The mean separation is $d \sim \sqrt{A/N} \propto \sqrt{M}$. For any macroscopic black hole, $d \gg L_{\text{min}}$: the shell is a *dilute network*, not a dense fluid.

View 1: External (Lattice Frequency). From outside, the shell is a 2D lattice of N point masses with spacing $d \propto \sqrt{M}$. The characteristic dynamical frequency is set by nearest-neighbour signal crossing: $\omega \sim c_s/d$, giving $a_{\text{char}} \sim c_s^2/d \propto 1/\sqrt{M}$.

View 2: Internal (Relativistic Compression). From a constituent’s own (downshifted) frame, the spacing d maps to a much larger proper distance $d_{\text{int}} = d/\varepsilon$ (where $\varepsilon \ll 1$ is the scale ratio between the constituent’s rulers and external rulers). Signals take a proportionally long proper time $\tau_{\text{int}} \sim d_{\text{int}}/c_s$ to cross the gap. However, because c_s is invariant across frames, the external observer maps this long internal proper time back to a short coordinate time:

$\tau_{\text{ext}} = \varepsilon \cdot \tau_{\text{int}} = d/c_s$. The relativistic compression that makes the internal “miles” appear as external “metres” is *exactly* compensated by the time dilation that makes the internal “hours” appear as external “seconds.” The ratio d/c_s is frame-invariant, so both views yield $a_{\text{char}} \propto 1/\sqrt{M}$.

Why they must agree. Both views compute the same invariant: the proper acceleration 4-vector magnitude of a shell constituent. Agreement is guaranteed by the framework’s requirement that c_s is constant across observation windows.

Since $T_{\text{Unruh}} \propto a_{\text{char}}$, we obtain the temperature law:

$$T_{\text{BH}} \propto \frac{1}{\sqrt{M}}. \quad (4)$$

This differs from the standard Hawking result ($T \propto 1/M$) because the shell model treats the horizon as a physical lattice with specific density-dependent dynamics, rather than a vacuum boundary condition.

3.5 Stochastic Cascade and Interior Structure

The cavity is thermally uniform only on large scales. On small scales ℓ , photon number fluctuations grow as $\ell^{-3/2}$. At some scale ℓ_* , a rare upward fluctuation satisfies a pinch-off condition $\Delta F < 0$. Defining the occupancy

$$N(\ell) = \frac{\rho_\gamma \ell^3}{M_P c^2}, \quad (5)$$

and comparing to the critical occupancy $N_* \approx 1$:

- $N(\ell) > N_*$: the fluctuation collapses into a **mini-shell** (black hole branch).
- $N(\ell) < N_*$: it relaxes into a **particle-type droplet** (matter branch).

Classification by Topology. Crucially, particle-branch droplets come in two populations:

1. **Scalar solitons** ($l = 0$): Simple density fluctuations with no phase winding. These are collisionless and form the dominant population—identified with *dark matter*.
2. **Topological defects** ($l \geq 1$): Rare fluctuations that trap a phase winding. These experience phase-friction and become dissipative—identified with *baryonic matter*.

Combinatorial statistics in the cavity favour the simpler $l = 0$ state by a factor of ~ 5 , naturally seeding the observed cosmic abundance ratio $\Omega_{\text{DM}}/\Omega_b \approx 5$.

The Morphological Amplifier. Droplets clustered in a gravitational caustic source a higher local noise background τ . This triggers a super-linear downshifting response in their neighbours ($\varepsilon \propto -\tau^\gamma$, $\gamma > 1$), deepening the potential well and drawing in more matter. This cooperative feedback transforms weak Brownian caustics into sharp, high-contrast filamentary structures—the sponge-like cosmic web observed in our universe.

4 Advanced Dynamics and Robustness

4.1 Prediction: Constant-Power Evaporation

The Stefan–Boltzmann law gives the radiated power $P \propto A \cdot T^4$. Using $A \propto M^2$ and (4):

$$P \propto M^2 \cdot \left(\frac{1}{\sqrt{M}} \right)^4 = M^2 \cdot \frac{1}{M^2} = \text{constant}. \quad (6)$$

Prediction: Black holes radiate at a constant power P_0 (of order 10^{15} W for standard parameters) throughout their lives, until they reach the Planck scale. This implies a linear lifetime $\tau \propto M$ —drastically shorter for supermassive black holes than the Hawking prediction (M^3). The rising temperature ($1/\sqrt{M}$) and shrinking area (M^2) have a perfect, compensatory relationship that keeps the total energy output constant.

4.2 Duality of Stiffness

A profound aspect of the particle–black hole duality lies in comparing their mechanisms for resisting collapse.

- **Microscopic:** A soliton is stabilized by a $|\Psi|^4$ potential term. Pressure $\propto |\Psi|^4$.
- **Macroscopic:** The shell is stabilized by radiation pressure $P_{\text{rad}} \propto T^4$. Since $T \propto \sqrt{\rho_{\text{surf}}}$ and we identify $\rho_{\text{surf}} \sim |\Psi|^2$, we find $P_{\text{rad}} \propto (\sqrt{\rho_{\text{surf}}})^4 = \rho_{\text{surf}}^2 \sim |\Psi|^4$.

The macroscopic equation of state mirrors the microscopic potential.

4.3 Window Rescaling and the Soliton Floor

Does the constant-power argument hold for an observer with a finer microscope? Yes. The “floor” at L_{min} is not an absolute limit of the vacuum but the limit of the *soliton’s* ability to downshift.

The Free-Energy Floor. The mechanism powering a soliton’s response to increased noise is *scale downshifting*: higher $\tau_{\text{eff}} \rightarrow$ smaller equilibrium size \rightarrow energy released along $\varepsilon_s(\tau) = -\alpha_s \tau^\gamma$.

- An ordinary soliton ($L_c \gg L_{\text{min}}$) sits partway up the hill. Plenty of downhill headroom remains.
- A shell constituent ($L_c \approx L_{\text{min}}$) has already rolled to the *bottom*. Further noise cannot drive further shrinkage.

Crucially, this floor is *relational*: L_{min} is the UV edge of the external observer’s window. The substrate has no absolute minimum scale. From the constituent’s own frame it has not “bottomed out”—it sees a perfectly normal window with a full spectrum and normal Unruh physics. But all of this self-consistent internal dynamics maps, via the sliding-window formalism, to the same fixed power P_0 that the external observer computes.

Covariant Power Balance. Let P_0 be both the inward Unruh power and the outward Stefan–Boltzmann power per unit proper shell area. Lorentz factors cancel: every observer measures the same P_0 . The constant-power evaporation law is therefore frame-independent.

4.4 Asymmetric Shells and Binary Recoil

In a binary system, the shell of one black hole is distorted and heated anisotropically by its companion’s noise field. This results in anisotropic thermal radiation from the shell’s outer surface, creating a continuous, non-gravitational **recoil force**. If the binary separation d is less than the ambient phase-coherence length L_{phase} , coherent phase-sheets can also form, adding an EM-like component to the interaction. These effects, including velocity-dependent retardation, could produce unique signatures in gravitational wave signals.

5 Duality with Cosmology

5.1 Strict Orientation-Duality

We promote the horizon duality to a strict identification. A single nucleation criterion defines a closed surface Σ that may be realised with two orientations:

- **Type I (Black Hole):** Trapped side points inward; a finite-thickness, optically thick shell forms and reprocesses flux to a photosphere. The observer is outside.
- **Type II (Cosmology):** Trapped side points outward; to interior observers there is no material shell, but the same thermal bath appears with the Gibbons–Hawking temperature.

These are opposite orientations of the same nucleation event. The interior of a Type I object (a black hole) *is* a Type II universe (a Hubble patch) to its inhabitants.

5.2 Type I / Type II dictionary (at a glance)

Item	Type I: Black hole (external view)	Type II: Cosmology (internal view)
Orientation of trapping	Trapped side points inward	Trapped side points outward
Material shell	Yes (finite-thickness reprocessor)	No privileged material wall for interior observers
Dominant temperature law (external units)	Photosphere set by shell micro-physics: $T \propto M^{-1/2}$	Gibbons–Hawking: $T \propto H$
Net luminosity (external units)	$P_{\text{out}} \approx \text{const}$ (reprocessed)	$P \propto H^2$ (bare geometric leakage)
Horizon radius identification	$R_S = 2GM/c_s^2$	$r_H = c_s/H$
Duality lock (late era)	$R_S = r_H$	$r_H = R_S$

Further dictionary items and robustness notes are collected in the SI (see SI Secs. ?? and ??).

5.3 Energy Conservation and Hubble Radius

The total mass-energy M of the parent must equal the energy in the child’s Hubble volume. For a flat FRW interior with density ρ and Friedmann equation $H^2 = (8\pi G/3)\rho$:

$$E_H = \frac{4}{3}\pi r_H^3 \rho c_s^2 = \frac{c_s^5}{2GH}. \quad (7)$$

Setting $Mc_s^2 = E_H$:

$$\boxed{H = \frac{c_s^3}{2GM}, \quad r_H = \frac{c_s}{H} = \frac{2GM}{c_s^2} = R_S.} \quad (8)$$

The child’s Hubble radius equals the parent’s Schwarzschild radius—*derived*, not assumed.

Local vs. Global Caveat. The identification $Mc_s^2 = E_H$ applies when the cavity is a single causal domain ($\mathcal{N} \rightarrow 1$; see SI Appendix ??). During earlier eras with many Hubble patches ($\mathcal{N} \gg 1$), $H = c_s^3/(2GM)$ describes energy per patch, not total M .

Epistemic Consequence. Because $H(\tau)$ is locally uniform (FRW symmetry), an observer has no local measurement revealing their distance to the wall. The late-time phantom phase ($w < -1$) causes r_H to *shrink*, making new causal contact with the wall harder, not easier.

5.4 Flux Reconciliation

A key apparent contradiction is the flux law:

- **Type I (BH):** $P \approx \text{const.}$
- **Type II (Cosmo):** $P \propto H^2$ (Gibbons–Hawking).

These are reconciled by the shell. The geometric (pre-reprocessing) intake on both sides scales $\propto H^2$ via $A \propto H^{-2}$, $T \propto H$. In the Type I case, the optically thick shell reprocesses this H^2 input to a photospheric output with $T_{\text{shell}} \propto M^{-1/2}$, $A_{\text{shell}} \propto M^2$, giving $P_{\text{shell}} \approx \text{const.}$ Without a reprocessor (Type II), the observer sees the bare H^2 leakage.

5.5 Resolving Apparent Contradictions

- **Shell presence/absence:** Type II lacks a material reprocessor for interior observers because they sit inside the asymptote; the same asymptote carries a real shell in the parent frame. Both satisfy the same nucleation and balance laws.
- **Expansion vs. downscaling:** Metric expansion is traded for matter downscaling via the sliding window; horizon thermodynamics and trapping are invariant under this gauge.
- **Isotropy:** Hyperbolic optics makes the asymptote effectively concentric; leading observables (redshift, dimming) are isotropic to first order. Observed anisotropies (CMB, filaments) arise from interior condensates.

6 Cosmological Implications

6.1 Observational Anchors

We estimate the parent mass using two numbers:

1. Internal CMB temperature at last scattering: $T_{\text{int}} \approx 3000$ K.
2. Internal look-back time to that epoch: $\tau_{\text{int}} \approx 13.7$ Gyr.

6.2 Parent Mass Estimate

The interior–exterior conformal factor F relates internal and external clocks and temperatures:

$$T_{\text{int}} = \frac{T_{\text{BH}}}{F}, \quad \tau_{\text{int}} = F \tau_{\text{ext}} = F \kappa M, \quad (9)$$

where $\kappa \equiv c_s^2/P_0$ is the inverse evaporation rate (units: s kg^{-1}) and $T_{\text{BH}} = k/\sqrt{M}$ defines the proportionality constant k . Eliminating F :

$$M_{\text{parent}} = \left(\frac{\tau_{\text{int}} T_{\text{int}}}{\kappa k} \right)^2. \quad (10)$$

Using provisional Hawking-normalised constants yields $M_{\text{parent}} \sim 10^{21}$ kg (comet scale), with an external lifetime of a few years. These are not final predictions but demonstrate self-consistency.

7 Dynamics: Time Mapping and the Heat Death

A critical test of the duality is the synchronization of clocks.

7.1 The Time Mapping Derivation

From $H = c_s^3/(2GM)$ and constant-power evaporation $M(t_p) = M_0 - (P/c_s^2)t_p$, we derive the parent-to-child clock ratio (detailed in SI Appendix ??):

$$\frac{dt_p}{d\tau} = -\frac{3c_s^5}{4GP_0}(1 + w_{\text{eff}}). \quad (11)$$

For both clocks to run forward ($dt_p/d\tau > 0$):

$$1 + w_{\text{eff}} < 0 \implies \boxed{w_{\text{eff}} < -1.} \quad (12)$$

Result: The framework *requires* a phantom equation of state during the DE era. Current observational constraints ($w \approx -1.03 \pm 0.03$) are consistent with—and mildly favour—this prediction.

7.2 Multi-Era Accounting

During radiation and matter eras, the cavity contains $\mathcal{N} \gg 1$ causally disconnected Hubble patches. The total BH mass distributes as $Mc_s^2 = \mathcal{N} \cdot E_H$. As the Hubble sphere grows into the cavity, \mathcal{N} shrinks until $\mathcal{N} \rightarrow 1$ at DE onset, when (8) locks in. The early-universe evolution is effectively *decoupled* from the parent’s evaporation (see SI Appendix ?? for details).

7.3 Predicted Value of $\epsilon = |1 + w|$

From (11), $P \sim (c_s^5/G) \cdot f$ where f is a dimensionless microphysical factor. Then $dt_p/d\tau \sim 3\epsilon/(4f)$. For the child to have a long lifetime ($dt_p/d\tau \ll 1$), we need $\epsilon \ll f$. If $f \sim \mathcal{O}(1)$ (natural for Planck-scale microphysics):

$$\boxed{|1 + w| = \epsilon \ll 1.} \quad (13)$$

7.4 The Endpoint: Asymptotic Heat Death

If P_0 and ϵ remained constant, the child would end in a Big Rip ($H \rightarrow \infty$ in finite time $\tau_{\text{end}} = \tau_* + 2/(3\epsilon H_*)$). However, the Planck pivot intervenes:

1. **Parent Side:** As $M \rightarrow M_P$, evaporation power switches to $P \propto M^4$ (quantum regime), taking infinite time to reach $M = 0$. A Planck-mass remnant forms.
2. **Child Side:** As $H \rightarrow H_P \equiv c_s/(2GM_P)$, the Hubble volume shrinks to a single Planck cell. Window drift runs out of UV headroom, forcing $\epsilon \rightarrow 0$. The simplest smooth parameterisation:

$$\epsilon(H) \approx \epsilon_0 \left(1 - \frac{H}{H_P}\right). \quad (14)$$

With this $\epsilon(H)$ and $\dot{H} = \frac{3}{2}\epsilon H^2$, linearising near H_P ($x \equiv 1 - H/H_P \ll 1$):

$$\dot{x} \approx -\frac{3\epsilon_0 H_P}{2} x \implies x(\tau) = x_0 e^{-\frac{3\epsilon_0 H_P}{2}\tau}. \quad (15)$$

H approaches H_P exponentially, taking *infinite* child cosmic time. The Big Rip is replaced by an **Asymptotic Heat Death**: the child universe dilutes toward an empty de Sitter state at $H = H_P$, approached but never reached.

8 The Infinite Hierarchy

The model implies a fractal structure: universes nested within black holes, nested within larger universes.

8.1 Impedance Matching

Standard Hawking radiation ($P \propto M^{-2}$) forbids nesting: a small inner shell would vaporize its parent (“infrared explosion”). **Constant power** ($P \approx \text{const}$) is the unique scaling law that allows impedance matching: $P_{\text{inner}} = P_{\text{outer}} = P_0$. The inner shell feeds the outer at exactly the rate the outer dissipates, permitting indefinite stable nesting.

8.2 Piecewise Arrows of Time

The infinite hierarchy yields a natural, local arrow of time at every level:

- **Local definition:** Within each cavity, the coarse-grained free energy has a monotone scale direction: $\dot{\sigma} < 0$ (solitons downshift). Thermodynamic dissipation (constant-power leakage) makes this irreversible.
- **Finite durations per level:** Constant-power emission drains each cavity’s UV reserve in finite local time, defining a local past boundary (entry from parent shell) and future boundary (Planck de Sitter asymptote).
- **No global beginning or end:** The hierarchy extends unboundedly in both directions. Concatenating piecewise arrows yields an eternal, globally past-and-future-unbounded structure with locally well-defined time everywhere.
- **Consistency with entropy:** At each level, coarse-grained entropy increases along the downshift. UV exhaustion at a shell resets the window and renormalises couplings. There is no global maximum entropy.

8.3 No Global Time Paradox

The hierarchy is eternal despite finite local lifetimes. The parent-to-child clock ratio from (11) gives $r \equiv dt_{\text{parent}}/d\tau_{\text{child}} \ll 1$.

- **Looking Down:** A level- n observer hosts an arbitrarily deep cascade inside its own finite lifetime, because the projected durations form a convergent geometric series: $\sum_{k \geq 1} r^k < \infty$.
- **Looking Up:** Projected upward, parent lifetimes scale by $\sim 1/r > 1$ per level. The upward chain diverges: there is no “top” level with a finite end.

Finite proper time at every level is compatible with an eternal hierarchy because “eternity” lives in the unbounded tower of scales and the compounding inter-level time dilation, not in an infinite duration of any single clock.

9 Conclusion

We have presented a self-consistent model where black holes are material shells and their interiors are expanding universes. The framework replaces the singularity with a blackbody cavity and standardizes the thermodynamics of horizons across scales.

Key falsifiable predictions include:

1. **Constant Power Evaporation:** Black holes radiate at $P \approx \text{const}$, not $P \propto 1/M^2$.
2. **Phantom Dark Energy:** The equation of state must be slightly phantom ($w \lesssim -1$) with $|1 + w| \ll 1$.
3. **Binary Signatures:** Anisotropic shell heating produces non-gravitational recoil forces detectable in precision waveform analyses.

4. **Shell Thickness:** A finite photosphere with $\delta R/R \propto M^{-1/2}$, potentially observable in horizon-scale imaging.

This duality suggests that our Big Bang was the formation of a shell in a parent universe, and our eventual heat death corresponds to that shell's relaxation to a Planck remnant. The cosmos is an infinite, self-similar cascade of such events, eternal and devoid of singularities.